Study of twist-2 distribution amplitudes and the decay constants of pseudoscalar and vector heavy mesons in light-front quark model

Nisha Dhiman

National Institute of Technology, Jalandhar, India

Ref.: Nisha Dhiman, Harleen Dahiya, Chueng-Ryong Ji and Ho-Meoyng Choi, Phys. Rev. D 100, 014026 (2019)

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Outline

- Light-front quark model
- Quark Distribution Amplitudes and Decay Constants
- Numerical Results
- Conclusions

Light-front quark model

- In this model, a meson bound state consisting of a quark q and an antiquark $\bar{q}$ with total momentum P and spin S is represented as

$$|M(P,S,S_z)\rangle = \int \frac{dp_q^+ d^2\mathbf{p}_{q\perp}}{16\pi^3} \frac{dp_{\bar{q}}^+ d^2\mathbf{p}_{\bar{q}\perp}}{16\pi^3} 16\pi^3 \delta^3(\tilde{P} - \tilde{p}_q - \tilde{p}_{\bar{q}}) \times \sum_{\lambda_q,\lambda_{\bar{q}}} \Psi^{SS_z}(\tilde{p}_q, \tilde{p}_{\bar{q}}, \lambda_q, \lambda_{\bar{q}}) \, |q(p_q,\lambda_q)\bar{q}(p_{\bar{q}},\lambda_{\bar{q}})\rangle,$$

— H.-M. Choi and C.-R. Ji, Phys. Rev. D 59, 074015 (1999)
— H.-M. Choi and C.-R. Ji, Phys. Lett. B 460, 461 (1999)

where $p_{q(\bar{q})}$ and $\lambda_{q(\bar{q})}$ are the on-mass shell light-front momentum and the light-front helicity of the constituent quark (antiquark).

- The momentum $\tilde{p}$ is defined as

$$\tilde{p} = (p^+,\ \mathbf{p}_\perp),\ \mathbf{p}_\perp = (p^1,\ p^2),\ p^- = \frac{m^2 + \mathbf{p}_\perp^2}{p^+}.$$

Light-front quark model

- The light-front momenta p_q and $p_{\bar{q}}$ in terms of light-front variables are defined as

$$\begin{aligned} p_q^+ &= x_1 P^+, \ \ p_{\bar{q}}^+ = x_2 P^+, \\ \mathbf{p}_{q_\perp} &= x_1 \mathbf{P}_\perp + \mathbf{k}_\perp, \ \ \mathbf{p}_{\bar{q}_\perp} = x_2 \mathbf{P}_\perp - \mathbf{k}_\perp, \end{aligned}$$

  where $x_{1(2)}$ is the longitudinal momentum fraction satisfying the relation $x_1 + x_2 = 1$ and $\mathbf{k}_\perp$ is the relative transverse momentum of the constituent.

- The momentum-space light-front wave function $\Psi^{SS_z}(\tilde{p}_q, \tilde{p}_{\bar{q}}, \lambda_q, \lambda_{\bar{q}})$ can be expressed as

$$\Psi^{SS_z} = \frac{\sqrt{p_q^+ p_{\bar{q}}^+}}{\sqrt{2}\sqrt{M_0^2 - (m_q - m_{\bar{q}})^2}} \, \bar{u}(p_q, \lambda_q)\, \Gamma\, v(p_{\bar{q}}, \lambda_{\bar{q}}) \sqrt{\frac{\partial k_z}{\partial x}}\, \Phi(x, \mathbf{k}_\perp),$$

  where $\Phi(x, \mathbf{k}_\perp)$ describes the momentum distribution of the constituents in the bound state with $x \equiv x_1$.

Light-front quark model

- M_0 is the invariant mass of the $q\bar{q}$ system,

$$M_0^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x_1} + \frac{m_{\bar{q}}^2 + \mathbf{k}_\perp^2}{x_2}.$$

