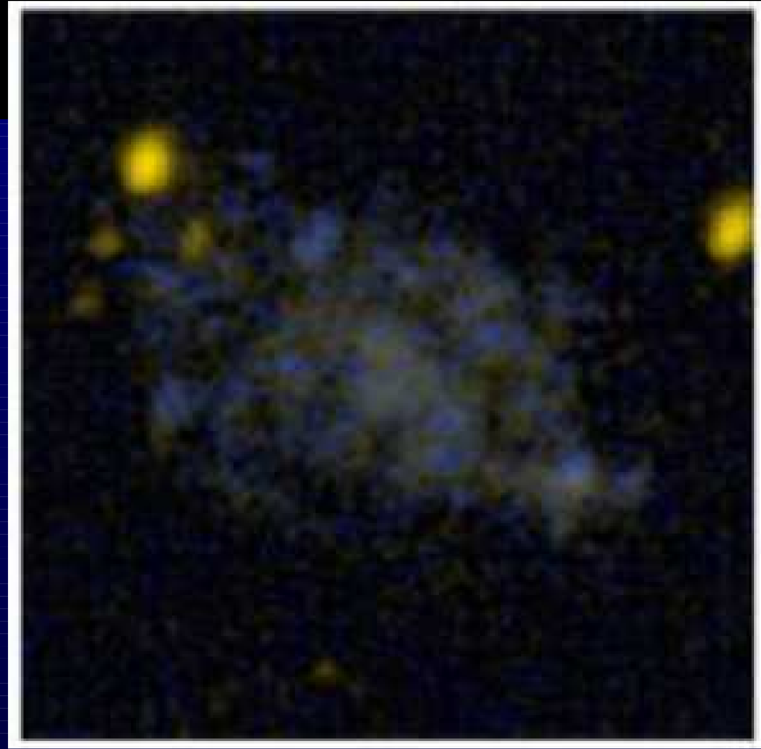
Dark matter, general relativity and the rotation curve of UGC 128



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(*In memorian*)

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FAPESP (Brazil; Thematic Project "Superdense Matter in the Universe"# 2013/26258-4) and CENTRA (Portugal)

A decades-old open problem

- The rotation curves of galaxies

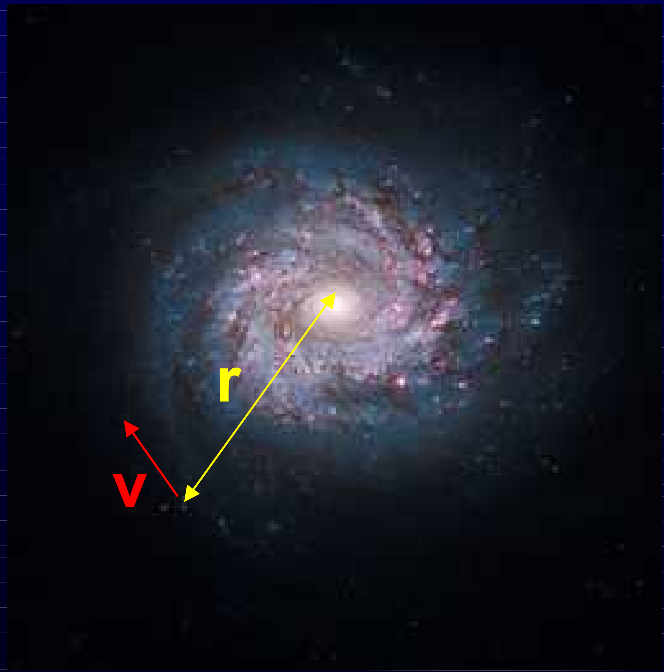
Typically:

Stars' orbital speeds are non-relativistic:

$V \sim 100 – 300$ km/s; $V/c \sim 10^{-3}$

A galaxy is not a strong gravity system:

$M/R \sim 10^{-8} << 1$



Why not apply general relativity

to find the rotation curves?

The main reasons are because:

Stars' orbital speeds are non-relativistic:

$V \sim 100 - 300$ km/s; $V/c \sim 10^{-3}$

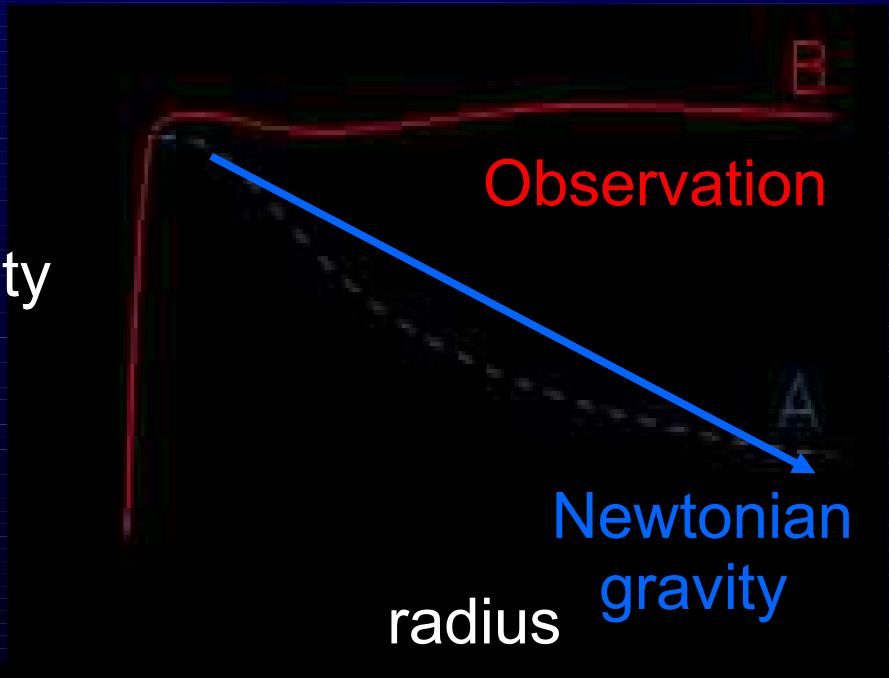
A galaxy is not a strong gravity system:

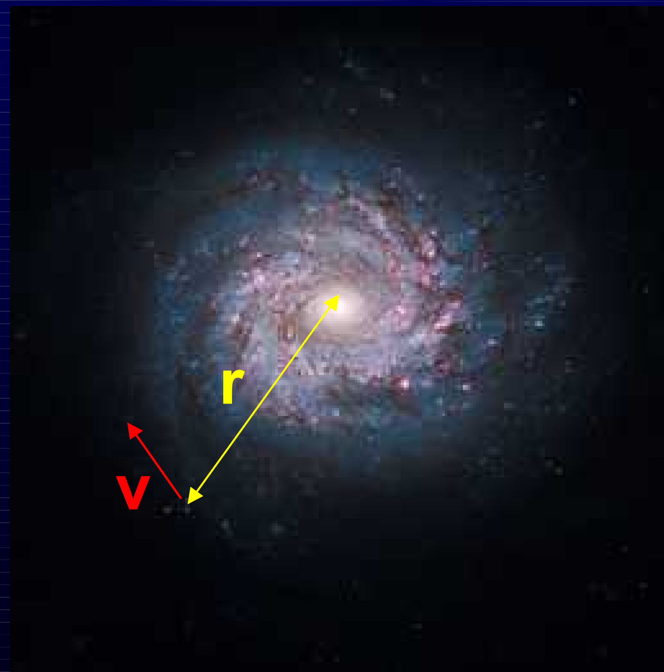
$M/R \sim 10^{-8} << 1$

Therefore Newtonian gravity is expected to work.

An open problem

- The flat rotation curves of galaxies





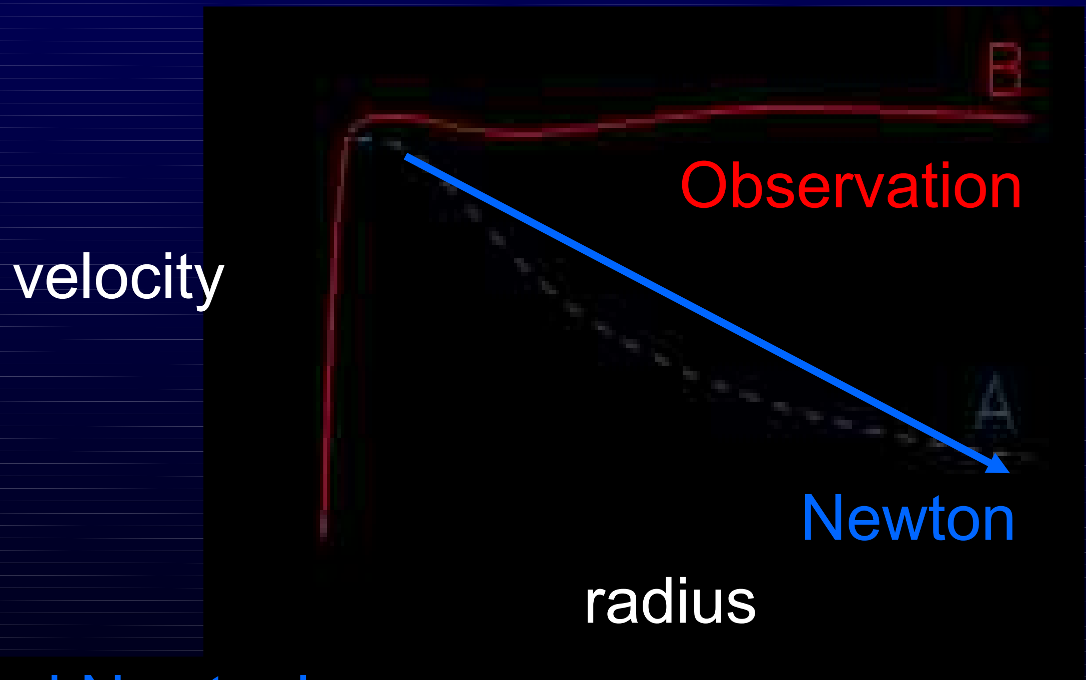
Newtonian gravity:

$$\rho(r) = \frac{3[v(r)]^2}{4\pi G r^2}$$

Tentative explanations

to the flat rotation curve problem

Dark matter halo



Newtonian gravity:

$$\rho(r) = \frac{3[v(r)]^2}{4\pi G r^2}$$

Modifyed Newtonian Dynamics (MOND):

$$\mu\left(\frac{a}{a_0}\right)a = \frac{GM}{r^2}$$

Outline of this talk

- A general relativistic model for a galaxy
- Application to NGC 2403
- Application to UGC 128
- Final remarks

