FCC-hh single beam stability

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Updates on:
- Beam pipe impedance
- Beam stability estimates
- Electron cloud buildup estimates and scaling
- Other collective effects
Resistive wall impedance: LHC and FCC-hh

Growth rate: \( \tau^{-1} = \omega_0 \Im \Delta Q \) \hspace{1cm} \text{(Sacherer 1974)}

\[
\frac{1}{\tau_k} = -\frac{1}{1 + k} \frac{\omega_0 qMI_b}{4\pi E_0} \beta_y \Re Z_y (\omega_{\min}) F'_k (\omega_{\min} - \frac{\chi}{\tau_b})
\]

\( \chi = Q' \gamma_i \omega_0 \tau_b \)

(Chromatic phase shift)

\( \omega_{\min} = (n - Q_y) \omega_0 \)

(lowest sideband)

\( \chi = 0: \)

growth time at 3.3 TeV:
approx. 50 turns
at 50 TeV:
approx. 500 turns
LHC at 7 TeV:
approx. 2000 turns

\( Z_\perp (\omega) = (1 - i) \frac{c}{\pi \omega b^3 \delta \sigma} \)

(Thick) resistive wall impedance

Transverse impedances (vertical real part)

\[ (n - Q)f_0 \]

FCC: kHz

LHC: few kHz
2D impedance code in frequency space

- Open source package FEniCS (A. Logg, K. Mardal, G. Wells et al.)
- Mesh from GMSH (C. Geuzaine, J. Remacle)

\[ \nabla \times \mu^{-1} \nabla \times \mathbf{E} - \omega^2 \varepsilon \mathbf{E} = -i\omega \mathbf{J}_s \]

U. Niedermayer et al., Space charge and resistive wall impedance computation in the frequency domain using the finite element method, Phys. Rev. ST-AB 18, 032001, 2015

BeamlImpedance2D (PYTHON): https://bitbucket.org/uniederm/beamimpedance2d.git
**FCC pipe: Vertical vs Horizontal**

Design Cu coating (d=0.3/0.1 mm)

'Full' Cu coating (d=0.3/0.1 mm)

Design vs Full: Increased horizontal impedance

\[
R_{x,y} \propto (n - Q) f_0
\]

\[
T \propto \frac{1}{f_0^b}
\]
\[ Z_{x,k}^{\text{eff}} = \sum_{p} \langle \hat{\beta}_x \rangle | \Delta_k (\omega_p - \omega_\xi) |^2 \]

\[ R \quad Q_{k=0} \approx Q_s \quad Q_s = \frac{E}{t^2_0 E_{0,b}} \]
(tune shift)  
(synchrotron tune)

TMCI threshold bunch intensity:

\[ N_{th}^{b} \approx \frac{4}{e^2} \frac{E}{\sum_{y} Z_{y,0}^{\text{eff}}} \]

Fully Cu coated: \[ \frac{N_{th}^{b}}{N_b} \approx 6 \]

Partially Cu coated: \[ \frac{N_{th}^{b}}{N_b} \approx 3 \]
FCC pipe: a-C coating

Pipe with a-C coating (1 $\mu m$)

**Remark:** For an SEY of about 1 the coating can be much thinner, for example only 30 nm. P.Pinto (CERN)
HTS coated FCC screen

Hybrid coating (HTS stripes):
- Possible reduction of the resistive wall instability growth rates by factor 5-6.
- TMCI thresholds!

Impedance contributions and database

\[ Z_{\text{eff}}^{x,k} = \sum_p \beta_x Z_x (\omega_p) \left| \Delta_k (\omega_p - \omega_\xi) \right|^2 \frac{\langle \hat{\beta}_x \rangle \left| \Delta_k (\omega_p - \omega_\xi) \right|^2}{\left( \Delta_k (\omega_p - \omega_\xi) \right)^2} \]

\( \Delta_k (\omega) \): Spectrum of head-tail modes

**Example:** Coupled bunch instability

-26 MΩ/m 100%  
-104 MΩ/m  
-1054 MΩ/m  
-145 MΩ/m  
-403 MΩ/m  
-1804 MΩ/m

**Example:** TMCI

\( \leq 0.1 \text{ MΩ/m}  \)
\( 2.4 \text{ MΩ/m}  \)
\( 1.8 \text{ MΩ/m}  \)
\( 0.4 \text{ MΩ/m}  \)
\( 1.3 \text{ MΩ/m}  \)
\( 3.8 \text{ MΩ/m}  \)

\( 46.4 \text{ MΩ/m}  \)

