High energy scattering in QCD: from low to high Bjorken x

Jamal Jalilian-Marian

Baruch College and CUNY Graduate Center

New York, NY

OUTLINE

QCD at high transverse momentum:

collinear factorization (twist expansion)

QCD at high energy (CGC):

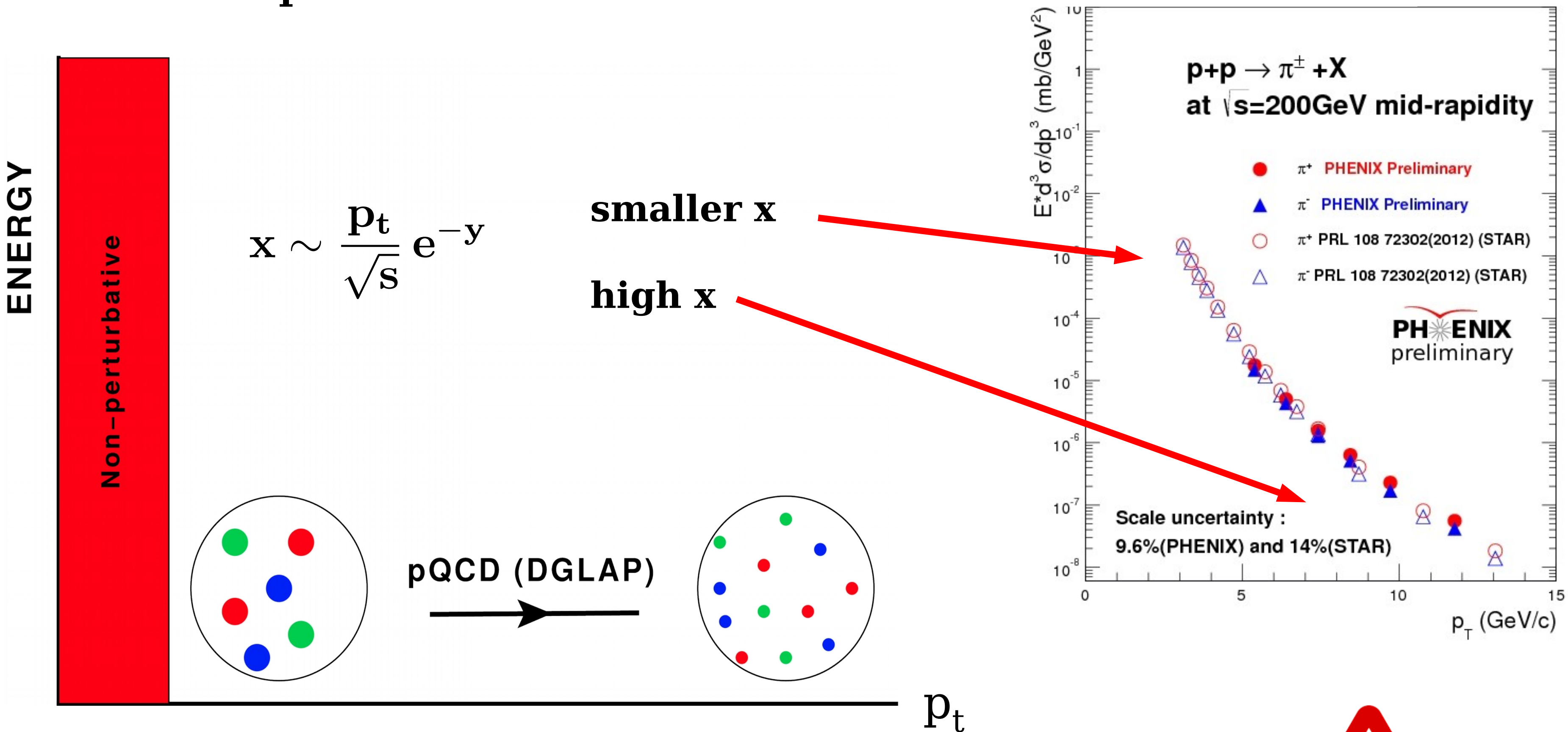
high gluon density effects

Toward a unified formalism:

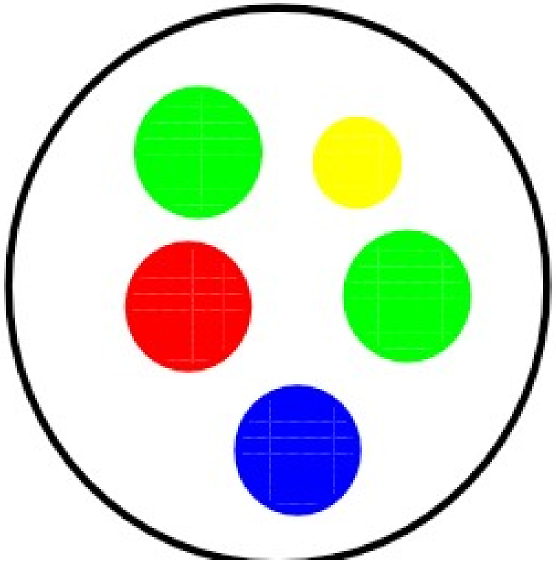
beyond eikonal approximation

pQCD: the standard paradigm

$$E\frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2) + \cdots\cdots$$



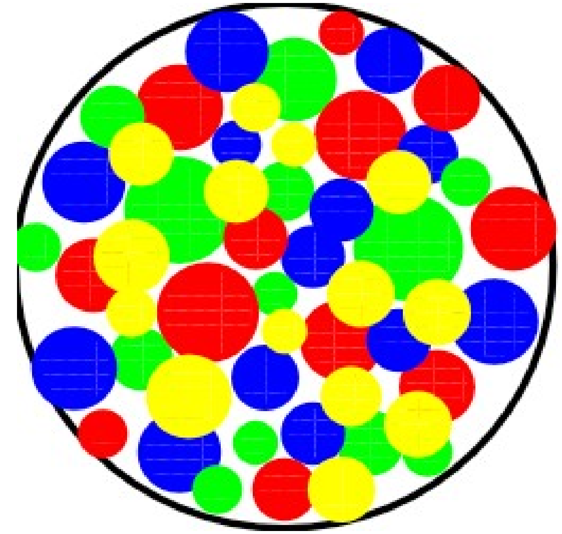
bulk of QCD phenomena happens at low p_t (small x)