- Radial wave function $\Phi(x, \mathbf{k}_\perp)$ is expanded in the three lowest order HO wave functions

$$\Phi(x, \mathbf{k}_\perp) = \sum_{n=1}^{3} c_n \phi_{nS},$$

Here

$$\phi_{1S}(x, \mathbf{k}_\perp) = \frac{1}{(\sqrt{\pi}\beta)^{3/2}} \exp(-\mathbf{k}^2/2\beta^2),$$

$$\phi_{2S}(x, \mathbf{k}_\perp) = \frac{1}{(\sqrt{\pi}\beta)^{3/2}} \left(\frac{2\mathbf{k}^2 - 3\beta^2}{\sqrt{6}\beta^2} \right) \exp(-\mathbf{k}^2/2\beta^2), \ \ and$$

$$\phi_{3S}(x, \mathbf{k}_\perp) = \frac{1}{(\sqrt{\pi}\beta)^{3/2}} \left(\frac{15\beta^4 - 20\beta^2\mathbf{k}^2 + 4\mathbf{k}^4}{2\sqrt{30}\beta^4} \right) \exp(-\mathbf{k}^2/2\beta^2).$$

Light-front quark model

- The internal momentum of the meson $\mathbf{k}^2 = \mathbf{k}_\perp^2 + k_z^2$, where the longitudinal component k_z is defined as

$$k_z = \left(x - \frac{1}{2}\right) M_0 + \frac{m_q^2 - m_{\bar{q}}^2}{2M_0}.$$

- For the variable transformation $(x, \mathbf{k}_\perp) \to \mathbf{k} = (\mathbf{k}_\perp, k_z)$, the Jacobian factor $\partial k_z / \partial x$ is given by

$$\frac{\partial k_z}{\partial x} = \frac{M_0}{4x(1-x)} \left[1 - \left(\frac{m_q^2 - m_{\bar{q}}^2}{M_0^2}\right)^2\right].$$

- We consider the radial wave function $\Phi(x, \mathbf{k}_\perp)$ as a trial wave function for the variational principle to the QCD-motivated Hamiltonian

$$H_{\rm c.m.} = \sqrt{\mathbf{k}^2 + m_q^2} + \sqrt{\mathbf{k}^2 + m_{\bar{q}}^2} + V_{q\bar{q}}.$$

– H.-M. Choi and C.-R. Ji, Phys. Rev. D 80, 054016 (2009)
– H.-M. Choi and C.-R. Ji, Phys. Rev. D 59, 074015 (1999)
– H.-M. Choi and C.-R. Ji, Phys. Lett. B 460, 461 (1999)

Light-front quark model

- $V_{q\bar{q}}$ is the combination of
  - Coulomb plus exponential-type potential, and
  - Hyperfine interaction

$$\begin{aligned} V_{q\bar{q}} &= V_0(r) + V_{\rm hyp}(r) \\ &= V_{\rm exp} + V_{\rm Coul} + V_{\rm hyp} \\ &= a + be^{\alpha r} - \frac{4\kappa}{3r} + \frac{2}{3}\frac{\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}}{m_q m_{\bar{q}}} \nabla^2 V_{\rm Coul}, \end{aligned}$$

where

- a, b and α are the parameters of the potential,
- κ is the strong coupling constant,
- $\langle \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \rangle = -3/4\,(1/4)$ for the pseudoscalar (vector) meson.

Light-front quark model

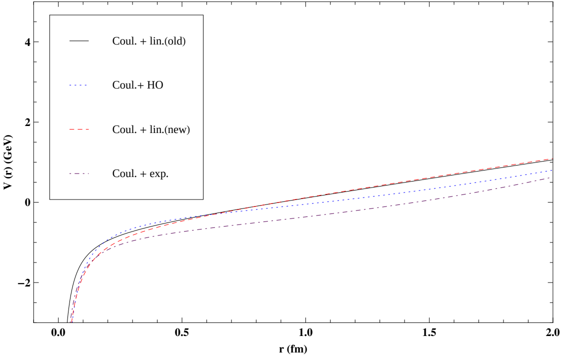

Figure: Variation of central potential $V_0(r)$ with respect to r in various potential models.

– H.-M. Choi, Phys. Rev. D 75, 073016 (2007)
– H.-M. Choi, C.-R. Ji, Z. Li, and H.-Y. Ryu, Phys. Rev. C 92, 055203 (2015)

Table: Constituent quark masses and the potential parameters (in units of GeV) obtained by the variational principle for the Hamiltonian Here $q = u$ and d.

m_q	m_s	m_c	m_b	σ	α	a
0.202	0.405	1.725	5.182	0.451	0.15	-1.075

Table: The Gaussian parameter β (GeV) for ground state pseudoscalar (D, D_s, B, B_s) and vector (D^*, D_s^*, B^*, B_s^*) mesons obtained by the variational principle. Here $q = u$ and d.

