

Review on the Hadronic Equation of State for Neutron Stars



Laura Tolós

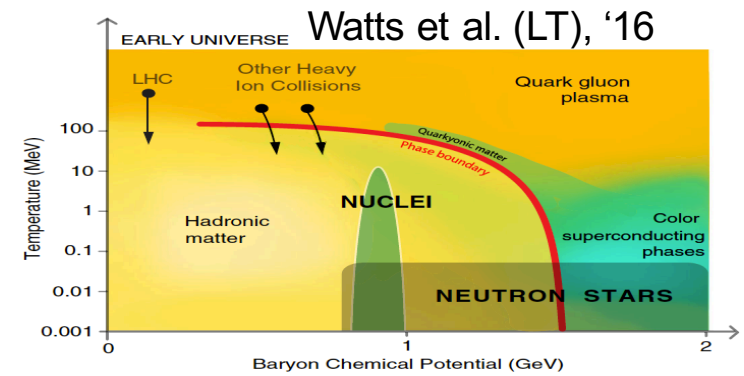
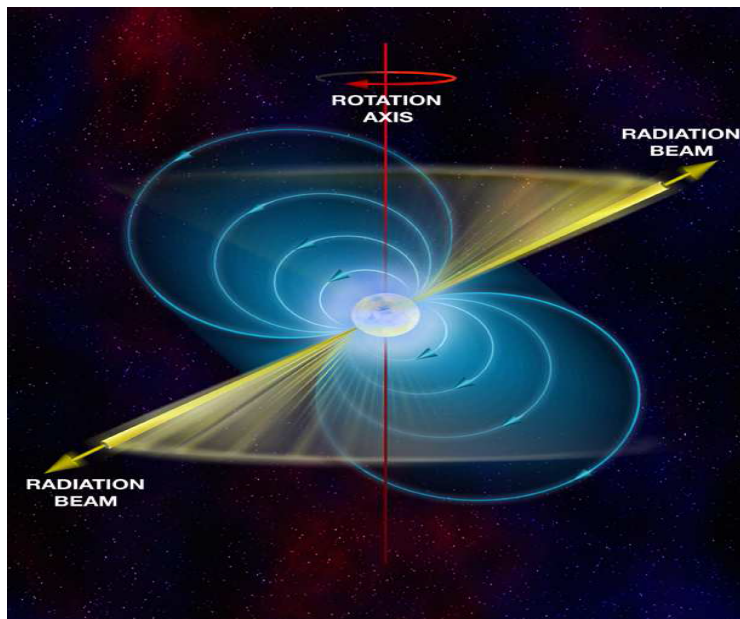
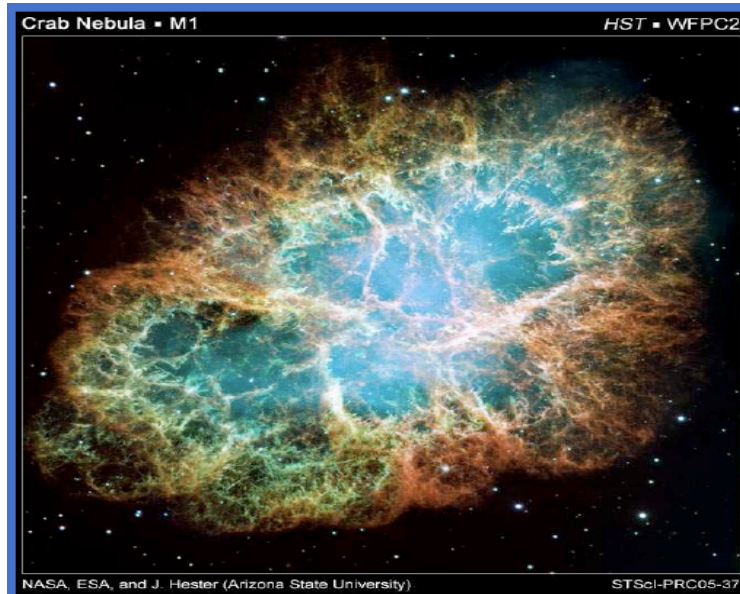
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Outline

- ✧ What is a Neutron Star?
- ✧ Observations: Mass, Radius & GW170817
- ✧ Internal Structure and Composition: the Inner Core
- ✧ Nuclear Equation of State for the Inner Core
- ✧ Constraints on the Nuclear Equation of State
- ✧ Ab-initio versus Phenomenological Models
- ✧ What about Hyperons?
- ✧ Structure of a Neutron Star: Mass and Radius
- ✧ Challenges and Future

What is a Neutron Star?

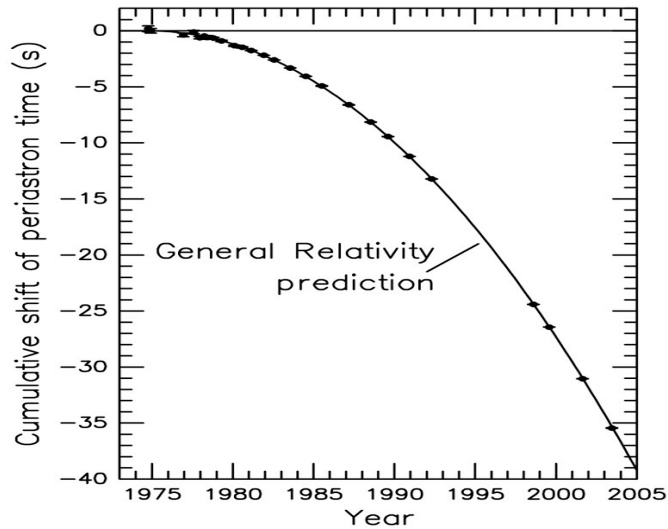


- produced in **core collapse supernova explosions**, usually observed as **pulsars**
- usually refer to compact objects with $M \approx 1-2 M_{\odot}$ and $R \approx 10-12 \text{ Km}$
- extreme densities up to $5-10 \rho_0$ ($n_0 = 0.16 \text{ fm}^{-3} \Rightarrow \rho_0 = 3 \cdot 10^{14} \text{ g/cm}^3$)
- magnetic field : $B \sim 10^{8..16} \text{ G}$
- temperature: $T \sim 10^{6..11} \text{ K}$
- observations: **masses, radius (?), gravitational waves, cooling...**

Observations: Mass

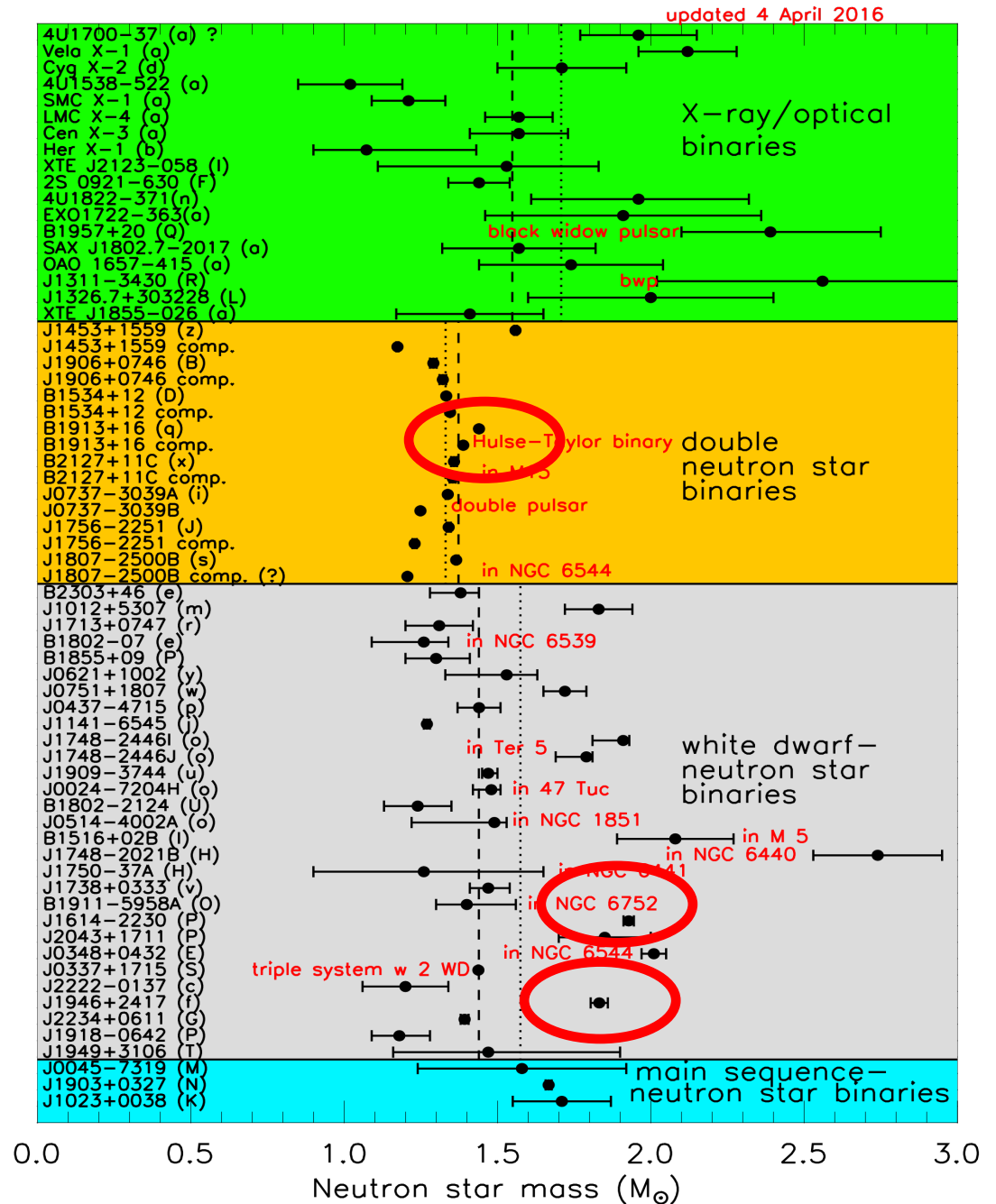
Lattimer '16

- > 2000 pulsars known
- best determined masses:
Hulse-Taylor pulsar
 $M = 1.4414 \pm 0.0002 M_{\odot}$
Hulse-Taylor Nobel Prize 94