The general relativistic (GR) model for a spiral galaxy: CT model

Developed by F. I. Cooperstock and S. Tieu

- 2005 astro-ph/0507619. astro-ph/0512048.
- 2006 *Mod. Phys. Lett. A* **21**, 2133 (2006)
- 2007 International Journal of Modern Physics A Vol. 22, No. 13 (2007) 2293–2325
- 2008 Mod. Phys. Lett. A **23**, 1745 (2008)
- 2009 Cooperstock, F.I.: General Relativistic Dynamics: Extending Einstein's Legacy Throughout the Universe. World Scientific, Singapore
- 2012: *J. D. Carrick and F. I Cooperstock:* Astrophys Space Sci (2012) 337:321–329
- 2017: *N. S. Magalhaes and F.I. Cooperstock* Astrophys Space Sci (2017) 362:210

The CT model: spiral galaxies

Thin, rotating disks

- Galactic components behave and interact as in a fluid:

Rotating globally dust cloud: it does not rotate rigidly as does a disk of steel

Low density ~ weak gravity

Freely gravitating

No pressure

Rotating

- In a stationary motion,

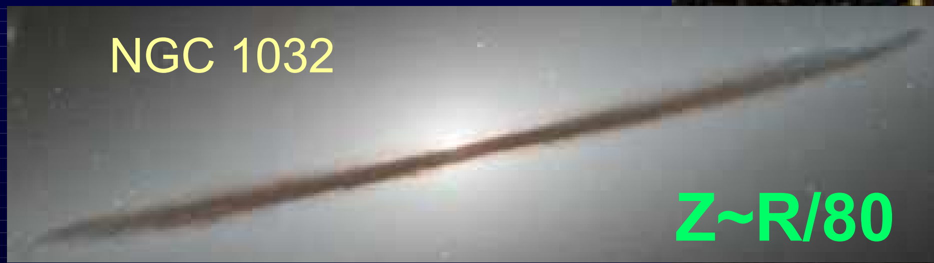
In an axially-symmetric motion

Image Credit: The COBE Project, DIRBE, NASA

The galaxy is a continuous gravitationally bound object


NGC 4302
Z~R/9


NGC 891
Z~R/35

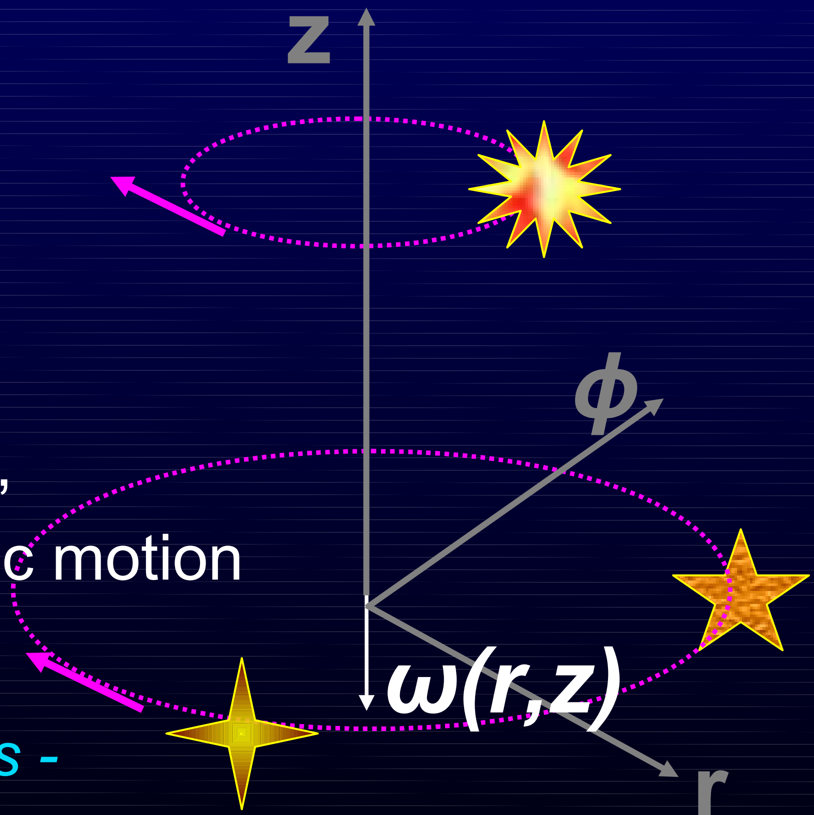

NGC 1032
Z~R/80


NGC 4565
Z~R/13

The GR model: spiral galaxies

Thin, rotating disks

- Galactic components behave and interact as in a fluid:
  - Low density ~ weak gravity
  - Freely gravitating
  - No pressure
  - Rotating
    - In a stationary motion,
    - In an axially-symmetric motion

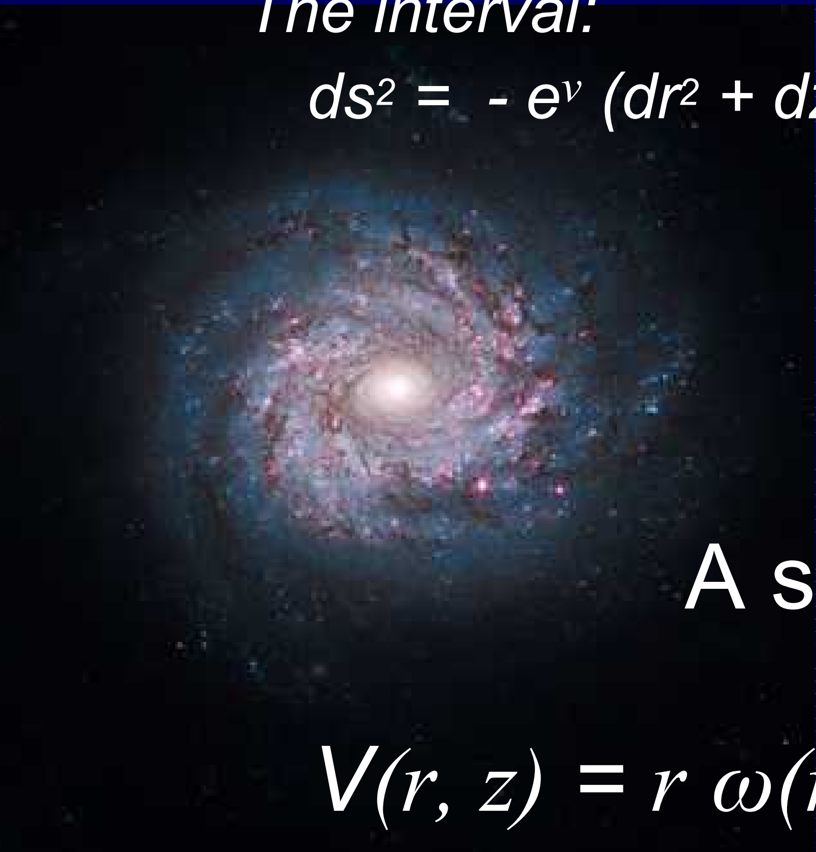


Assumption on physical grounds -
In the interior of the galaxy:
(Lanczos 1924; van Stockum 1937)

$$ds^2 = -e^{\nu}(dr^2 + dz^2) - (r^2 - N^2)\,d\phi^2 - 2\,N\,c\,d\phi\,dt + c^2\,dt^2$$

The CT model: speed

The interval:



$$ds^2 = - e^{\nu} (dr^2 + dz^2) - (r^2 - \mathbf{N}^2)\, d\phi^2 - 2\, \mathbf{N}\, c\, d\phi\, dt + c^2\, dt^2$$

$N(r, z)$: rotation component

A star's orbital speed is:

$$V(r, z) = r\, \omega(r, z) \quad \rightarrow \quad V(r, z) \approx c\, N(r, z) / r$$

Einstein's equations

Approximation:

Terms are kept only up to the order of

the first power of the gravitation constant, G

Not a post-Newtonian perturbation scheme

$$\rho(r, z) = [c^2 / (8\pi G)]\,(N_r^2 + N_z^2) / r^2$$

Non-linear in the gravitational field: non-Newtonian source

$$N_{rr} + N_{zz} - N_r / r = 0$$

Linear equation, with no dependency on the source of the gravitational field

C&T do not aim at a detailed analysis of the exact van Stockum space–time, but at finding approximate solutions, with sufficient accuracy for the physical situation at hand

Finding the rotation curve - 1

$$V(r, z) \approx c\, N(r, z) / r$$

One of the Einstein's equations:

$$\rho(r, z) = [c^2 / (8\pi G)]\, (N_r^{\,2} + N_z^{\,2}) / r^2$$

Non-linear in the gravitational field: non-Newtonian source

If we knew the galactic density function we could look for the metric function *N* and then determine a star's velocity.

We will take another path.

