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INVESTIGATING TRANSVERSE MOMENTUM DISTRIBUTIONS WITH JETS

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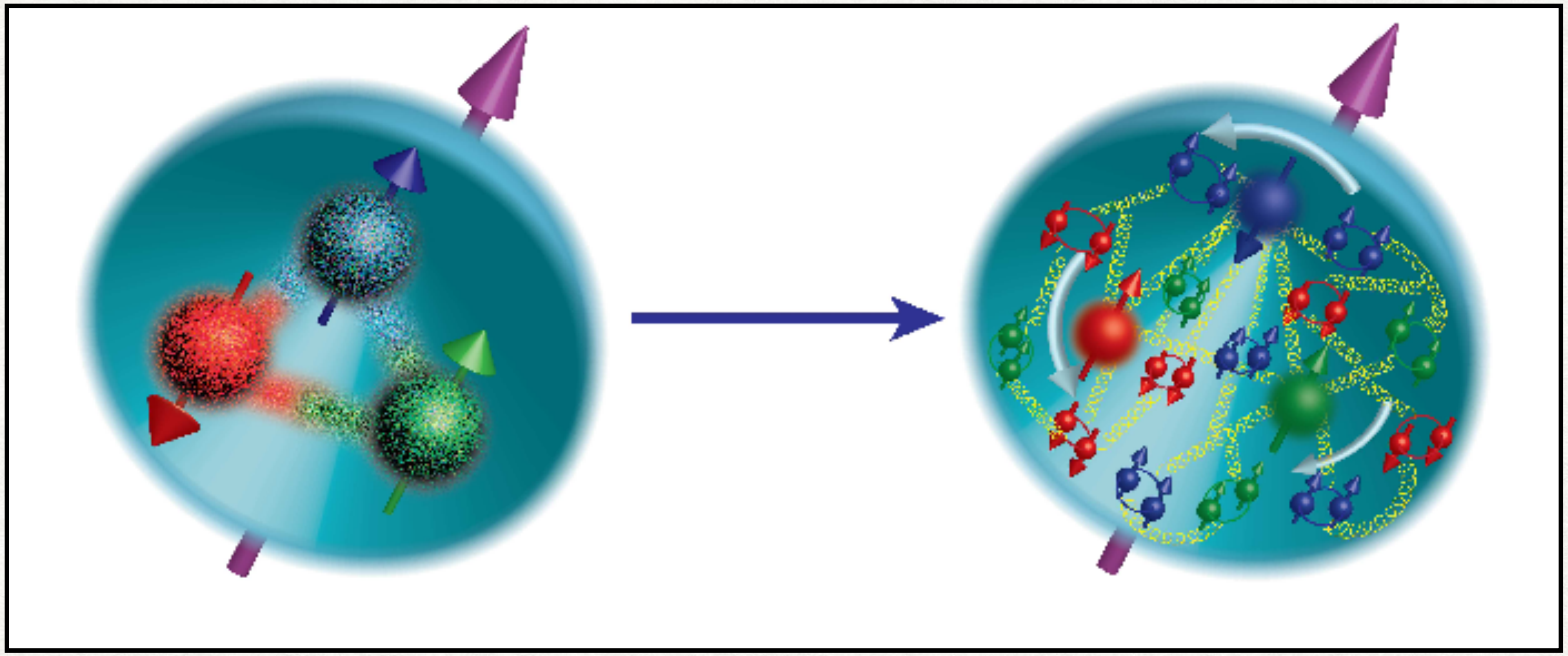
IPARCOS

UNIVERSITAS COMPLVTENSIS MATRITENSIS

STRONG 2020

Based on collaborations with:
DGR, I.Scimemi, W. Waalewijn, L. Zoppi PRL **121**, 162001(2018) arXiv: 1807.07573
DGR, I.Scimemi, W. Waalewijn, L. Zoppi arXiv: 1904.04259 (accepted in JHEP)
DGR, Y. Makris, I.Scimemi, V. Vaidya, L. Zoppi JHEP **1908** (2019) 161 arXiv: 1907.0589

The knowledge about the hadron structure has been increased a lot in the last years

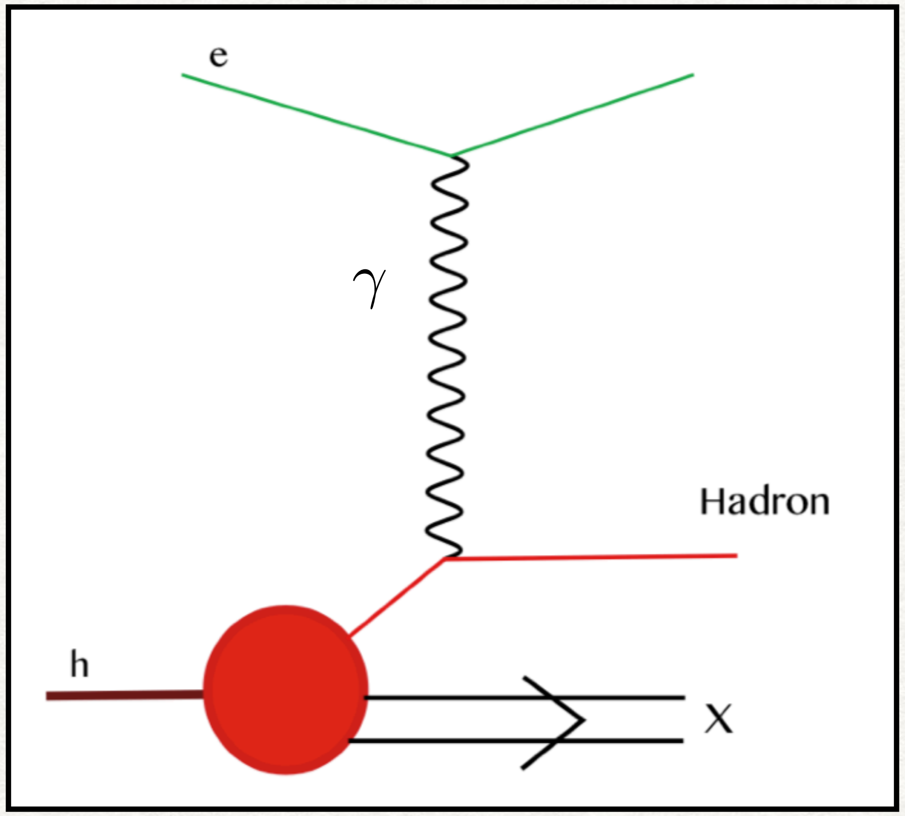


But the 3D mapping of hadrons is still a challenging topic…

↓

Let us use jets to investigate hadron structure!

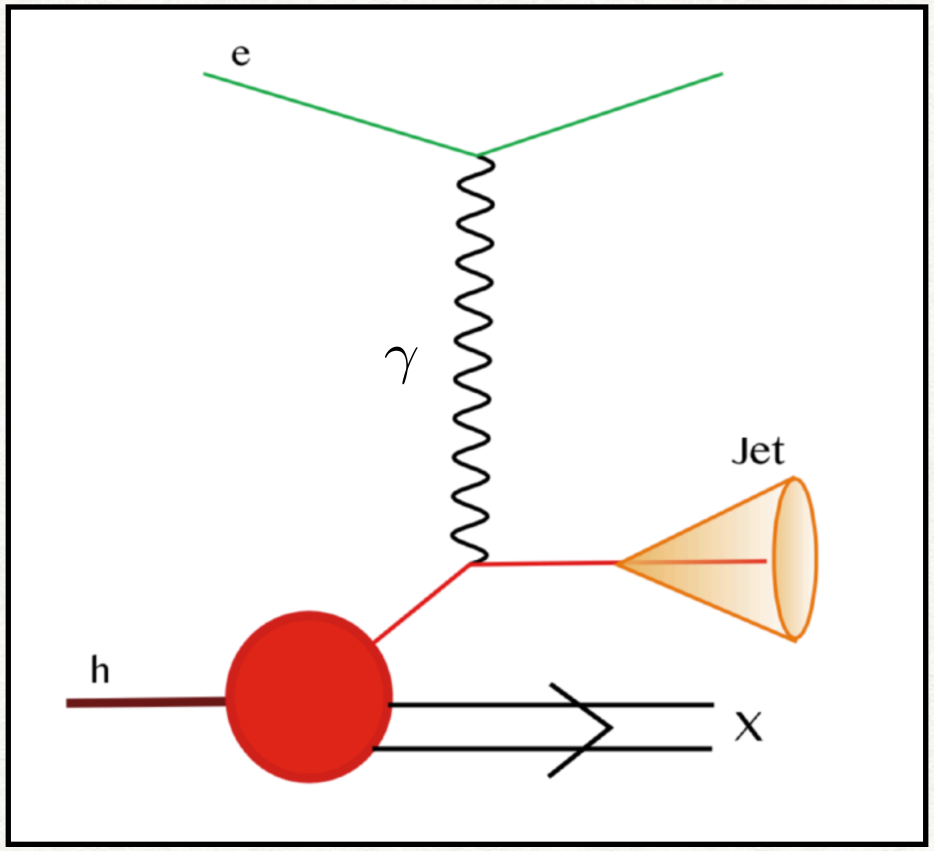
TMDS WITHOUT JETS



Factorization theorem for SIDIS

$$\frac{d\sigma_{eN\to eN'X}}{dQ^2dxdzd\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu)\int\frac{d\boldsymbol{b}}{(2\pi)^2}e^{-i\boldsymbol{bq}}\underbrace{f_{a/N}(x,\boldsymbol{b},\mu,\zeta)}_{\text{TMDPDF}}\underbrace{d_{a/N'}(z,\boldsymbol{b},\mu,\zeta)}_{\text{TMDFF}}$$

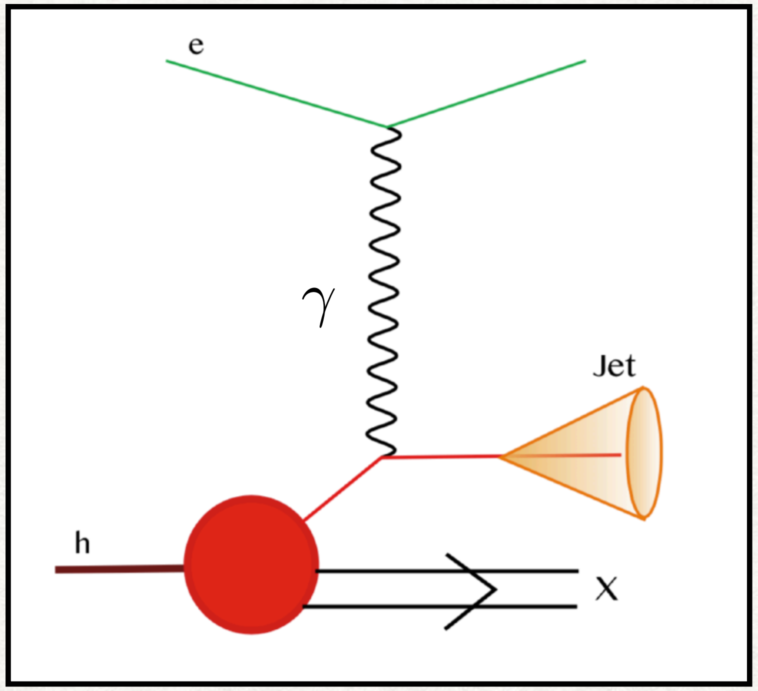
TMDS WITH JETS



Factorization theorem for SIDIS

$$\frac{d\sigma_{eN\to eJX}}{dQ^2 dx dz d\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} \underbrace{f_{a/N}(x,\boldsymbol{b},\mu,\zeta)}_{\text{TMDPDF}} \underbrace{J_q^{\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta)}_{\text{TMD jet function}}$$

TMDS WITH JETS



Questions:
Can we write TMD factorization theorems for processes with jets?

Can we write factorization theorems for jets regardless of the size of the jet?

Nonperturbative effects for jets are in principle more suppressed that for nuclear TMDs!