- PSR J1614-2230¹
 $M = (1.97 \pm 0.04) M_{\odot}$;
- PSR J0348+0432²
 $M = (2.01 \pm 0.04) M_{\odot}$

¹Demorest et al '10; ²Antoniadis et al '13



Radius

analysis of X-ray spectra from neutron star (NS) atmosphere:

- RP-MSP: X-ray emission from radio millisecond pulsars
- BNS: X-burst from accreting NSs
- QXT: quiescent thermal emission of accreting NSs

theory + pulsar observations:

$$R_{1.4M} \sim 11-13 \text{ Km}$$

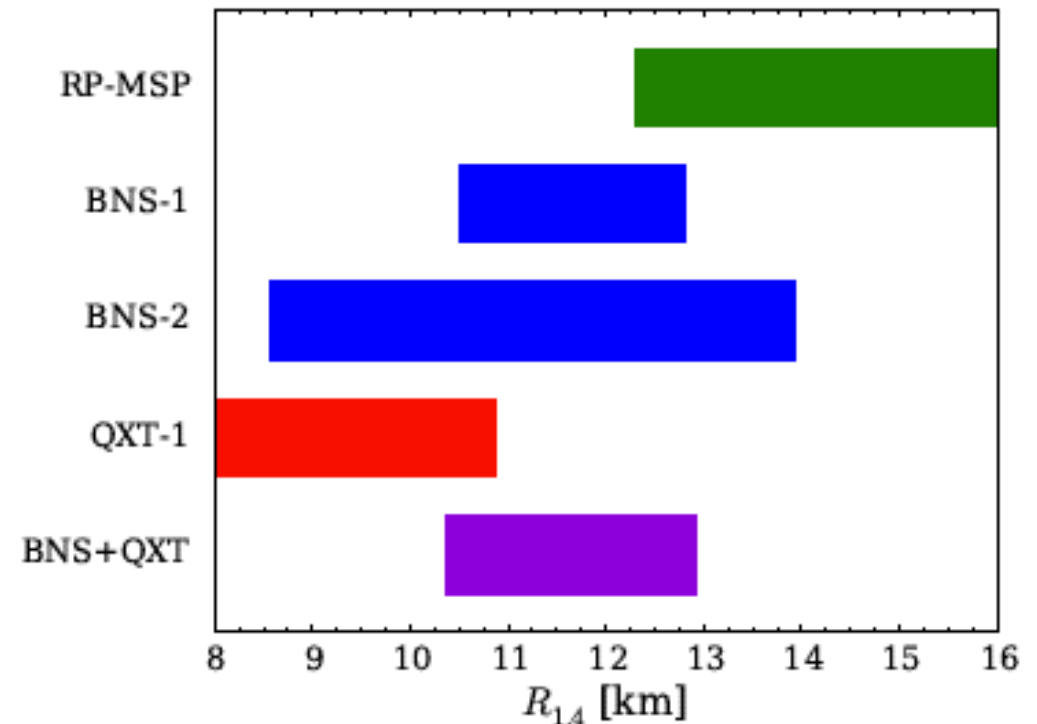
Lattimer and Prakash '16

Some conclusions:

- ✓ marginally consistent analyses, favored $R < 13 \text{ Km}$ (?)
- ✓ future X-ray telescopes (NICER, eXTP) with precision for M-R of $\sim 5\%$
- ✓ what about GW170817?

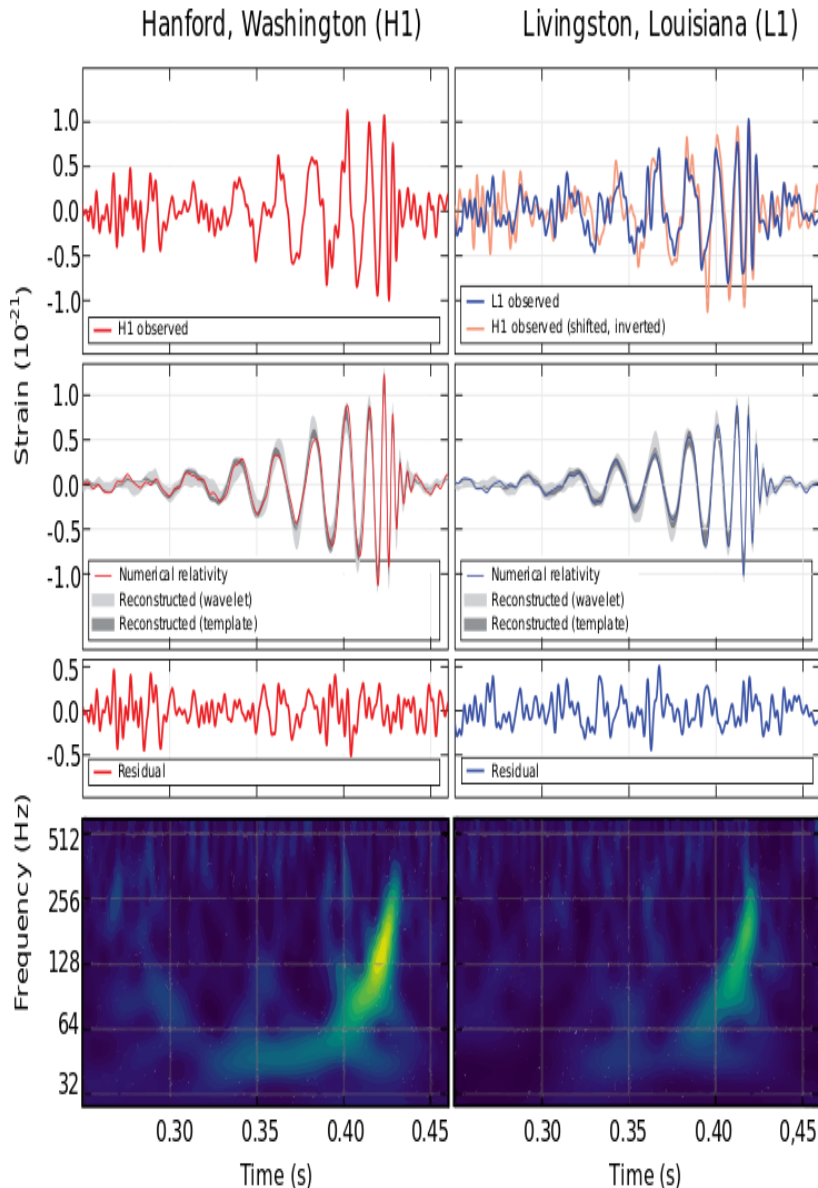
Fortin et al '15:

- RP-MSP: Bodganov '13
- BNS-1: Nattila et al '16
- BNS-2: Guver & Ozel '13
- QXT-1: Guillot & Rutledge '14
- BNS+QXT: Steiner et al '13



adapted from Fortin's talk
@ NewCompstar Annual Meeting '16

GW170817



Abbot et al. (LIGO-VIRGO) '17

	Low-spin prior ($\chi \leq 0.05$)	High-spin prior ($\chi \leq 0.89$)
Binary inclination θ_{JN}	146^{+25}_{-27} deg	152^{+21}_{-27} deg
Binary inclination θ_{JN} using EM distance constraint [105]	151^{+15}_{-11} deg	153^{+15}_{-11} deg
Detector frame chirp mass \mathcal{M}^{det}	$1.1975^{+0.0001}_{-0.0001} M_{\odot}$	$1.1976^{+0.0004}_{-0.0002} M_{\odot}$
Chirp mass \mathcal{M}	$1.186^{+0.001}_{-0.001} M_{\odot}$	$1.186^{+0.001}_{-0.001} M_{\odot}$
Primary mass m_1	$(1.36, 1.60) M_{\odot}$	$(1.36, 1.89) M_{\odot}$
Secondary mass m_2	$(1.16, 1.36) M_{\odot}$	$(1.00, 1.36) M_{\odot}$
Total mass m	$2.73^{+0.04}_{-0.01} M_{\odot}$	$2.77^{+0.22}_{-0.05} M_{\odot}$
Mass ratio q	$(0.73, 1.00)$	$(0.53, 1.00)$
Effective spin χ_{eff}	$0.00^{+0.02}_{-0.01}$	$0.02^{+0.08}_{-0.02}$
Primary dimensionless spin χ_1	$(0.00, 0.04)$	$(0.00, 0.50)$
Secondary dimensionless spin χ_2	$(0.00, 0.04)$	$(0.00, 0.61)$
Tidal deformability $\tilde{\Lambda}$ with flat prior	300^{+500}_{-190} (symmetric) / 300^{+420}_{-230} (HPD)	$(0, 630)$

quadrupole moment $Q_{ij} = -\lambda \mathcal{E}_{ij}$ tidal field

$l=2$ Love number $k_2 = \frac{3}{2} \lambda R^{-5}$ tidal deformability radius

dimensionless tidal deformability $\Lambda = \frac{2k_2}{3C^5}$ compactness: $C=M/R$

$\tilde{\Lambda} ::=$ weighted tidal deformability (Λ_1 and Λ_2)

using tidal deformability sets constraints on

$M_{\text{TOV}} \sim 2.16-2.17 M_{\odot}$

Margalit and Metzger '17, Rezzolla, Most and Weih '18

$9-10 \text{ Km} \lesssim R_{1.4M_{\odot}} \lesssim 13 \text{ Km}$

Annala et al '18, Kumar et al '18, Abbott et al '18, Fattoyev et al '18, Most et al '18, Lim et al '18, Raithel et al '18, Burgio et al '18, Tews et al '18, De et al '18, Abbott et al '18, Malik et al '18, ..

