

Three Higgs doublet model with horizontal S_3 symmetry

A. Alvarez-Cruz², M. Gómez-Bock¹, M. Mondragón² and A. Pérez-Martínez²

¹ DAFM, Universidad de las Américas Puebla, Ex. Hda. Sta. Catarina Mártir s/n, San Andrés Cholula, Puebla. México

² Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-364, Cd. México 01000, México.

Abstract. We present a brief overview of a three-Higgs-doublet Model CP-invariant with S_3 -symmetry as a flavor symmetry. This model contains three Higgs $SU(2)$ doublets as well as three right handed neutrinos, besides the usual Standard Model particles. We discuss some features of the Yukawa, Higgs and neutrino sectors of this model, and present some new results on the trilinear couplings of the Higgs sector, plus a fit of all parameters in the neutrino one.

1. Introduction

The organization of the fermions into generations or families has helped to understand the relations among the elementary particles and the nature of their interactions, but we still have not fully understood why nature chose this structure. In addition, now with the well measured neutrino mixing angles new puzzles arise, since the form of the V_{PMNS} matrix differs considerably from the V_{CKM} one. Many efforts have been done towards unravelling the flavor symmetry of the model, as well as on studying possible processes which may allow to understand the origin and development of the flavor structure in order to reproduce the data we obtain from experiments.

Our interest and motivation in the S_3 symmetry extension of the SM is that the difference in masses between the third and first two generations, as well as the structure of the V_{CKM} mixing matrix suggest that the fermions belong to a $\mathbf{2} + \mathbf{1}$ irreducible representation of a flavour symmetry. The smallest non-Abelian flavour symmetry with such characteristics is the permutation group of three objects, S_3 . This flavour symmetry has been proposed a long time ago [1,2], and has been extensively studied in the last years (see for instance [3–15]) in different settings, due to its simplicity and predictivity. We will concentrate on a low energy model ($S_3 - 3H$), which we will try to keep minimal in the sense of not adding extra flavons, and not breaking explicitly the flavour symmetry. Many other discrete symmetries have been proposed at low and high energies, which are also interesting for different reasons (see for instance [16], and references therein).

2. 3HDM with S_3 symmetry

S_3 is the smallest non-Abelian discrete group, and it corresponds to the permutations of three objects or the symmetries (rotations and reflections) of an equilateral triangle. Its irreducible

representations (irrep) are a symmetric singlet $\mathbf{1}$, an anti-symmetric singlet $\mathbf{1}_A$ and one doublet $\mathbf{2}$ [3]. We will work here on the two-dimensional matrix representation, D_i presented in [3].

The field content of the model consists of the usual $SU(2)$ doublets and singlets for SM quarks and leptons as well as three Higgs doublets and three right-handed neutrinos. We will assume all fields to belong to a reducible $\mathbf{2} + \mathbf{1}$ representation of S_3 , where all the fields in the first two generations will be in a doublet, and the ones in the third generation in the symmetric singlet. The same assignment follows for the three Higgs doublets and right-handed neutrinos. The S_3 extension of the SM has been previously used to calculate neutrino masses and mixings with interesting results [5], as well as lepton masses and flavour changing neutral currents [17, 18]. The scalar sector is also interesting since there is an economy of parameters compared to a more generic 3HDM.

The S_3 Higgs potential is constructed with three $SU(2)$ complex Higgs doublets H_1, H_2, H_s which can be written as

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \quad H_s = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}. \quad (1)$$

We introduce the following variables as in [19, 20]

$$\begin{aligned} x_1 &= H_1^\dagger H_1, & x_4 &= \text{Re}(H_1^\dagger H_2), & x_7 &= \text{Im}(H_1^\dagger H_2), \\ x_2 &= H_2^\dagger H_2, & x_5 &= \text{Re}(H_1^\dagger H_s), & x_8 &= \text{Im}(H_1^\dagger H_s), \\ x_3 &= H_s^\dagger H_s, & x_6 &= \text{Re}(H_2^\dagger H_s), & x_9 &= \text{Im}(H_2^\dagger H_s). \end{aligned} \quad (2)$$

