Partonic orbital angular momentum in the nucleons chiral periphery

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Physics Letters B 797 (2019) 134847

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Light Cone 2019
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Outline

- Motivation
- EMT,AM and Peripheral Dynamics
- Transverse Densities
  Definitions, properties
- Angular momentum in impact parameter space
  Spectral Functions, ChPT calculation
- Light front formulation
  Wave functions and χGPDs
- Transverse densities of orbital angular momentum

Motivation

We compute distributions of angular momentum in the nucleon from first principles in the peripheral region and present them in the light front formalism.

- Studying the nucleon's partonic content at peripheral transverse distances $b = \mathcal{O}(M_\pi^{-1})$, Chiral Dynamics.
- Explore definitions of orbital angular momentum
- Provide a mechanical picture of AM distributions. Light-front formulation of dynamics in the chiral periphery.

Angular Momentum and Energy Momentum Tensor

- The total (quark plus gluon) AM tensor of QCD

$$J^{\mu\alpha\beta}(x) = S_q^{\mu\alpha\beta}(x) + \widetilde{J}^{\mu\alpha\beta}(x).$$

- S_q is of the flavor-singlet axial current of the quark field,

$$S_q^{\mu\alpha\beta}(x) = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma}\sum\nolimits_f \bar{\psi}_f(x)\gamma_\gamma\gamma^5\psi_f(x).$$

- $\widetilde{J}$ is given in terms of the total (quark plus gluon) kinetic EMT of QCD,

$$\widetilde{J}^{\mu\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x).$$

E. Leader and C. Lorce Phys. Rep., 541 (3) (2014)

C. Lorce, L. Mantovani and B. Pasquini Phys. Lett. B, 776 (2018)

EMT Form Factors

Kinetic EMT transition matrix between two nucleon states at $x = 0$:

$$\begin{aligned}\langle N_2|T_{\mu\nu}(0)|N_1\rangle &= \bar{u}_2 \Big[A(\Delta^2)\gamma_{\{\mu}p_{\nu\}} + B(\Delta^2)p_{\{\mu}i\sigma_{\nu\}\alpha}\frac{\Delta^\alpha}{2M_N} \\ &+C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M_N}\right) + \tilde{C}(\Delta^2)M_N g_{\mu\nu} \\ &+ D(\Delta^2)p_{[\mu}i\sigma_{\nu]\alpha}\frac{\Delta^\alpha}{2M_N}\Big] u_1,\end{aligned}$$

- Not symmetric. However in peripheral dynamics $D\left(\Delta^2\right)$ can be neglected.

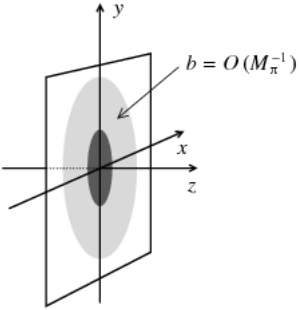
EMT form factors

$$\begin{aligned}\langle N_2|T_{\mu\nu}(0)|N_1\rangle \quad = \quad & \bar{u}_2 \Bigg[A(\Delta^2)\gamma_{\{\mu}\, p_{\nu\}} + B(\Delta^2) p_{\{\mu}\, i\sigma_{\ \nu\}\alpha}\frac{\Delta^\alpha}{2M_N} \\ & + C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M_N}\right) + \tilde{C}(\Delta^2) M_N g_{\mu\nu} \\ & + D(\Delta^2) p_{[\mu}\, i\sigma_{\ \nu]\alpha}\frac{\Delta^\alpha}{2M_N}\Bigg] u_1 ,\end{aligned}$$

Working in the $\Delta^+ = 0$ reference frame, one can write the EMT form factors in terms of matrix elements of light front components of the EMT. For the combination $A + B$,

$$A(-\boldsymbol{\Delta}_T^2) + B(-\boldsymbol{\Delta}_T^2) = \frac{4}{2p^+}\, \frac{i(\boldsymbol{\Delta}_T \times \boldsymbol{e}_z)}{\boldsymbol{\Delta}_T^2} \cdot \sum_{\sigma_1\sigma_2} \langle N_2|\, T^{+T}(0)\, |N_1\rangle S_z(\sigma_1, \sigma_2)$$

Transverse Densities



$$\rho(b) = \int \frac{d^2\mathbf{\Delta_T}}{(2\pi)} F(-\Delta_T^2)$$

Connect Form Factors and GPDs to nucleon intrinsic spacial structure

G.A. Miller,ARNPS 60 (2010)
M.Burkardt,PRD62(2000)

- Boost invariants;
- Light front wave function overlap for multiparticle systems.
- Distance b as parameter

$$[b \to M_\pi^{-1}]$$

Chiral periphery.