Internal structure and composition:....

- **Atmosphere**

few tens of cm, $\rho \leq 10^4$ g/cm³ made of atoms

- **Outer crust or envelope**

few hundred m's, $\rho = 10^4 - 4 \cdot 10^{11}$ g/cm³ made of free e⁻ and lattice of nuclei

- **Inner crust**

1-2 km, $\rho = 4 \cdot 10^{11} - 10^{14}$ g/cm³ made of free e⁻, neutrons and neutron-rich atomic nuclei

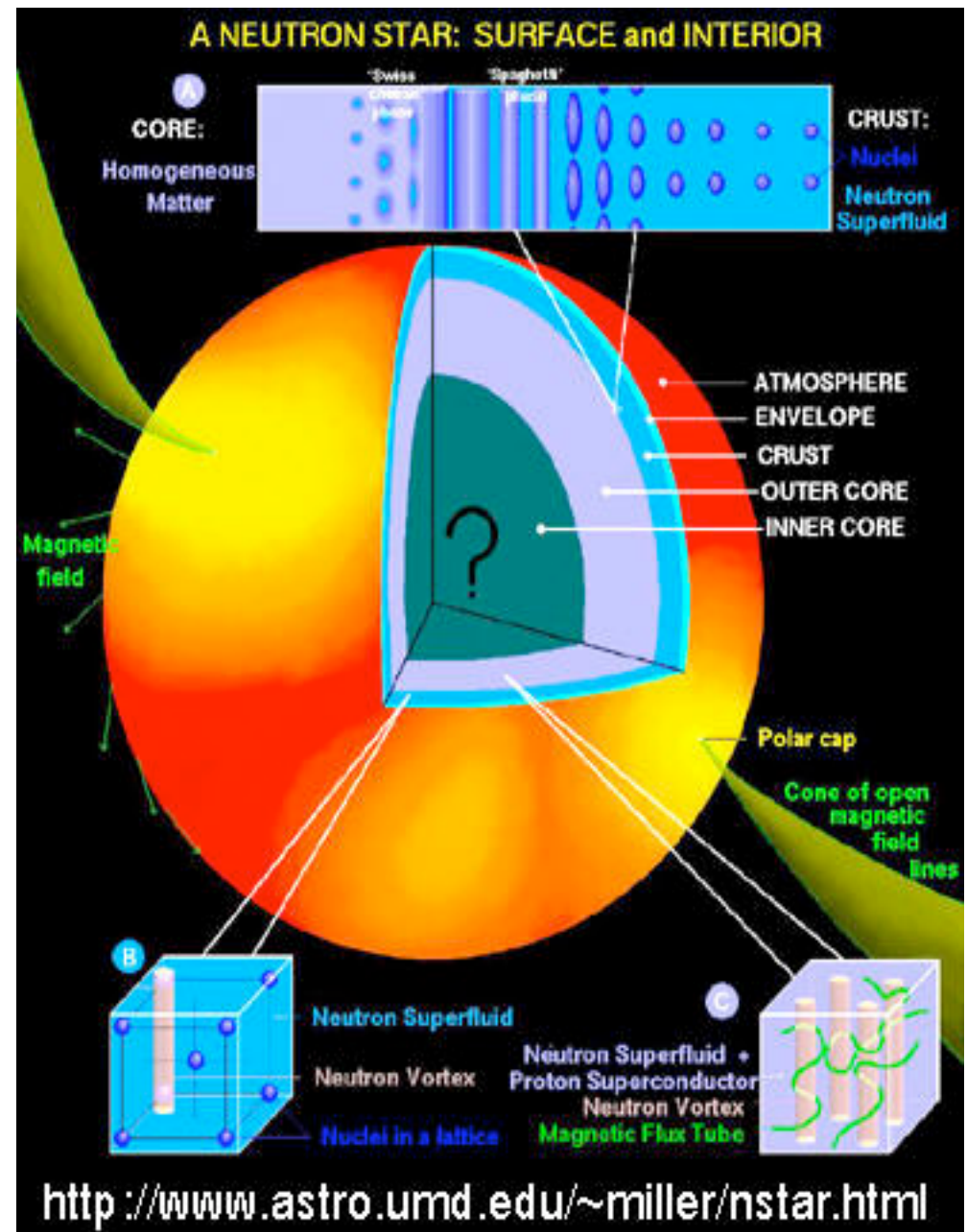
$\sim \rho_0/2$: uniform fluid of n, p, e⁻

- **Outer core**

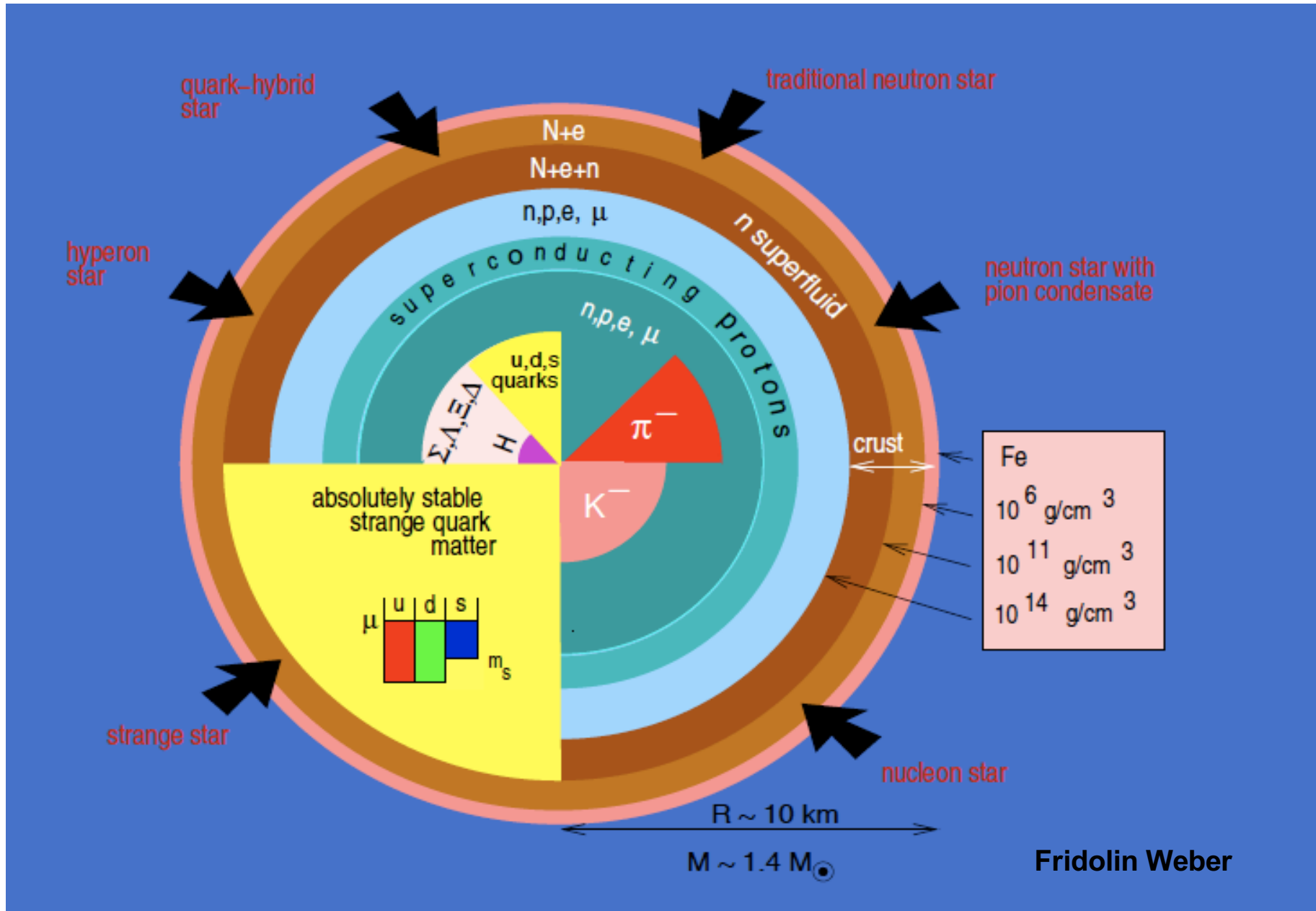
$\rho_0/2 - 2\rho_0$ is a soup of n, e⁻, μ and possible neutron 3P_2 superfluid or proton 1S_0 superconductor