J^{PC}	β_{qc}	β_{cs}	β_{qb}	β_{bs}
0^{-+}	0.2980	0.3010	0.3191	0.3290
1^{--}	0.2818	0.2926	0.3115	0.3250

Table: Ground state mass spectra (in units of GeV) of pseudoscalar ($D_{(s)}$) and vector ($D^*_{(s)}$) mesons obtained from the exponential type potential and their comparison with the experimental data and the LFQM results obtained from the linear and HO potentials.

	M_D	M_{D^*}	M_{D_s}	$M_{D_s^*}$
Present work	1.803	1.884	1.929	1.971
Exp.	1.869	2.010	1.968	2.112
LFQM, Lin	1.836	1.998	2.011	2.109
LFQM, HO	1.821	2.024	2.005	2.150
LFQM	1.875	1.962	1.981	2.031

Light-front quark model

Table: Ground state mass spectra (in units of GeV) of pseudoscalar ($B_{(s)}$) and vector ($B^*_{(s)}$) mesons obtained from the exponential type potential and their comparison with the experimental data and the LFQM results obtained from the linear and HO potentials.

	M_B	M_{B^*}	M_{B_s}	$M_{B_s^*}$
Present work	5.212	5.242	5.313	5.329
Exp.	5.279	5.325	5.367	5.415
LFQM, Lin	5.235	5.315	5.375	5.424
LFQM, HO	5.235	5.349	5.378	5.471
LFQM	5.233	5.268	5.314	5.333

- The quark DAs are defined in terms of the matrix elements of non-local operators that are sandwiched between the vacuum and the meson states

$$
\begin{aligned}
\langle 0|\bar{q}(0)\gamma^{\mu}\gamma_5 q(0)|P(P)\rangle &= i f_P P^{\mu}\int_0^1 \phi_P(x)dx, \\
\langle 0|\bar{q}(0)\gamma^{\mu} q(0)|V(P,\lambda=0)\rangle &= f_V M_V \epsilon^{\mu}(\lambda)\int_0^1 \phi_{V\|}(x)dx, \\
\langle 0|\bar{q}(0)\sigma^{\mu\nu} q(0)|V(P,\lambda=\pm 1)\rangle &= i f_V^{\perp}[\epsilon^{\mu}(\lambda)P_{\nu}-\epsilon^{\nu}(\lambda)P_{\mu}]\int_0^1 \phi_{V\perp}(x)dx.
\end{aligned}
$$

Here ϕ_P, $\phi_{V\|}$ and $\phi_{V\perp}$ are the twist-2 DAs of pseudoscalar, longitudinally and transversely polarized vector mesons, respectively.

– C.-W. Hwang, Phys. Rev. D 81, 114024 (2010)

- The explicit forms of quark DAs in our LFQM are given by

$$\phi_P(x) = \frac{2\sqrt{6}}{f_P}\int \frac{d^2\mathbf{k}_\perp}{\sqrt{16\pi^3}}\sqrt{\frac{\partial k_z}{\partial x}}\Phi(x,\mathbf{k}_\perp)\frac{\mathcal{A}}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}},$$

$$\phi_{V\parallel}(x) = \frac{2\sqrt{6}}{f_V}\int \frac{d^2\mathbf{k}_\perp}{\sqrt{16\pi^3}}\sqrt{\frac{\partial k_z}{\partial x}}\frac{\Phi(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}}\left\{\mathcal{A}+\frac{2\mathbf{k}_\perp^2}{M_0+m_q+m_{\bar{q}}}\right\},$$

$$\phi_{V\perp}(x) = \frac{2\sqrt{6}}{f_V^\perp}\int \frac{d^2\mathbf{k}_\perp}{\sqrt{16\pi^3}}\sqrt{\frac{\partial k_z}{\partial x}}\frac{\Phi(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2+\mathbf{k}_\perp^2}}\left\{\mathcal{A}+\frac{\mathbf{k}_\perp^2}{M_0+m_q+m_{\bar{q}}}\right\},$$

where $\mathcal{A} = (1-x)m_q + x m_{\bar{q}}$.

