Leptogenesis in CPT violating backgrounds

T Bossingham

Theoretical Particle Physics and Cosmology Group, Department of Physics, King's College London, Strand, London WC2R 2LS, UK

E-mail: thomas.bossingham@kcl.ac.uk

Abstract. In a *temperature dependent* CPT-Violating (CPTV) axial time-like background (induced by the Kalb-Ramond tensor field of string theory) we discuss leptogenesis by solving the Boltzmann equation. The current work non-trivially modifies the framework of a previous phenomenological approach (where the author was involved) where the CPTV *axial* background was considered to be a constant. The constant background approximation is shown to capture the main phenomenological features of leptogenesis.

1. Introduction and Motivation

It has been shown [1, 2, 3] that matter-antimatter asymmetry (through leptogenesis), can occur in appropriate *constant* backgrounds in the cosmological (Robertson-Walker) frame of the early universe. Such constant backgrounds were associated with postulated axial current condensates. In the model leptogenesis originates from tree-level decays of a heavy sterile (right-handed, Majorana) neutrino (RHN) into Standard Model (SM) leptons, in the presence of a generic CPTV time-like axial background [2, 3]. The relevant Lagrangian is given by:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}\partial\!\!\!/ N - \frac{m_N}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - \sum_k y_k \overline{L}_k \tilde{\varphi} N + h.c. \qquad (1)$$

where $\mathcal{L}_{\rm SM}$ denotes the SM Lagrangian; B_μ is a CPTV background field, associated with physics beyond the SM; N is the RHN spinor field, with (Majorana) mass m_N; N^c is the charge conjugate spinor; $\tilde{\varphi}$ is the adjoint ($\tilde{\varphi}_i = \varepsilon_{ij}\varphi_j$) of the Higgs field φ; L_k is a lepton (doublet) field of the SM sector, with k a generation index; y_k is a Yukawa coupling, which is non-zero and provides a non-trivial ("Higgs portal") interaction between the RHN and the SM sectors. For simplicity [2, 3] we restrict ourselves to the first generation ($k = 1$), and set $y_1 = y$.

In [2, 3], the model assumed that B_μ has only a non-zero temporal component with no time or space dependence (compatible with spatial homogeneity):

$$B_0 = \text{constant} \neq 0 \ , \ B_i = 0 \ , i = 1, 2, 3 \ . \qquad (2)$$

The Lagrangian (1) then reduces to a Standard Model Extension (SME) Lagrangian in a Lorentz and CPTV background [5].

In the presence of the background (2) and the Higgs portal Yukawa interactions of (1) [2, 3], a lepton asymmetry is generated due to the CP and CPTV tree-level decays of the RHN N into

SM leptons,:

$$\begin{aligned} \text{Channel I} &: \qquad N \to l^- h^+ \;,\; \nu\, h^0 \;, \\ \text{Channel II} &: \qquad N \to l^+ h^- \;,\; \overline{\nu}\, h^0 \;. \end{aligned} \tag{3}$$

where $\ell^\pm$ are charged leptons, ν ($\overline{\nu}$) are light "active" neutrinos (antineutrinos) in the SM sector, h^0 is the neutral Higgs field, and $h^\pm$ are the charged Higgs fields. As a result of the non-trivial $B_0 \neq 0$ background (2), the decay rates of the Majorana RHN between the channels I and II are different, resulting in a lepton asymmetry, ΔL^{TOT}, which then freezes out at a temperature T_D. In [3], a detailed study of the associated Boltzmann equations for the processes in (3), and their reciprocals, led to the result:

$$\frac{\Delta L^{TOT}}{s} \simeq (0.016,\, 0.019)\, \frac{B_0}{m_N}, \qquad m_N/T_D \simeq (1.44,\, 1.77), \tag{4}$$

where s is the entropy density of the universe. This implies that the phenomenologically acceptable values of the lepton asymmetry of $\mathcal{O}(8 \times 10^{-11})$, can then be communicated to the baryon sector through (B-L) conserving sphaleron processes in the SM (where B is baryon number and L is lepton number). The observed amount of baryon asymmetry (baryogenesis) in the universe, occur for values of

$$\frac{B_0}{m_N} \sim 10^{-9}, \qquad \text{at freezeout temperature} \quad T = T_D: \quad m_N/T_D \simeq (1.77, 1.44), \tag{5}$$

With a value of the Yukawa coupling $y \sim 10^{-5}$, and for $m_N = \mathcal{O}(100)$ TeV [2, 3] we thus obtain a $B_0 \sim 0.1$ MeV, for phenomenologically relevant leptogenesis to occur at $T_D \simeq (56-69)$ TeV, in our scenario. In [2, 3] the microscopic justification of the background B_0 was based on speculation.