$S \to \infty, \;\; Q^2 \;\; fixed$

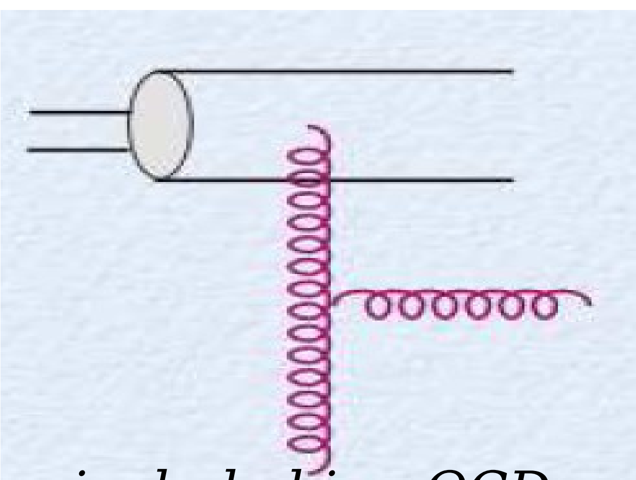
$x_{Bj} \equiv \frac{Q^2}{S} \to 0$

gluon radiation $\mathbf{P_{gg}(x) \sim \frac{1}{x}}$



<u>collinear factorization breaks down at small x</u>

“attractive” bremsstrahlung *vs.* *“repulsive” recombination*

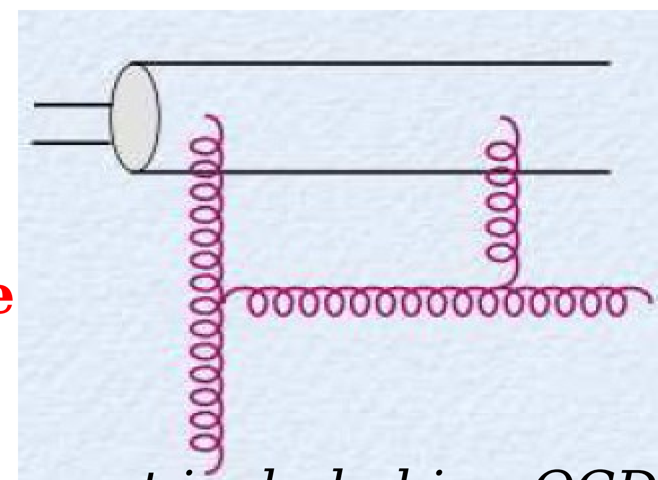


included in pQCD

$$\frac{\alpha_s}{\mathbf{Q^2}} \frac{\mathbf{xG(x, Q^2)}}{\pi \mathbf{r^2}} \sim \mathbf{1}$$

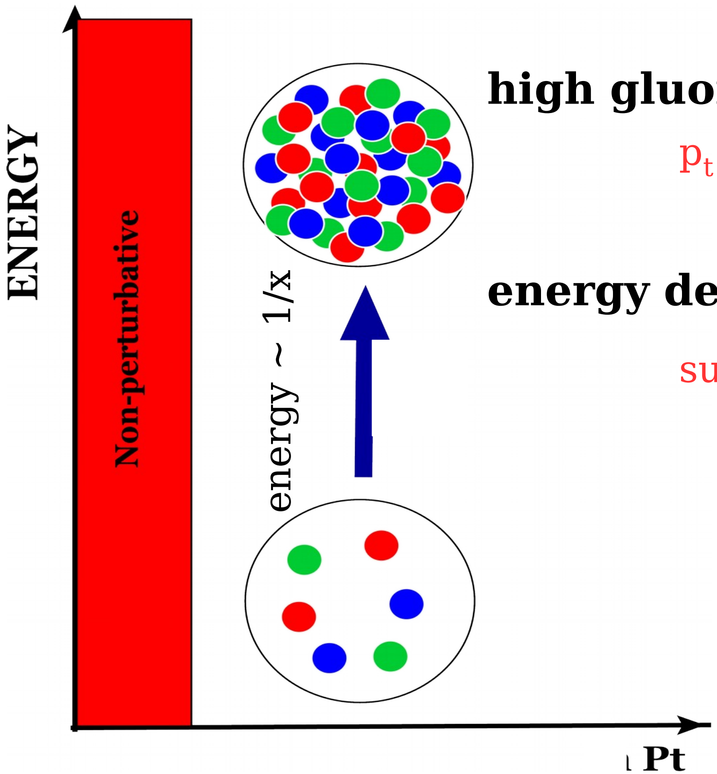
saturation scale

$$\mathbf{Q_s^2(x, b_t, A)}$$



not included in pQCD

A hadron/nucleus at high energy: gluon saturation



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

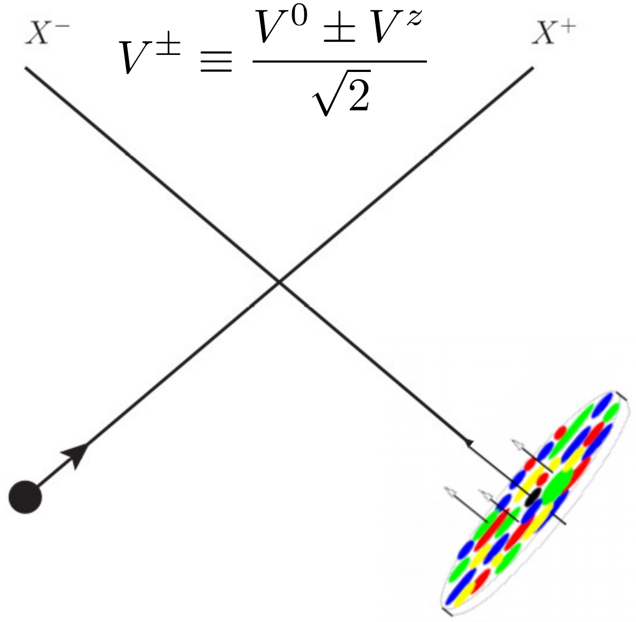
Shadowing/Nuclear modification factor
Azimuthal angular correlations (di-jets,...)
Long range rapidity correlations (ridge,...)
Initial conditions for hydro
Thermalization (?)