This would be a clean channel to measure nonperturbative effects in the initial nucleon

Hadronization effects should be considered (ungroomed/groomed jets)

OUTLINE

- Building a jet
    - Radius and jet algorithms
    - Jet axis
- TMD factorization
- Factorization: ungroomed/groomed jets. Phenomenology
    - Ungroomed jets: Factorization for dijet decorrelation and SIDIS processes
    - Groomed jets: Factorization for dijet decorrelation and SIDIS processes
- Conclusions

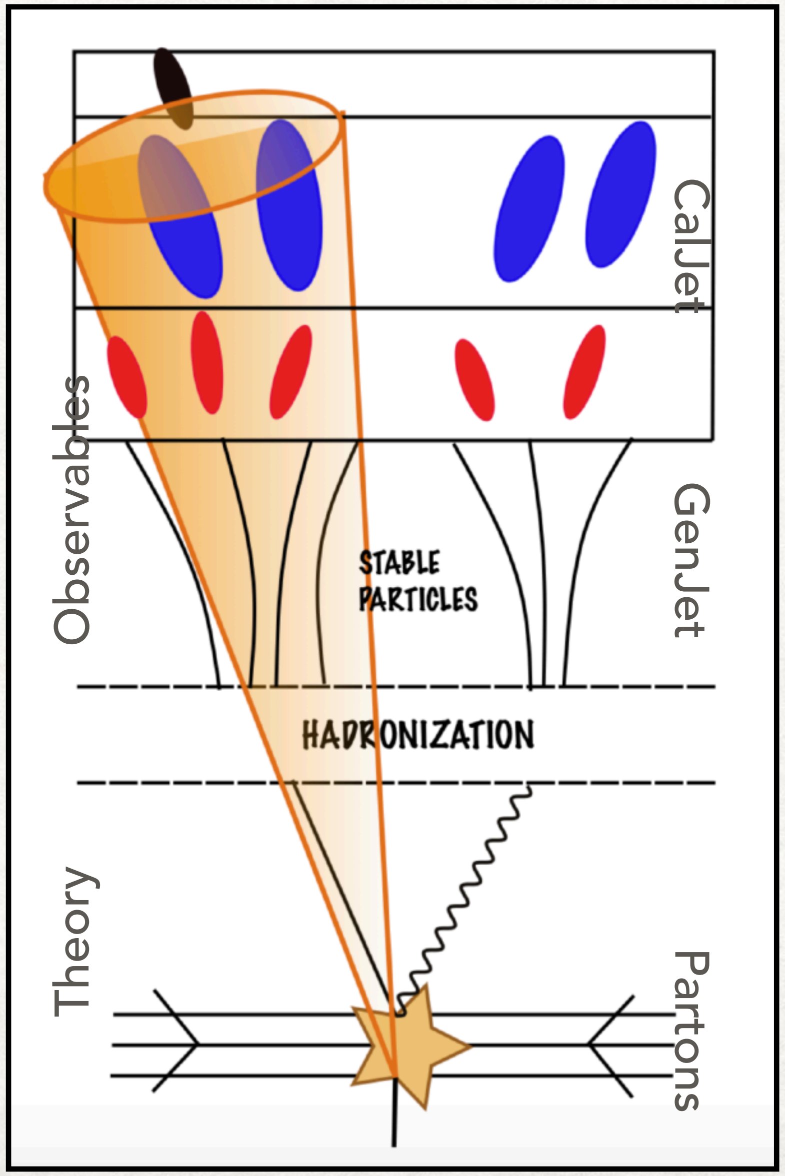
BUILDING A JET

RADIUS AND JET ALGORITHMS

Standard kt-type algorithms

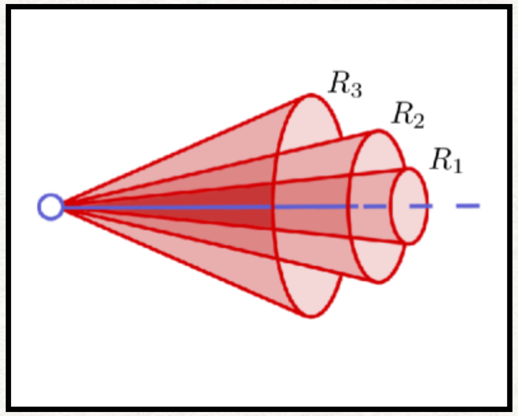
$$d_{ij} = \min\left(k_{T,1}^{2w}, k_{T,2}^{2w}\right)\frac{\Delta R_{ij}}{R}$$

$$w \in \{-1, 0, 1\}$$

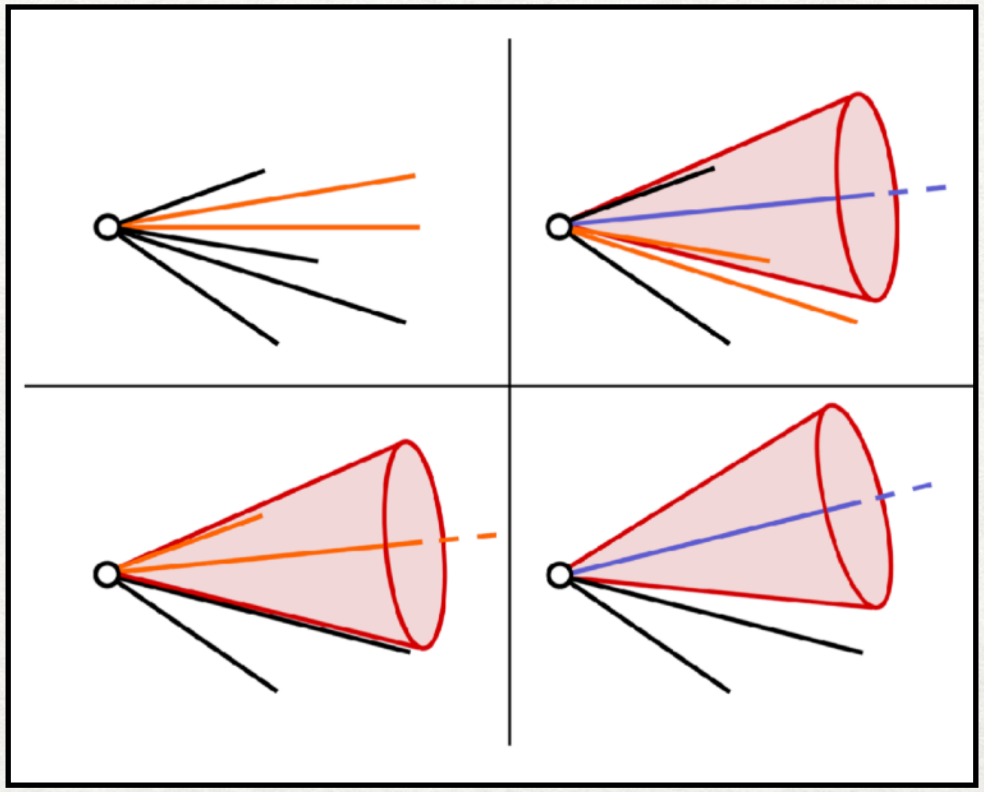


Building a jet…

Step 1:
Set a size
(Radius)



Step 2:
Run a jet
algorithm



JET AXIS

Larkoski, Neill, Thaler `14
arXiv: 1401.2158

Standard jet axis (SJA)

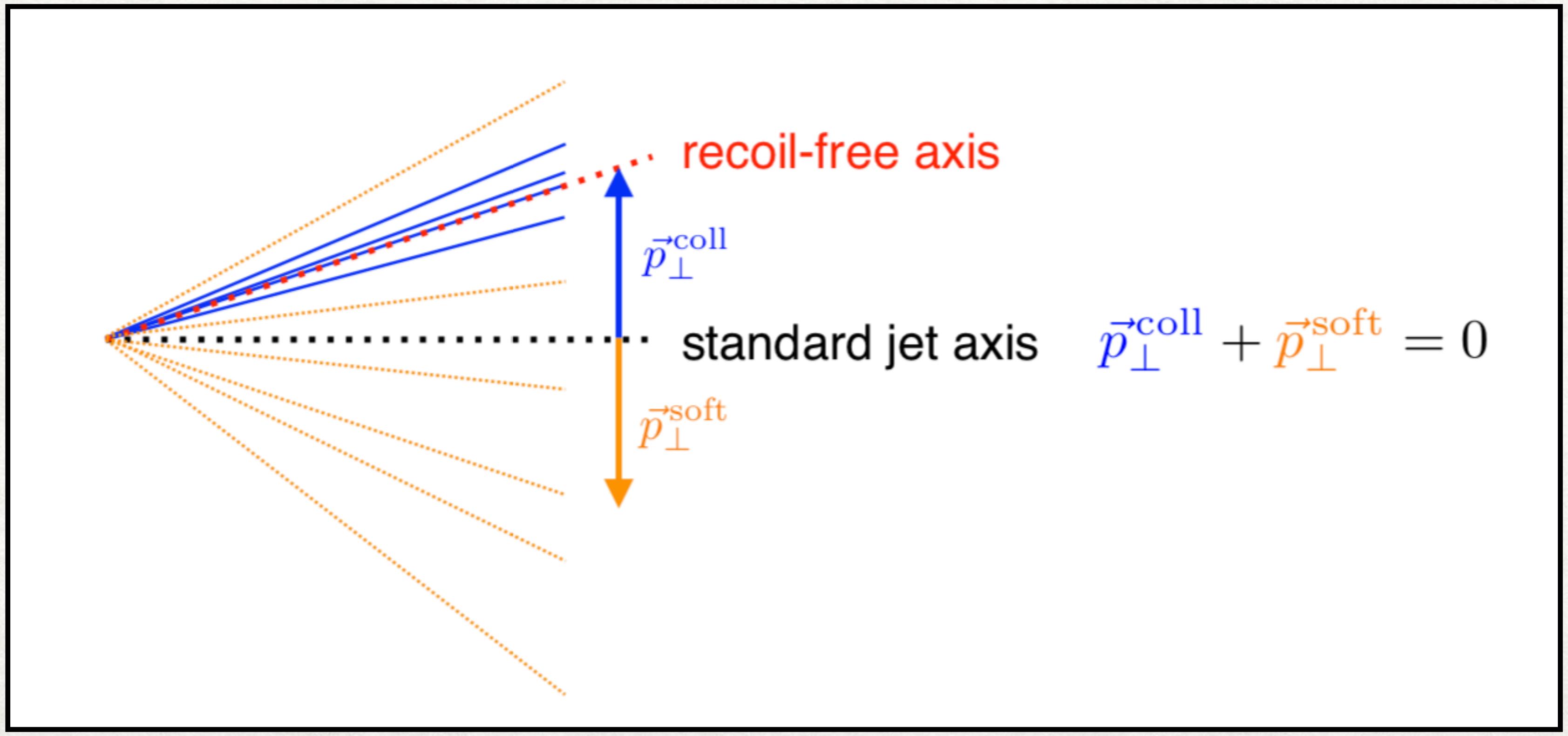
The sum of the momentum of collinear and soft particles is zero

Introduces soft-sensitivity to the axis definition. Important with unintegrated transverse momentum

Winner-take-all (WTA)

It always follows the direction of the most energetic particle

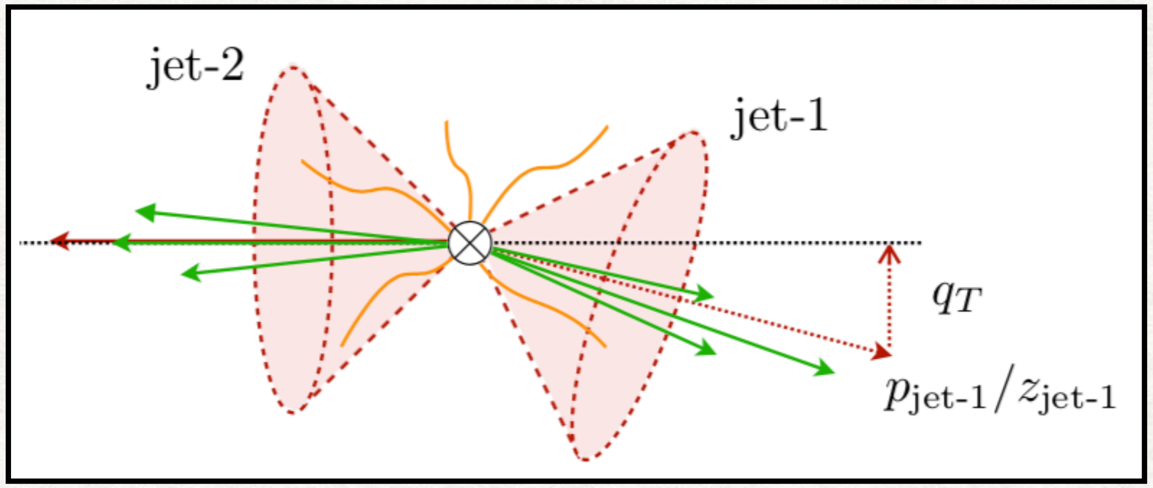
Recoil invariant. It is not sensitive to soft radiation