3. Higgs potential and masses

We take all the possible terms in the potential which preserve the discrete S_3 permutational symmetry, as reported in [21, 22]. The most general Higgs potential invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y \times S_3$ in the symmetry adapted basis is given as

$$\begin{aligned} V &= \mu_1^2 \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right) + \mu_0^2 \left(H_s^\dagger H_s \right) + \frac{a}{2} \left(H_s^\dagger H_s \right)^2 + b \left(H_s^\dagger H_s \right) \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right) \\ &+ \frac{c}{2} \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right)^2 + \frac{d}{2} \left(H_1^\dagger H_2 - H_2^\dagger H_1 \right)^2 + e f_{ijk} \left(\left(H_s^\dagger H_i \right) \left(H_j^\dagger H_k \right) + h.c. \right) \\ &+ f \left\{ \left(H_s^\dagger H_1 \right) \left(H_1^\dagger H_s \right) + \left(H_s^\dagger H_2 \right) \left(H_2^\dagger H_s \right) \right\} + \frac{g}{2} \left\{ \left(H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1 \right)^2 \right\} \\ &+ \frac{h}{2} \left\{ \left(H_s^\dagger H_1 \right) \left(H_s^\dagger H_1 \right) + \left(H_s^\dagger H_2 \right) \left(H_s^\dagger H_2 \right) + \left(H_1^\dagger H_s \right) \left(H_1^\dagger H_s \right) + \left(H_2^\dagger H_s \right) \left(H_2^\dagger H_s \right) \right\} \quad (3) \end{aligned}$$

where $f_{112} = f_{121} = f_{211} = -f_{222} = 1$. This same potential has also been analyzed in refs. [19, 20, 23, 24] without CP violation and in ref. [25] also with spontaneous CP violation.

We will rewrite the *vevs* in spherical coordinates

$$v_1 = v \cos \varphi \sin \theta, \quad v_2 = v \sin \varphi \sin \theta, \quad v_3 = v \cos \theta. \quad (4)$$

The use of this spherical parametrization is helpful to visualize the relation within the *vevs*. The angle θ gives the amount of mixing between the *vev* of the singlet and the *vevs* of the doublets. We may obtain a relation between v_1 and v_2 and v_3 as

$$\tan \varphi = \frac{v_2}{v_1}, \quad \tan \theta = \frac{v_2}{v_3 \sin \varphi} \quad (5)$$

Applying the minimization conditions of the potential there will lead to a relation between $v_1^2 = 3v_2^2$, *i.e.* fixing also the value of $\varphi = \pi/6$.

In order to get the Higgs boson masses it is necessary to perform the usual form for the rotation matrix R_i , to obtain the mass matrix and physical states [23, 24]. In our parameterization the pseudoscalar and charged Higgs boson masses can be written as

$$m_{A_1}^2 = -2(d + g)v^2 \sin^2 \theta - 5ev^2 \sin \theta \cos \theta - 2hv^2 \cos^2 \theta, \quad (6)$$

$$m_{A_2}^2 = -v^2(e \tan \theta + 2h), \quad (7)$$

$$m_{H_1^\pm}^2 = -[5ev^2 \sin \theta \cos \theta + (f + h)v^2 \cos^2 \theta] - 2gv^2 \sin^2 \theta, \quad (8)$$

$$m_{H_2^\pm}^2 = -v^2 [e \tan \theta + (f + h)]. \quad (9)$$

From these expressions it can be seen that when the value of $\tan \theta$ is high some masses will be naturally heavy.

In terms of the potential parameters, considering also the spherical parameterization we have

$$\begin{aligned} M_a^2 &= \left[(c + g)v^2 \sin^2 \theta + \frac{3}{2}ev^2 \sin \theta \cos \theta \right] \\ M_b^2 &= [3ev^2 \sin^2 \theta + 2(b + f + h)v^2 \sin \theta \cos \theta] \\ M_c^2 &= av^2 \cos^2 \theta - \frac{ev^2 \tan \theta \sin^2 \theta}{2} \end{aligned} \quad (10)$$

where the mixing angle of the neutral scalars α is

$$\tan(2\alpha) = -\frac{M_b^2}{M_a^2 - M_c^2}. \quad (11)$$

This leads to the following expressions for the neutral scalar Higgs bosons masses as follows:

$$m_{h_0}^2 = -9ev^2 \sin \theta \cos \theta, \quad (12)$$

$$m_{H_1, H_2}^2 = (M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}, \quad (13)$$

Clearly, when the parameter $e = 0$ we find a massless Goldstone boson, corresponding to an $SO(2)$ symmetry, as first pointed out in [20]. But also, it is possible that either $\cos \theta = 0$ or $\sin \theta = 0$, with $e \neq 0$, which also gives a massless boson, but now due to the fact that either both $v_{1,2}$ or v_3 are zero.