2. Microscopic (string-inspired) framework

A physically interesting and simple microscopic scenario for B_0 is one in which the CPT-Violating background (CPTV) is provided by the field strength of the spin-1 antisymmetric tensor (Kalb-Ramond (KR)) field which is part of the massless (bosonic) gravitational multiplet of strings [1, 2]. The bosonic gravitational multiplet of a generic string theory consists of three fields [6]: a traceless, symmetric, spin-2 tensor field $g_{\mu\nu}$, that is uniquely identified with the graviton, a spin 0 (scalar) field, the dilaton Φ (identified with the trace of the graviton), and the spin-1 antisymmetric tensor (Kalb-Ramond) field $B_{\mu\nu} = -B_{\nu\mu}$. In this work we restrict ourselves to the closed string sector, where there is a $U(1)$ gauge symmetry $B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$ which characterises the target-space effective action; so in the action it is the gauge-invariant three-form field strength of the field $B_{\mu\nu}$, with components

$$H_{\mu\nu\rho} = \partial_{[\mu}\, B_{\nu\rho]}, \tag{6}$$

which appears; the symbol $[\dots]$ denotes complete antisymmetrisation of the respective indices. The 3-form $H_{\mu\nu\rho}$ satisfies, by construction, the Bianchi identity

$$\partial_{[\mu}\, H_{\nu\rho\sigma]} = 0. \tag{7}$$

The bosonic part of the (3+1)-dimensional effective action, S_B, in the Einstein frame is [7]

$$S_B = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \Big(R - e^{-4\Phi} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \Omega \Big) + \dots, \tag{8}$$

where $G = M_P^{-2}$ is the (3+1)-dimensional Newton constant (with M_P the four-dimensional Planck mass), and is related to the string mass scale M_s via [6]: $G^{-1} = \mathcal{V}^{(n)}\, M_s^{2+n}$, with $\mathcal{V}^{(n)}$ a compactification volume (or appropriate bulk volume factor, in brane universe scenarios). For standard (ten space-time dimensional) superstrings n=6. The last term Ω on the rhs of (8) represents a vacuum energy term. It can arise either in non-critical-dimension string models [8], or from bulk contributions in brane universe scenarios; in the latter case, it includes anti-de-Sitter-type (negative) contributions [9]. The $\dots$ represent terms containing derivatives of the dilaton field, Φ; Φ is assumed [2, 3] to be slowly varying at epochs of the Universe, relevant for leptogenesis. As a first approximation we take $\Phi \simeq$ constant, and absorb it in an appropriate normalisation of the KR field. In this approximation, the vacuum energy term Ω is treated as a constant that is determined phenomenologically by requiring appropriately suppressed vacuum energy contributions.

It is known [6, 7] that the KR field strength terms H^2 in (8) can be absorbed into a generalised curvature scheme with a "torsionful connection", with the contorsion proportional to $H^\rho_{\mu\nu}$ field strength, $\overline{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + H^\rho_{\mu\nu} \neq \overline{\Gamma}^\rho_{\nu\mu}$, where $\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\nu\mu}$ is the torsion-free Christoffel symbol. Fermion fields, of mass m, are minimally coupled to the contorsion (which is proportional to $H^\rho_{\mu\nu}$). The corresponding Dirac term for fermions reads [10, 2, 3]:

$$\begin{aligned} S_{Dirac} &= \int d^4x\sqrt{-g}\,\Big[\frac{\imath}{2}\Big(\overline{\psi}\gamma^\mu\overline{\mathcal{D}}(\overline{\omega})_\mu\,\psi - (\overline{\mathcal{D}}(\overline{\omega})_\mu\,\overline{\psi})\,\gamma^\mu\,\psi\Big) - m\,\overline{\psi}\,\psi\Big],\\ &= \int d^4x\sqrt{-g}\bar{\psi}\Big(\imath\gamma^\mu\partial_\mu - m\Big)\psi + \int d^4x\sqrt{-g}\,(\mathcal{F}_\mu + B_\mu)\,\bar{\psi}\gamma^5\gamma^\mu\psi\ ,\\ \overline{\mathcal{D}}_a &= \partial_a - \frac{\imath}{4}\,\overline{\omega}_{bca}\,\sigma^{bc}, \quad \sigma^{ab} = \frac{\imath}{2}[\gamma^a,\gamma^b]\ , \quad \omega_\mu^{\ ab} \equiv e_\nu^{\ a}[\partial_\mu e^{\nu b} + \Gamma^\nu_{\ \mu\sigma}e^{\sigma b}]\\ \mathcal{F}^\mu &= \varepsilon^{abc\mu}\,e_{b\lambda}\,\partial_a\,e^\lambda_c\ , \quad B^\mu = -\frac{1}{4}e^{-2\phi}\varepsilon_{abc}^{\ \ \ \mu}H^{abc}, \quad J^{5\mu} = \bar{\psi}\gamma^\mu\gamma^5\psi, \end{aligned} \tag{9}$$

where $e^a_\mu(x)$ are the vielbeins; $g_{\mu\nu}(x) = e^a_\mu(x)\,\eta_{ab}\,e^b_\nu(x)$; η_{ab} is the Minkowski metric of the tangent space at a space-time point with coordinates x^μ; the generalised spin-connection is: $\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$; $K_{abc} = \frac{1}{2}\,(H_{cab} - H_{abc} - H_{bca}) = -\frac{1}{2}\,H_{abc}$; $\omega_{ab\mu}$ is the standard torsion-free spin connection. Our convention is that Latin letters denote tangent-space indices, while Greek letters refer to space-time indices. In (9), we used standard properties of the γ-matrices. For a Robertson-Walker metric $g_{\mu\nu}$ background, of relevance to us here, $\mathcal{F}_\mu = 0$, and thus we can write the action (9) in the form:

$$S_{Dirac} = S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\,B_\mu\,\bar{\psi}\gamma^5\gamma^\mu\psi \ \equiv\ S^{Free}_{Dirac} - \int d^4x\sqrt{-g}B_\mu J^{5\mu}, \tag{10}$$

thus yielding a minimal coupling of the $H_{\mu\nu\rho}$ field to the fermion axial current.

In four space-time dimensions, the KR three-form H can be expressed in terms of its dual pseudoscalar $b(x)$ (KR "axion") field [8, 10]

$$\partial^\mu b = -\frac{1}{4}e^{-2\phi}\varepsilon_{abc}^{\ \ \ \mu}H^{abc}, \tag{11}$$

where $\varepsilon^{0123} = +1$, $\varepsilon_{0123} = -1$, $etc.$ is the gravitationally covariant totally antisymmetric Levi-Civita tensor. From the definition of B_μ in (9), we deduce that

$$B^\mu = \partial^\mu b(x)\ . \tag{12}$$

The full effective action S_{eff} is given by

$$S_{eff} = S_B + S_{Dirac} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + \frac{8}{3}\partial_\sigma b\,\partial^\sigma b - \Omega\right) + S^{Free}_{Dirac} - \int d^4x\sqrt{-g}\partial_\mu b\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} \tag{13}$$

Upon splitting the *quantum* field into a background, $\bar{b}(x)$, and fluctuations, $\tilde{b}(x)$,

$$b(x) = \bar{b}(x) + \tilde{b}(x), \tag{14}$$

we find the equations of motion for the KR background field $\bar{b}(x)$ to be,

$$\partial_\alpha\left[\sqrt{-g}\left(\frac{8}{3\kappa^2}\partial^\alpha\bar{b} - J^{5\,\alpha}\right)\right] = 0. \tag{15}$$

It was assumed that the background field $\bar{b}(x)$ is linear in cosmic time t, so that $\dot{\bar{b}}$ is constant and prevents any mixing terms $(\bar{b}\tilde{b})$ appearing in the effective action. On expanding the current about the condensate $J^5_0 = \langle J^5_0\rangle$ + quantum fluctuations (and ignoring the fluctuations) and setting $B_\mu = \partial_\mu\bar{b}$ we obtain,

$$\partial_t\left[\sqrt{-g}\left(\frac{8}{3\kappa^2}B_0 - \langle J^5_0\rangle\right)\right] = 0, \tag{16}$$

from this expression we are able to obtain the temperature dependence for the CPT violating background field $B_0(T)$.