S. Arsenyev (IPAC 18)
Beam stability: LHC vs FCC

Growth rate for transverse coupled bunch instabilities:

\[ \mu \frac{q^2 N_b}{m l_{bb}} Z \]

TMCI single bunch threshold bunch intensity:

\[ N_{th}^b = \frac{4 E}{\gamma^2 \sqrt{2 \tilde{\gamma} Z_{y,0}^{\text{eff}}}} \]

- Larger circumference (5:1) -> lower frequency: **1 kHz vs 8 kHz**
- Smaller screen diameter (2:3) -> **larger impedance** (factor 3)
- Screen temperature: 50 K (5:2), maximum field 16 T (2:1) -> **changed conductivities**
- Larger average \( \beta \)-function (2:1) -> **growth rates**
- Smaller beams (1:3) -> weaker **Landau damping, e-cloud thresholds**
- LHC-like bunches and 25 ns spacing (1:1)
Space charge in the FCC ?

\[ Q_{sc} \propto \frac{q^2 N_b}{b_{n,y}} \]

(space charge tune shift)

\[ Q_s = \frac{E}{2 t} \]

(synchrotron tune shift)

\[ \frac{Q_k}{Q_s} \approx \frac{1}{2} \frac{Q_{sc}}{Q_s} + k \]

with

\[ \frac{Q_{sc}}{Q_s} \propto \frac{2}{E} \]

\[ \Delta Q_k = Q_k - Q_0 = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{(\Delta Q_{sc} / 2)^2 + (k Q_s)^2} \]

Head-tail tune shift with space charge:
Multibunch mode coupling vs Sacherer's formula

Klinkenberg, Arsenyev, Schulte
Landau damping of head-tail modes

Head-tail modes (including the k=0 mode) are not rigid-bunch modes:

Particle tracking with octupoles and resistive wall wake.
Landau damping with octupoles: Results of particle tracking

- Octupoles provide a similar stabilization for higher-order modes
- The 2D “rigid-bunch” dispersion relation can be applied to “non-rigid” modes
- Octupoles: reliable well-understood damping mechanism
- For k>0: fewer octupoles needed (lower growth rates)
- The k=0 mode will be stabilized by feedback systems.

V. Kornilov
Landau damping: Electron lenses

Tune shift: \( \Delta Q_x^e(J_x, J_y) \approx 2\Delta Q_e (1 - a J_x - a J_y) \)

\[ \Delta Q_x^e(J_x, J_y) = 2\Delta Q_e \int_0^{1/2} \frac{I_0(\tilde{J}_x u) - I_1(\tilde{J}_x u)}{\exp(\tilde{J}_x u + \tilde{J}_y u)} I_0(\tilde{J}_y u) \, du \]

Electron lenses

Proof of principle experiment in LHC required!

Example: One lens (l=2 m, \( I_e = 1 \text{ A} \)) in LHC would provide a tune spread similar to the 168 octupoles.

V. Shiltsev et al., PRL (2017)

Injection energy

T. Pieloni, C. Tambasco (2018)
Electron cloud buildup and scaling

Example from openE CLOUD:
Saturated e-cloud density in FCC drift section without a-C coating

Questions (to the simulations):
- Minimum a-C coated area
- Scaling of buildup / heat-load with beam energy (and pipe radius)

Scaling with pipe radius.

Scaling with beam energy (beam radius):
A new gridless ecloud-code code has been developed!
Summary and outlook

- Impedance contributions and database
- Landau damping and requirements: Octupoles and electron lens
- Electron cloud buildup thresholds (SEY requirements)

-> CDR input

To do:

- Role of mode coupling for coupled bunches in FCC
- Proof of principle experiments with electron lenses
- Electron cloud and other sources of tune shifts: Beam stability

- ....
Backup
Stability with octupoles: FCC top energy

\[ I_{\text{oct}}^F = +500 \text{ A} \]
\[ I_{\text{oct}}^D = +500 \text{ A} \]

\[ \Delta Q_{\text{coh}} - \text{Damping as in LHC:} \]
\[ 3554 \text{ LHC-octupoles.} \]
\[ 508 \text{ Advanced-technology octupoles.} \]

V.Kornilov
Octupoles and Landau damping

Tune shifts from octupoles:

$$\Delta Q_x = a_x J_x - b_{xy} J_y$$
$$\Delta Q_y = a_y J_y - b_{xy} J_x$$

($J_{x,y}$: actions variables)