TMD FACTORIZATION

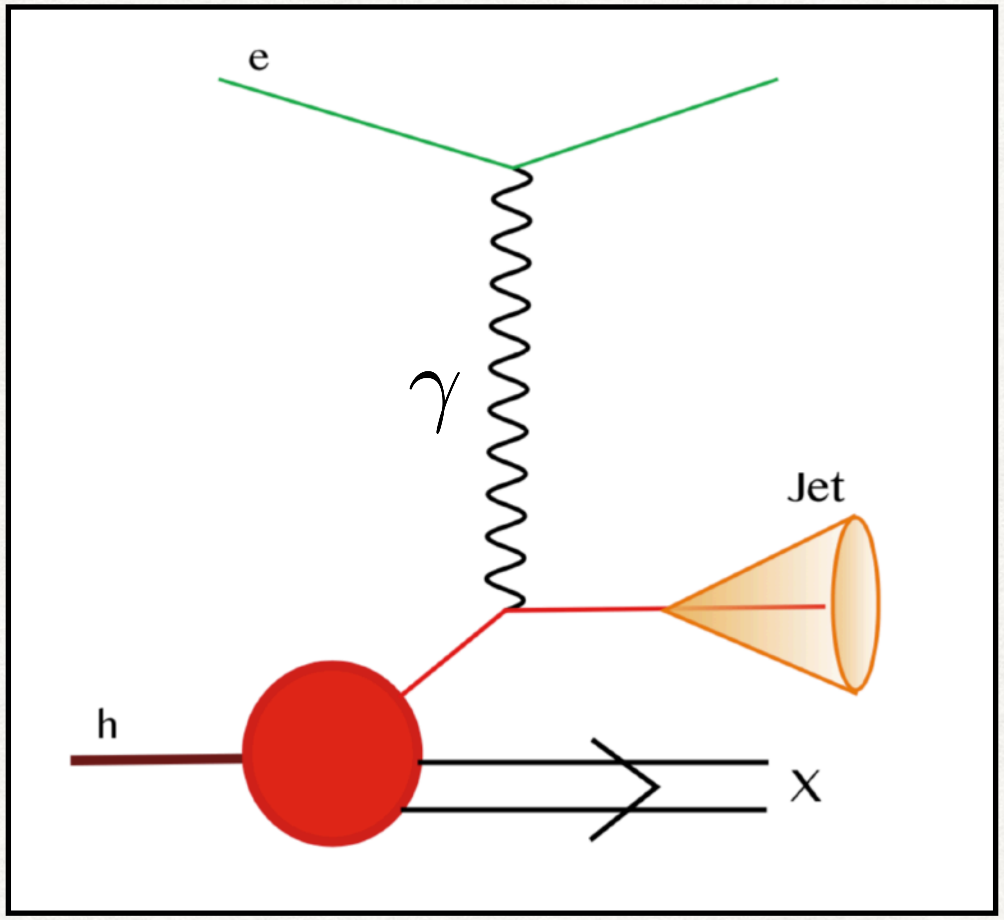
INTERESTING PROCESSES TO FACTORIZE

$e^+e^- \to \text{dijet} + X$



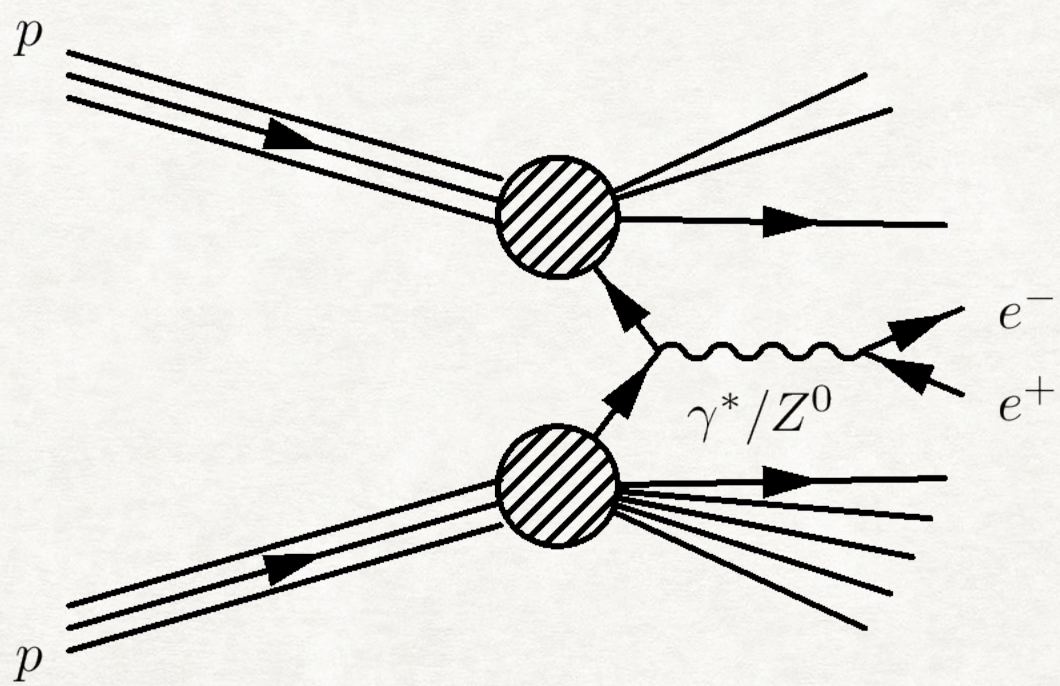
$$d\sigma \sim H \int d\mathbf{b} J_1 J_2 \quad ?$$

SIDIS with jet



$$d\sigma \sim H \int d\mathbf{b} F J_1 \quad ?$$

TMD FACTORIZATION IN A NUTSHELL



Cross-section written as a product of two TMDs

Similar formulas are valid for SIDIS (EIC) and e+e-

TMDs have a double-scale evolution, associated to a particular kind of divergences: rapidity divergences.

We have new nonperturbative effects which cannot be included in PDFs.

See M. Echevarria talk on Wednesday

$$\frac{d\sigma}{dQ^2dyd(q_T^2)} = H(Q^2,\mu)\int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{bq})} \boxed{F^{\mathrm{BARE}}_{f\leftarrow h_1}(x_1,\boldsymbol{b};\mu,\delta^+)F^{\mathrm{BARE}}_{f\leftarrow h_2}(x_2,\boldsymbol{b};\mu,\delta^-)S(\boldsymbol{b},\mu,\delta^+\delta^-)}$$

Hard modes

Collinear modes + zero bin (overlap c/s)

ill defined! Rapidity divergences

Soft modes

Collins, Soper, Sterman `85
Nucl.Phys. B250, 199 (1985)

Collins `11
Foundations of perturbative QCD

Echevarría, Idilbi, Scimemi `11
arXiv:1111.4996

FACTORIZATION: UNGROOMED/GROOMED JETS PHENOMENOLOGY

UNGROOMED/GROOMED JETS

UNGROOMED JETS

Different definitions

Dependence on the radius

Standard: small radii

Large jet-radii: new def.

Even if NP effect are reduced from hadronic TMDs

Final state NP hadronization effects

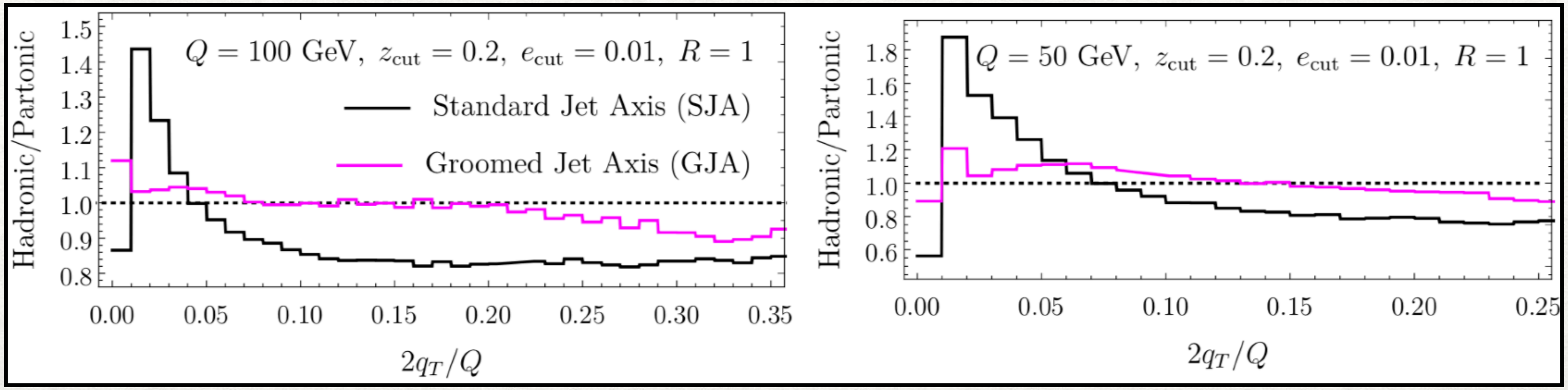
GROOMED JETS

SOFT-DROP → Removes contaminating soft radiation

1. Constructing an angular tree of the jet
2. Removing branches that fail an energy requirement

Only narrow energetic core remains from the original jet

NP hadronization effects are highly reduced!



FACTORIZATION: UNGROOMED JETS

Dijet decorrelation

$e^+e^- \to \text{dijet} + X$

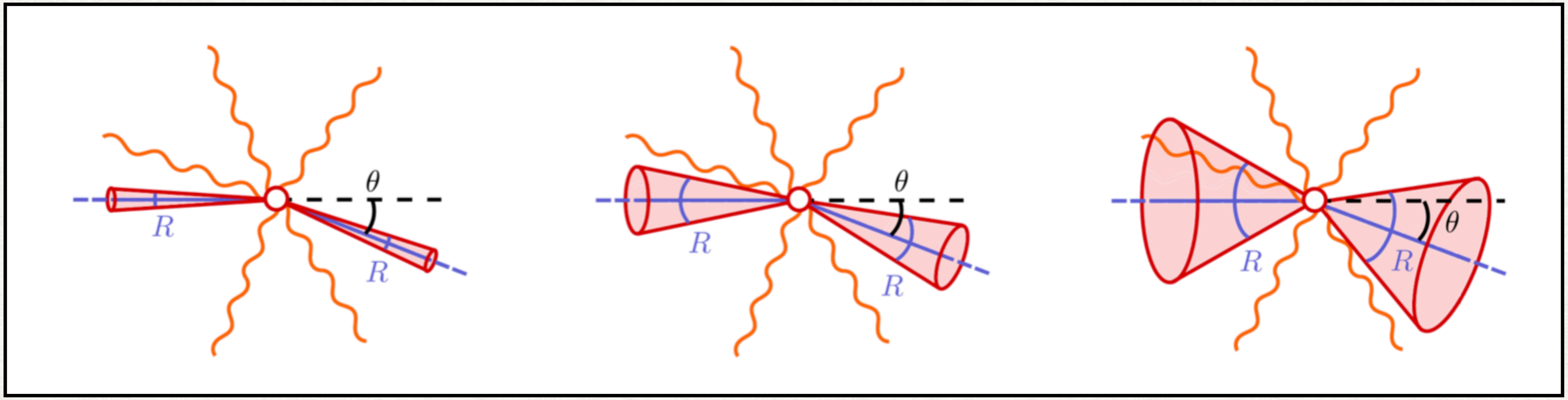
$$\boldsymbol{q} = \frac{\boldsymbol{p}_1}{z_1} + \frac{\boldsymbol{p}_2}{z_2}$$