M.Strikman, C.Weiss PRC82(2010)
CG, C.Weiss JHEP 1401 (2014)

Transverse density of angular momentum

- Orbital contribution to longitudinal spin component

$$\langle L_z(\boldsymbol{b})\rangle = \frac{1}{2p^+}\langle \boldsymbol{b} \times T^{+T}(\boldsymbol{b})\rangle \cdot \boldsymbol{e}_z,$$

- Matrix elements

$$\langle L_z(\boldsymbol{b})\rangle = -\frac{\langle\sigma_z\rangle}{2} b \frac{d}{db}\left(\rho_A(b) + \rho_B(b) + \rho_D(b)\right).$$

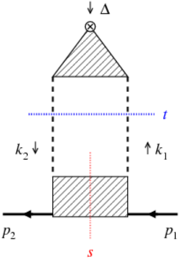
L. Adhikari and M. Burkardt, PRD **94**, no. 11, 114021 (2016)

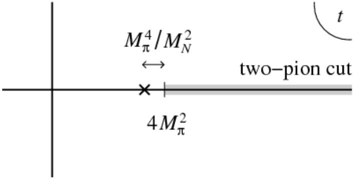
- Including quark spin contribution $S_z(b)$ leads to sum rule ,

$$\int d^2b\left[\langle L_z\rangle + \langle S_z\rangle\right](b) = S_z[rest].$$

From onset of Chiral dynamics, in the nucleons periphery $\langle S_z\rangle$ is suppressed. Axial current vanishes between two-pion states.

TD from dispersive representations of form factors in χPT





- Two-pion cut dispersion relation

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'-t} \frac{\mathrm{Im}F(t')}{\pi}$$

- Filter high momentum contributions

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\mathrm{Im}F(t+i0)}{\pi}$$

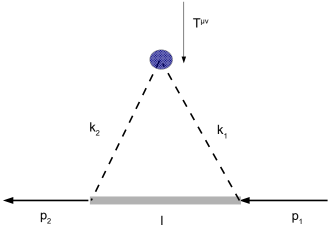
At large $b \sim O(M_\pi^{-1})$:

$$\mathcal{L}_{QCD}(\text{quarks,gluons}) \rightarrow \mathcal{L}_{\text{ChEFT}}(\pi)$$

- The isoscalar axial current in ChEFT does not have terms quadratic in the pion field, then ,

$$\langle S_z \rangle \text{ vanishes for } b \sim O(M_\pi^{-1}) \quad (1$$

Energy momentum tensor and AM in χPT



- From $\mathcal{L}_\chi(\pi)$ and Noether's theorem

$$T_{\mu\nu} = \mathrm{Tr}\left(\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}g_{\mu\nu}(\partial_\sigma\pi)^2 + \frac{1}{2}g_{\mu\nu}M_\pi^2\pi\pi\right) + \text{terms } \pi^4, \ldots$$

to obtain

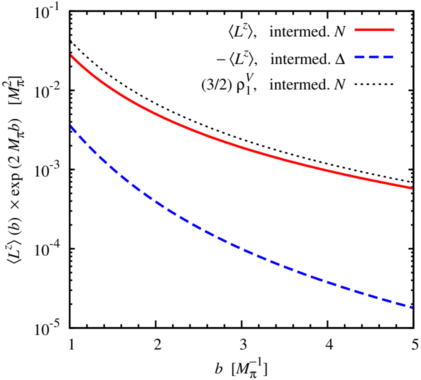
$$\left\langle N_2|T^{N\pi}_{\mu\nu}(0)|N_1\right\rangle = -\frac{3}{4}\frac{g_A^2}{F_\pi^2}\int\frac{d^4l}{(2\pi)^4}\left[-i\bar{u}_2\not{k}_2\gamma^5(\not{l}+M_N)\not{k}_1\gamma^5u_1D_N(l)\right.$$
$$\left.(k_{2\mu}k_{1\nu}+k_{1\mu}k_{2\nu}-g_{\mu\nu}(k_2k_1-M_\pi^2))D_\pi(k_2)D_\pi(k_1)\right].$$