Finding the rotation curve - 2

$$V(r, z) \approx c\, N(r, z) / r$$

Another of the Einstein's equations:

$$N_{rr} + N_{zz} - N_r / r = 0$$

Linear equation, with no dependency on the source of the gravitational field

By finding a solution to this equation we have means to look for both a star's velocity and the galactic mass density that generates the gravitational field.

$$\rho(r, z) = [c^2 / (8\pi G)]\,(N_r^2 + N_z^2) / r^2$$

An expression for *N(r,z)*

$$N_{rr} + N_{zz} - N_r / r = 0$$

Our solution (an expansion in "*n*"):

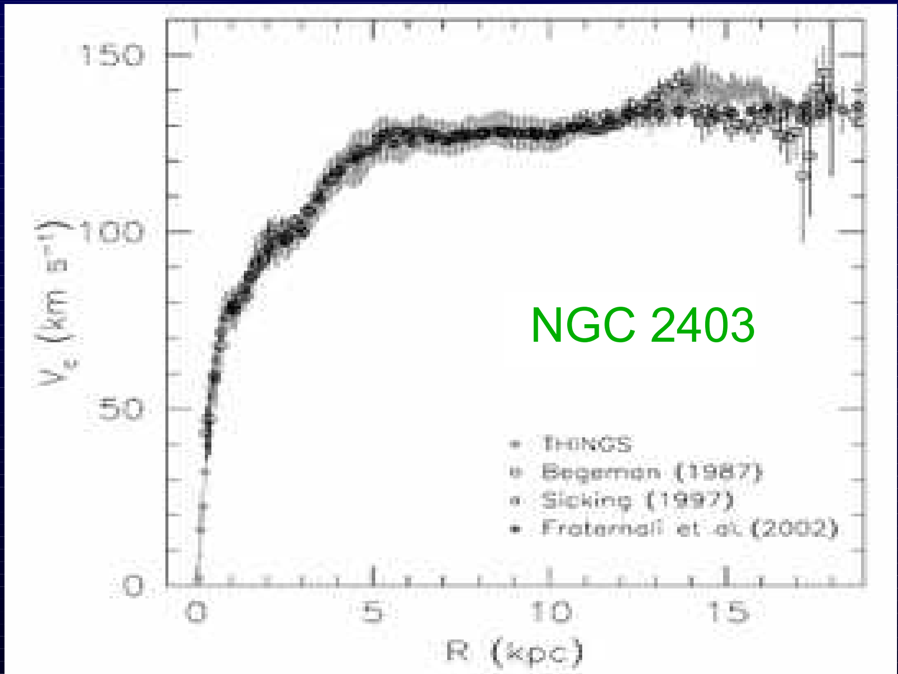
$$N(r, z) = -r \sum k_n C_n \exp(-k_n|z|) J_1 (k_n r)$$

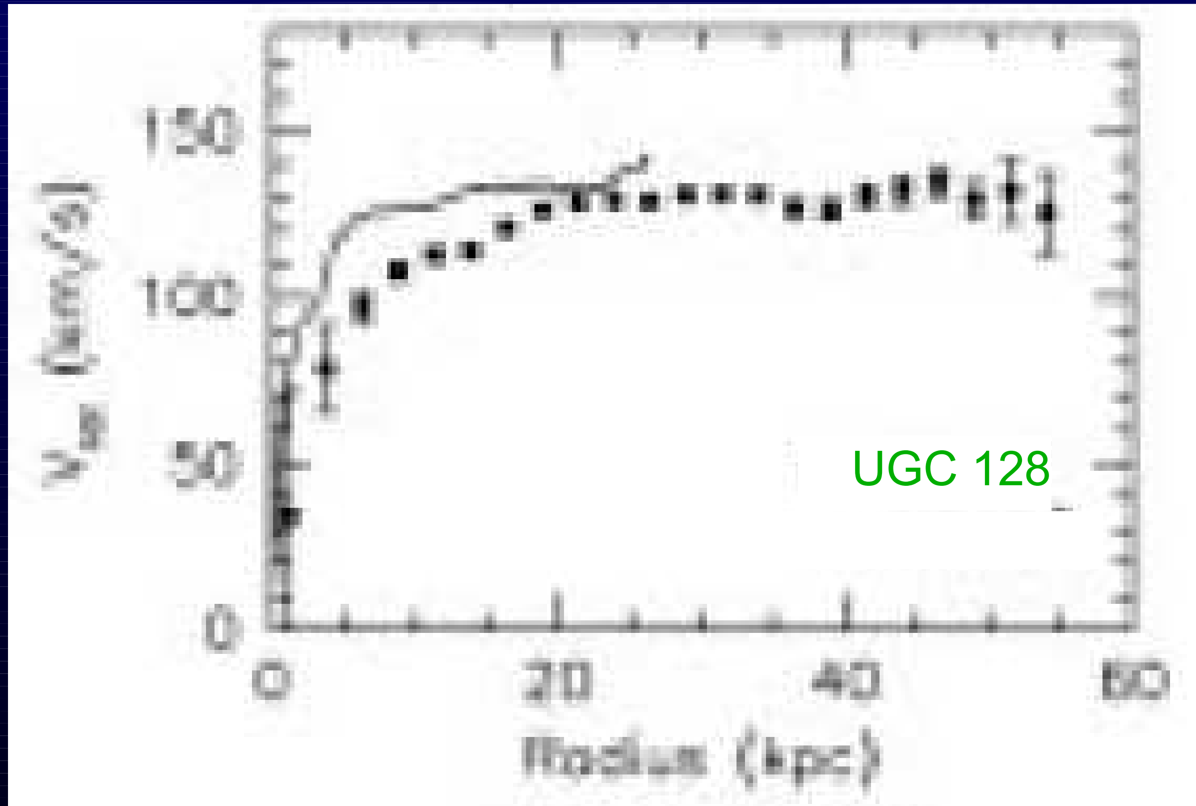
Approach - Determine the coefficients k_n and C_n for a galaxy using actual *(V, r)* data sets with $z = 0$:

$$V(r, 0) = c\, N(r, 0) / r$$

The solution is asymptotically flat yielding a well-defined galactic mass

Applications:
NGC 2403 and UGC 128



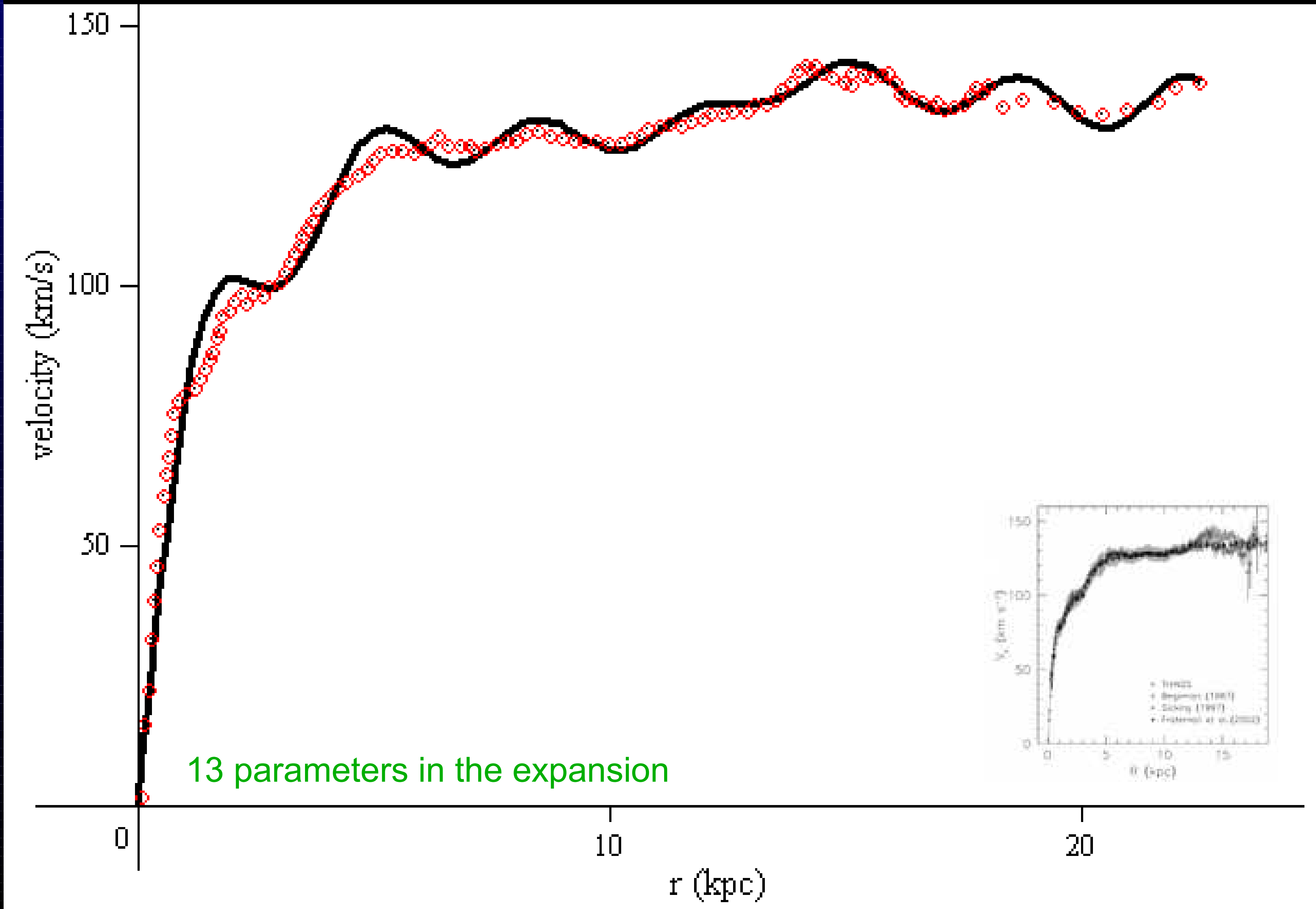


They have almost identical luminosities, almost identical maximum rotation velocities and are morphologically very similar.