- **Inner core (?)**

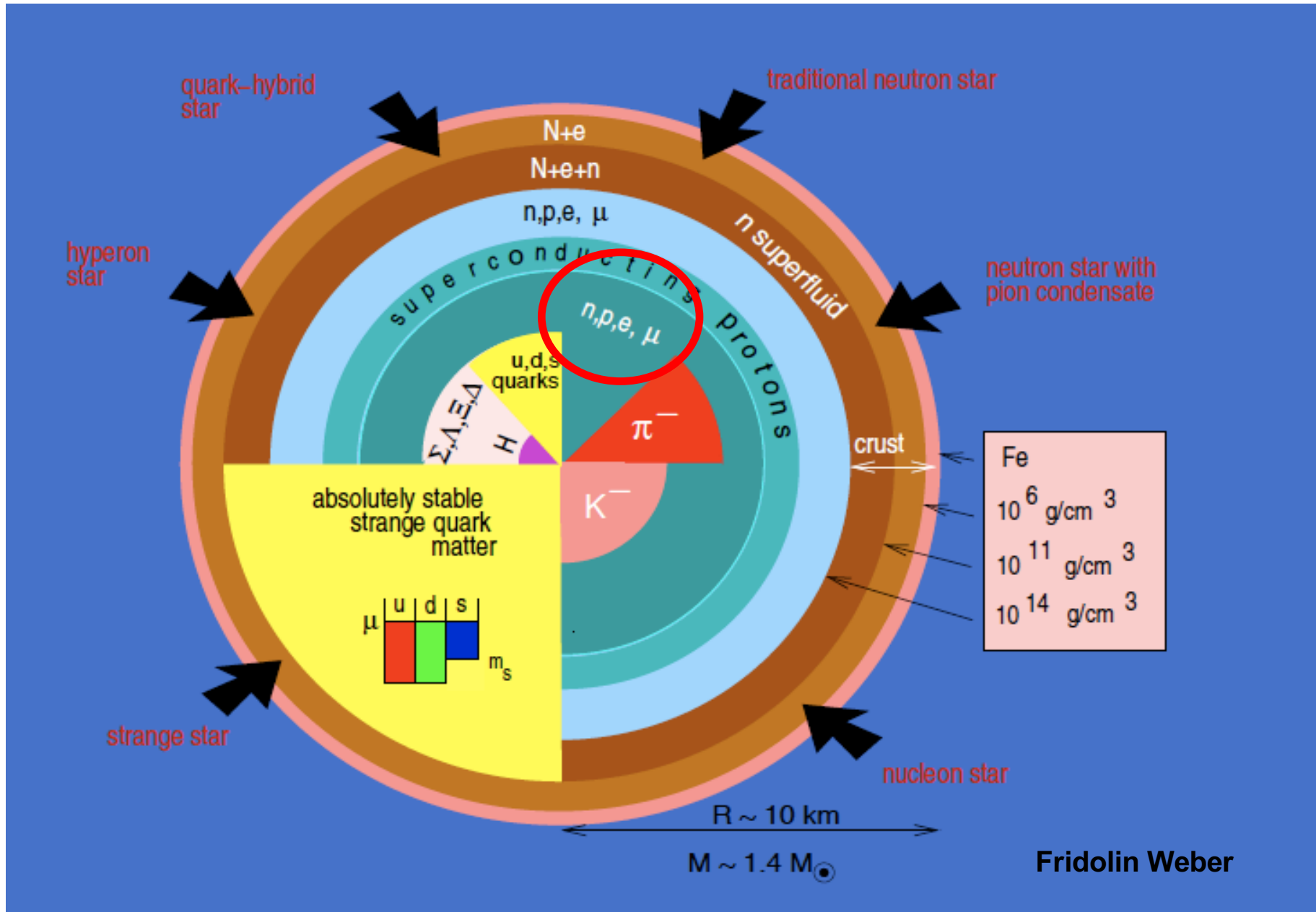
2-10 ρ_0 with unknown interior made of hadronic, exotic or deconfined matter



.... the Inner Core

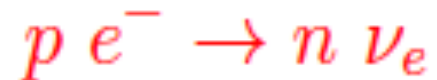
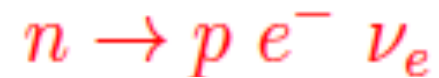


.... the Inner Core: n, p, e, μ



Nuclear Equation of State for the Inner Core

Neutrons, protons and electrons are in β -equilibrium



This equilibrium can be expressed in terms of the **chemical potentials**.
Since the mean free path of the ν_e is $\gg 10$ km, neutrinos freely escape

$$\mu_n = \mu_p + \mu_e$$

Charge neutrality is also ensured by demanding

$$n_p = n_e$$

Note that **baryon number is conserved** too: $n = n_n + n_p$

The **nuclear Equation of State (EoS)** is a relation between thermodynamic variables describing the state of nuclear matter.

At $T=0$,

$$E(n_B, \delta) = E(n_B, 0) + S(n_B)\delta^2$$

with

$$\delta = (N - Z)/A$$

$$A = N + Z$$

neutron
number
proton
number

baryon density

mass number

energy of
symmetric nuclear matter

$$E(n_B, 0) = E(n_0) + \frac{1}{18} K_0 \epsilon^2,$$

$$\epsilon = (n_B - n_0)/n_0$$

symmetry energy

$$S(n_B) = S_0 + \frac{1}{3} L \epsilon + \frac{1}{18} K_{\text{sym}} \epsilon^2$$

$$E(n_0)/A \equiv E_0/A$$

$$n_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$$

$$E_0/A = -16.0 \pm 1.0 \text{ MeV}$$

binding energy per nucleon at
saturation density n_0

$$S_0 \equiv \frac{1}{2} \left(\frac{\partial^2 E}{\partial \delta^2} \right)_{n_B=n_0, \delta=0}$$

symmetry energy at n_0

$$L \equiv 3n_0 \left(\frac{\partial S(n_B)}{\partial n_B} \right)_{n_B=n_0}$$

$$K_0 \equiv 9n_0^2 \left(\frac{\partial^2 E}{\partial n_B^2} \right)_{n_B=n_0, \delta=0}$$

incompressibility at n_0

$$K_{\text{sym}} \equiv 9n_0^2 \left(\frac{\partial^2 S(n_B)}{\partial n_B^2} \right)_{n_B=n_0}$$

Constraints on Nuclear Equation of State

Constraints from Nuclear Physics Experiments

- E/A from experimentally measured nuclear masses

$$n_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$$

$$E_0/A = -16.0 \pm 1.0 \text{ MeV}$$

- K_0 from isoscalar giant monopole resonances in heavy nuclei and HiCs (difficult experimentally)

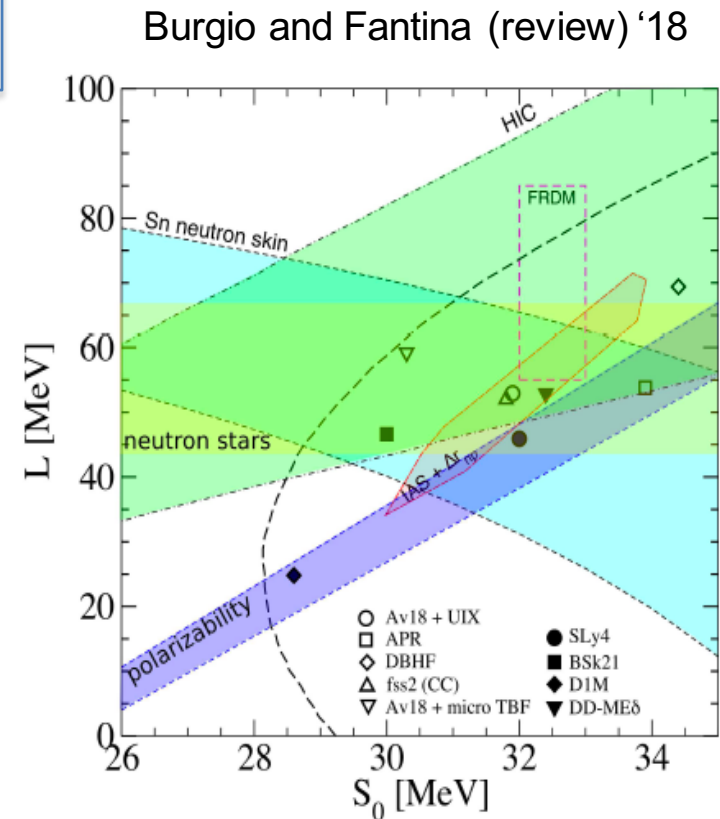
$$? 180 \text{ MeV} < K_0 < 270 \text{ MeV} ?$$

- S_0 from nuclear masses, isobaric analog state phenomenology, neutron skin thickness and HiCs; additionally from NS data (fairly well constrained)

$$S_0 \sim 30\text{-}32 \text{ MeV}$$

- L from dipole resonances, electric dipole polarizability and neutron skin thickness (very uncertain)

- Other higher order coefficients are very uncertain, such as K_{sym}



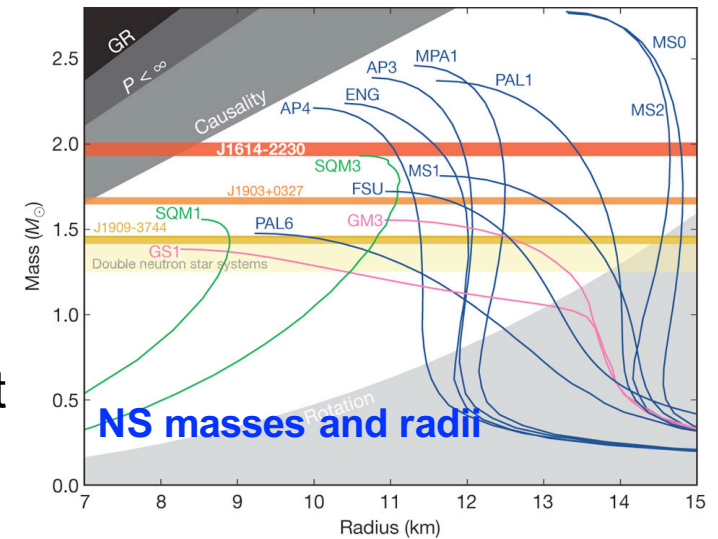
Constraints on Nuclear Equation of State