From imaginary part of $T^{\pi N}$,

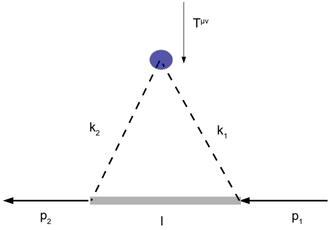
$$\frac{1}{\pi}\mathbf{Im}(A(t)+B(t)) = \frac{3}{4}\frac{g_A^2}{F_\pi^2}M_N^2\frac{(t/2-M_\pi^2)^3}{(4\pi)^2\sqrt{P^2}^5\sqrt{t}}\left(\frac{2}{3}x^3+x-\left(x^2+1\right)\mathbf{arctan}(x)\right) \tag{2}$$

Energy momentum tensor and AM in χPT

$$\langle L_z \rangle(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} \frac{\sqrt{t}b}{2} K_1(\sqrt{t}b) \frac{\textbf{\textit{Im}}[A+B](t+i0)}{\pi}$$



Energy momentum tensor and AM in χPT



- From $\mathcal{L}_\chi(\pi)$ and Noether's theorem

$$T_{\mu\nu} = \mathrm{Tr}\left(\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}g_{\mu\nu}(\partial_\sigma\pi)^2 + \frac{1}{2}g_{\mu\nu}M_\pi^2\pi\pi\right) + \text{terms } \pi^4, \ldots$$

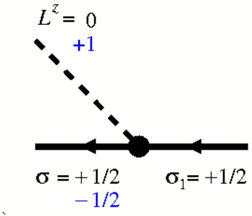
to obtain

$$\left\langle N_2|T^{N\pi}_{\mu\nu}(0)|N_1\right\rangle = -\frac{3}{4}\frac{g_A^2}{F_\pi^2}\int\frac{d^4l}{(2\pi)^4}\left[-i\bar{u}_2\not{k}_2\gamma^5(\not{l}+M_N)\not{k}_1\gamma^5u_1D_N(l)\right.$$
$$\left.(k_{2\mu}k_{1\nu}+k_{1\mu}k_{2\nu}-g_{\mu\nu}(k_2k_1-M_\pi^2))D_\pi(k_2)D_\pi(k_1)\right].$$

In LF variables ($v^\pm = v^0 \pm v^z$),

$$\left\langle N_2|\frac{T^{++}_{N\pi}(0)}{2p^+}|N_1\right\rangle = \frac{3}{4}p^+\int\frac{dy}{y\bar{y}}\frac{d^2\boldsymbol{k}_T}{(2\pi)^3}y\Psi^*\left(y,\boldsymbol{k}_T+\bar{y}\frac{\boldsymbol{\Delta}_T}{2};\right)\Psi\left(y,\boldsymbol{k}_T-\bar{y}\frac{\boldsymbol{\Delta}_T}{2}\right),$$
$$\left\langle N_2|\frac{T^{+T}_{N\pi}(0)}{2p^+}|N_1\right\rangle = \frac{3}{4}\int\frac{dy}{y\bar{y}}\frac{d^2\boldsymbol{k}_T}{(2\pi)^3}\boldsymbol{k}_T\Psi^*\left(y,\boldsymbol{k}_T+\bar{y}\frac{\boldsymbol{\Delta}_T}{2};\right)\Psi\left(y,\boldsymbol{k}_T-\bar{y}\frac{\boldsymbol{\Delta}_T}{2}\right)$$

Light front wave functions



$$\Psi(y, \widetilde{\boldsymbol{k}}_T = \boldsymbol{k}_T + y p_{1T}) \equiv \frac{\Gamma(y, \widetilde{\boldsymbol{k}}_T)}{\underbrace{\Delta \mathcal{M}^2(y, \widetilde{\boldsymbol{k}}_T)}_{\text{Inv. Mass difference}}}$$

while in transverse coordinate space,

$$\begin{aligned} \Phi(y, r_T) &= \int \frac{d^2 \widetilde{\boldsymbol{k}}_T}{(2\pi)^2} e^{\widetilde{\boldsymbol{k}}_T \cdot \boldsymbol{r}_T} \Psi(y, \widetilde{\boldsymbol{k}}_T) \\ &= -2i \left[U_0(y, r_T) S^z + i \frac{U_1(y, r_T) \boldsymbol{r}_T \cdot \boldsymbol{S}_T}{r_T} \right] \end{aligned}$$

Eigenfunctions of LF Hamiltonian.

Allow quantum mechanical description of peripheral dynamics.