$$\mathbf{x \leq 0.01}$$

Scattering at high energy (small x) *(proton-nucleus)*

Eikonal approximation

$$V^{\pm} \equiv \frac{V^0 \pm V^z}{\sqrt{2}}$$



$$J_a^{\mu} \simeq \delta^{\mu -} \rho_a$$

$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0$$ (in $A^+ = 0$ gauge)

does not depend on x^-

solution to EOM:

$$A_a^-(x^+, x_t) \equiv n^- \, S_a(x^+, x_t)$$

with

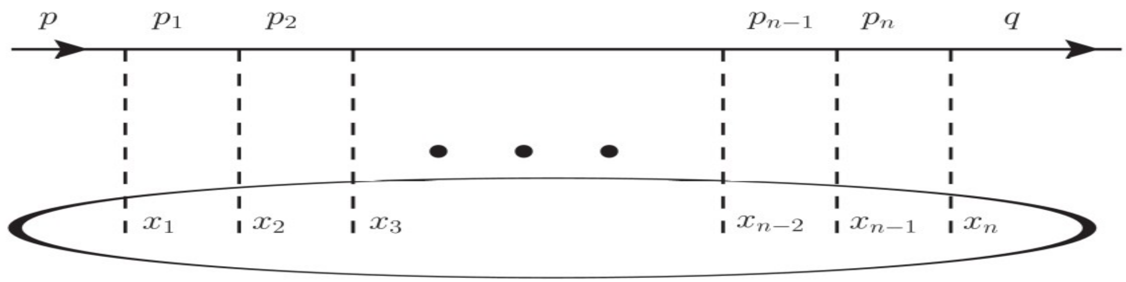
$$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$
$$n^2 = 2\, n^+ n^- - n_\perp^2 = 0$$

recall (eikonal limit): $\bar{u}(q)\gamma^\mu u(p) \rightarrow \bar{u}(p)\gamma^\mu u(p) \sim p^\mu$

$$\bar{u}(q) \not{A} \, u(p) \rightarrow p \cdot A \sim p^+ A^-$$

multiple scattering of a quark from background color field $S_a(x^+, x_t)$

$$A_a^-(x^+, x_\perp) \equiv n^- \, S_a(x^+, x_\perp)$$

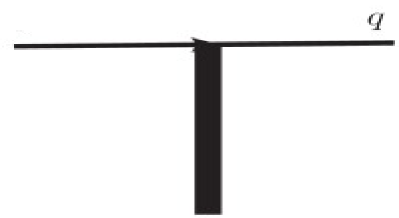


$$i\mathcal{M}_n = 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t \, e^{-i(q_t - p_t)\cdot x_t} \Big\{ (ig)^n \, (-i)^n (i)^n \int dx_1^+ \, dx_2^+ \cdots dx_n^+ \, \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \big[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t) \cdots S(x_2^+, x_t) S(x_1^+, x_t) \big] \Big\} u(p)$$

$$i\mathcal{M} = \sum_n i\,\mathcal{M}_n$$

$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t \, e^{-i(q_t - p_t)\cdot x_t} \, [V(x_t) - 1] \; u(p)$$

with $V(x_t) \equiv \hat{P} \, \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+ \, n^- \, S_a(x^+, x_t)\, t_a \right\}$



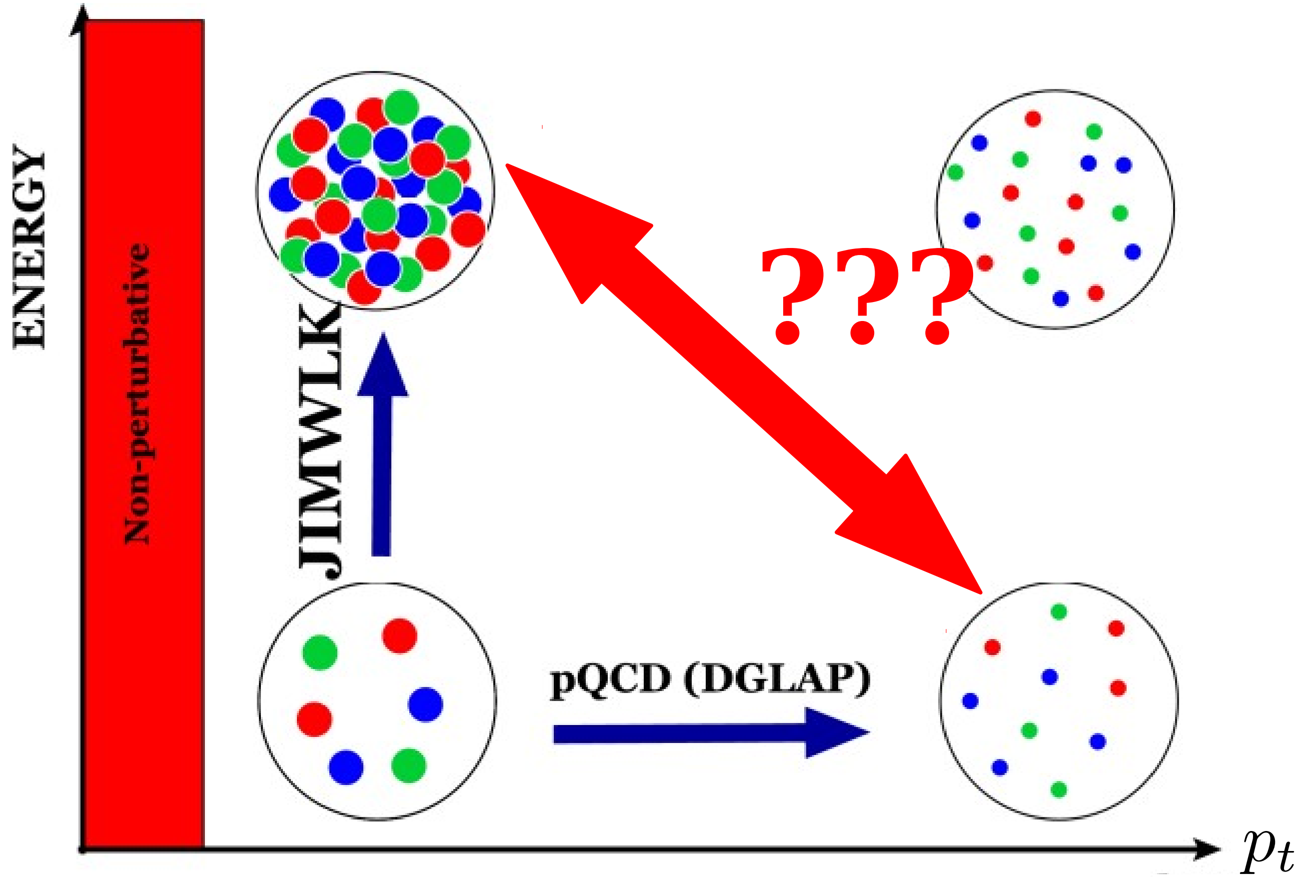
DIS, proton-nucleus collisions involve dipoles (<u>toward precision: NLO evolution</u>) $< Tr\, V(x_\perp)\, V^\dagger(y_\perp) >$