Scaling with energy:

$$\Rightarrow N_{oct} L_m \propto E_0^2$$

From LHC to FCC-hh: $7^2 \times 168$ octupoles

$E_0 = \gamma_0 mc^2$

$L_m$: length of magnet

$N_{oct}$: # of magnets

2D dispersion relation [1-3]:

$$1 = \Delta Q_{coh} \int \frac{1}{\Delta Q_{oct} - \Omega/\omega_0} J_x \frac{\partial \psi_\perp}{\partial J_x} dJ_x dJ_y$$


2D beam distribution:

$$\psi_\perp (J_x, J_y) = e^{-(J_x + J_y)}$$

rms tune spread:

$$\delta Q_{x,y} = \langle \Delta Q_{x,y} (J_x, J_y) \rangle$$

$$| \Delta Q | \lesssim \delta Q$$

$$\tau^{-1} = \omega_0 \mathcal{S} \Delta Q$$

Very approximate stability condition
Finite chromaticity

Example case $Q'=15$: the growth rates for the $k=0$ coupled bunch mode would be lower by a factor 0.6, but the $k=1$ mode would be present in addition (growth rate corresponding to 650 turns at injection).

(Sacherer 1974)
Landau damping: Scaling with energy

The good news: \[ \frac{1}{\tau} \propto \frac{1}{E_0} \] (instability growth rate)

The OK news: \[ \delta Q_{oct} \approx (\omega_0 \tau)^{-1} \propto \frac{L}{E_0} \] (tune spread required for LD)

The bad news: \[ \delta Q_{oct} \approx N_{oct} L_m \frac{\varepsilon}{E_0^2} \] (tune spread provided by octupoles)

\[ E_0 = \gamma_0 mc^2 \]

\( L \): circumference

\( L_m \): length of magnet

\( N_{oct} \): # of magnets

\( \varepsilon \): normalized emittance

\[ \Rightarrow N_{oct} L_m \propto E_0^2 \]

From LHC to FCC-hh: \( 7^2 \times 168 \) octupoles
Landau damping: Possible alternative schemes

**FCC-hh**: Active feedback for \( k=0 \) modes, Landau damping for \( k>1 \).
Still, additional Landau damping concepts are helpful!

**LHC**: 10 x larger stability area then with octupoles

\[
\Delta Q_{\text{max}} = 0.01
\]
\[
l_e = 2 \text{ m}
\]
\[
I_e = 0.8 \text{ A}
\]

\[
\Delta Q_{x,y}(z) = \pm \frac{q\hat{\beta}_{x,y}k^{(2)}}{4\pi m\gamma_0} \cos \left( \frac{\omega_{rf} z}{c} + \phi \right)
\]
\[
\delta Q_{x,y} \propto J_z \quad \text{(longitudinal action)}
\]

No local spread (in \( z \))!
Dispersion relation and Landau damping?

Detailed particle tracking studies are ongoing (see also V. Kornilov).

Shiltsev et al, PRL 2017

\[
\Delta Q_{\text{max}} = 0.01
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\[
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\[
I_e = 0.8 \text{ A}
\]

Radio-Frequency Quadrupole (RFQ)

Grudiev PRAB 2014
Schenk et al, IPAC17

\[
\Delta Q_{x,y}(z) = \pm \frac{q\hat{\beta}_{x,y}k^{(2)}}{4\pi m\gamma_0} \cos \left( \frac{\omega_{rf} z}{c} + \phi \right)
\]
\[
\delta Q_{x,y} \propto J_z \quad \text{(longitudinal action)}
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No local spread (in \( z \))!
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**Electron cloud: Tune shift and relativistic limit**

**Tune shift** induced by the pinch along the bunch

\[ \Delta Q_x(z) = \frac{r_p L b_y}{\gamma_0 (b_x + b_y)} \bar{\rho} \lambda_z(z) \]

\[ \bar{\rho} \approx \frac{E_s}{\pi m_e c^2 r_e b^2} \]

(b: pipe radius)

Furman, Zholents, PAC 1999
Petrov, Boine-Frankenheim, PRAB 2014

Tune shift potentially effects Landau damping (see for example Burov 2013).

Electron space charge field \( \mathbf{E} \) and instability thresholds in the **ultrarelativistic limit** \( \alpha \to 0 \)

\[ \rho_b(r, z) \to \delta(r) \lambda(z) \]

\[ \epsilon_0 \nabla \cdot \mathbf{E} = \rho_e \]

\[ \omega_e = \frac{\sqrt{\lambda_b r_e c^2}}{a} \to \infty \]

D. Astapovych