Constraints from Astrophysical Observations

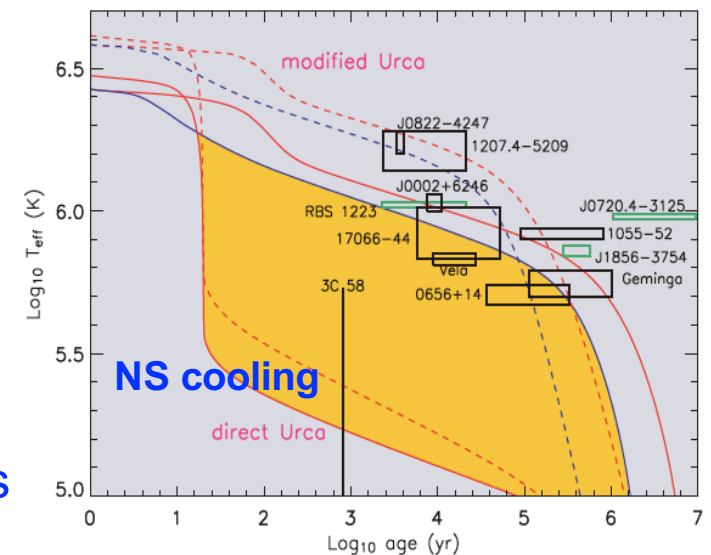
- **NS masses**
 precise values for 2NSs in binary system
 with $\sim 2M_{\odot}$

$$1.5M_{\odot} \lesssim M_{\text{max}} \lesssim 2.5M_{\odot}$$
- **NS radii**
 - precise estimations of NS radii are very difficult because observations are indirect
 - need of simultaneous mass-radius measurement
 - future: NICER, ATHENA+, eXTP
- **NS cooling**
 depends on composition and on occurrence of superfluidity, thus giving complementary information on EoS
- **NS moment of inertia**
 mass and radius constrained by determination of moment of inertia, but not yet measured
- **Gravitational waves and quasi-periodic oscillations**

Ozel et al '16



Lattimer and Prakash '04



Ab-initio versus Phenomenological Models

Microscopic Ab-initio Approaches

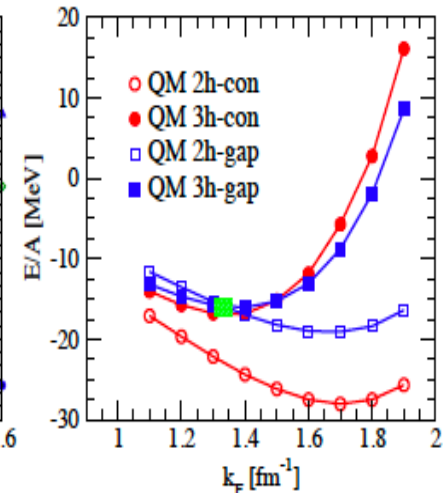
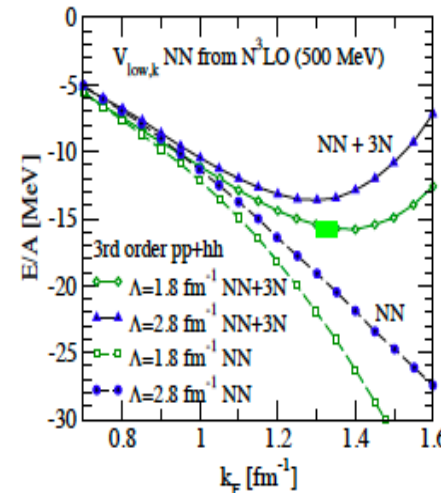
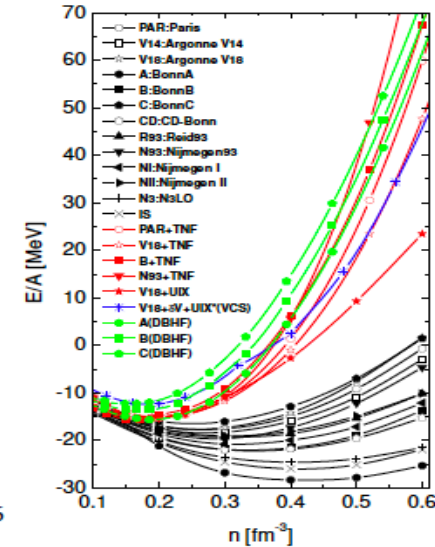
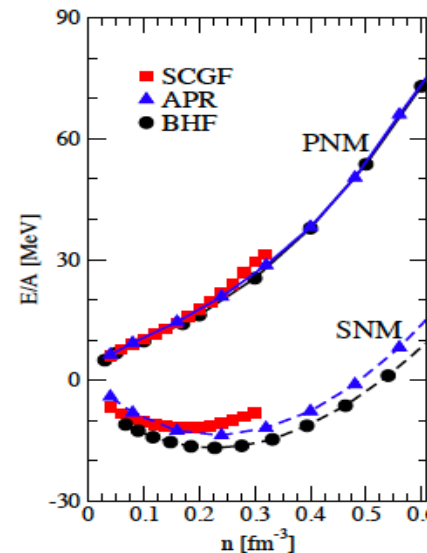
Based on solving the many-body problem starting from two- and three-body interactions

- Variational method: APR, CBF,..
- Quantum Montecarlo : VMC, AFDMC, GFDMC..
- Coupled cluster expansion
- Diagrammatic: BBG (BHF), SCGF..
- Relativistic DBHF
- RG methods: SRG from χ EFT..
- Lattice methods

Advantage: systematic addition of higher-order contributions

Disadvantage: applicable up to?
(SRG from χ EFT \sim 1-2 n_0)

Burgio and Fantina (review) '18



Ab-initio versus Phenomenological Models

Phenomenological Models

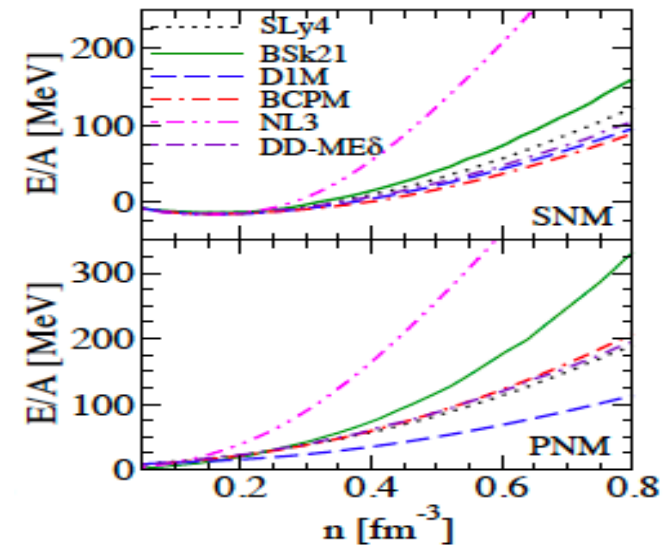
Based on density-dependent interactions adjusted to nuclear observables and neutron star observations

- *Non-relativistic EDF: Gogny, Skyrme..*
- *Relativistic Mean-Field (RMF) and Relativistic Hartree-Fock (RHF)*
- *Liquid Drop Model: BPS, BBP,..*
- *Thomas-Fermi model: Shen*
- *Statistical Model: HWN, RG, HS..*

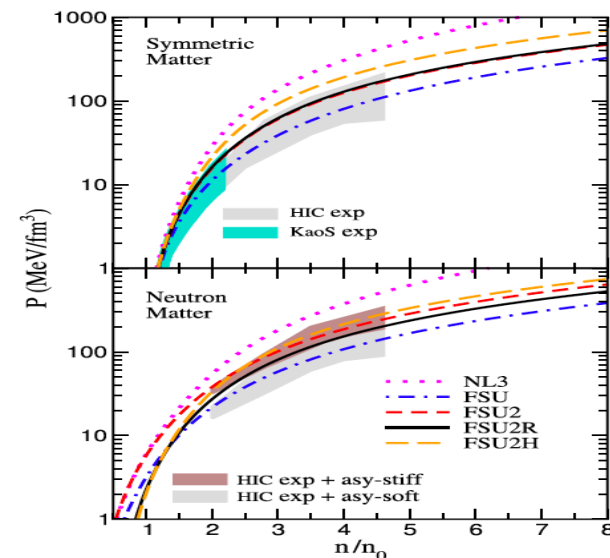
Advantage: applicable to high densities beyond n_0