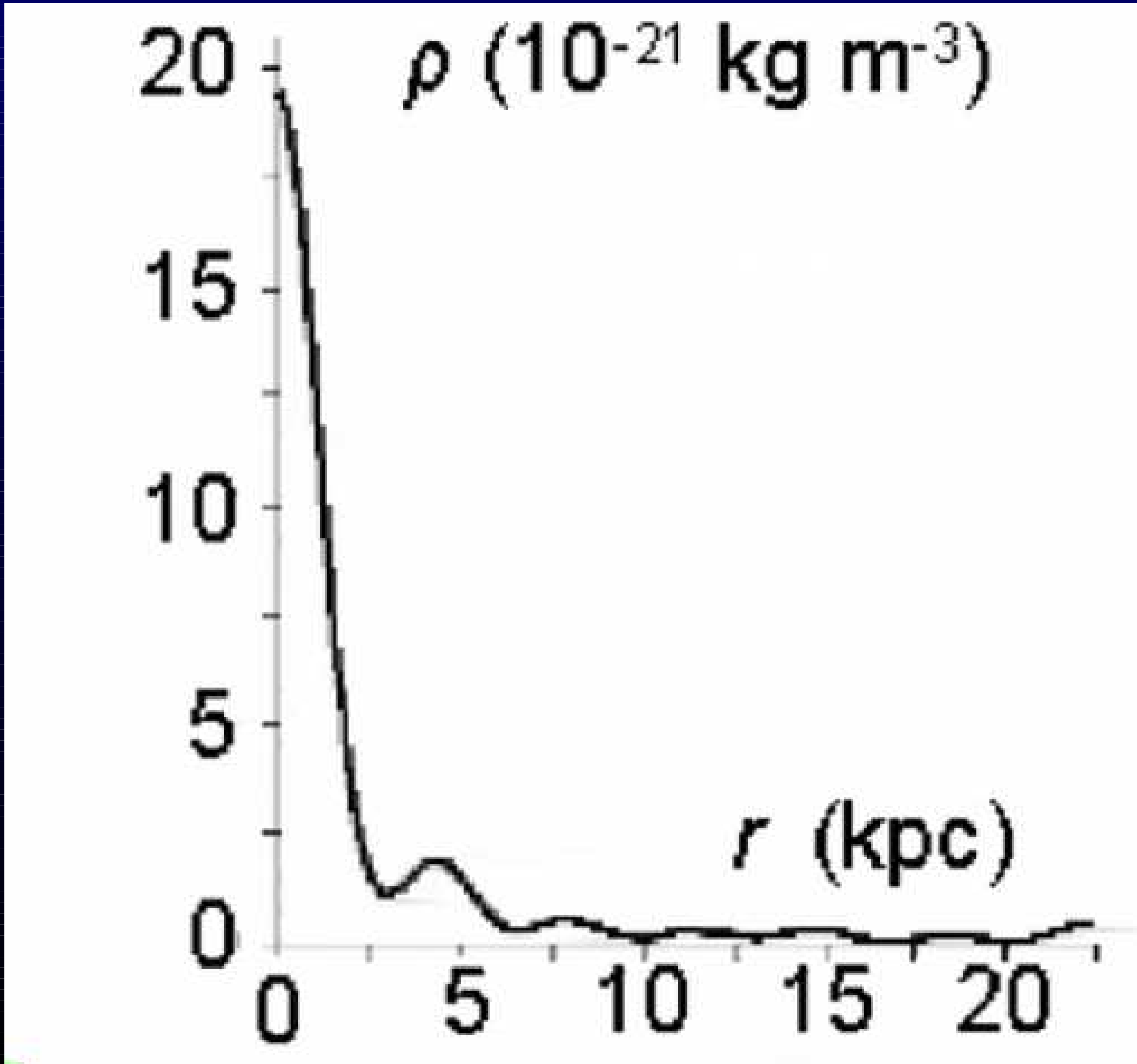
But they have a very large difference in surface brightness: UGC 128 has much lower surface brightness than NGC 2403.