We replace the condensate of the axial current in (16) by its thermal counterpart, $\langle J^{0\,5}\rangle_T$, since we assume thermal equilibrium for (high) temperatures above decoupling of the heavy sterile neutrinos $T \geq T_D$. It is found that the thermal current expectation value $\langle J^{0\,5}\rangle_T$ sums to zero and therefore does not contribute [4]. In our cosmological scenario, the backgrounds depend at most on cosmic time, which in turn can be related to temperature, $t = t(T)$. The relationship between the cosmic time t and temperature T depends on the cosmological era. If we assume that the decoupling temperature $T_D = \mathcal{O}(100)$ TeV (as was the case for the constant-B_0 case of [3]) the relevant cosmological era is the radiation era. This assumption will, a posteriori, be shown to be consistent. In the radiation era, for which the scale factor $a(t)$ of the universe scales as follows :

$$a(t)_{\rm rad} \sim t^{1/2} \sim T^{-1} \quad \Rightarrow \quad t \sim T^{-2}. \tag{17}$$

The metric determinant then scales in that era as $\sqrt{-g} \propto a(t)^3 \propto T^{-3}$. The most general solution of (16) then, when expressed in terms of T reads:

$$B_0(T) \simeq A\,T^3, \tag{18}$$

where the constant A will be determined by the boundary condition $B_0(T = T_D \sim 100$ TeV), which in turn is given by the requirement of the production of phenomenologically acceptable values for lepton asymmetry.

3. Leptogenesis

In this section we proceed to solve the lepton asymmetry Boltzmann equation that will allow a determination of the asymmetry abundance for the decay processes into the charged leptons $N \leftrightarrow l^- h^+$ and $N \leftrightarrow l^+ h^-$, as well as the decay processes into the neutral leptons $N \leftrightarrow \nu h^0$ and

$N \leftrightarrow \bar{\nu} h^0$. For details on the analysis of the abundance of the heavy neutrino see [3, 4]. The lepton asymmetry Boltzmann equation is given by,

$$\frac{d\mathcal{L}}{dx} + J(x)\mathcal{L} = K(x), \qquad x = \frac{m_N}{T} < 1, \qquad \mathcal{L} = Y_{l(-)} - Y_{l(+)}, \tag{19}$$

$$J(x) = \omega^2 x^{10/3}(1 - 0.5668x^2 + 0.3749x^4),$$

$$K(x) = \Big[\nu^2 x^{13/3}(1 - 0.2385x^2 - 0.3538x^4)\bar{Y}_N(x) - \sigma^2 x^{13/3}(1 - 0.1277x^2 - 1.4067x^4) - \delta^2\Big]\frac{B_0(x)}{m_N}$$

$$\omega^2 \simeq 1.1381, \quad \nu^2 \simeq 6.5459, \quad \sigma^2 \simeq 0.0281, \quad \delta^2 \simeq 0.038.$$

where for $0 < x < 1$ the solution for $B_0(x)$ is given by:

$$B_0 = A\,T^3 = \Phi\,x^{-3}, \qquad \Phi \equiv A\,m_N^3. \tag{20}$$

This is the modification from the analysis given in [3] to include the temperature dependence of the CPT violating background field. The integrating factor method (for a linear first order differential equation) is used to solve the Boltzmann equation. The general solution is given by,

$$\mathcal{L}(x) = I_{\mathcal{L}}^{-1}(x)\int^x d\tilde{x} K(\tilde{x}) I_{\mathcal{L}}(\tilde{x}). \tag{21}$$

where the integrating factor is given by,

$$I_{\mathcal{L}}(x) = \exp\Big[\int^x d\tilde{x} J(\tilde{x})\Big] = \tilde{D}\exp\Big[0.2626x^{13/3} - 0.1019x^{19/3} + 0.0512x^{25/3}\Big]. \tag{22}$$