Disadvantage: not systematic

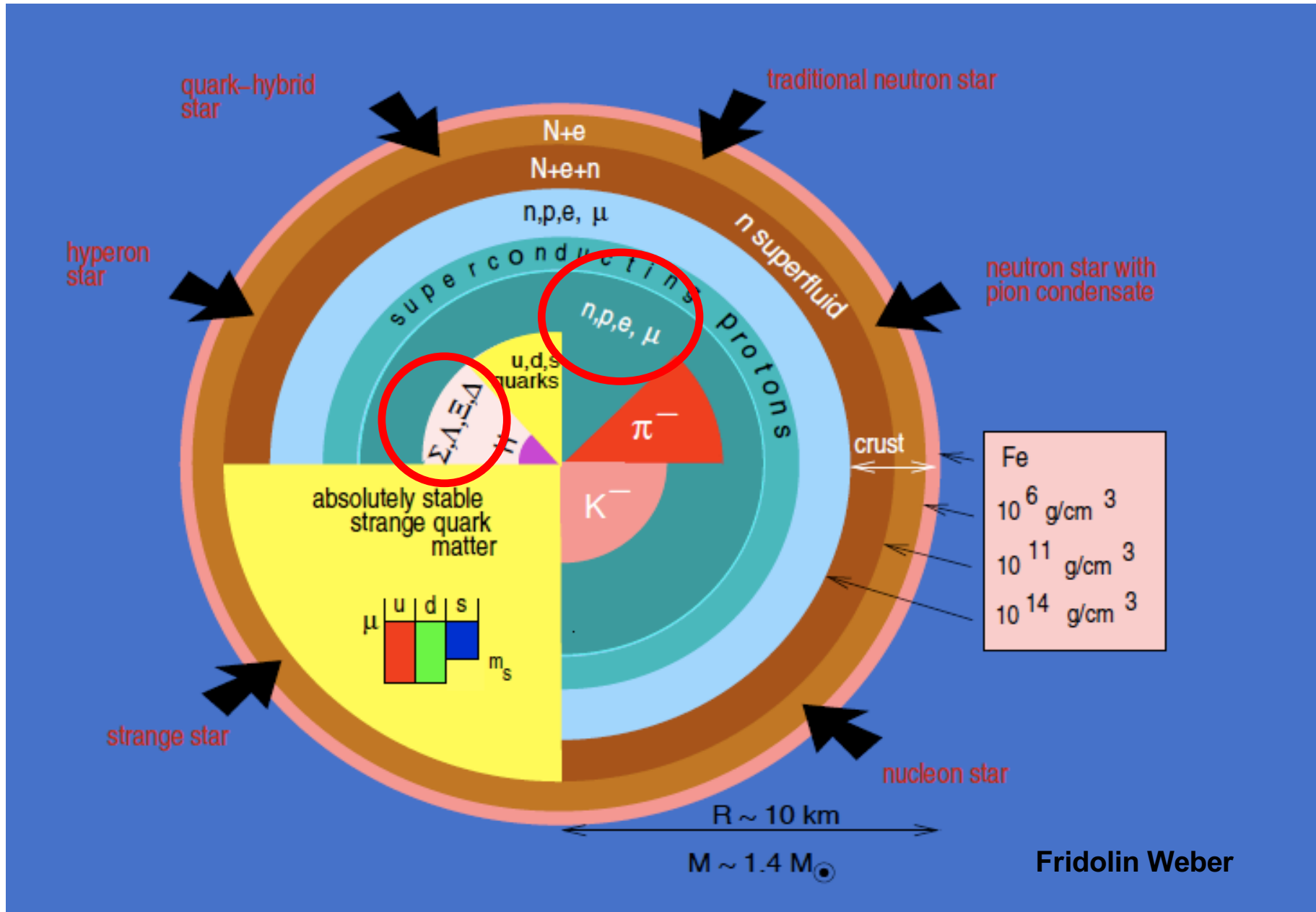
Burgio and Fantina (review) '18



LT, Centelles and Ramos '17



.... the Inner Core: n, p, e, μ , Υ



What about Hyperons?

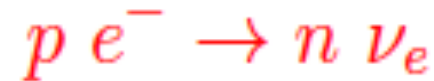
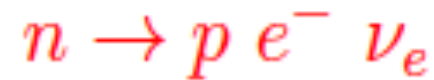
credit: Vidana

A **hyperon** is a baryon containing one or more strange quarks

First proposed in 1960 by Ambartsumyan & Saakyan

Hyperon	Quarks	$I(J^P)$	Mass (MeV)
Λ	uds	$0(1/2^+)$	1115
Σ^+	uus	$1(1/2^+)$	1189
Σ^0	uds	$1(1/2^+)$	1193
Σ^-	dds	$1(1/2^+)$	1197
Ξ^0	uss	$1/2(1/2^+)$	1315
Ξ^-	dss	$1/2(1/2^+)$	1321
Ω^-	sss	$0(3/2^+)$	1672

Traditionally neutron stars were modeled by a **uniform fluid of neutron rich matter in β -equilibrium**



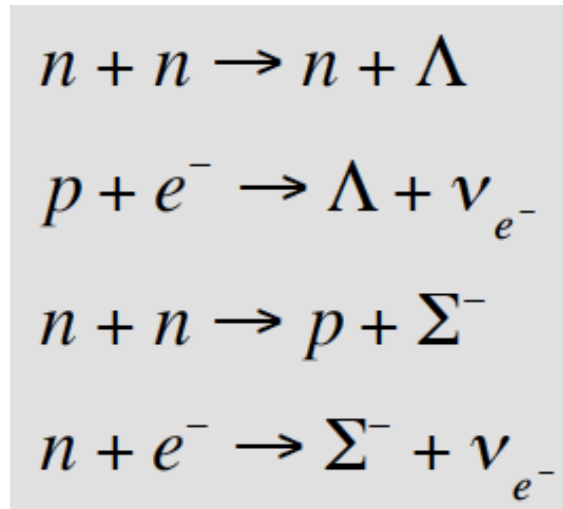
but more exotic degrees of freedom are expected, such as **hyperons**, due to:

- high value of density at the center and
- the rapid increase of the nucleon chemical potential with density

Hyperons might be present at $n \sim (2-3)n_0$!!!

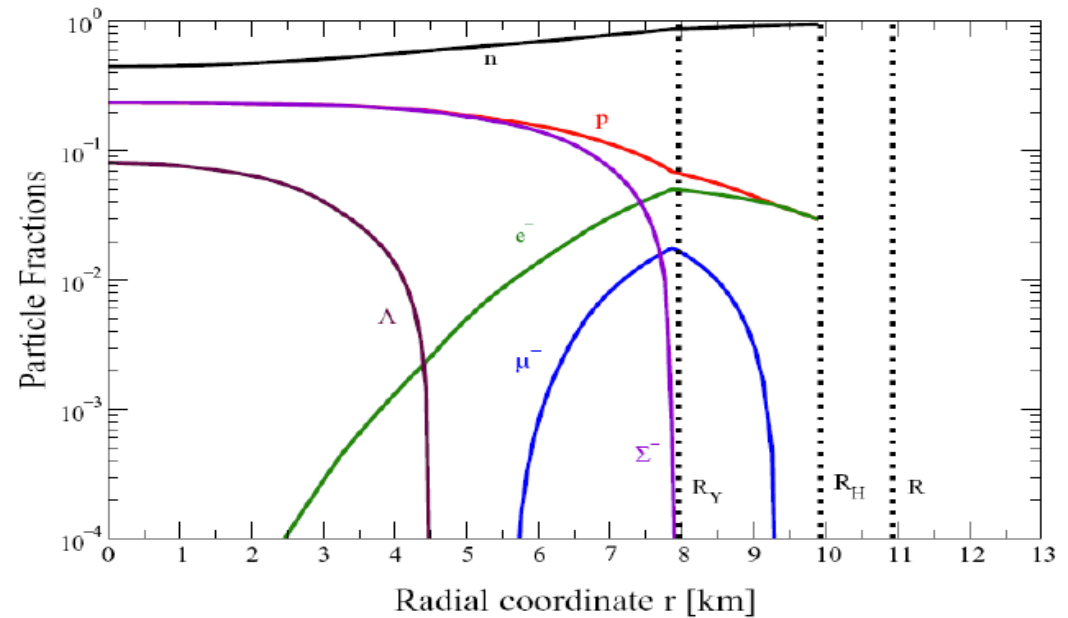
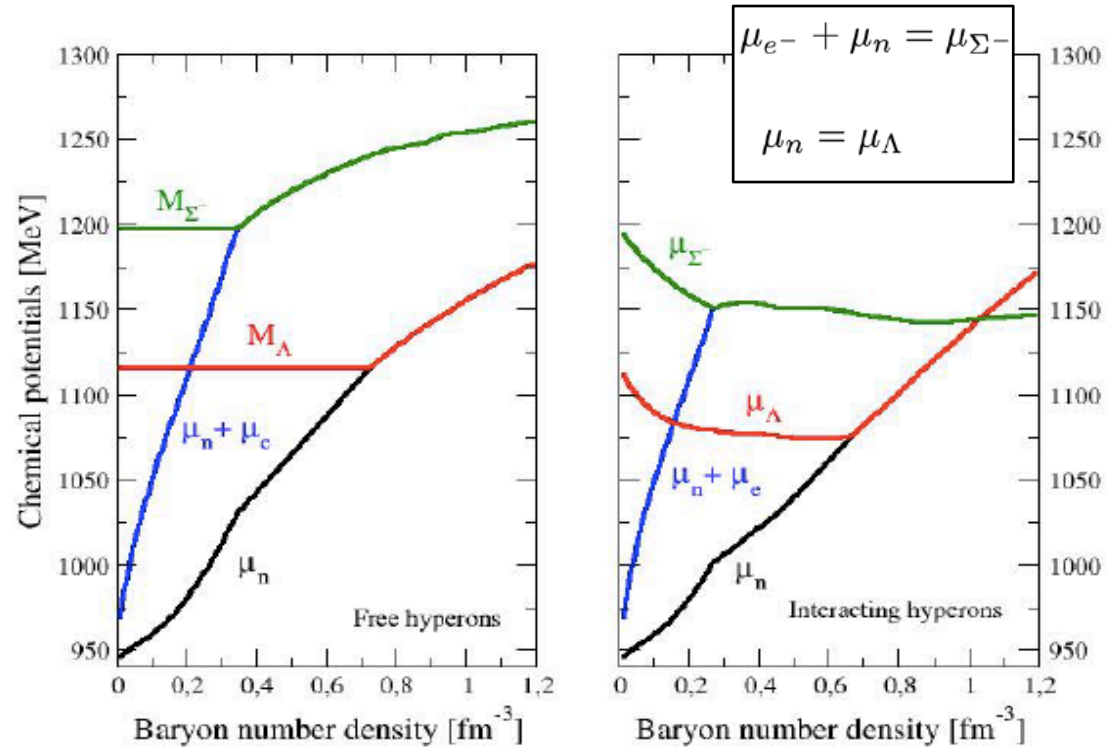
β -stable hyperonic matter