Application 1: NGC 2403

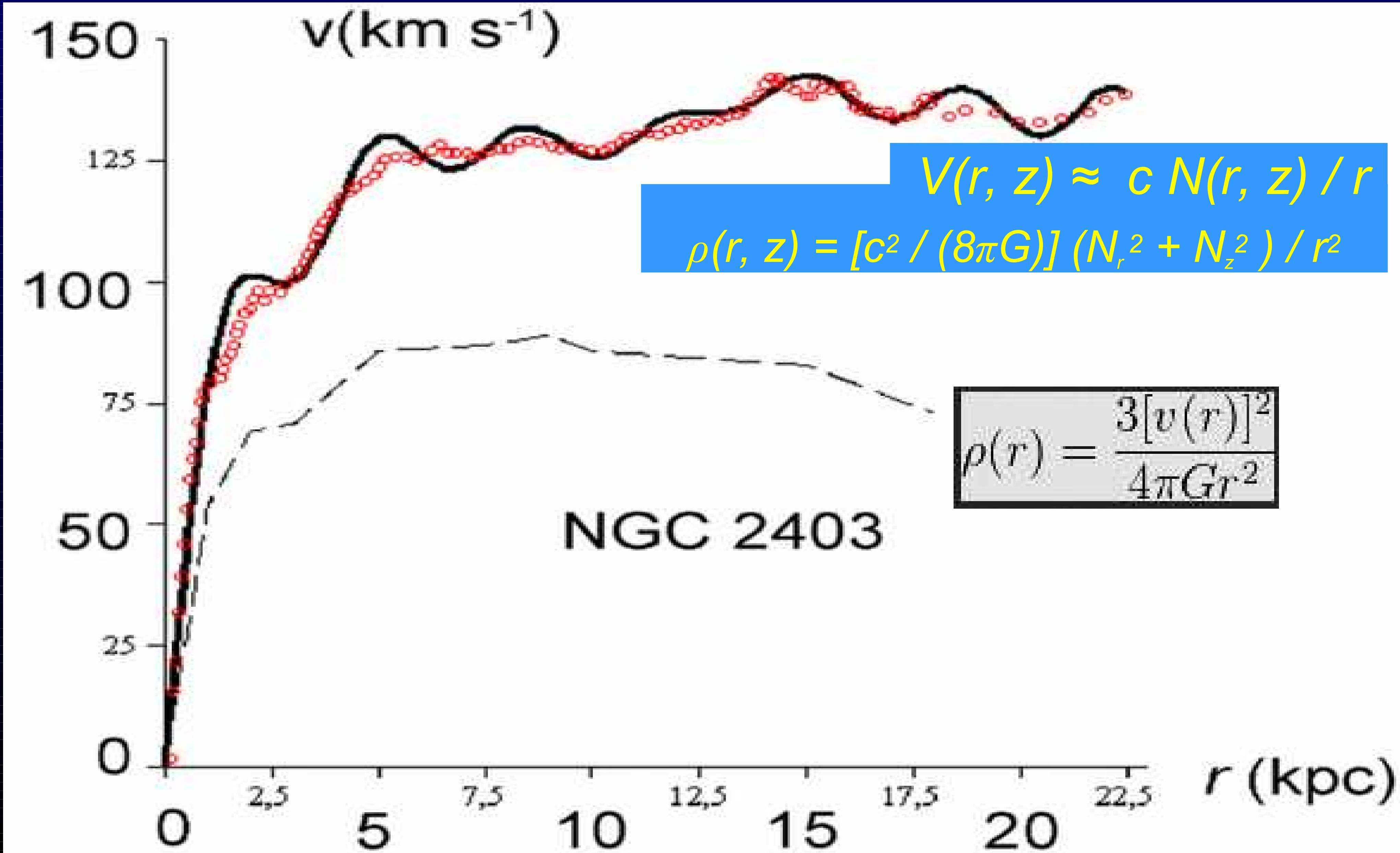
NGC 2403: CT model RC fit



NGC 2403: mass density curve



NGC 2403: Newtonian curve

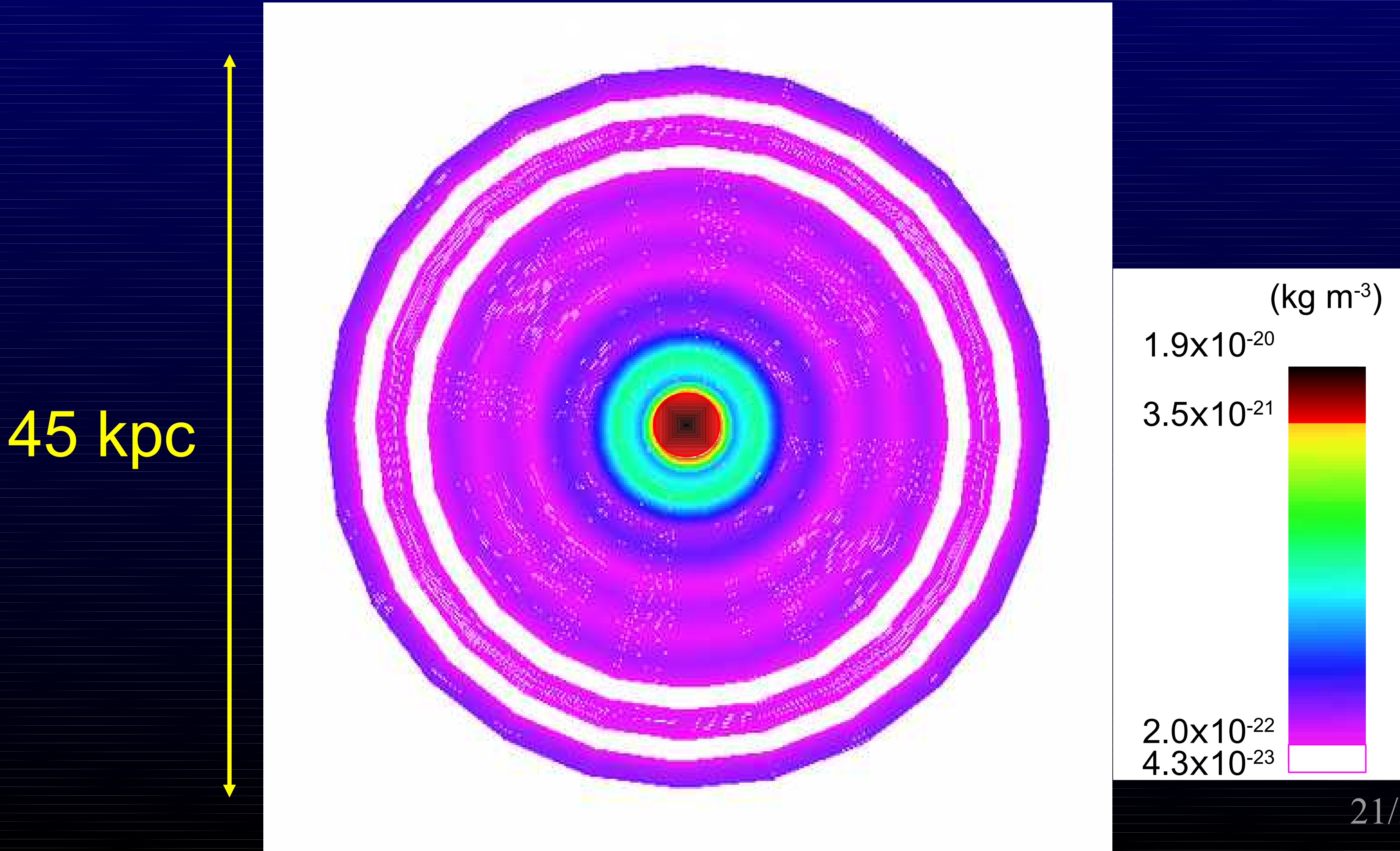


$$V(r, z) \approx c\, N(r, z) / r$$

$$\rho(r, z) = [c^2 / (8\pi G)]\, (N_r^2 + N_z^2) / r^2$$

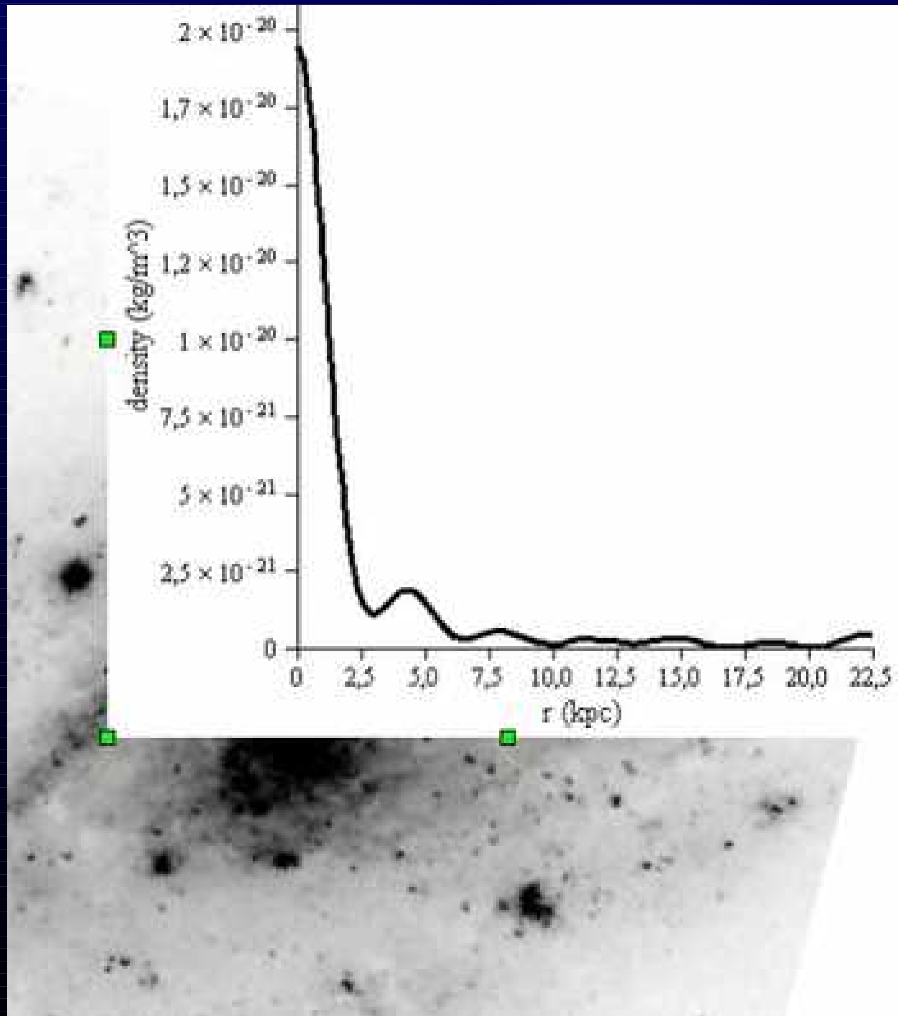
$$\rho(r) = \frac{3[v(r)]^2}{4\pi G r^2}$$

NGC 2403: source's face-on density



NGC 2403: light × mass density

NGC 2403: light × mass density



Infrared , 7900A

NGC 2403: Total mass

Integrating over a disk with $z \sim r/15$:

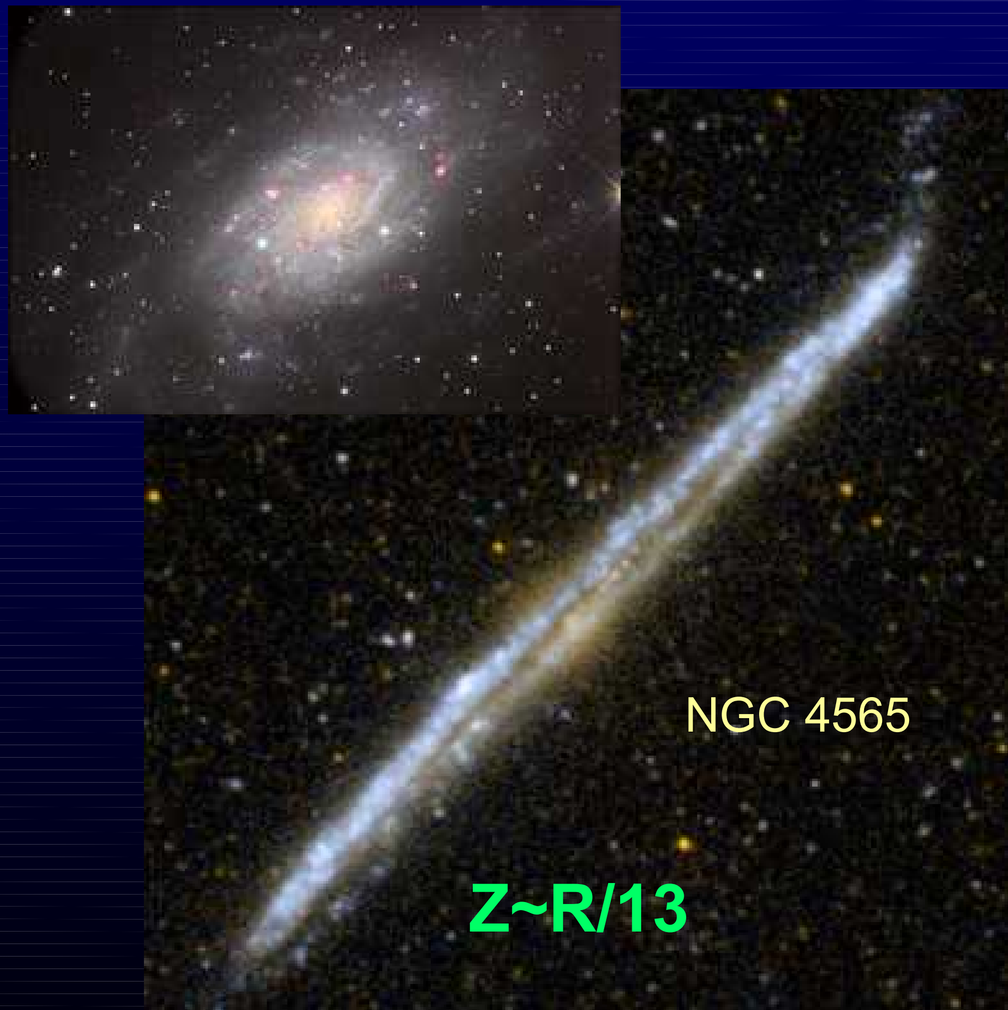
$M_{CT} = 1.83 \times 10^{10}\ M_{\odot}$

Other authors:

$1.3 - 10.2 \times 10^{10}\ M_{\odot}$

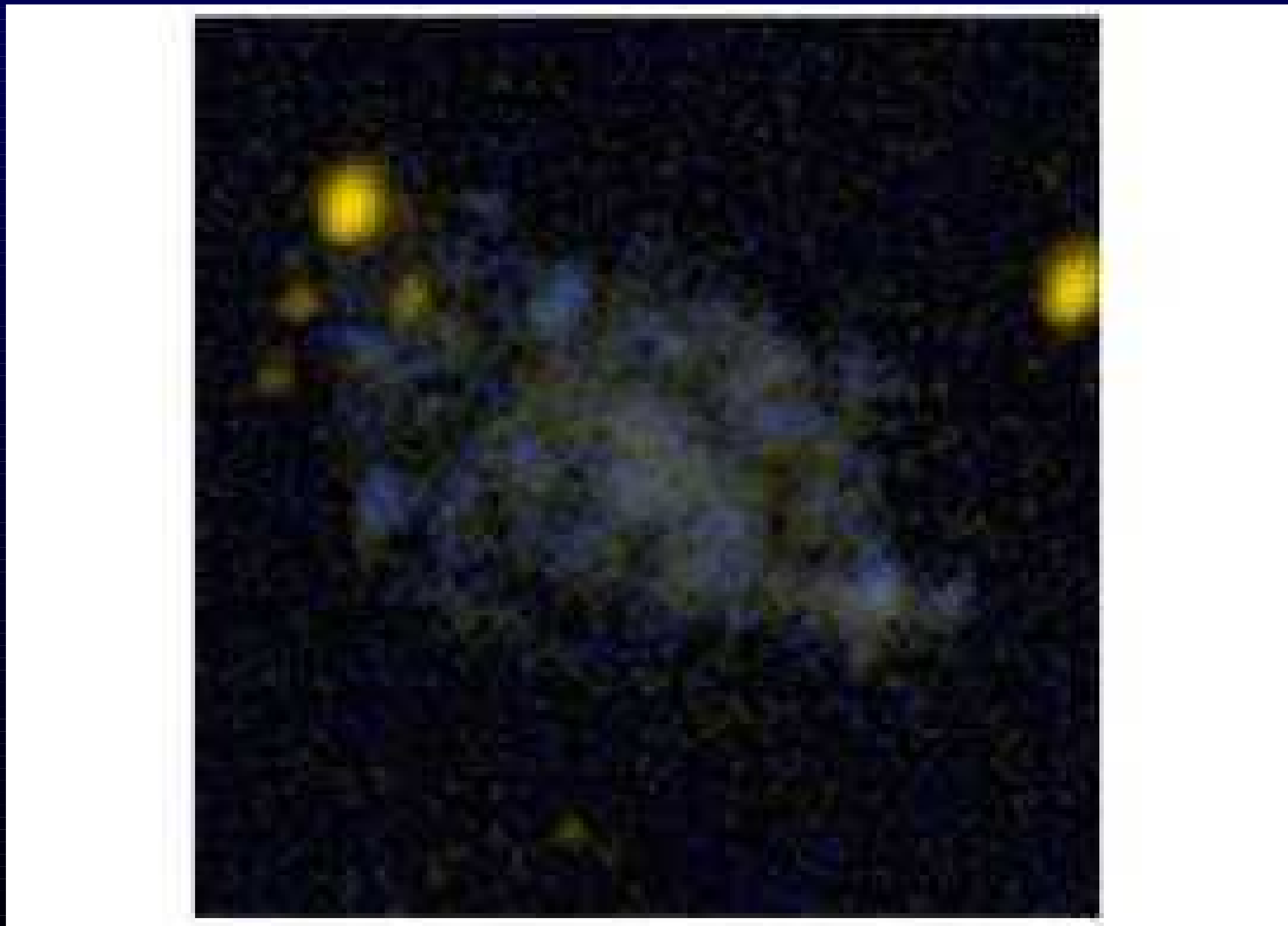
Total barionic (stars+gas) mass (de Blok & McGaugh 1996):