When using the integrating factor, the expression is simplified by expanding the exponential to first order in the calculation of the integral appearing in the general solution. The general solution is given by,

$$\begin{aligned}\mathcal{L}(x<1) \simeq\ & \frac{\Phi}{m_N}\Big[1 - x^{13/3}(0.2626 - 0.1019x^2 + 0.0512x^4)\Big] \\ & \times \Big[0.019x^{-2} + 0.0182x^{7/3} - 0.0027x^{13/3} + 0.0014x^{19/3} + 0.002x^{20/3} \\ & - 0.0023x^{26/3} + 0.0013x^{32/3} - 0.004x^{11} + 0.0001x^{38/3} + 0.0028x^{13} \\ & - 0.0001x^{44/3} - 0.0005x^{15} - 0.0038x^{46/3} + C_0\Big].\end{aligned} \tag{23}$$

The final constant of integration (C_0) is found by equating the lepton asymmetry equilibrium abundance to the Boltzmann solution ($\mathcal{L}^{eq}(x_P) \simeq \mathcal{L}(x_P)$) evaluated at a point $x_P < 1$.

$$\mathcal{L}^{eq}(x) \simeq 0.0455\frac{B_0(x)}{m_N}x, \qquad B_0(x) \simeq \Phi x^{-3}, \tag{24}$$

where the lepton asymmetry equilibrium abundance is the difference between the leptons of helicity $\lambda = -1$ and the anti-leptons of helicity $\lambda = +1$. Once a point x_P is chosen we perform a

Table 1. A table showing the different values of the constant Φ and the background field B_0 with respect to different expansion points x_P at the decoupling value of $x_D = 1$.

x_P	$\frac{\Phi}{m_N}$	Φ(keV)	$B_0(x_D)$(keV)
0.50	3.6×10^{-12}	0.36	0.36
0.75	5.9×10^{-12}	0.59	0.59
0.90	7.4×10^{-12}	0.74	0.74

$(7,7)$ diagonal Padé expansion around this point for the Boltzmann equation solutions ($\mathcal{L}(x<1)$ and $\bar{Y}_N(x<1)$) in order to study the regime of $x = x_D \geq 1$ where decoupling orders. The observable lepton asymmetry is then calculated by,

$$\frac{\Delta L^{Total}}{s} = \frac{\mathcal{L}^P(x_D = 1,\ x_P < 1)}{2\bar{Y}_N^P(x_D = 1,\ x_P < 1)} = q\frac{\Phi}{m_N} \simeq 8 \times 10^{-11}, \tag{25}$$

where q is a number that depends on the expansion point x_P. Equating this result to the phenomenological value we obtain a range of values for the constant Φ depending on the chosen point x_P, the value of decoupling is $x_D = 1$. The results are given in table 1.

Compared to the case of constant B_0 studied in [3], we observe that the LV and CPTV value of the background field B_0 at decoupling $T_D = \mathcal{O}(100)$ TeV, which yields phenomenologically acceptable lepton asymmetry in the universe is smaller, being in the keV range. Using the temperature dependence of the background field and the final constant being known from the point of decoupling ($\Phi = (0.36 - 0.74)$ keV), we can get an estimate of the magnitude of the background field today. Today we have $x^{today} \simeq 4.2 \times 10^{17}$ which leads to,

$$B_0^{today} = B_0(x^{today}) \simeq (4.9 - 10.0) \times 10^{-51} \text{ eV} \tag{26}$$

These values indicate that the current value of the "torsion" LV and CPTV field B_0 lies comfortably within the current bounds [11], $B_0 < 0.01$ eV and (for the spatial components) $B_i < 10^{-31}$ GeV; so even a boost by small velocities, due to a difference of the laboratory frame with respect to the cosmological frame, will still yield spatial components within the above limits.

4. Conclusions

In this work we have generalised our previous study of leptogenesis based on a *constant* LV and CPTV time-like axial background to the case of a torsion background varying with the temperature of the early universe. The torsion is provided here by the antisymmetric tensor (Kalb-Ramond) spin-one field of the massless bosonic multiplet of closed string theory. The *phenomenology* of our leptogenesis, remains largely unchanged from the constant background case, and is consistent with the stringent current epoch constraints on LV and CPTV, as well as cosmological constraints on the vacuum energy density. The order of magnitude estimate of the CPT violating background field required to produce the observed lepton asymmetry is calculated to be $B_0(x_D) \simeq (0.36 - 0.74)$ keV.

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