$$\theta \approx \tan\theta = 2|\boldsymbol{q}|/Q$$

competes with R

In all cases

$\theta \ll 1$



$\theta \gg R$ $\theta \sim R$ $\theta \ll R$

FACTORIZATION: UNGROOMED JETS

Dijet decorrelation

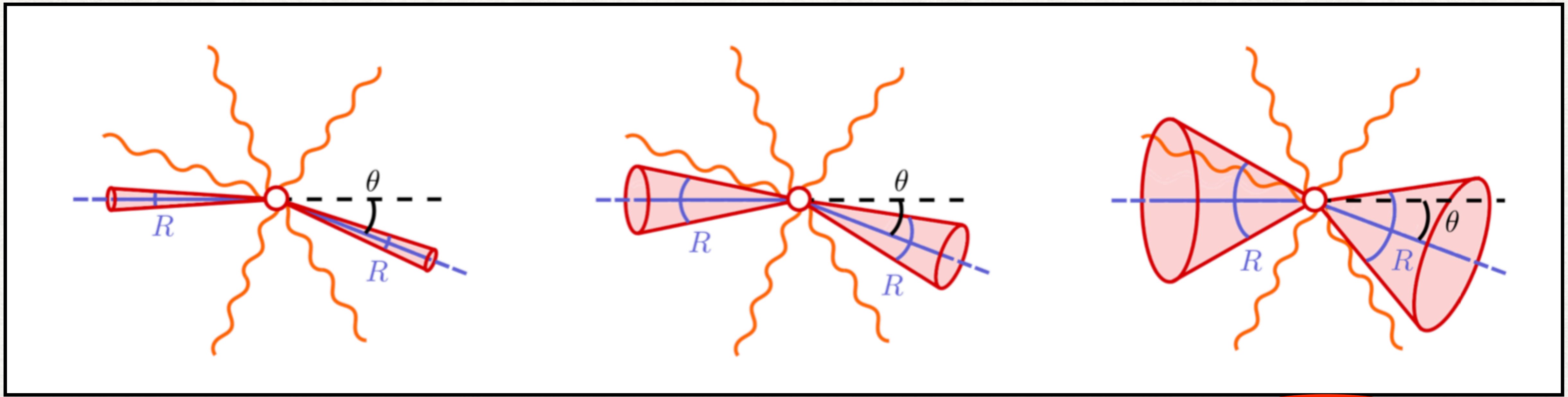
$$e^+e^- \to \text{dijet} + X$$

$$\boldsymbol{q} = \frac{\boldsymbol{p}_1}{z_1} + \frac{\boldsymbol{p}_2}{z_2}$$ competes with R

$$\theta \approx \tan\theta = 2|\boldsymbol{q}|/Q$$

In all cases

$$\theta \ll 1$$

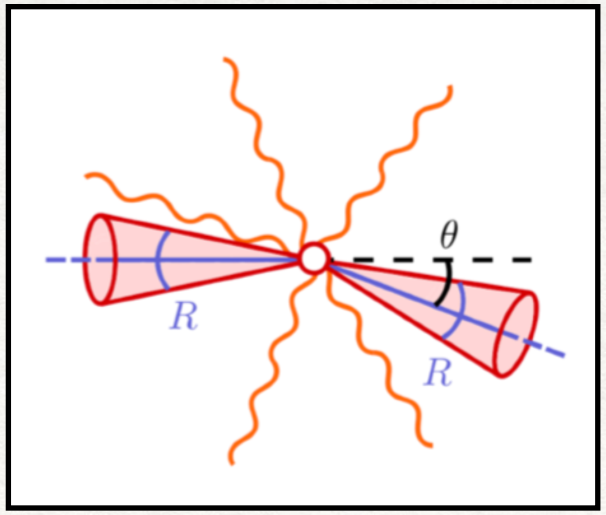


$\theta \gg R$ | $\theta \sim R$ | $\theta \ll R$

Most interesting case for current and future experiments!

FACTORIZATION: UNGROOMED JETS

$$\theta \sim R \ll 1$$



$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H(s,\mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q^{\text{axis}}\left(z_1, \boldsymbol{b}, \frac{\sqrt{s}}{2}R, \mu, \zeta_1\right) J_q^{\text{axis}}\left(z_2, \boldsymbol{b}, \frac{\sqrt{s}}{2}R, \mu, \zeta_2\right)\left[1 + \mathcal{O}\left(\frac{\boldsymbol{q}^2}{s}\right)\right]$$

$$J_q^{\text{axis}} = \frac{z}{2N_c}\sum_X \text{Tr}\left\{\frac{\bar{\not n}}{2}\langle 0|\delta\left(\bar{n}\cdot p_J/z - \bar{n}\cdot P\right) e^{-i\boldsymbol{b}\boldsymbol{P}_\perp}\chi_n(0)|J^{\text{axis}}_{\text{alg},R}X\rangle\langle J^{\text{axis}}_{\text{alg},R}X\rangle|\bar{\chi}_n(0)|0\rangle\right\}$$

TMD semi-inclusive Jet function

Refactorized!

$$R \ll \theta \ll 1$$

$$J_i^{\text{axis}} = \sum_j \int \frac{dz'}{z'}\left[(z')^2 \mathbb{C}(z', \boldsymbol{b}, \mu, \zeta)\right] \mathcal{J}_i\left(\frac{z}{z'}, \frac{\sqrt{s}}{2}R, \mu\right)\left[1 + \mathcal{O}\left(b^2 s^2 R^2/4\right)\right]$$

TMDFFs Coefficients $\otimes$ Coll. jet Function

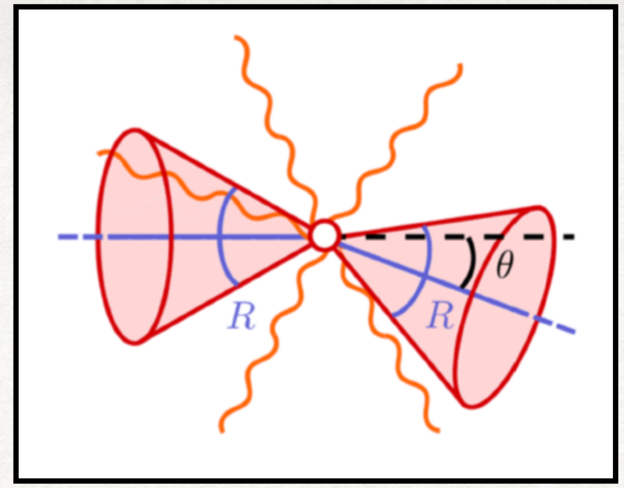
Echevarría, Scimemi, Vladimirov `16

Kang, Ringer, Vitev `16
arXiv:1606.06732

The wide angle soft radiation does not resolve individual collinear emissions in the jet → **The soft function is the same That for TMD fragmentation**

FACTORIZATION: UNGROOMED JETS

$$\theta \ll 1 \quad \theta \ll R$$



Interesting case for Belle, BaBar. At low energies jets with big radius appear

Strong dependence on the axis election!

SJA

The SJA is aligned with the total momentum of the jet

↓

Hard splittings with typical angle R are allowed inside the jet, generating additional soft radiation

↓

Factorization broken!

$$\boxed{\frac{d\sigma^{\mathrm{SJA}}_{(ee\to JJX)}}{d\boldsymbol{q}} = \sum_{m=2}^{\infty} \mathrm{Tr}_c\left[\mathcal{H}_m(\{n_i\}) \otimes \mathcal{S}_m(\boldsymbol{q}, \{n_i\})\right]}$$

WTA

The soft radiation does resolve the jet boundary, but this fact does not affect to the position of the axis

↓

No distinction between soft radiation inside and outside the jet!

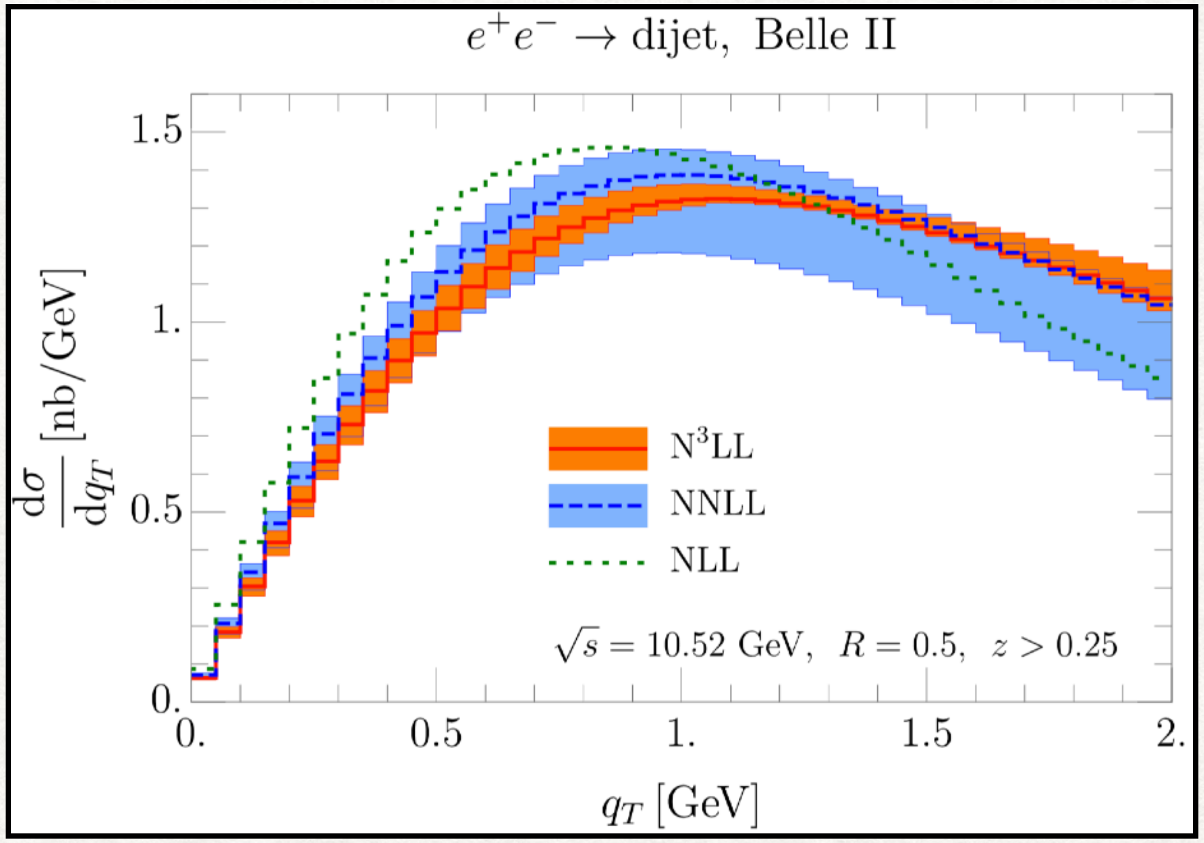
↓

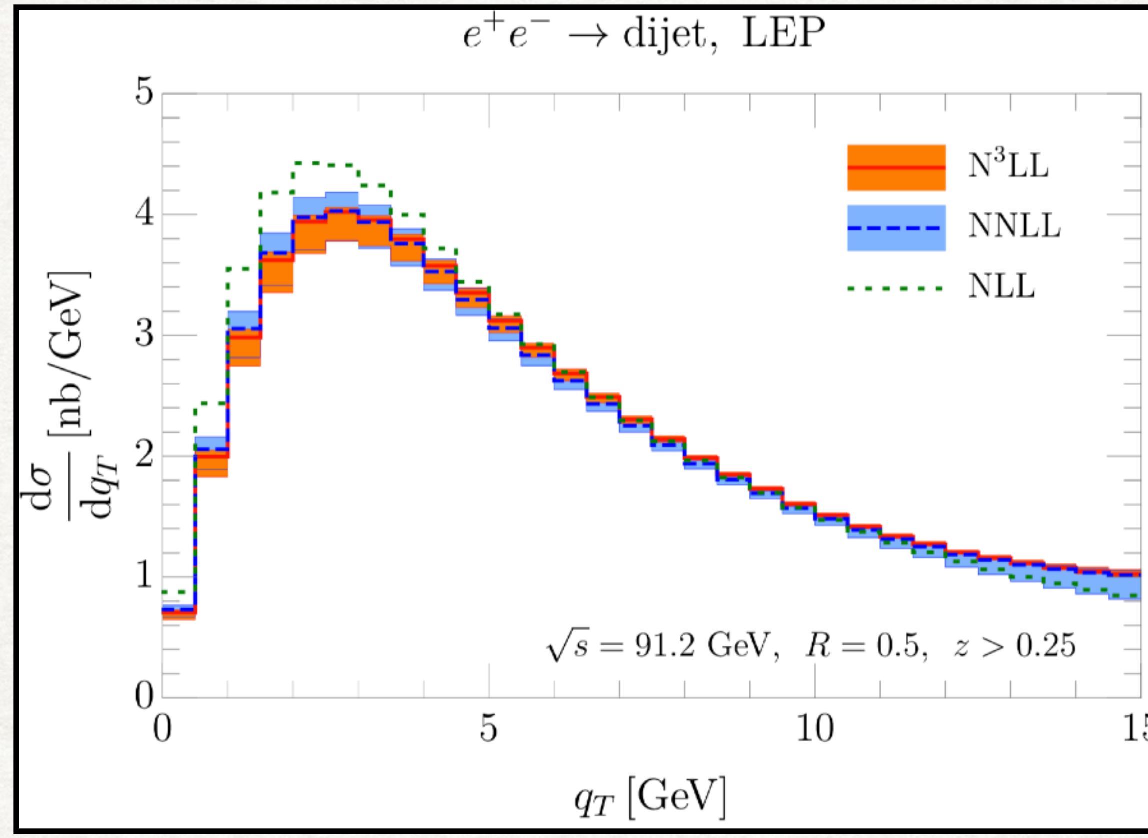
TMD Soft function is conserved!
And factorization holds!