scattering from small x modes of the target can cause only a <u>*small angle deflection*</u>

large x partons of target are needed to cause a large angle deflection of quark

$$A_a^\mu(x^+, x^-, x_\perp)$$

QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

<u>kinematics of saturation: *where is saturation applicable?*</u>

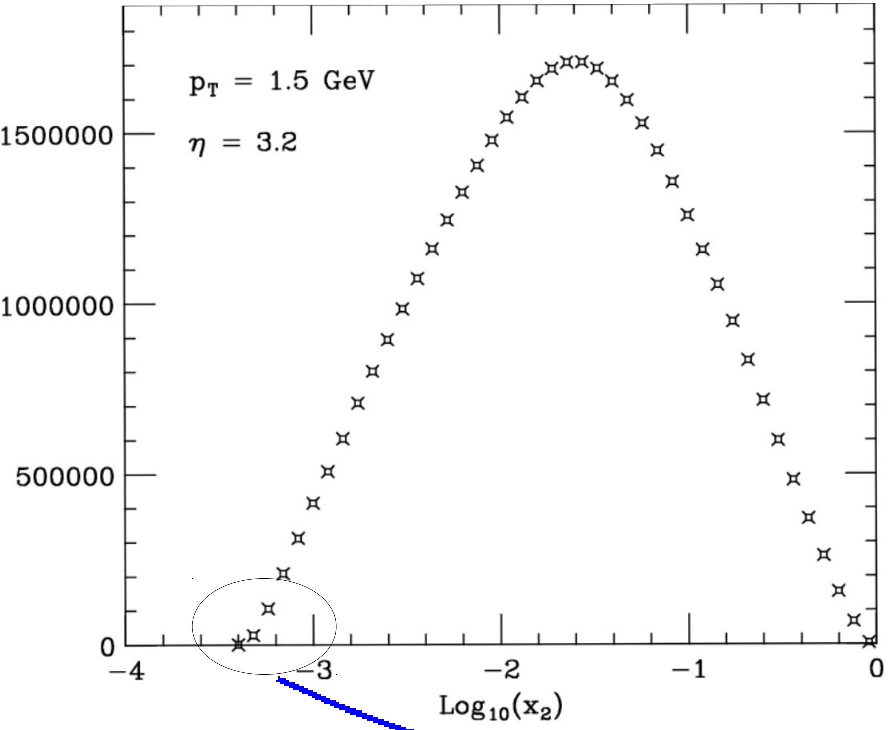
jet physics, high p_t (polar and azimuthal) angular correlations

cold matter energy loss, spin physics, ...

Pion production at RHIC: kinematics

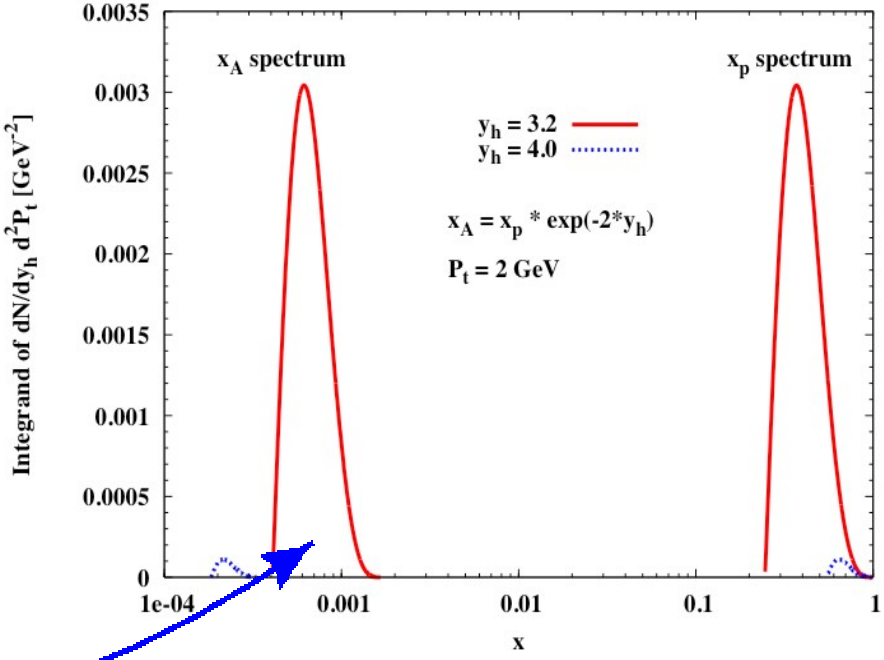
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x,Q^2)\cdots\cdots \longrightarrow x_{min}G(x_{min},Q^2)\cdots$$

this is an extreme approximation with potentially severe consequences!