μ_N is large enough to make $N \rightarrow Y$ favorable



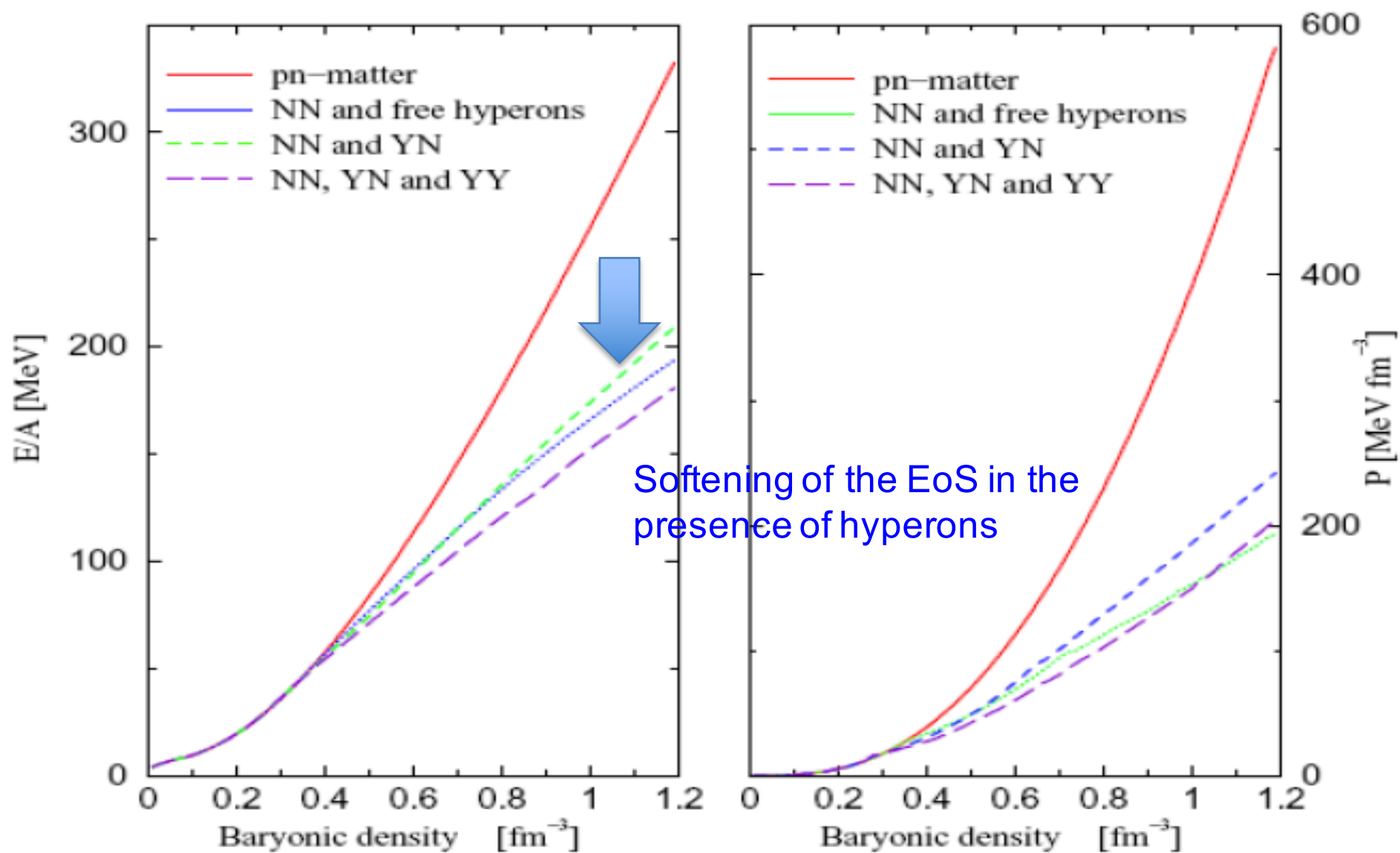
$$\mu_i = b_i \mu_n - q_i \mu_e$$

$$\sum_i x_i q_i = 0$$



credit: Vidana

Equation of State of Hyperonic Matter



Structure of a Neutron Star: Mass and Radius

“Recipe” for neutron star structure calculation

- **Energy density** $\epsilon = \frac{E}{V}; \quad \epsilon(n, x_e, x_p, x_\Lambda, \dots); \quad x_i = \frac{n_i}{n}$

- **Chemical potentials** $\mu_i = \frac{\partial \epsilon}{\partial n_i}$

- **β -equilibrium and charge neutrality** $\begin{cases} \mu_i = b_i \mu_n - q_i \mu_e \\ \sum_i x_i q_i = 0 \end{cases}$

- **Composition/EoS** $x_i(n); \quad P(n) = n^2 \frac{d(\epsilon/n)}{dn}(n, x_i(n))$

- TOV equations (hydrostatic equilibrium)

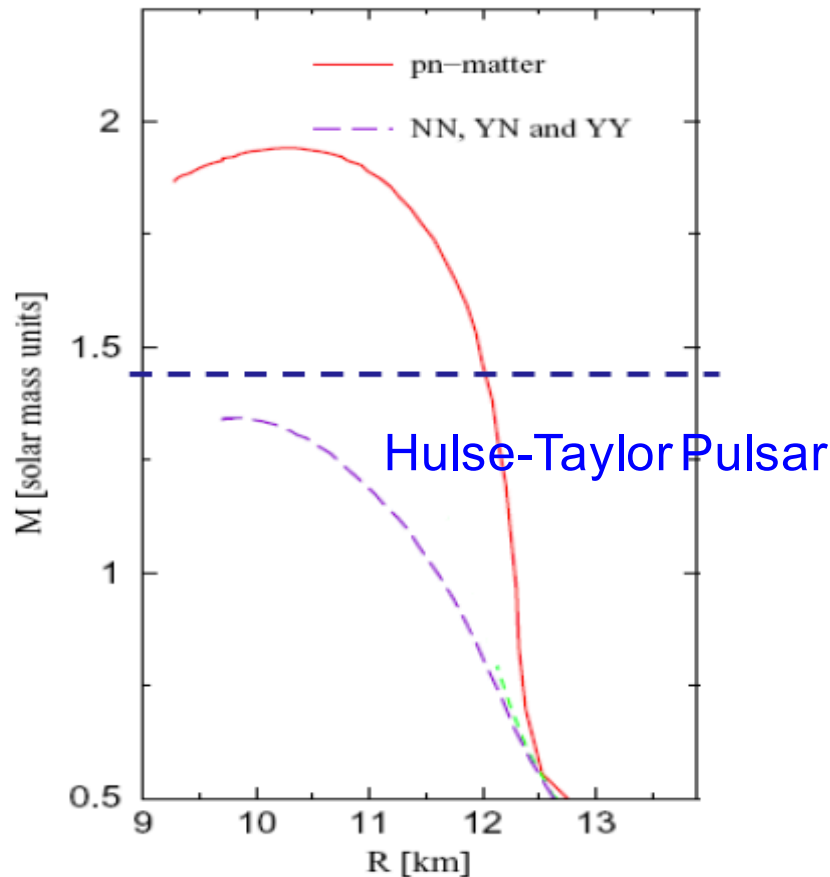
$$\frac{dP}{dr} = -\frac{Gm\epsilon}{r^2} \left(1 + \frac{P}{\epsilon}\right) \left(1 + \frac{4\pi r^3 P}{m}\right) \left(1 - \frac{2Gm}{r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

$$m(r=0) = 0 \quad P(r=0) = P(\epsilon_c)$$

$$m(r=R) = M \quad P(r=R) = 0$$

- **Structure of the neutron star** $n(r), M(R), \dots$ adapted from Shulze@Compstar07



Vidana, Polls, Ramos, Engvik & Hjorth-Jensen '00

Scenario	Maximum mass (M_{\odot})	R (km)
pn-matter	1.9 – 2.2	10.3
NN, NY	1.5 – 1.6	10.2
NN, NY, YY	1.3 – 1.4	10.0

Hadronic model too soft

Hyperons induce a softer EoS than when only nucleons are considered and, hence, a smaller maximum mass that the neutron star can sustain.

EoS is too soft!! Need of extra pressure at high density to compare with observations of $2 M_{\text{sun}}$

The Hyperon Puzzle

The Hyperon Puzzle



Scarce experimental information:

- data from 40 single and 3 double Λ hypernuclei
- few YN scattering data (~ 50 points) due to difficulties in preparing hyperon beams and no hyperon targets available

Chatterjee and Vidana '16

The presence of hyperons in neutron stars is energetically probable as density increases. However, it induces a strong softening of the EoS that leads to **maximum neutron star masses $< 2M_{\text{sun}}$**

Solution?