We have calculated the trilinear self-coupling of the Higgs bosons in terms of their masses, and we have reaffirmed the residual \mathcal{Z}_2 symmetry reported in ref. [24], where h_0 is odd under this symmetry. We present below particular trilinear self-couplings and the couplings with the gauge boson, of the type HVV.

$$h_0 h_0 h_0 = 0, \quad (14)$$

$$H_2 H_2 H_2 = \frac{i}{v \sin \theta \cos \theta} \left(m_{h_0}^2 \left(\frac{\cos^3(\alpha - \theta)}{9 \cos^2 \theta} \right) + m_{H_2}^2 \left(\cos(\alpha - \theta) - \frac{\sin 2\alpha \sin(\alpha - \theta)}{2} \right) \right), \quad (15)$$

$$H_1 H_1 H_1 = -\frac{i}{v \sin \theta \cos \theta} \left(m_{h_0}^2 \left(\frac{\sin^3(\alpha - \theta)}{9 \cos^2 \theta} \right) + m_{H_1}^2 \left(\sin(\alpha + \theta) - \frac{\sin 2\alpha \cos(\alpha - \theta)}{2} \right) \right),$$

The coupling with the vector bosons are the following

$$H_1 W^\pm W^\mp = i \frac{4M_W^2 \cos(\alpha - \theta) g^{\mu\nu}}{v} \quad (16)$$

$$H_2 W^\pm W^\mp = i \frac{4M_W^2 \sin(\alpha - \theta) g^{\mu\nu}}{v} \quad (17)$$

$$h_0 W^\pm W^\mp = 0 \quad (18)$$

$$H_1 Z Z = \frac{i2M_Z^2 \cos(\alpha - \theta) g^{\mu\nu}}{v} \quad (19)$$

$$H_2 Z Z = \frac{i2M_Z^2 \sin(\alpha - \theta) g^{\mu\nu}}{v} \quad (20)$$

4. 3HDM with S_3 Yukawa Lagrangian

As with the scalar part of the Lagrangian, we look for a structure that preserve the discrete S_3 permutational symmetry for couplings within Higgs fields and fermions. Then, the general structure of the Yukawa part of the Lagrangian is

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \quad (21)$$

where U, D, E, ν correspond to the up and down quarks, leptons and neutrinos respectively. The expressions for each term can be found in ref. [3].

There is also research done with the S_3 symmetry without extending the Higgs sector [21, 26, 27]. In this case it is necessary to break explicitly the symmetry to give appropriate masses to the fermions and reproduce the mixing. Thus, it is natural to assume that there should be a flavor structure also in the Higgs sector, leading to the necessity of its extension. A rich flavor structure in the Yukawa sector could not only explain the different fermion masses, but will also lead to FCNC processes. This flavour structure in the S_3 symmetry Yukawa with a 3HDM sector has been studied already in [3], where the following universal mass structure for all fermions was found

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix}. \quad (22)$$

In the quark sector the $S_3 - 3H$ model gives rise to the Fritzsch and generalized Fritzsch textures, as well as the NNI interactions [27–29]. It is also possible to reparameterize the V_{CKM} mixing matrix in terms of the quark masses, showing that it has good compatibility with the experimental data [27].

In the interaction basis we can write the general Yukawa Lagrangian of $S_3 - 3H$ implying that all three doublets will couple with all fermions, leading to

$$\mathcal{L}_{H_f f_j}^Y = \bar{Q}_{iL} H_1 Y_{1ij} d_{jR} + \bar{Q}_{iL} H_2 Y_{2ij} d_{jR} + \bar{Q}_{iL} H_3 Y_{3ij} d_{jR}, \quad (23)$$

where we have the usual fermionic $SU(2)$ doublets \bar{Q}_{iL} , the $SU(2)$ singlet d_{jR} and $i, j = 1, 2, 3$ are flavor indexes. As for H_k , they are the three $SU(2)$ doublets given in eq. (1). Thus we see potentially a flavour violation structure in the Yukawa couplings, unless Yukawa matrices in the flavour basis are set to be diagonal or nearly diagonal.