Starting point/expression/operator?

pQCD: quark and gluon operators

$$\overline{\Psi}(y^-, 0_t)\gamma^+\Psi(0^-, 0_t)$$

renormalization lead to DGLAP evolution eq.

CGC: correlators of Wilson lines (DIS, Hybrid,....)

$$F_2 \sim Tr\, V(x_t)\, V^\dagger(y_t)$$

renormalization leads to JIMWLK/BK evolution eq.

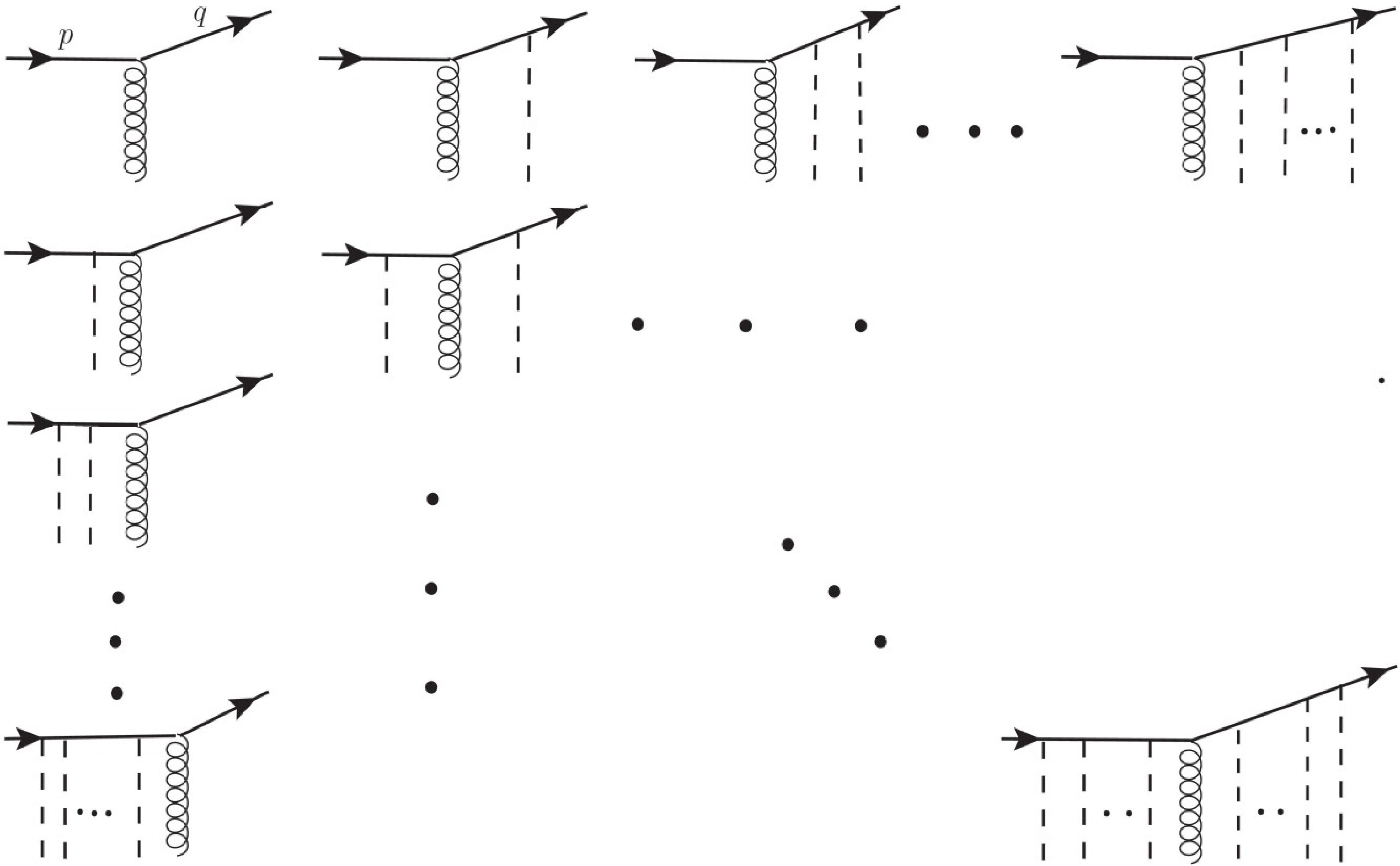
toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0)$$
$$S = S(p^+ \sim 0, p^-/P^- \ll 1, p_t)$$

allow hard scattering by including one hard field $A_a^\mu(x^+, x^-, x_t)$ where there is large momenta exchanged and quark can get deflected by a large angle.

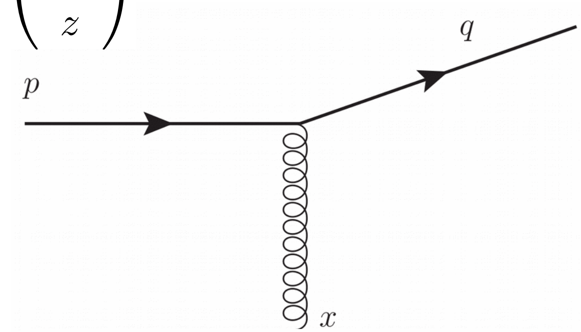
include eikonal multiple scattering before and after (along a different direction) the hard scattering


p
q

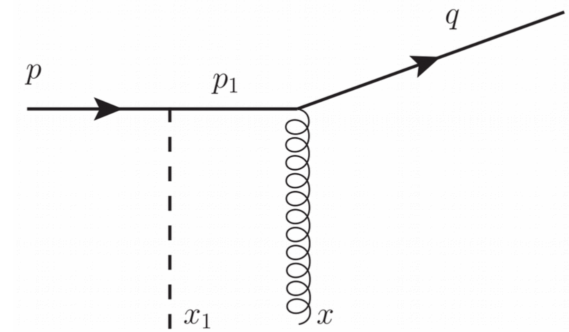
hard scattering: large deflection
scattered quark travels in the new “z” direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