- stiffer YN and YY interactions
- hyperonic 3-body forces
- push of Y onset by Δ or meson condensates
- quark matter below Y onset

Mass and Radius

- General relativity arguments: neutron stars are not black holes

$$R > 2GM$$

- Compressibility (stability) of matter: $dP/d\rho > 0$ (from TOV equations)

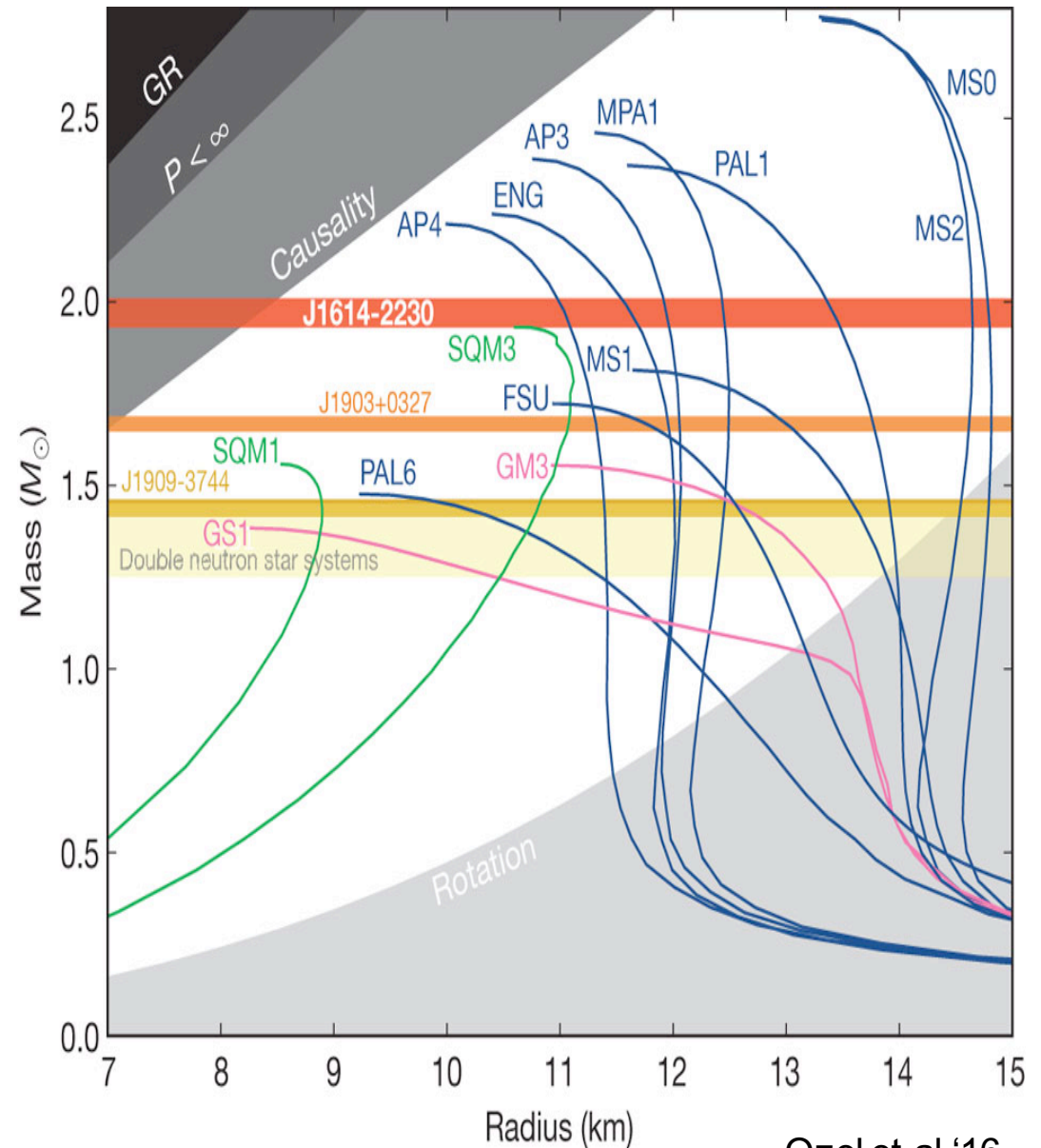
$$R > \frac{9}{4}GM$$

- Causality constraint: speed of sound must be smaller than the speed of light

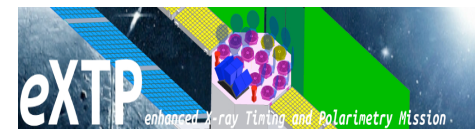
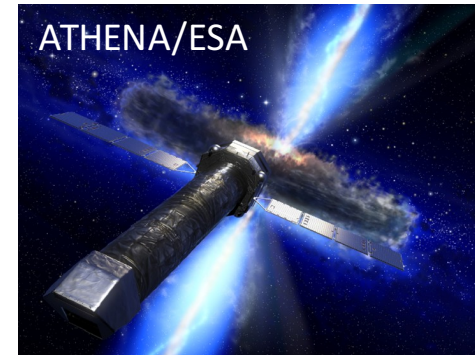
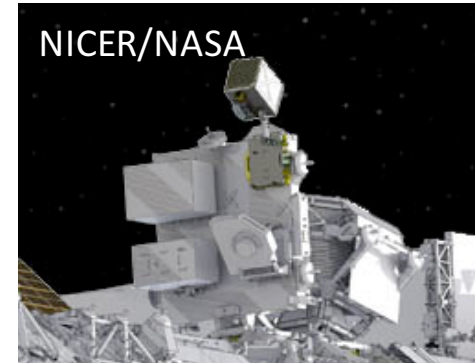
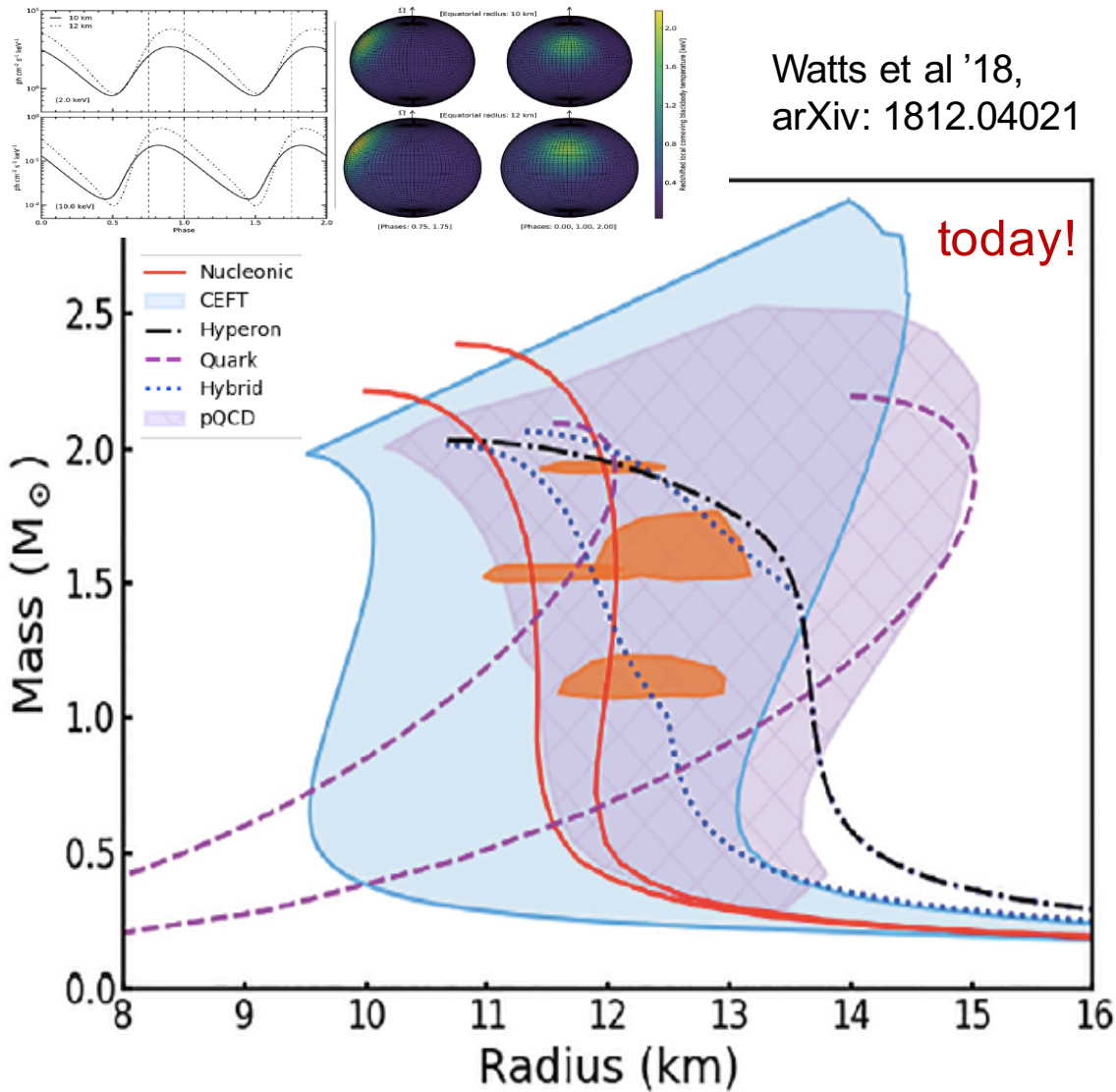
$$R > 2.9GM$$

- Rotation must not pull the star apart (the centrifugal force for a particle on the surface cannot exceed the gravitational force)

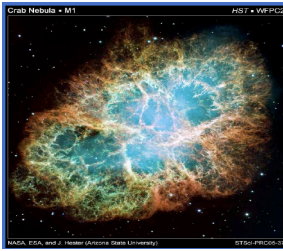
$$R < \left(\frac{GM}{2\pi}\right)^{1/3} \frac{1}{\nu^{2/3}}$$



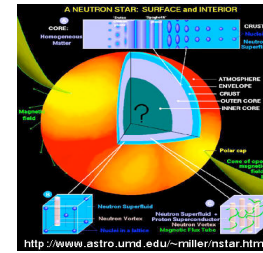
Future: space missions to study the interior of NS



Constraints from pulse profile modelling of rotation-powered pulsars with eXTP



Challenges/Future



The EoS is the crucial input for describing the statical (and dynamical) properties of neutron stars

There has been a lot of progress over the years in modelling the EoS

There are still some challenges for the determination of the EoS, such as:

- the model dependence of the experimental and astrophysical constraints
- the treatment and role of the three-body forces
- the presence of hyperons
- the search for a unified EoS for neutron stars and supernovae

Future: NICER, ATHENA, eXTP.. and GW observations

There exists an online EoS database developed by COST “NewCompstar” and being improved by PHAROS: [CompOSE](https://compose.obspm.fr/), <https://compose.obspm.fr/>

Reviews to read:

Oertel, Hempel, Klaehn and Typel ‘17; Burgio and Fantina ‘18; Vidana ‘18