$$\boxed{\frac{d\sigma^{\mathrm{WTA}}_{(ee\to JJX)}}{dz_1 dz_2 d\boldsymbol{q}} = H \int \frac{d\boldsymbol{b}}{(2\pi)^2} J_q^{WTA}(z_1, \boldsymbol{b}) J_q^{WTA}(z_2, \boldsymbol{b})}$$

Jet functions should be written in this limit…

PERTURBATIVE CONVERGENCE: LARGE RADIUS





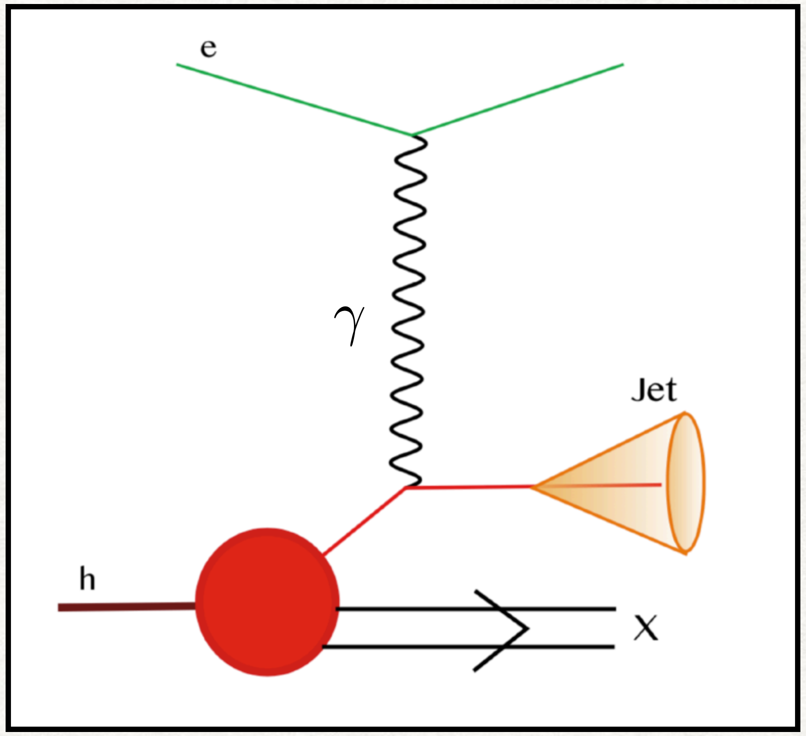
As the large-R jet function does not depend on radius or z, we can predict the two loop jet function by RG + Numerical constant (Event2)

We use an improved NLO + NNLO large-R jet functions
Resummation up to N3LL!

Theoretical errors are reduced when the perturbative order is increased!

FACTORIZATION: UNGROOMED JETS SIDIS

Factorization in SIDIS with a jet is analogous to the one already studied!

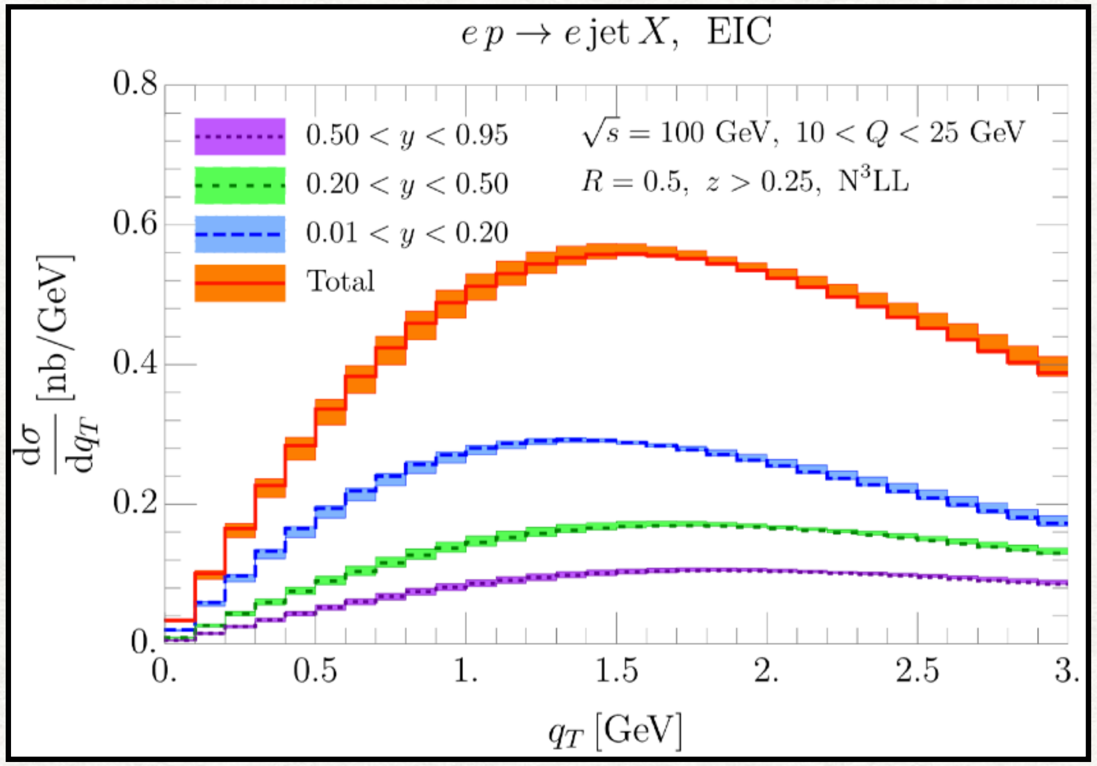


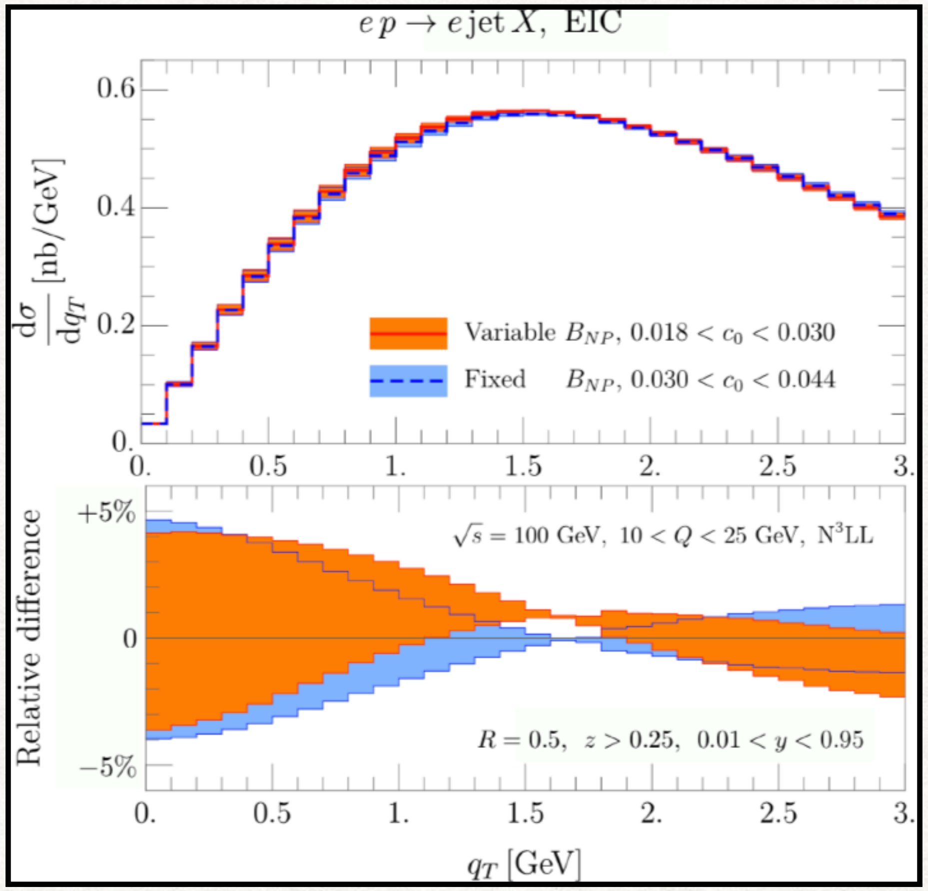
Factorization for SIDIS with jet in Breit frame

$$\frac{d\sigma_{eN\to eJX}}{dQ^2 dx dz d\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} \underbrace{f_{a/N}(x,\boldsymbol{b},\mu,\zeta)}_{\text{TMDPDF}} \underbrace{J_q^{\text{axis}}(z,\boldsymbol{b},QR/2,\mu,\zeta)}_{\text{TMD jet function}}$$

It has dependence on the jet radius. For all R we use WTA axis

PHENO RESULTS FOR SIDIS WITH UNGROOMED JET





We include an effect of the two-loop jet function

Most of the cross section comes from low elasticity region

Theoretical errors from the Hard scale and the OPE scale are shown and are small

Non perturbative model for the TMDPDF taken from Bertone, Scimemi, Vladimirov `19 arXiv:1902.08474

$$f_{NP}(x,\boldsymbol{b}) = \exp\left(-\frac{(\lambda_1(1-x)+\lambda_2 x+\lambda_3 x(1-x))\boldsymbol{b}^2}{\sqrt{1+\lambda_4 x^{\lambda_5}\boldsymbol{b}^2}}\right)$$

But the cross section is dominated by nonperturbative parameter of evolution

$$\mathcal{D}(\mu,\boldsymbol{b}) = \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma + \mathcal{D}_{\text{pert}}(\mu_0,\boldsymbol{b}) + c_0\boldsymbol{b}\boldsymbol{b}^*$$

FACTORIZATION: GROOMED JETS

Two extra scales for groomed jets

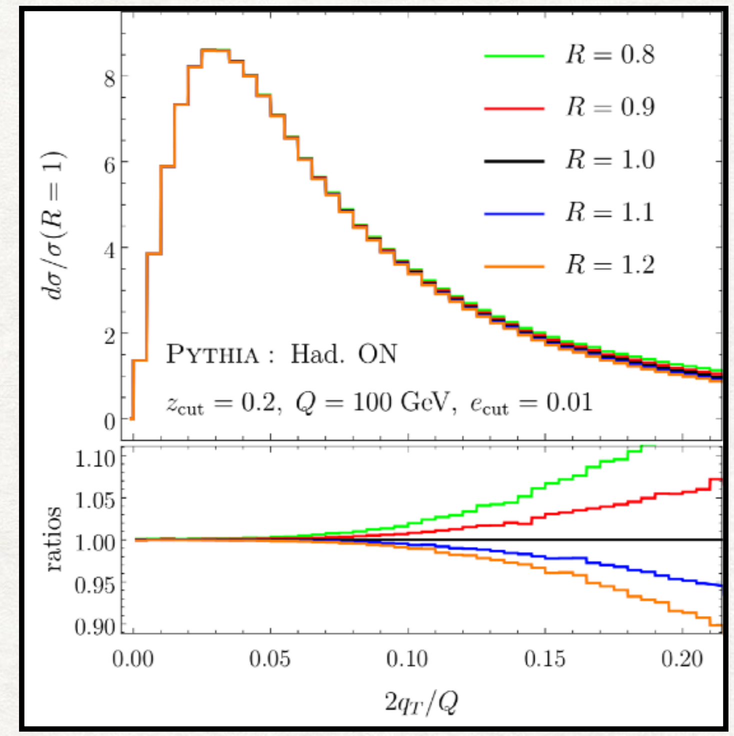
Grooming parameter → $\frac{\min\{\mathrm{E_i}, \mathrm{E_j}\}}{E_i + E_j} > z_{\text{cut}} \left(\frac{\theta_{ij}}{R}\right)^\beta$