$M_{bar} \sim 1.82 \times 10^{10}\ M_{\odot}$



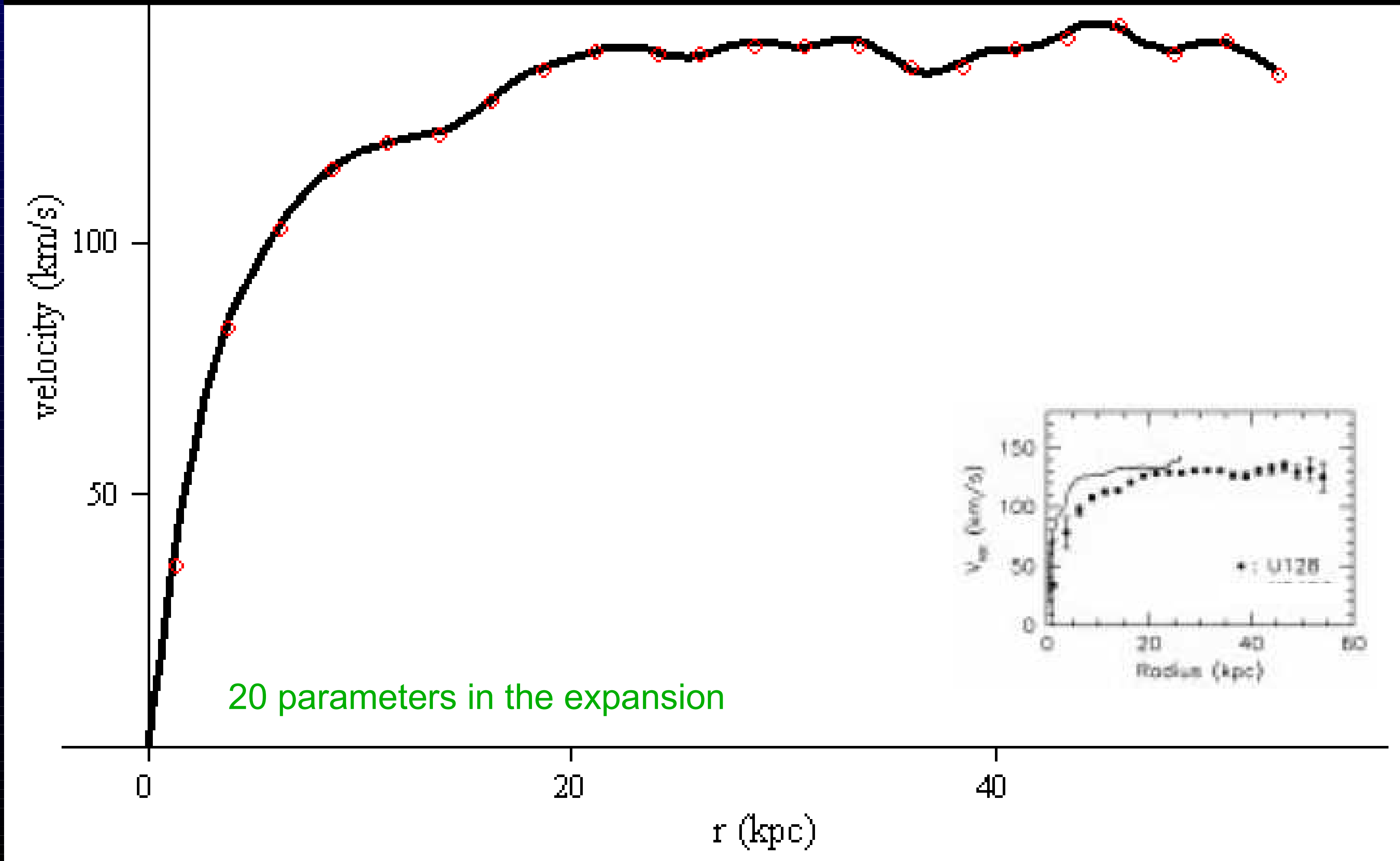
Application 2: UGC 128

Luminosity is the total amount of energy emitted per unit of time by an astronomical object.

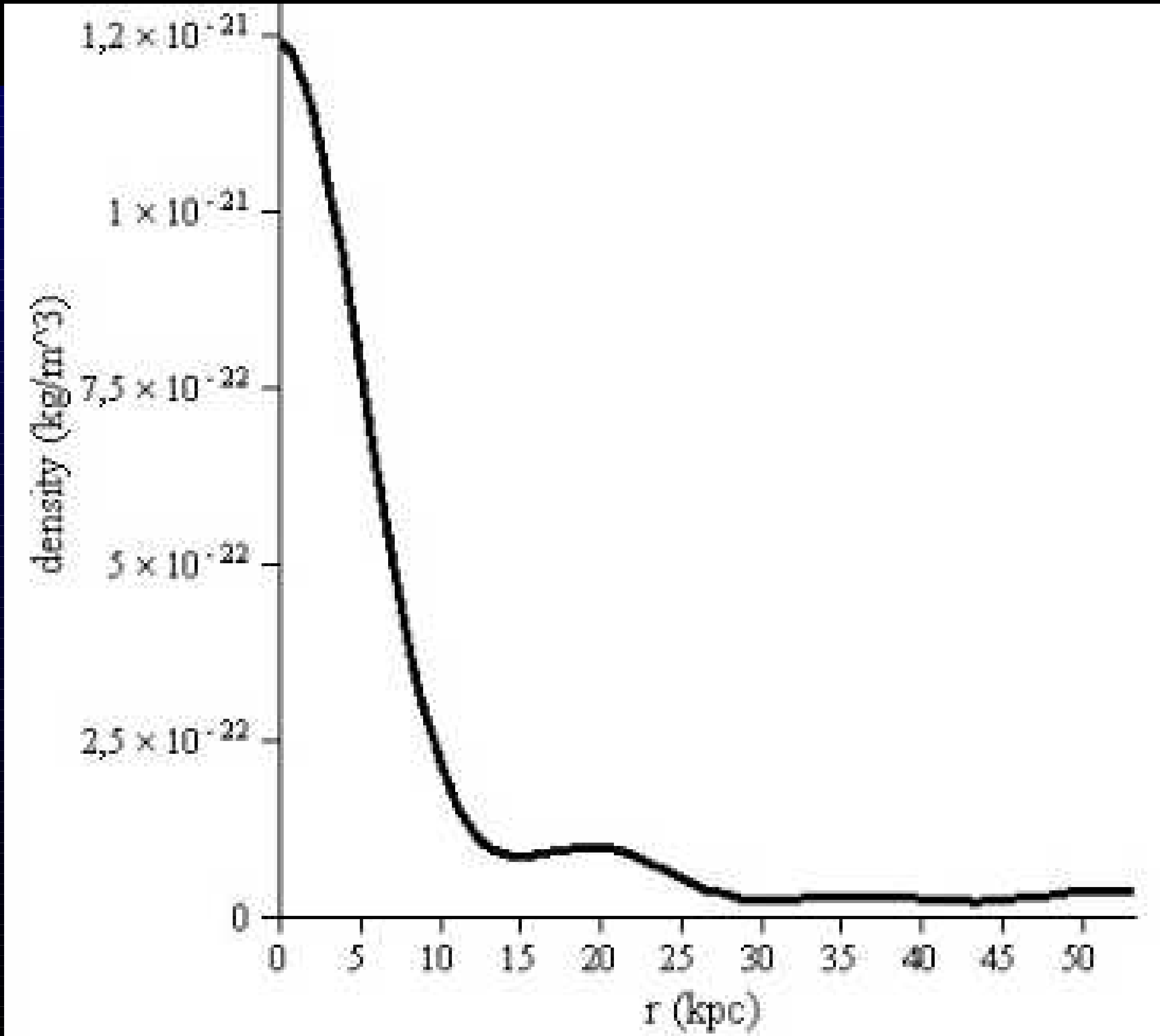


A galaxy's brightness is defined in terms of its apparent magnitude — how bright it appears