– H.-M. Choi and C.-R. Ji, Phys. Rev. D 75, 034019 (2007)

- We can define the decay constants for the pseudoscalar and the vector mesons as

$$\begin{aligned}\langle 0|\bar{q}\gamma^{\mu}\gamma_5 q|P(P)\rangle &= if_P P^{\mu},\\ \langle 0|\bar{q}\gamma^{\mu} q|V(P,\lambda=0)\rangle &= f_V M_V \epsilon^{\mu}(\lambda),\\ \langle 0|\bar{q}\sigma^{\mu\nu} q|V(P,\lambda=\pm 1)\rangle &= if_V^{\perp}[\epsilon^{\mu}(\lambda)P_{\nu}-\epsilon^{\nu}(\lambda)P_{\mu}].\end{aligned}$$

Numerical Results

- For the present calculations of the decay constants, we included the systematic errors in our analysis obtained both from the $\pm 10\%$ variation of β values for the fixed quark masses and the $\pm 10\%$ variation of quark masses for the fixed β values.

Numerical Results

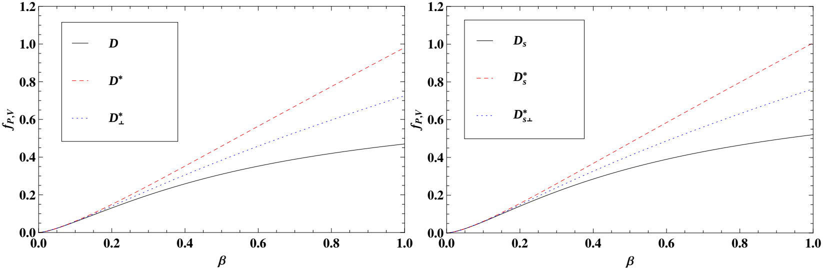

Figure: The decay constants $f_{D_{(s)}}$, $f_{D^*_{(s)}}$ and $f^{\perp}_{D^*_{(s)}}$ of pseudoscalar, longitudinally and transversely polarized vector D (left panel) and D_s (right panel) mesons as functions of the parameter β (in GeV).

Numerical Results

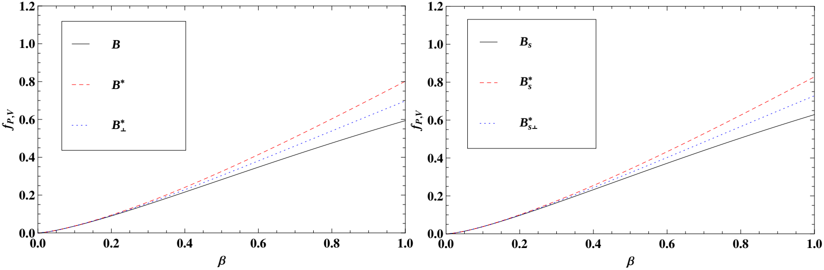

Figure: The decay constants $f_{B_{(s)}}$, $f_{B^*_{(s)}}$ and $f^{\perp}_{B^*_{(s)}}$ of pseudoscalar, longitudinally and transversely polarized vector B (left panel) and B_s (right panel) mesons as functions of the parameter β (in GeV).

Numerical Results

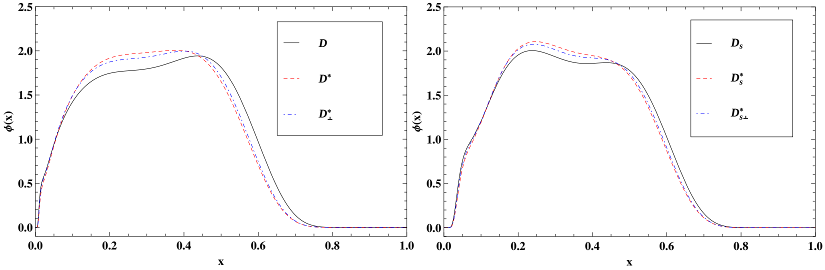

Figure: Normalized DAs for the heavy pseudoscalar (solid line), longitudinally (dashed line) and transversely (dotted dashed line) polarized vector D (left panel) and D_s (right panel) mesons.