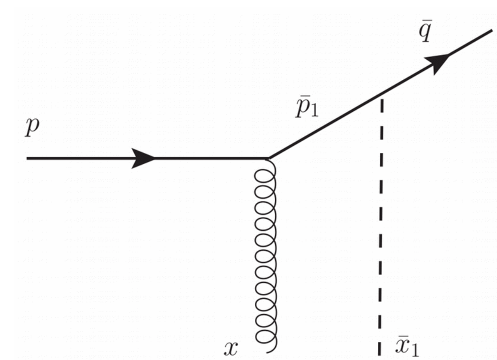
$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \; [\not{A}(x)] \; u(p)$$



$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \\ &\quad \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1) \right] u(p) \end{aligned}$$

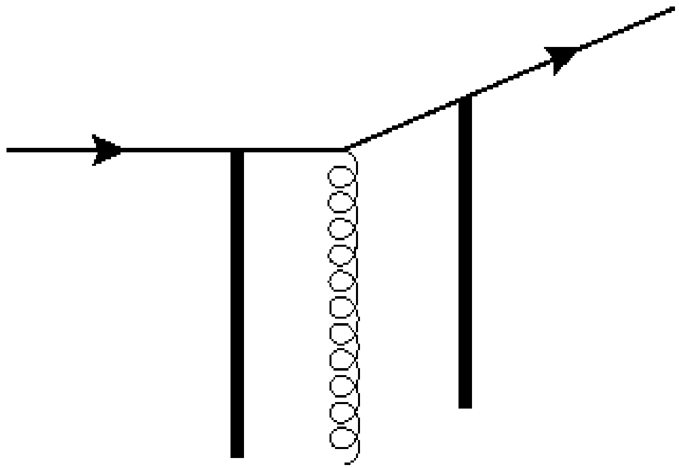


$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \\ &\quad \bar{u}(\bar{q}) \left[\bar{\not{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\bar{\not{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x) \right] u(p) \end{aligned}$$



with $\quad \vec{\bar{v}} = \mathcal{O} \, \vec{v}$

summing all the terms gives:



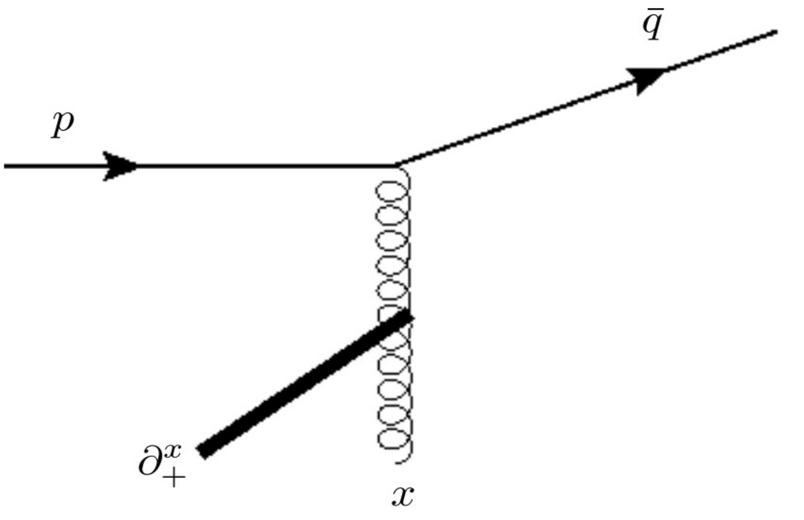
$$
\begin{aligned}
i\mathcal{M}_1 \quad = \quad & \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\, \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\; e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\; e^{-i(k_t-p_t)\cdot z_t} \\
& \bar{u}(\bar{q}) \left[\overline{V}_{AP}(x^+,\bar{z}_t)\, \not{\bar{n}}\, \frac{\not{\bar{k}}}{2\bar{k}^+}\, \left[ig\not{A}(x)\right]\, \frac{\not{k}}{2k^+}\, \not{n}\, V_{AP}(z_t,x^+)\right] u(p)
\end{aligned}
$$

with

$$
\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\, \bar{S}_a^-(\bar{z}_t,\bar{z}^+)\, t_a\right\}
$$

$$
V_{AP}(z_t,x^+) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{x^+} dz^+\, S_a^-(z_t,z^+)\, t_a\right\}
$$

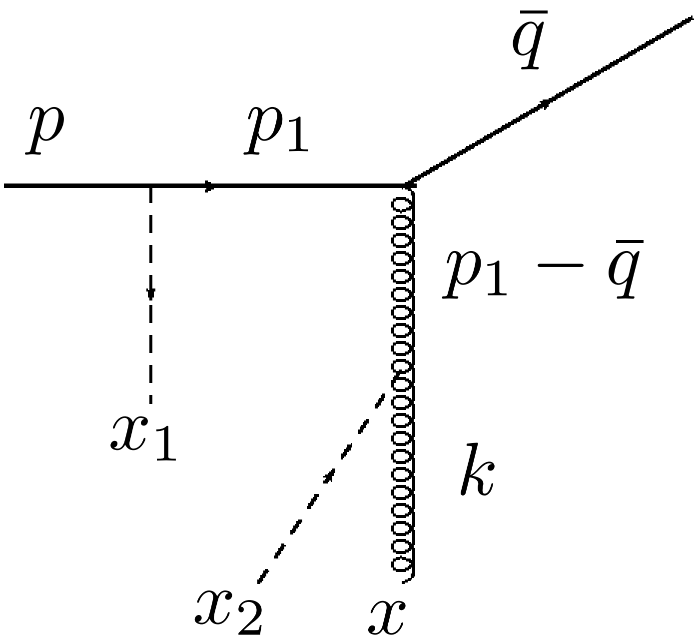
all re-scatterings of hard gluon can be re-summed



$$i\mathcal{M}_2 \quad = \quad \frac{2\,i}{(p-\bar{q})^2}\int d^4x\, e^{i(\bar{q}-p)x}\,\bar{u}(\bar{q})\left[(ig\,t^a)\;\left[\partial_{x^+}\,U^\dagger_{AP}(x_t,x^+)\right]^{ab}\right.$$
$$\left.\left[n\cdot(p-\bar{q})\not{A}_b(x)-(p-\bar{q})\cdot A_b(x)\,\not{n}\right]\;\right]u(p)$$

with $$U_{AP}(x_t,x^+)\equiv\hat{P}\,exp\left\{ig\int_{-\infty}^{x^+}dz^+\,S_a^-(z^+,x_t)\,T_a\right\}$$

but there is more!