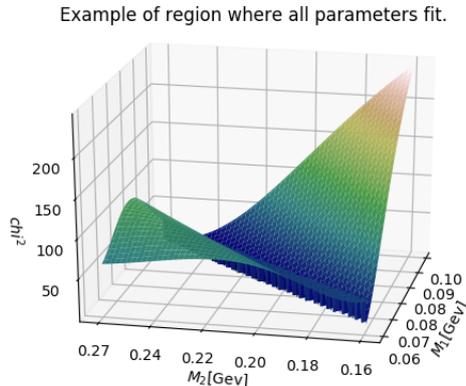


Figure 1. Right-handed neutrino masses in a χ^2 analysis, where experimental results on neutrino masses in the best fit are in 2σ agreement, see text.

5. Neutrino mixing and masses

In the leptonic sector the $S_3 - 3H$ model shows also interesting features. As already mentioned, there is a universal mass matrix structure, which also has interesting consequences for the neutrino sector. It is assumed that the left-handed neutrino masses are generated through a seesaw type I mechanism, and that there is an extra \mathcal{Z}_2 symmetry in the leptonic sector to achieve a further reduction in the free parameters [3]. The charged lepton mass matrices and neutrinos, as well as the V_{PMNS} mixing matrix was obtained in [17], showing that is close to the tri-bimaximal form if the right-handed neutrinos are assumed mass degenerate. This leads to a non-vanishing reactor mixing angle θ_{13} , albeit very small, and both the atmospheric and solar angle within the experimental range. Some flavour lepton violating processes have been calculated, showing that they are suppressed in the model [18] due to an interplay between the mass hierarchy of the charged leptons and the $S_3 \times \mathcal{Z}_2$ symmetry. If all the right-handed neutrinos are assumed to have different masses then it is possible to obtain also the reactor mixing angle in good agreement with the experimental value [30].

A new parameterization of the neutrino mass matrix leads to more interesting results in this sector, showing that it is also possible to generate enough leptogenesis to get the observed baryon asymmetry in the Universe [31]. Even more recently, in order to reduce free parameters and constrain the model, we have used a form of Powell's optimization in a multivariable space to fit the experimental results of the differences of the neutrino masses and the neutrino mixing parameters. With the χ^2 square analysis we can ensure that there are regions in parameter space where the $S_3 - 3H$ model fits all the experimental constraints in the neutrino sector, and make some predictions about leptogenesis, the heavy neutrino masses and a closed Dirac mass matrix. We even have regions where we can overfit the values of the parameters to get a $\chi^2 \sim 0.1$, allowing for some freedom to search for leptogenesis or adjust forthcoming experimental results, see figure. In the example shown, the left-handed neutrino masses are in the range

$$m_1 = 1.116 \times 10^{-8} \text{ eV}, \quad m_2 = 1.51810^{-8} \text{ eV}, \quad m_3 = 1.02210^{-8} \text{ eV} \quad (24)$$

with an inverted hierarchy.

6. Conclusions

The $S3 - 3H$ model has proven to be very successful not only in accommodating the hierarchy and structure of the quark and neutrino mixing matrices, but also in providing interesting results in the Higgs and leptonic sectors. We have shown some known results on the quark, leptonic and scalar sector of the model, plus some new results involving the trilinear self-couplings of the Higgs bosons and some of their couplings to the vector bosons. In the neutrino sector a new parameterization of the neutrino mass matrix allows to fix two of the right-handed neutrino masses and search for viable leptogenesis. To continue studying the feasibility of the model more detailed analysis in the scalar, quark and leptonic sectors are under way.

Acknowledgements

This work was partially supported by a Consejo Nacional de Ciencia y Tecnología (CONACyT) PhD Fellowship and SNI México. M. G-B acknowledges partial support from Universidad de las Américas Puebla. This work was also partially supported by grant UNAM PAPIIT IN111518.