Computable at leading order in chiral periphery

C.G , C. Weiss, JHEP **1507**, 170 (2015)

$$\begin{aligned} \Gamma(y, \widetilde{\boldsymbol{k}}_T) &\approx \frac{g_A M_N}{F_\pi} \bar{u}(y, \boldsymbol{k}_T) i \gamma_5 u(p_{1T}) \\ &= \frac{2 i g_A M_N^2}{F_\pi \sqrt{\bar{y}}} \left[y \boldsymbol{S}_z + \frac{\widetilde{\boldsymbol{k}}_T \cdot \boldsymbol{S}_T}{M_N} \right] \end{aligned}$$

with radial functions,

$$\left. \begin{array}{l} U_0(y, r_T) \\ U_1(y, r_T) \end{array} \right\} = \frac{g_A M_N \, y \sqrt{\bar{y}}}{2\pi F_\pi} \left\{ \begin{array}{l} y M_N \; K_0(M_T r_T) \\ M_T \; K_1(M_T r_T) \end{array} \right. ,$$

and transverse mass

$$M_T^2 = \bar{y}^2 M_\pi + y^2 M_N^2$$

AM in Chiral Periphery

For a given baryon $-\pi$ intermediate state

$$\langle L_z\rangle(b) = \frac{D_B}{2}\frac{1}{2\pi}\int \frac{dy}{y\bar{y}^3}\bar{y}\mathrm{Tr}\left[S_z\Phi^\dagger(y,\boldsymbol{r}_T)\left(\boldsymbol{r}_T\times(-i)\frac{\partial}{\partial \boldsymbol{r}_T}\right)\Phi(y,\boldsymbol{r}_T)\right]|_{\boldsymbol{r}_T=\boldsymbol{b}/\bar{y}},$$

Validates a probabilistic quantum-mechanical interpretation of $\rho_{lz}(b)$ as a density of OAM in the nucleon's periphery! This cannot be achieved with other definitions of $\langle L_z\rangle(b)$.

- For $N-\pi$ -intermediate state,

$$\langle L_z\rangle_N(b) \quad = \quad \frac{D_N}{2}\frac{1}{2\pi}\int \frac{dy}{y\bar{y}^2}U_1(y,\boldsymbol{b}/\bar{y})^2. \tag{3}$$

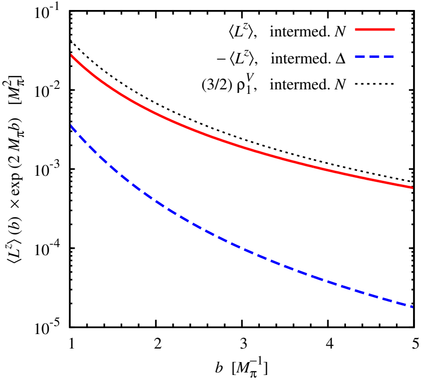
  with$D_N = 3$.

- For $\Delta-\pi$ -intermediate state,

$$\begin{aligned}\langle L_z\rangle_\Delta(b) \quad = \quad & \frac{D_\Delta}{2}\frac{1}{6\pi}\int \frac{dy}{y\bar{y}^2} \\ & \left(2V_1(y,\boldsymbol{b}/\bar{y})^2 + 2V_1(y,\boldsymbol{b}/\bar{y})W_1(y,\boldsymbol{b}/\bar{y})\right. \\ & \left.-W_1(y,\boldsymbol{b}/\bar{y})^2 + 3W_2(y,\boldsymbol{b}/\bar{y})^2\right).\end{aligned}$$

  with $D_\Delta = 2$

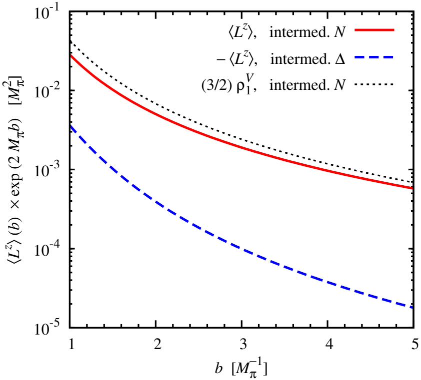
AM in Chiral Periphery



- Relative (-) sign between intermediate N and *Delta*
- Large N_c cancellation

$$\begin{aligned} \langle L_z \rangle_\Delta(b) &\approx -\frac{D_\Delta}{2} \frac{1}{6\pi} \int \frac{dy}{y\bar{y}^2} W_1(y, \boldsymbol{b}/\bar{y})^2 \\ &\approx -\frac{D_\Delta}{2} \frac{1}{6\pi} \int \frac{dy}{y\bar{y}^2} \left(\frac{3}{2} U_1(y, \boldsymbol{b}/\bar{y}) \right)^2 \\ &\approx -\langle L_z \rangle_N(b) \end{aligned}$$