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\,[(p_1 - \bar{q})^2 + i\epsilon]}$$

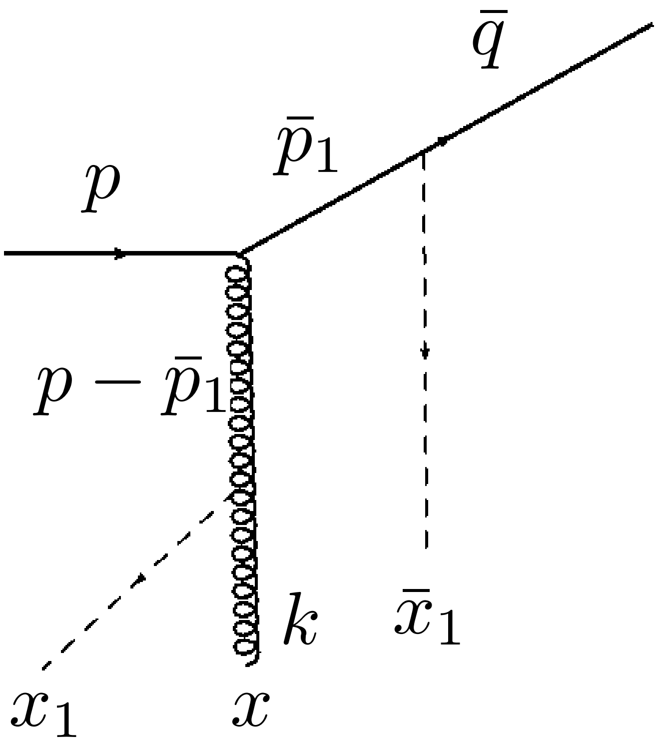
both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^-(\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\ [(p_1 - \bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

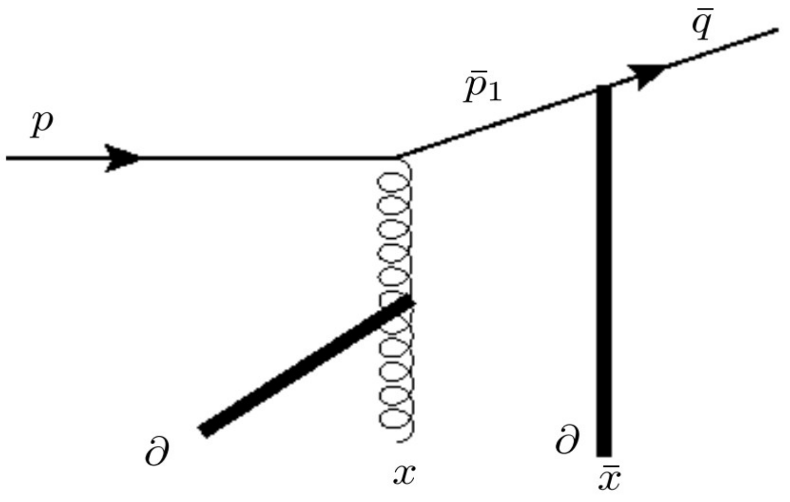
$\theta(x^+ - \bar{x}_1^+)$ and $\theta(\bar{x}_1^+ - x^+)$

ignoring the phases the contribution of the two poles add!

path ordering is lost!

however further re-scatterings are still path-ordered before/after $\mathbf{x_1^+}, \mathbf{\bar{x}_1^+}$