References

- [1] Pakvasa S and Sugawara H 1978 *Phys. Lett.* **B73** 61–64
- [2] Derman E 1979 *Phys. Rev.* **D19** 317–329
- [3] Kubo J, Mondragon A, Mondragon M and Rodriguez-Jauregui E 2003 *Prog. Theor. Phys.* **109** 795–807 [Erratum: *Prog. Theor. Phys.*114,287(2005)] (*Preprint hep-ph/0302196*)
- [4] Chen S L, Frigerio M and Ma E 2004 *Phys. Rev.* **D70** 073008 [Erratum: *Phys. Rev.*D70,079905(2004)] (*Preprint hep-ph/0404084*)
- [5] Mondragon A, Mondragon M and Peinado E 2008 *AIP Conf. Proc.* **1026** 164–169 (*Preprint 0712.2488*)
- [6] Dicus D A, Ge S F and Repko W W 2010 *Phys. Rev.* **D82** 033005 (*Preprint 1004.3266*)
- [7] Kubo J 2013 *Fortsch.Phys.* **61** 597–621 (*Preprint 1210.7046*)
- [8] Cárcamo Hernández A E, Martínez R and Ochoa F 2016 *Eur. Phys. J.* **C76** 634 (*Preprint 1309.6567*)
- [9] Cárcamo Hernández A E, Cataño Mur E and Martínez R 2014 *Phys. Rev.* **D90** 073001 (*Preprint 1407.5217*)
- [10] Ma E and Srivastava R 2015 *Phys. Lett.* **B741** 217–222 (*Preprint 1411.5042*)
- [11] Das D, Dey U K and Pal P B 2016 *Phys. Lett.* **B753** 315–318 (*Preprint 1507.06509*)
- [12] Cárcamo Hernández A E, de Medeiros Varzielas I and Schumacher E 2016 *Phys. Rev.* **D93** 016003 (*Preprint 1509.02083*)
- [13] Ge S F, Kusenko A and Yanagida T T 2018 *Phys. Lett.* **B781** 699–705 (*Preprint 1803.03888*)
- [14] Espinoza C, Garcés E A, Mondragón M and Reyes-González H 2019 *Phys. Lett.* **B788** 185–191 (*Preprint 1804.01879*)
- [15] Gómez-Izquierdo J C and Mondragón M 2018 (*Preprint 1804.08746*)
- [16] Ishimori H, Kobayashi T, Ohki H, Shimizu Y, Okada H and Tanimoto M 2010 *Prog. Theor. Phys. Suppl.* **183** 1–163 (*Preprint 1003.3552*)
- [17] Mondragon A, Mondragon M and Peinado E 2007 *Phys. Rev.* **D76** 076003 (*Preprint 0706.0354*)
- [18] Mondragon A, Mondragon M and Peinado E 2008 *J. Phys.* **A41** 304035 (*Preprint 0712.1799*)
- [19] Emmanuel-Costa D, Felix-Beltran O, Mondragon M and Rodriguez-Jauregui E 2007 *AIP Conf. Proc.* **917** 390–393 [390(2007)]
- [20] Beltran O F, Mondragon M and Rodriguez-Jauregui E 2009 *J. Phys. Conf. Ser.* **171** 012028
- [21] Derman E and Tsao H S 1979 *Phys. Rev.* **D20** 1207
- [22] Kubo J, Okada H and Sakamaki F 2004 *Phys. Rev.* **D70** 036007 (*Preprint hep-ph/0402089*)
- [23] Barradas-Guevara E, Félix-Beltrán O and Rodríguez-Jáuregui E 2015 (*Preprint 1507.05180*)
- [24] Das D and Dey U K 2014 *Phys. Rev.* **D89** 095025 [Erratum: *Phys. Rev.*D91,no.3,039905(2015)] (*Preprint 1404.2491*)
- [25] Emmanuel-Costa D, Ogreid O M, Osland P and Rebelo M N 2016 *JHEP* **02** 154 [Erratum: *JHEP*08,169(2016)] (*Preprint 1601.04654*)
- [26] Mondragon A and Rodriguez-Jauregui E 1999 *Phys. Rev.* **D59** 093009 (*Preprint hep-ph/9807214*)
- [27] González Canales F, Mondragón A, Mondragón M, Saldaña Salazar U J and Velasco-Sevilla L 2013 *Phys. Rev.* **D88** 096004 (*Preprint 1304.6644*)
- [28] Meloni D, Morisi S and Peinado E 2011 *J. Phys.* **G38** 015003 (*Preprint 1005.3482*)
- [29] Barranco J, Gonzalez Canales F and Mondragon A 2010 *Phys. Rev.* **D82** 073010 (*Preprint 1004.3781*)
- [30] Gonzalez Canales F, Mondragon A and Mondragon M 2013 *Fortsch. Phys.* **61** 546–570 (*Preprint 1205.4755*)
- [31] Cruz A A and Mondragón M 2017 (*Preprint 1701.07929*)