Jet mass → $e \equiv \left(\frac{m_J}{2E_J}\right)^2$

Hierarchy

$$Q \gg Q z_{\text{cut}} \gg q_T \geq Q\sqrt{e} \gg Q\sqrt{e z_{\text{cut}}}$$

Remove contaminating Soft radiation → Independence on the radius → Pythia check



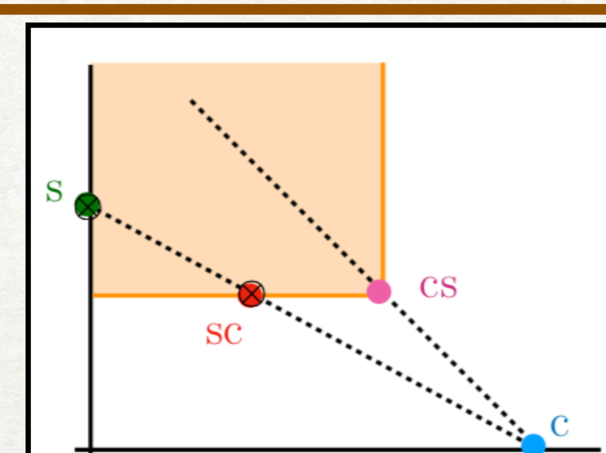
New modes in factorization!

Soft: $p_s^\mu \sim q_T(1,1,1)$

Collinear: $p_c^\mu \sim Q(\lambda_c^2, 1, \lambda_c),\ \lambda_c = \sqrt{e}$

Soft-collinear: $p_{sc}^\mu \sim Q z_{\text{cut}}(\lambda_{sc}^2, 1, \lambda_{sc}),\ \lambda_{sc} = q_T/(Q z_{cut})$

Collinear-soft: $p_{cs}^\mu \sim Q z_{\text{cut}}(\lambda_{cs}^2, 1, \lambda_{cs}),\ \lambda_{cs} = \sqrt{e/z_{cut}}$



FACTORIZATION: GROOMED JETS

The cross-section is factorized as

$$\frac{d\sigma}{de_1 de_2 d^2\boldsymbol{q}_T} = H^{ij}(Q;\mu) \times \mathcal{J}_i^{\perp}(e_1, Q, z_{\text{cut}}, \boldsymbol{q}_T; \mu, \zeta_A) \otimes \mathcal{J}_j^{\perp}(e_2, Q, z_{\text{cut}}, \boldsymbol{q}_T; \mu, \zeta_B)$$

Groomed jet function

$$\mathcal{J}_i^{\perp}(e, Q, z_{\text{cut}}, \boldsymbol{b}; \mu, \zeta) = \sqrt{S(\boldsymbol{b})}\, \mathcal{J}_i^{\perp}(e, Q, z_{\text{cut}}, \boldsymbol{b})$$

$$\zeta_A \zeta_B = Q^4 z_{\text{cut}}^4$$

Energy needed is higher!

Soft radiation absorbed
same as TMDs

New modes → Refactorization!

$$\mathcal{J}_i^{\perp}(e, Q, z_{\text{cut}}, \boldsymbol{b}) = \underbrace{S_{sc,i}^{\perp}(Qz_{\text{cut}}, \boldsymbol{b})}_{\text{Soft-collinear}} \times \int de' \underbrace{S_{cs,i}(e - e', Qz_{\text{cut}})}_{\text{Collinear-soft}} \underbrace{J_i(e', Q)}_{\text{Jet}}$$

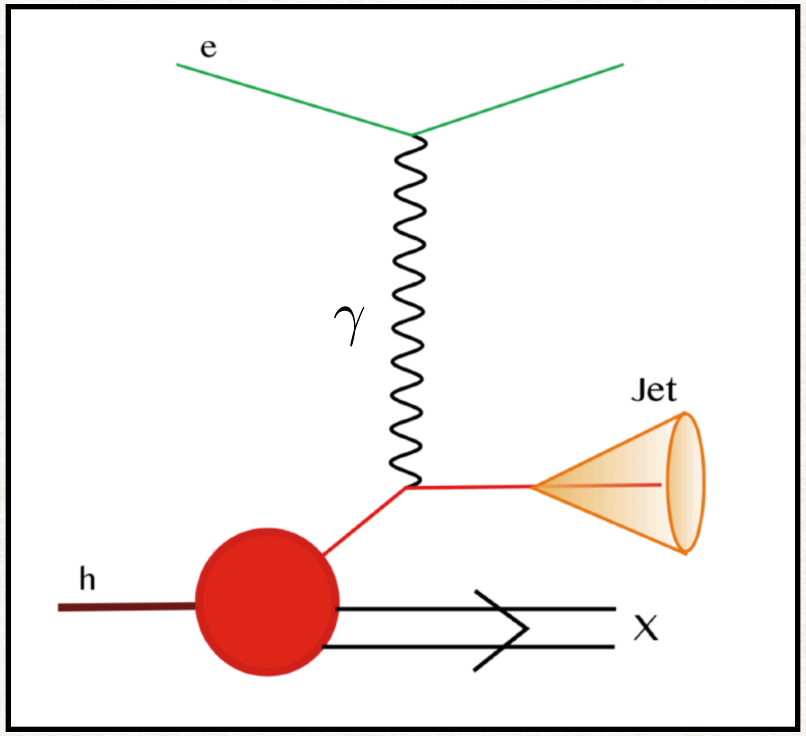
All at NLO!

Integrate parameters
as in experiments

$$\frac{d\sigma}{d^2\boldsymbol{q}_T}(e_{\text{cut}}) = H^{ij}(Q;\mu) \int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\boldsymbol{q})_T} \mathcal{J}_i^{\perp}(e_{\text{cut}}, Q, z_{\text{cut}}, \boldsymbol{b}; \mu, \zeta) \mathcal{J}_j^{\perp}(e_{\text{cut}}, Q, z_{\text{cut}}, \boldsymbol{b}; \mu, \zeta)$$

FACTORIZATION: GROOMED JETS SIDIS

Factorization in SIDIS with a groomed jet introduces now the groomed jet function



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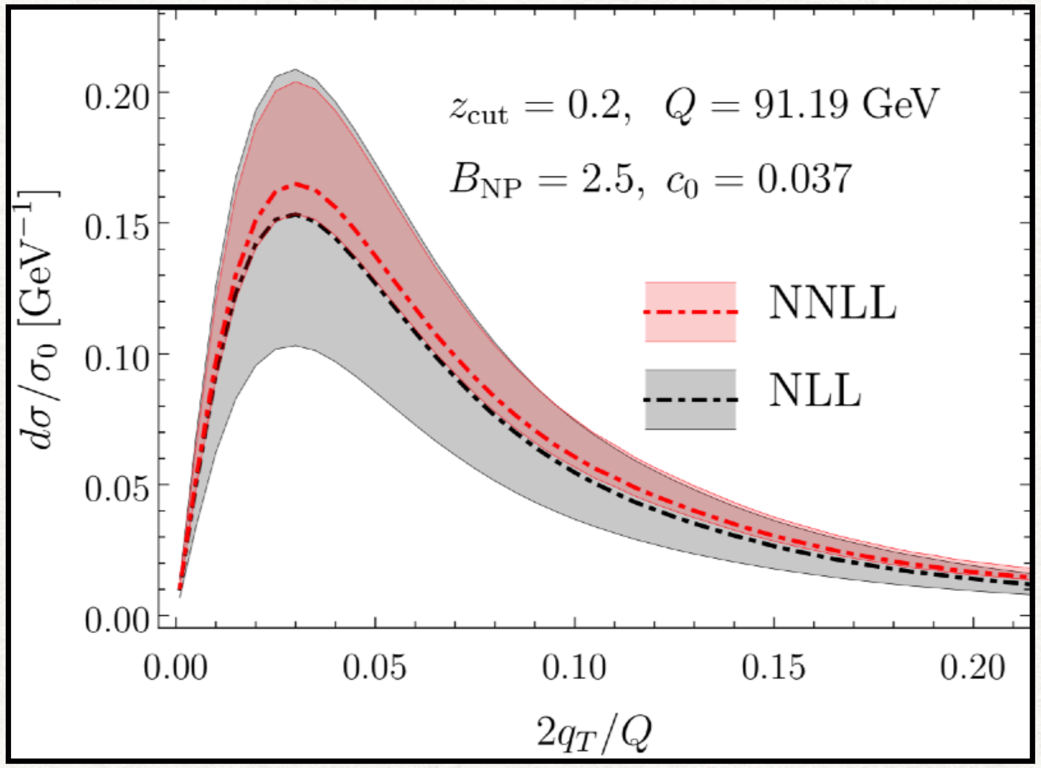
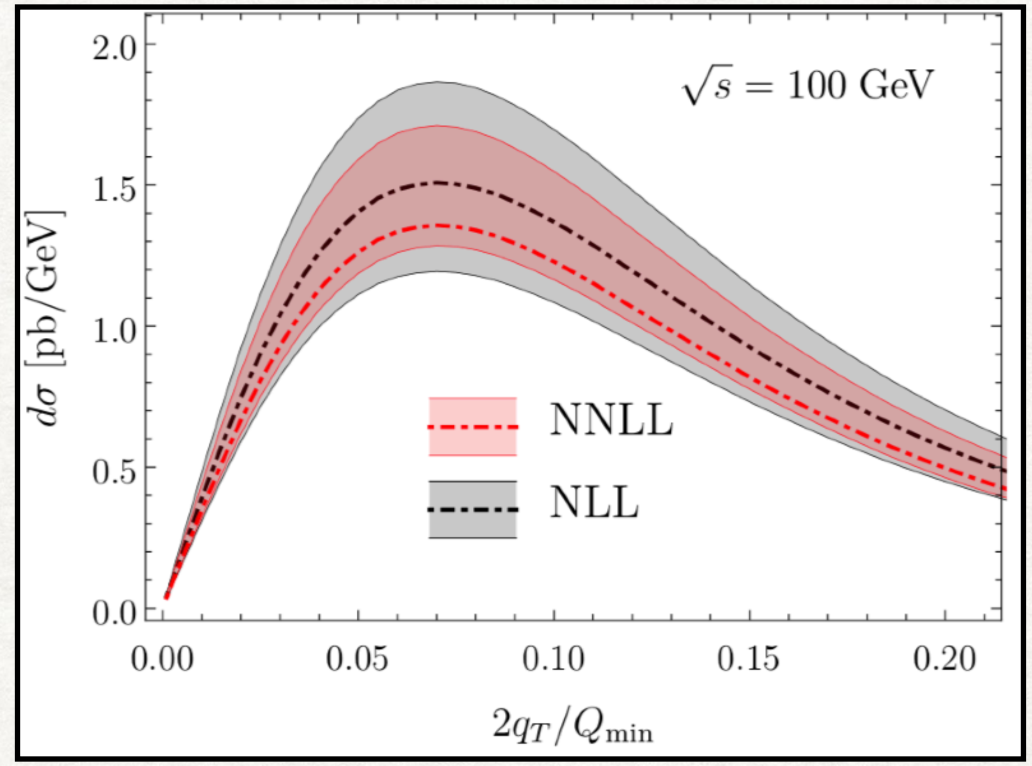
PHENO: GROOMED JETS. PERTURBATIVE CONVERGENCE