AM in Chiral Periphery



$$
\begin{aligned}
\rho_{1,N}(b) &= \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^3} \left(U_0(y, \boldsymbol{b}/\bar{y})^2 + U_1(y, \boldsymbol{b}/\bar{y})^2 \right), \\
\langle L_z \rangle_N(b) &= \frac{3}{2} \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^2} U_1(y, \boldsymbol{b}/\bar{y})^2.
\end{aligned}
$$

Summary and Outlook

Transverse densities computed in Chiral EFT were used in a model independent approach to quantify the distribution of angular momentum in the periphery of the nucleon. It is observed that in this region,

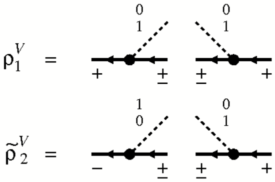
- the Energy Momentum tensor is symmetric, and that
- the angular momentum density is orbital.

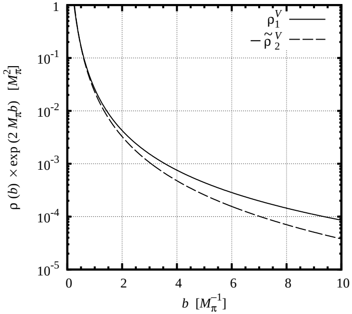
By using LF variables, the density of angular momentum is written as a proper density in LF-quantum mechanics of a pion-nucleon fluctuation of a nucleon.

This framework opens the possibility of fully exploring the role that chiral dynamics plays constraining the nucleons internal structure.

- Transverse densities associated with form factors of the energy momentum tensor.Distributions of matter in impact parameter space.
- Expand on different intermediate baryons. (Ongoing) work on intermediate Δ probes large Nc limit properties of LCWF and allows the study of higher orbital modes

Charge and magnetization densities from LF dynamics





C.G , C. Weiss, JHEP **1507**, 170 (2015)

Light front current matrix as wavefunction overlap,

$$\frac{J(b)}{2p^+} = \frac{1}{2\pi}\int \frac{dy}{y\bar{y}}\Phi^\dagger\left(y, \frac{b}{\bar{y}}\right)\Phi\left(y, \frac{b}{\bar{y}}\right),$$

then in terms of radial functions

$$\left.\begin{array}{c}\rho_1^V(b)\\ \tilde{\rho}_2^V(b)\end{array}\right\} = \frac{1}{2\pi}\int\frac{dy}{y\bar{y}^3}\left\{\begin{array}{c}[U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2\\ -2\,U_0(y, b/\bar{y})\,U_1(y, b/\bar{y})\end{array}\right\}.$$

Inequality and positive definiteness of light front current,

$$|\rho_1(b)| > \tilde{\rho}_2(b) \Rightarrow J^+(b) > 0$$

weakly bound pions.

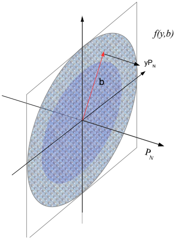
Near parametric equality,

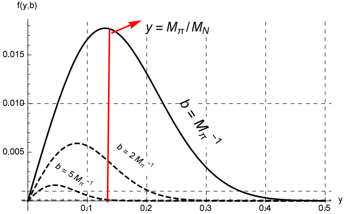
$\rho_1(b) \approx -\tilde{\rho}_2(b) \Rightarrow$ relativistic pion-nucleon system

Explain through left right asymmetry in Transverse densities .

CG, C. Weiss, PRC **92**, no. 2, 025206 (2015)

Charge magnetization densities and χGPDs





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then in terms of radial functions

$$\left.\begin{array}{c} \rho_1^V(b) \\ \widetilde{\rho}_2^V(b) \end{array}\right\} = \frac{1}{2\pi} \int \frac{dy}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y})]^2 \\ -2\, U_0(y, b/\bar{y})\, U_1(y, b/\bar{y}) \end{array} \right\}.$$

Integrands correspond to Fourier trans. of GPDs, $H(y,t)$ and $E(y,t)$, i.e., $f_1(y,b)$, $f_2(y,b)$ respectively,

$$\left.\begin{array}{c} f_1^V(y,b) \\ \widetilde{f}_2^V(y,b) \end{array}\right\} = \frac{1}{2\pi} \frac{1}{y\bar{y}^3} \left\{ \begin{array}{c} [U_0(y, b/\bar{y})]^2 + [U_1(y, b/\bar{y} \\ -2 U_0(y, b/\bar{y}) U_1(y, b/\bar{y} \end{array} \right.$$