Numerical Results

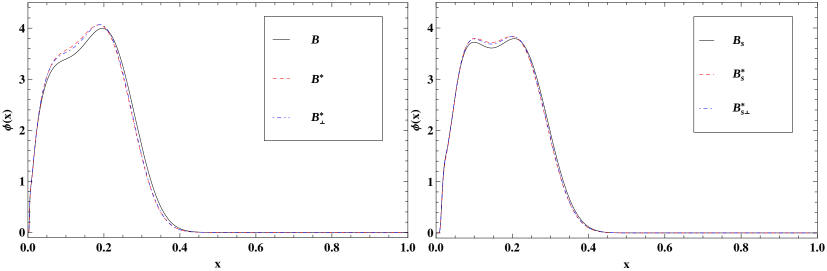

Figure: Normalized DAs for the heavy pseudoscalar (solid line), longitudinally (dashed line) and transversely (dotted dashed line) polarized vector B (left panel) and B_s (right panel) mesons.

Numerical Results

Table: Pseudoscalar, longitudinally and transversely polarized vector D meson decay constants (in units of MeV) in the present work and their comparison with the available experimental data and the previous work.

	f_D	f_{D^*}	$f^{\perp}_{D^*}$
Present work	$197^{+19+0.2}_{-20-1.0}$	230^{+29-5}_{-28+6}	208^{+24-3}_{-24+3}
Exp.	203.7 ± 4.7	–	–
LFQM, Lin	197	239	–
LFQM, HO	180	212	–
LFQM, Lin	208	230	–

Table: Pseudoscalar, longitudinally and transversely polarized vector D_s meson decay constants (in units of MeV) in the present work and their comparison with the available experimental data and the previous work.

	f_{D_s}	$f_{D_s^*}$	$f_{D_s^*}^{\perp}$
Present work	$219^{+21-0.2}_{-22-0.8}$	253^{+31-6}_{-31+6}	233^{+26-3}_{-26+3}
Exp.	257.8 ± 4.1	–	–
LFQM, Lin	233	274	–
LFQM, HO	218	252	–
LFQM, Lin	231	260	–

Table: Pseudoscalar, longitudinally and transversely polarized vector B meson decay constants (in units of MeV) in the present work and their comparison with the available experimental data and the previous work.

	f_B	f_{B^*}	$f_{B^*}^{\perp}$
Present work	163^{+21-4}_{-20+4}	172^{+24-6}_{-23+6}	165^{+22-5}_{-21+5}
Exp.	188 ± 25	–	–
LFQM, Lin	171	186	–
LFQM, HO	161	173	–
LFQM, Lin	181	188	–

Numerical Results

Table: Pseudoscalar, longitudinally and transversely polarized vector B_s meson decay constants (in units of MeV) in the present work and their comparison with the available experimental data and the previous work.

	f_{B_s}	$f_{B_s^*}$	$f_{B_s^*}^{\perp}$
Present work	184^{+23-4}_{-23+4}	194^{+26-6}_{-25+7}	187^{+24-5}_{-24+6}
Exp.	–	–	–
LFQM, Lin	205	220	–
LFQM, HO	208	223	–
LFQM, Lin	205	216	–

Conclusions

- We have calculated the mass spectra of the ground state pseudoscalar and vector heavy mesons as well as the decay constants of the corresponding mesons using the mixed wave function Φ of $1S$, $2S$ and $3S$ HO states as the trial wave function. We also compared our results with available experimental data and the previous LFQM results with the linear and HO potentials as well as other theoretical model predictions.
- Our LFQM predictions are comparable to each other regardless of the confining potential type as far as the efficacy of model prediction is achieved by allowing sufficient number of HO basis functions for the trial wave function.

Conclusions

- It appears however that we need more HO basis functions for the trial wave function in the case of the exponential-type confining potential compare to the case of the linear and HO confining potentials.
- For the present analysis with the exponential-type confining potential, we used the larger number of HO basis functions ($1S$, $2S$, and $3S$) to achieve the efficacy of the model calculations, while we achieved the efficacy of LFQM with the linear and HO confining potentials using only up to the two lowest order HO wave functions in our previous analyses

Conclusions

- Not only for the mass sprectra but also for the decay constants of pseudoscalar and vector light and heavy mesons, our results are in reasonable agreement with the available experimental data as well as comparable with other theoretical model predictions.
- The quark DAs of D, D^*, D_s and D_s^* mesons show much broader shapes than those of B, B^*, B_s and B_s^* mesons due to the large mass difference between b and c quarks.

Thank you