$e^+e^- \to \text{dijet} + X$



SIDIS with jet

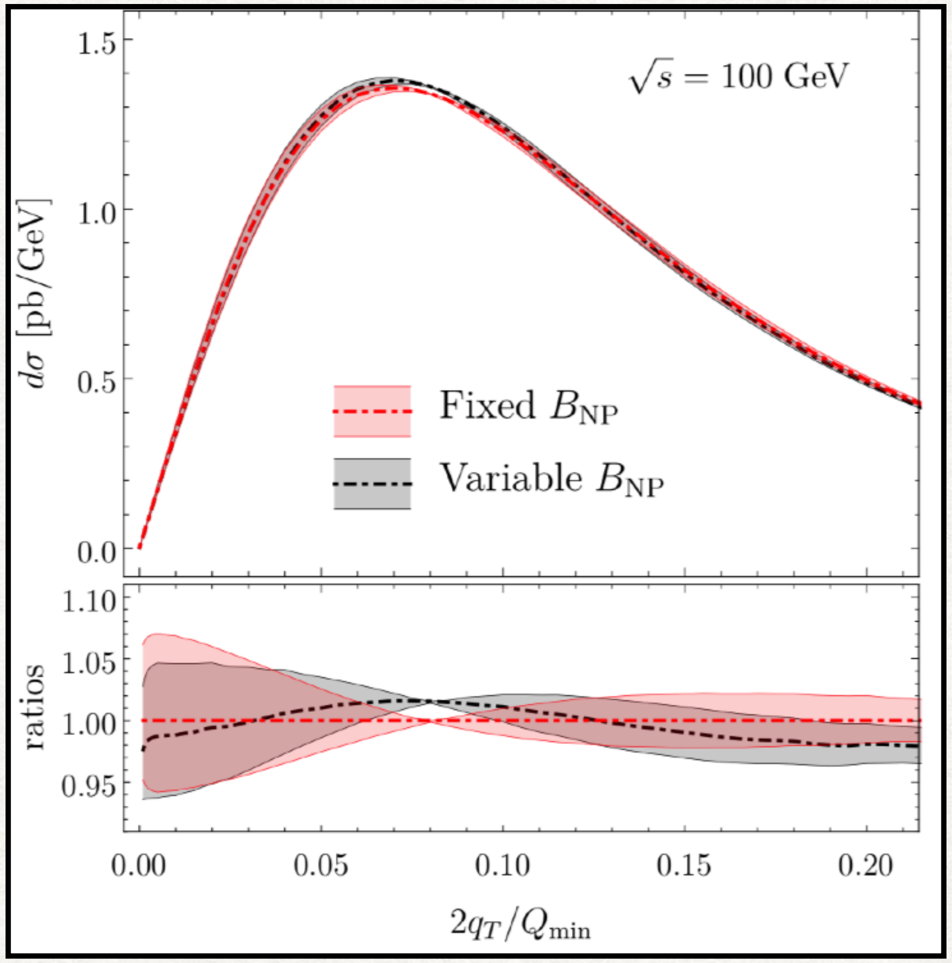


Scale variations are reduced when we go to NNLL calculation

But they are somewhat larger for a NNLL calculation → $\mu_{cs} \sim Q\sqrt{e_{\text{cut}} z_{\text{cut}}}$ approaches to NP-regime → Profiles

Increase Pert. order

PHENO: GROOMED JETS. SIZE OF NP EFFECTS



Non perturbative model for the TMDPDF taken from
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$$f_{NP}(x,\boldsymbol{b}) = \exp\left(-\frac{(\lambda_1(1-x)+\lambda_2 x+\lambda_3 x(1-x))\boldsymbol{b}^2}{\sqrt{1+\lambda_4 x^{\lambda_5}\boldsymbol{b}^2}}\right)$$

But the cross section is dominated by nonperturbative parameter of evolution

$$\mathcal{D}(\mu,\boldsymbol{b}) = \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\Gamma + \mathcal{D}_{\rm pert}(\mu_0,\boldsymbol{b}) + c_0 \boldsymbol{b}\boldsymbol{b}^*$$

NP effects are smaller for groomed-jets than for ungroomed-jets!

CONCLUSIONS

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Our aim is to extract information about NP contributions to hadronic TMDs

↓

We use jets, in order to reduce NP effects in final state with hadrons (TMDFFs)

TWO APPROACHES

UNGROOMED JETS (N3LL)	GROOMED JETS (NNLL)
Factorization depends on the radius of the jet	Factorization does not depend on the radius of the jet
WTA axis election the Soft function is the same that for hadronic TMD ↓ We find a way to study big radius jets	New modes
	Contaminating soft radiation is out!
	Due to grooming parameter energy needed is higher

Hadronic TMDs and TMD jet functions share the same double-scale RG evolution

arTeMiDe → Pheno

https://teorica.fis.ucm.es/artemide/

↓

Needs future comparison with data!

CONCLUSIONS

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TWO APPROACHES

Thanks!!!

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↓

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BACKUP SLIDES

DOUBLE-SCALE RENORMALIZATION GROUP EVOLUTION

Same RG evolution for hadronic TMDs and for TMD jet functions!

TMDs

$$\mu^2 \frac{d}{d\mu^2} D_i(z, \boldsymbol{b}, \mu, \zeta) = \frac{1}{2} \gamma_F^i(\mu, \zeta) D_i(z, \boldsymbol{b}, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} D_i(z, \boldsymbol{b}, \mu, \zeta) = -\mathcal{D}^i(\mu, \boldsymbol{b}) D_i(z, \boldsymbol{b}, \mu, \zeta)$$

Jets

$$\mu^2 \frac{d}{d\mu^2} J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta) = \frac{1}{2} \gamma_F^i(\mu, \zeta) J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta) = -\mathcal{D}^i(\mu, \boldsymbol{b}) J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta)$$

DOUBLE-SCALE RENORMALIZATION GROUP EVOLUTION

They have a common evolution factor

TMDs

$$D_i(z,\boldsymbol{b},\mu_f,\zeta_f) = \exp\left[\int_{(\mu_i,\zeta_i)}^{(\mu_f,\zeta_f)} \left(\gamma_F^i(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^i(\mu,\boldsymbol{b})\frac{d\zeta}{\zeta}\right)\right] D_i(z,\boldsymbol{b},\mu_i,\zeta_i)$$

Jets

$$J_i^{\text{axis}}(z,\boldsymbol{b},QR,\mu_f,\zeta_f) = \exp\left[\int_{(\mu_i,\zeta_i)}^{(\mu_f,\zeta_f)} \left(\gamma_F^i(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^i(\mu,\boldsymbol{b})\frac{d\zeta}{\zeta}\right)\right] J_i^{\text{axis}}(z,\boldsymbol{b},QR,\mu_i,\zeta_i)$$

This fact makes phenomenological analysis simpler!

Scimemi, Vladimirov `17

https://teorica.fis.ucm.es/artemide/

TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

$$\theta \ll 1$$
$$\theta \ll R$$

Collinear radiation of typical angle **θ** sees the jet boundary infinitely far away

Independence of the radius of the jet!

The collinear radiation is mostly inside the jet

The dependence on z is power suppressed!

$$J_i^{\rm WTA}(z, \boldsymbol{b}, ER, \mu, \zeta) = \delta(1-z)\,\mathscr{J}_i^{\rm WTA}(\boldsymbol{b}, \mu, \zeta)\left[1 + \mathcal{O}\left(\frac{1}{\boldsymbol{b}^2 E^2 R^2}\right)\right]$$

where

$$\mathscr{J}_i^{\rm WTA}(\boldsymbol{b}, \mu, \zeta) = \frac{1}{2N_c(\bar{n}\cdot p_J)}\mathrm{Tr}\left\{\frac{\bar{\not{n}}}{2}\langle 0|e^{-i\boldsymbol{b}\boldsymbol{P}_\perp}\chi_n(0)|J_{\rm alg}^{\rm WTA}\rangle\langle J_{\rm alg}^{\rm WTA}|\bar{\chi}_n(0)|0\rangle\right\}$$

$$\mathscr{J}_i^{[0]\rm WTA}(\boldsymbol{b}, \mu, \zeta) = 1$$

$$\mathscr{J}_i^{[1]\rm WTA}(\boldsymbol{b}, \mu, \zeta) = 2\left\{N_i + L_\mu\left[\mathcal{C}_i' + \mathcal{C}_i\left(\mathbf{l}_\zeta - \frac{1}{2}L_\mu\right)\right]\right\}$$

$$N_q = C_F\left(\frac{7}{2} - \frac{5\pi^2}{12} - 3\ln 2\right)$$

$$N_g = C_A\left(\frac{131}{36} - \frac{5\pi^2}{12}\right) - \frac{17}{18}n_f T_R - \beta_0 \ln 2$$

TMD SEMI-INCLUSIVE JET FUNCTION AT NNLO

We know the evolution of the TMD jet function and in this limit it does not depend on radius or z

↓

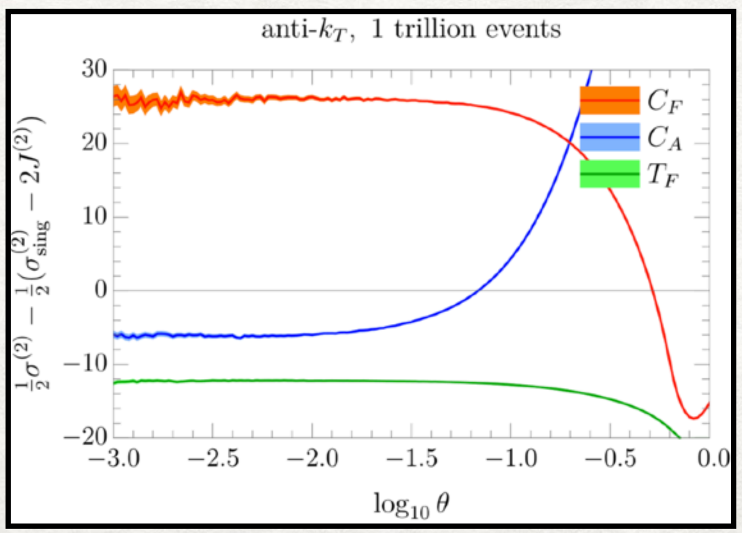
We can predict the log behavior of the two-loop jet function solving renormalization group equations and the constant can be numerically calculated (EVENT2)

$$\theta \ll 1$$
$$\theta \ll R$$

Predicted by RG equations

Predicted by EVENT2

$$\mathscr{J}^{[2]\mathrm{WTA}}(\boldsymbol{b}, \mu, \zeta) = \sum_{k=1}^{4} \sum_{l=0}^{k} C_{kl} L_\mu^k \mathbf{l}_\zeta^l + C_0$$



$$C_0 = j_{C_F} + j_{C_A} + n_f j_{T_F}$$

TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

The Soft function is the same that for TMDs in some cases for SJA and in ALL cases for WTA

The Semi-inclusive jet function is renormalized as a TMD

$$\boxed{J_q^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta) = Z_{UV}(\mu, \epsilon) R_q(\delta, \zeta, \epsilon) J_q^{\text{axis},B}(z, \boldsymbol{b}, QR, \mu, \delta)}$$

$$\theta \ll 1$$
$$\theta \ll R$$

We have a new definition of the operator only valid for WTA axis!

Collinear radiation of typical angle **θ** sees the jet boundary infinitely far away

Independence of the radius of the jet!