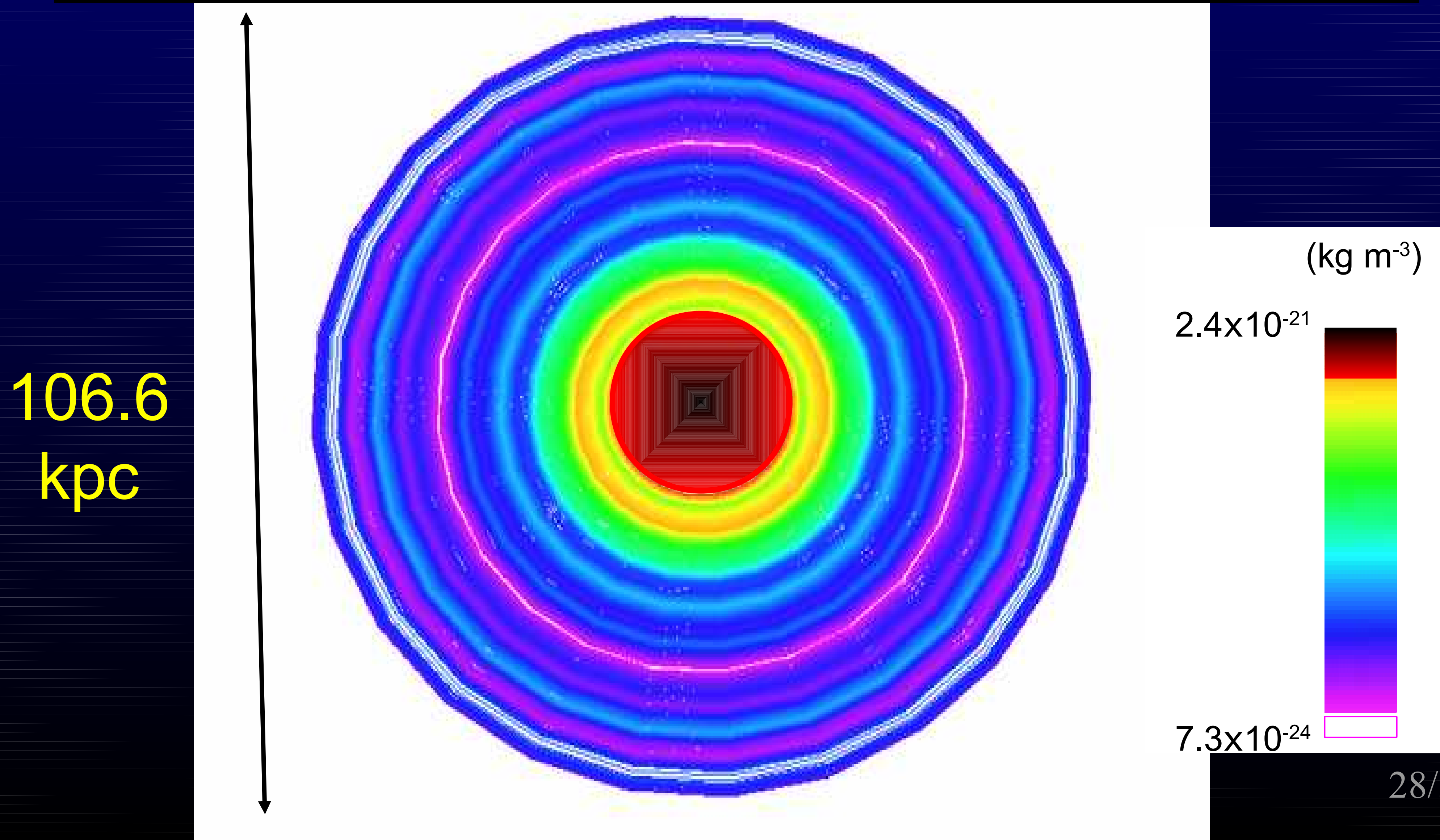
UGC 128: CT model RC fit



UGC 128: mass density curve

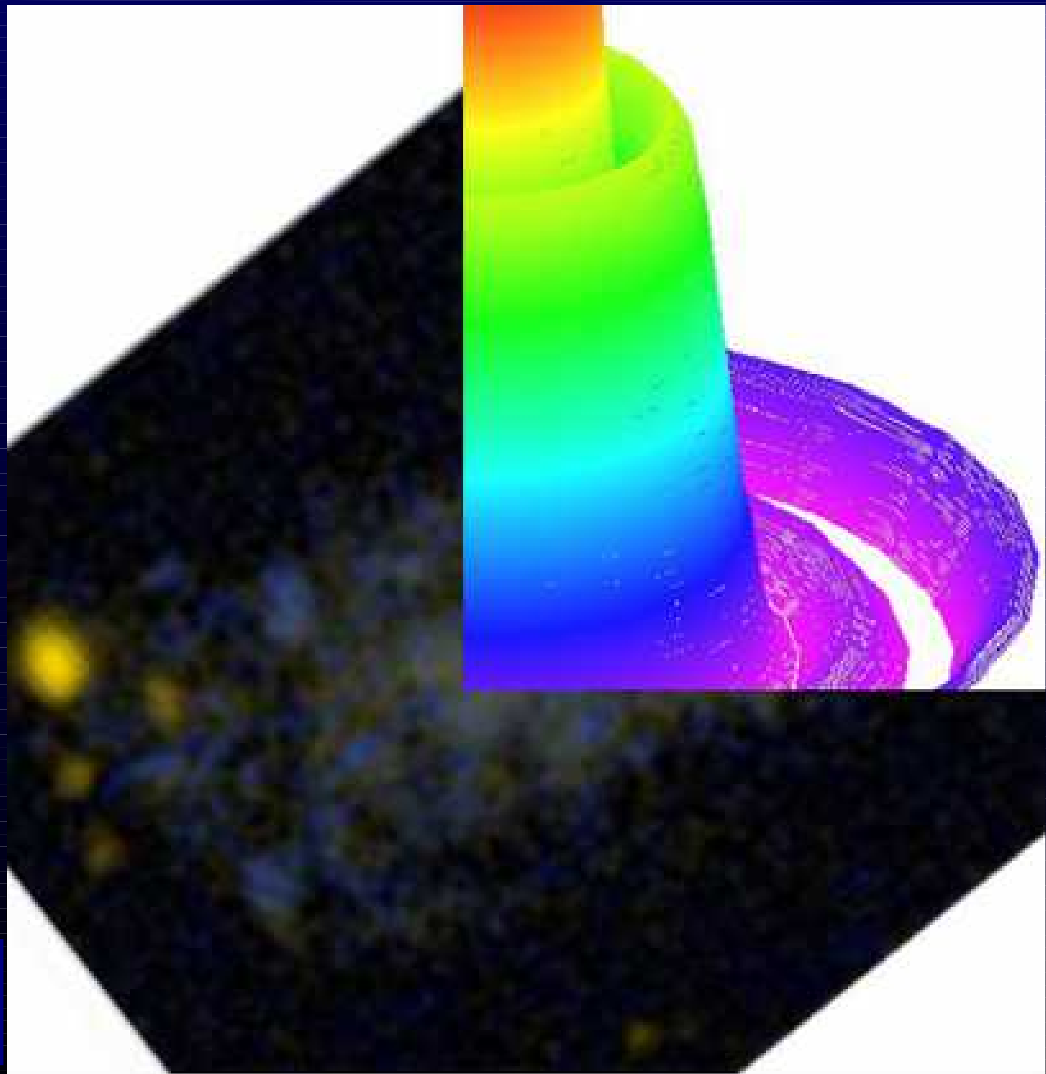


UGC 128: source's face-on density

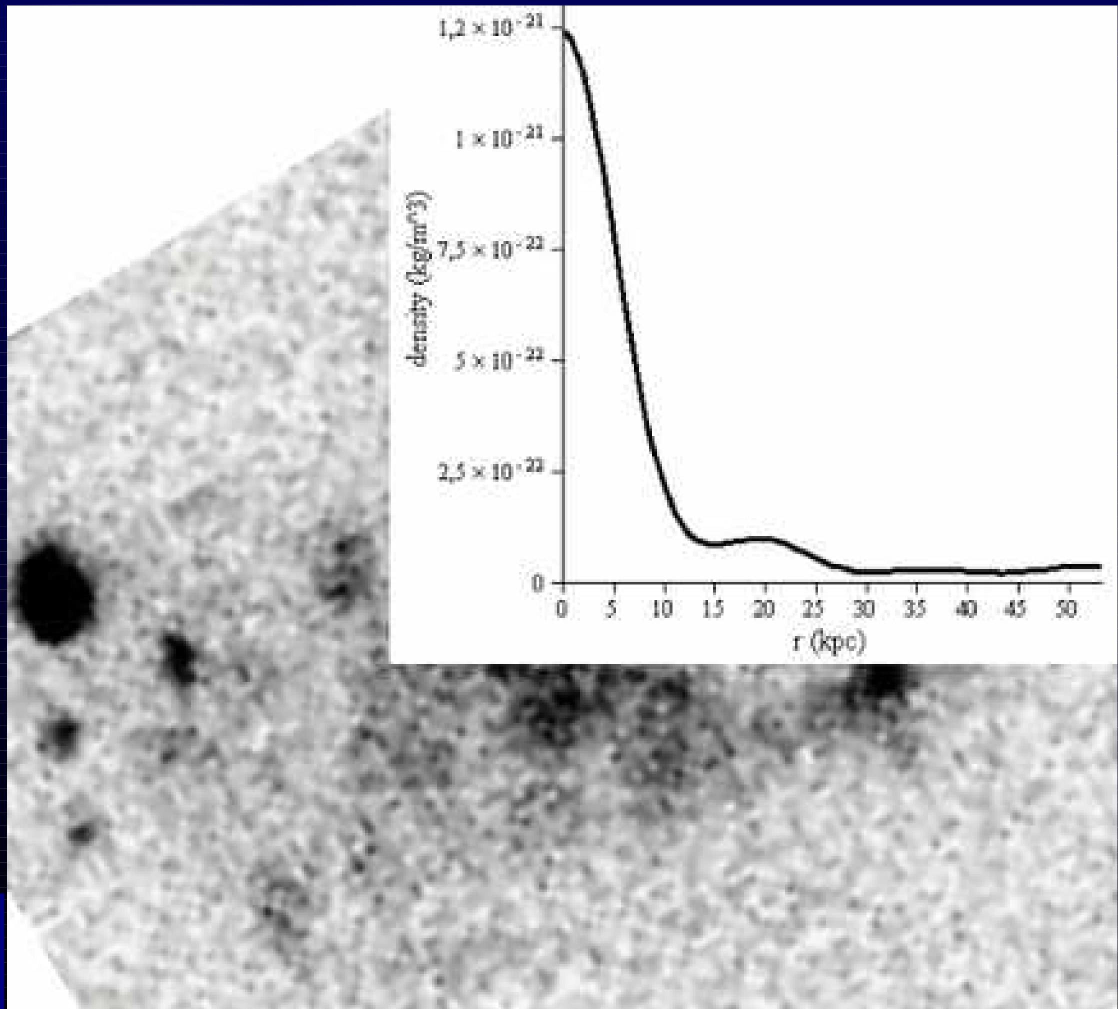


UGC 128: light × mass density

FUV-NUV ,
1516-2267A



UGC 128: light × mass density



NUV , 2267A

UGC 128 total mass

Integrating over a disk with $z \sim r/27$:

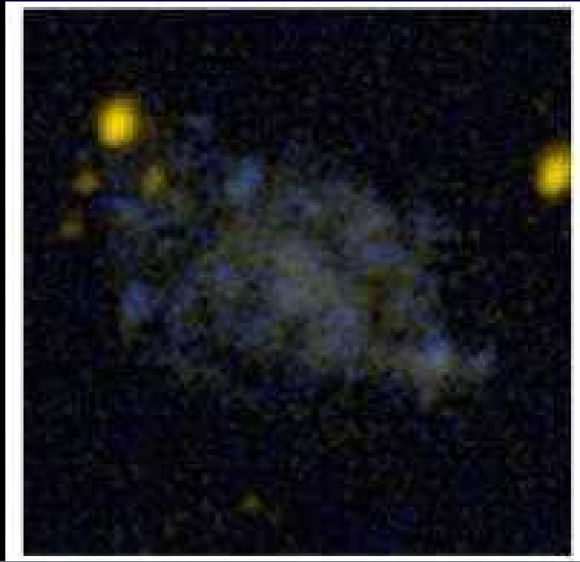
$M_{CT} = 2.71 \times 10^{10}\ M_{\odot}$

Results from others:

$1.5 - 17.5 \times 10^{10}\ M_{\odot}$

Total barionic (stars+gas) mass (de Blok & McGaugh 1996):

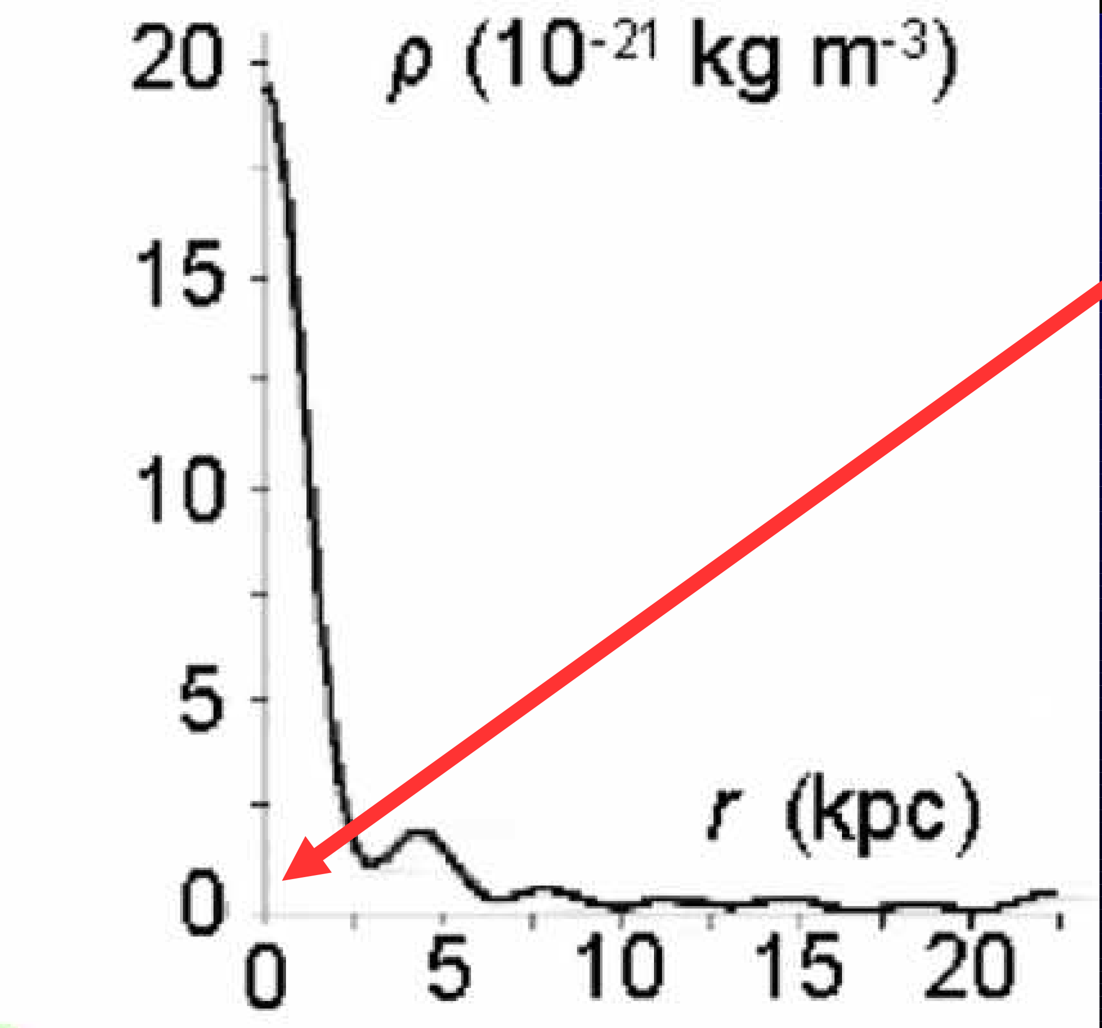
$M_{bar} \sim 2.72 \times 10^{10}\ M_{\odot}$

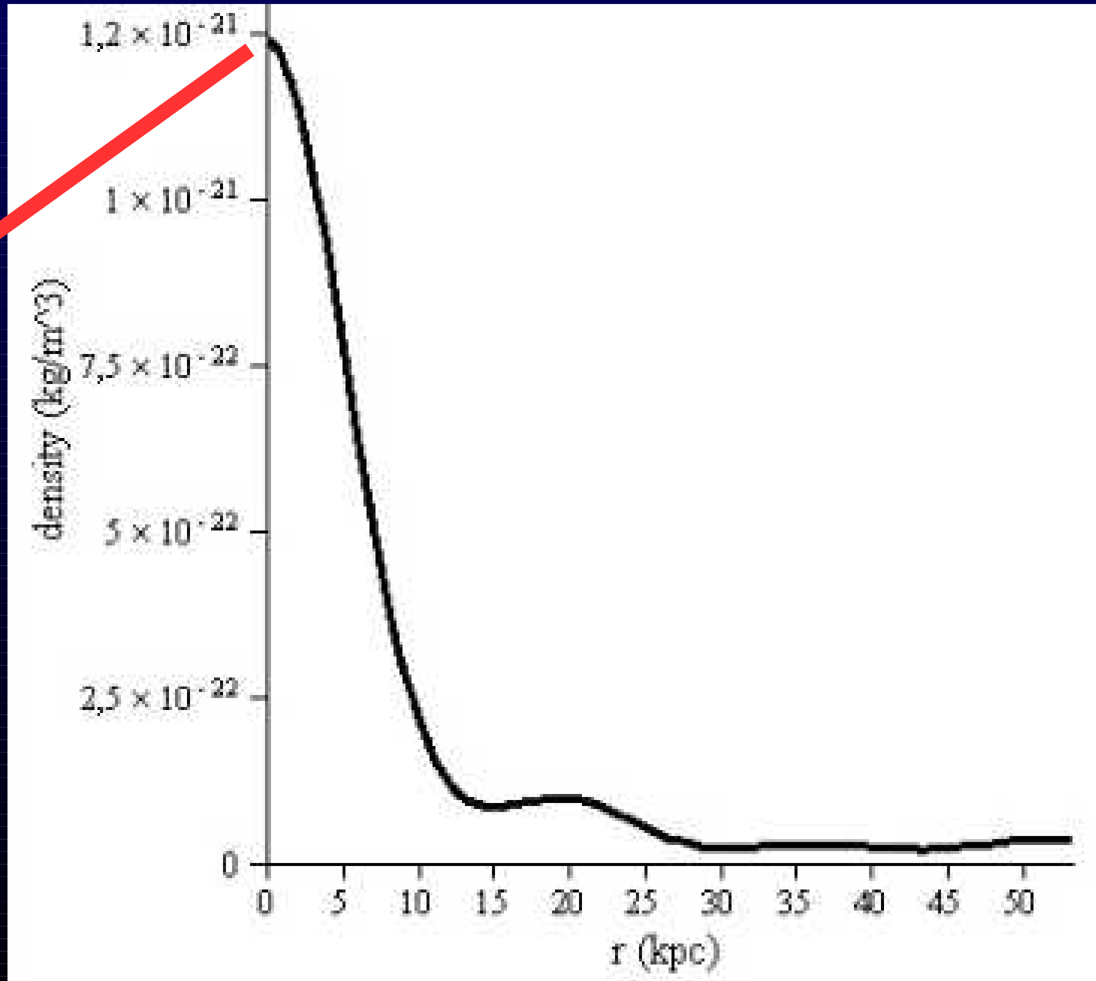


Comparing UGC 2403 and UBC 128

CT mass density curves at z=0

NGC 2304 UGC 128





Estimated thickness

NGC 2403

$z \sim r/15$

$M = 1.8 \times 10^{10}\, M_\odot$

UGC 128

$z \sim r/27$

$M = 2.7 \times 10^{10}\, M_\odot$

Final remarks

The good rotation curve fits resulted from the application of general relativity to a metric chosen in view of general galactic features.

- The fact that the galactic components rotate in a stationary way yielded the gravitational conditions responsible for flat rotation curves.
- The model suggests that there may be no dark matter in galaxies as far as rotation curves are concerned.
- The CT model may perhaps be used to estimate galactic thicknesses.
- Our results suggest that NGC 2403 may be considerably thicker than UGC 128.

Final remarks

- The model does not differentiate between stars and gas, as the galactic composition is assumed to be a fluid of dust particles.
- As a natural continuation of this work more realistic metrics could be used to model the spiral galaxies, for example.
- The model illustrates how general relativity may yield relevant results on galactic scales due to the non-linear characteristic of rotating gravitational fields.

Thank you for your attention.

Obrigada por sua atenção.