The collinear radiation is mostly inside the jet

The dependence on z is power suppressed!

$$J_i^{\text{WTA}}(z, \boldsymbol{b}, QR, \mu, \zeta) = \delta(1-z)\, \mathscr{J}_i^{\text{WTA}}(\boldsymbol{b}, \mu, \zeta) \left[1 + \mathcal{O}\left(\frac{1}{\boldsymbol{b}^2 Q^2 R^2}\right)\right]$$

where

$$\mathscr{J}_i^{\text{WTA}}(\boldsymbol{b}, \mu, \zeta) = \frac{1}{2N_c(\bar{n}\cdot p_J)} \text{Tr}\left\{ \frac{\bar{\not{n}}}{2} \langle 0| e^{-i\boldsymbol{b}\boldsymbol{P}_\perp} \chi_n(0) | J_{\text{alg}}^{\text{WTA}} \rangle \langle J_{\text{alg}}^{\text{WTA}} | \bar{\chi}_n(0) |0\rangle \right\}$$

TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

The Soft function is the same that for TMDs in some cases for SJA and in ALL cases for WTA

The Semi-inclusive jet function is renormalized as a TMD

$$\boxed{J_q^{\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) = Z_{UV}(\mu,\epsilon) R_q(\delta,\zeta,\epsilon) J_q^{\text{axis},B}(z,\boldsymbol{b},QR,\mu,\delta)}$$

$$\theta \ll 1$$
$$\theta \sim R$$

The definition of the operator is the usual one

$$J_i^{\text{axis}} = \sum_{n=0}^{\infty} a_s^n J_i^{[n]\text{axis}}$$

$$J_i^{[0]\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) = \delta(1-z)$$

$$J_i^{[1]\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) = 2\left(\sum_j c_{ji} p_{ji}\right)\Big[L_R - L_\mu - 2\ln(1-z) + \frac{1}{4}|\boldsymbol{b}|^2 Q^2 R^2 (1-z)^2$$
$$\times {}_2F_3\left(\{1,1\},\{2,2,2\};-\frac{1}{4}|\boldsymbol{b}|^2 Q^2 R^2(1-z)^2\right)\Big] + \delta(1-z)\left[2\mathcal{C}_i' L_R - \mathcal{C}_i L_\mu^2 + 2\mathcal{C}_i L_\mu \mathbf{l}_\zeta + 2\tilde{d}_i^{\text{axis}}(\boldsymbol{b}QR)\right]$$

The dependence on the axis is only here!

DELTA REGULARIZATION

$$W_n = P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma) e^{-\delta\sigma x}\right)$$

$$S_n = P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma) e^{-\delta\sigma}\right)$$

At diagram level → Eikonal propagators

$$\frac{1}{(k_1^+ + i0)(k_1^+ + k_2^+ + i0)...(k_1^+ + ... + k_n^+ + i0)} \to \frac{1}{(k_1^+ + i\delta)(k_1^+ + k_2^+ + 2i\delta)...(k_1^+ + ... + k_n^+ + ni\delta)}$$

This regularization makes zero-bin equal to soft factor

R-factor is scheme dependent!

$$R = \frac{\sqrt{S(\boldsymbol{b})}}{\text{zero-bin}} \xrightarrow{\delta-\text{reg.}} R_{\delta-\text{reg.}} = \frac{1}{\sqrt{S(\boldsymbol{b})}}$$

Non-abelian exponentiation satisfied at all orders!

δ-regularization violates gauge properties of WL by power suppressed in δ terms

Only calculation at $\delta \to 0$ is legitimate!

THE CROSS SECTION

Ingredients to build cross-section

$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H_{ee\to q\bar{q}}(s,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q\left(z,\boldsymbol{b},\frac{\sqrt{s}R}{2},\mu,\zeta\right) J_q\left(z,\boldsymbol{b},\frac{\sqrt{s}R}{2},\mu,\zeta\right) R^2\left[\boldsymbol{b};(\mu_i,\zeta_i)\to(\mu_f,\zeta_f)\right]$$

Hard factor: Same that for DY. Known and introduced in arTeMiDe up to 2-loops.

Evolution kernel: Same that for TMDs. Known and introduced in arTeMiDe up to 3-loops.

TMD jet functions: Calculated at 1-loop. New arTeMiDe module built.

NUMERICAL RESULTS: LARGE RADIUS

The cross-section is simplified!

The jet functions do not depend on the radius size

The dependence in z is power suppressed (cross-section is less differential)

In the case of big radius factorization is only held for WTA axis!

$$\frac{d\sigma_{ee\to JJX}}{d\boldsymbol{q}} = H(Q^2,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}}\, \mathscr{J}_q^{\mathrm{WTA}}(\boldsymbol{b},\mu,\zeta)\, \mathscr{J}_q^{\mathrm{WTA}}(\boldsymbol{b},\mu,\zeta) R^2[\boldsymbol{b};(\mu_i,\zeta_i)\to(\mu_f,\zeta_f)]$$

NUMERICAL RESULTS

Ingredients to build cross-section

$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H(Q^2,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q^{\text{axis}}(z_1,\boldsymbol{b},QR,\mu,\zeta) J_q^{\text{axis}}(z_2,\boldsymbol{b},QR,\mu,\zeta) R^2[\boldsymbol{b};(\mu_i,\zeta_i)\to(\mu_f,\zeta_f)]$$

Hard factor: Same that for DY. Known and introduced in arTeMiDe up to 2-loops.

Evolution kernel: Same that for TMDs. Known and introduced in arTeMiDe up to 3-loops.

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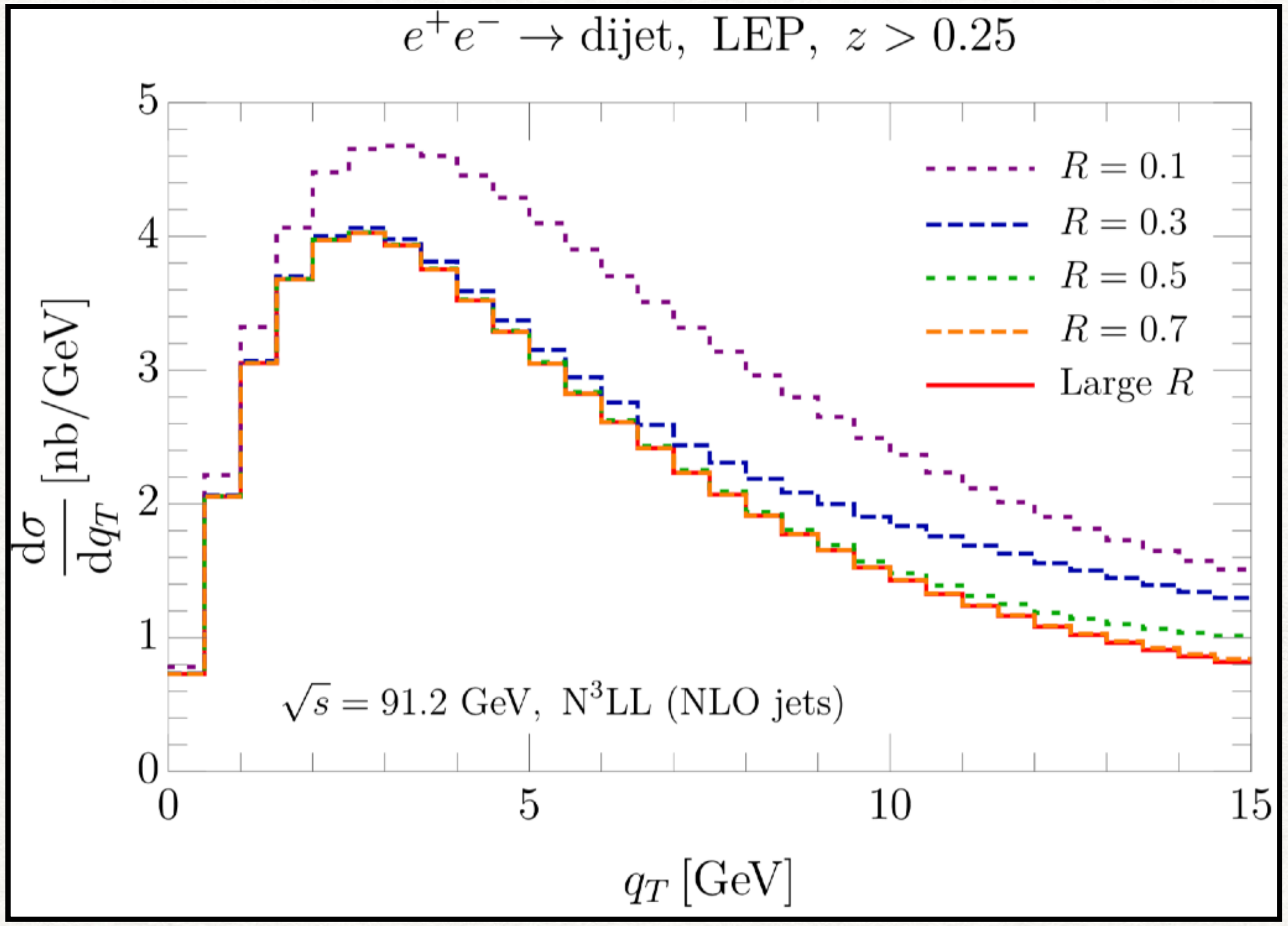
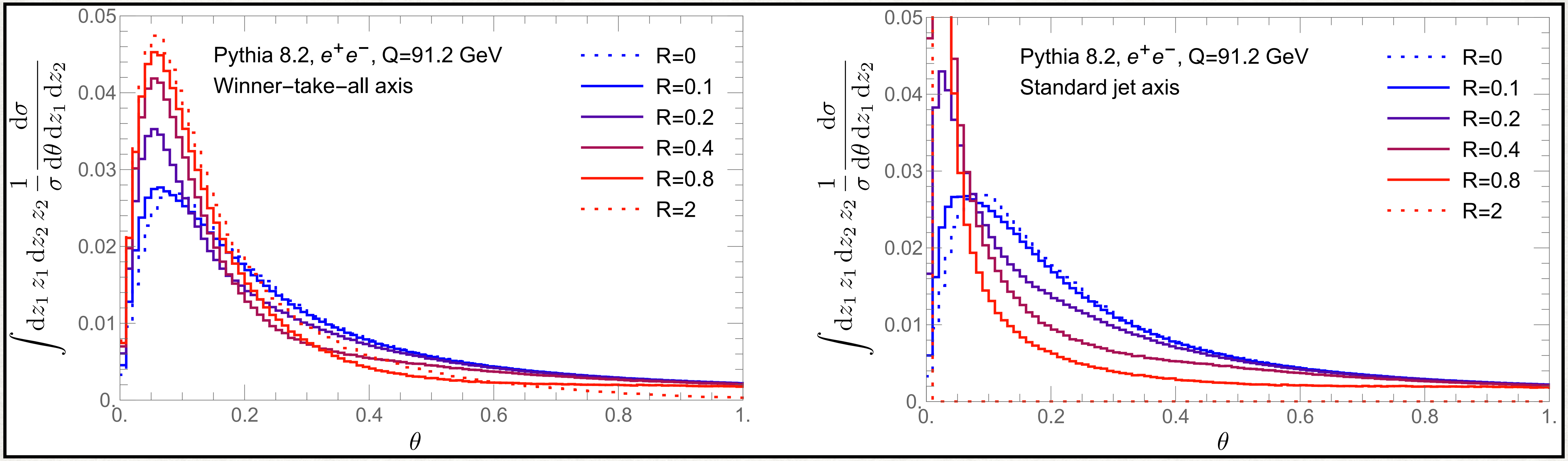
VARY RADIUS: LARGE-R VS FINITE R



The large radius approximation is a very accurate approximation for jet functions with finite radius (but not so small)

This fact allow us to skip some of the technical complications of the finite radius jet function

CHECKING WITH PYTHIA 8.2.



Cross-section of angular decorrelation for different values of the radii of the jets

For small values of R the cross-section for both axis elections agrees!

For big values of R the cross section in SJA is inconsistent!

Factorization is broken

WTA axis solves the problems!

CHOOSING SCALES AND ζ-PRESCRIPTION

The election of the final scales is dictated by the hard scales of the process

$$\mu_f = Q \qquad\qquad \zeta_f = Q^2$$

Some of the logs in the jet function (or in the coefficient function) in TMDs are dangerous!

$$L_\mu^2, L_\mu \mathrm{l}_\zeta$$

Related to TMD evolution.

They make the cross-section blowing up!

These logs are cancelled by a particular choice of $\zeta = \zeta_\mu$ $\boxed{\zeta\text{-prescription}}$

$$\mu^2 \frac{dJ(z, \boldsymbol{b}, \mu, \zeta_\mu)}{d\mu^2} = 0 \longrightarrow \frac{\gamma_F(\mu, \zeta_\mu(\boldsymbol{b}))}{2\mathcal{D}(\mu, \boldsymbol{b})} = \frac{\mu^2}{\zeta_\mu(\boldsymbol{b})} \frac{d\zeta_\mu(\boldsymbol{b})}{d\mu^2} \longrightarrow \mathrm{l}_{\zeta_\mu} = \frac{L_\mu}{2} - \frac{3}{2} + \mathcal{O}(a_s)$$

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