Re-scatterings of hard gluon and final state quark re-sum to

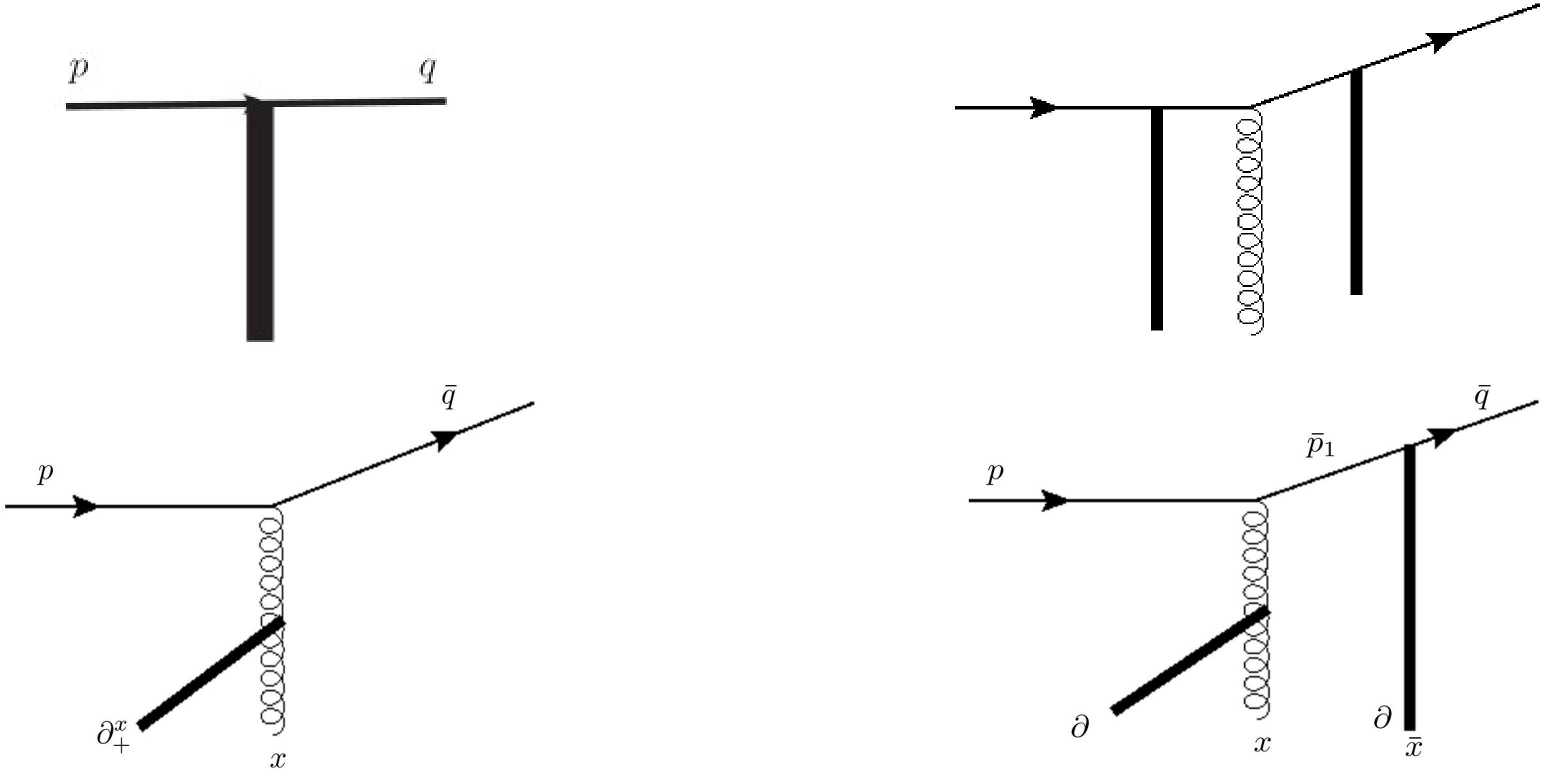


$$i\mathcal{M}_3 \; = \; -2\,i\int d^4x\, d^2\bar{x}_t\, d\bar{x}^+ \,\frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+-p^+)x^-}\, e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\, e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot\bar{x}_t}$$

$$\bar{u}(\bar{q})\left[\left[\partial_{\bar{x}^+}\overline{V}_{AP}(\bar{x}^+,\bar{x}_t)\right]\,\not{n}\,\not{\bar{p}}_1\,(igt^a)\,\left[\partial_{x^+}U^\dagger_{AP}(x_t,x^+)\right]^{ab}\right.$$

$$\left.\frac{\left[n\cdot(p-\bar{q})\not{A}^b(x)-(p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\,2n\cdot(p-\bar{q})\,p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]}\right]u(p)$$

full amplitude: $\mathbf{i}\mathcal{M} = \mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}$



soft (eikonal) limit:

$$\begin{aligned} A^\mu(x) &\rightarrow n^- S(x^+, x_t) \\ n \cdot \bar{q} &\rightarrow n \cdot p \end{aligned}$$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

cross section: $|\mathbf{i}\mathcal{M}|^{\mathbf{2}} = |\mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}|^{\mathbf{2}}$

tedious, long expressions,...

spinor helicity formalism for Dirac Algebra: light-front spinors

spin asymmetries

illustration:

$$|\mathcal{N}_2^{++}|^2 \;\sim\; g^2 \frac{q^+}{p^+} \frac{1}{q_\perp^4} \int d^4x \, d^4y \, e^{i(q^+ - p^+)(x^- - y^-)} \, e^{-i(q_t - p_t)\cdot(x_t - y_t)}$$

$$\Big\{ \Big[(p^+ - q^+)^2 \, q_\perp^2 \, A_\perp^b(x) \cdot \, A_\perp^c(y) + 4 \, p^+ q^+ \, q_\perp \cdot A_\perp^b(x) \, q_\perp \cdot A_\perp^c(y) \Big]$$

$$+ i \, \epsilon^{ij} \, [(p^+)^2 - (q^+)^2] \Big[q_i \, A_j^b(x) \, q_\perp \cdot A_\perp^c(y) - q_i \, A_j^c(y) \, q_\perp \cdot A_\perp^b(x) \Big] \Big\}$$

$$\left[\partial_{y^+} \, U_{AP}(y_t, y^+)\right]^{ca} \left[\partial_{x^+} \, U_{AP}^\dagger(x_t, x^+)\right]^{ab}$$

SUMMARY

CGC is a systematic approach to high energy collisions

- strong hints from RHIC, LHC,...
- connections to TMD,...
- toward precision: NLO,...

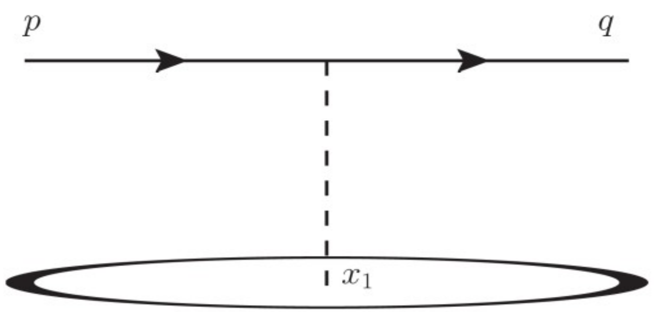
CGC breaks down at large x (high p_t)

- a significant portion of EIC phase space is at large x
- transition from DGLAP physics to CGC

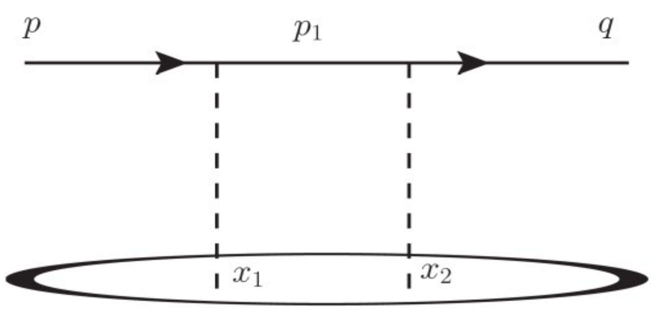
Toward a unified formalism:

- particle production in both small and large x (p_t) kinematics
- spin, azimuthal asymmetries in intermediate p_t region
- one-loop correction to cross section: from JIMWLK to DGLAP ?

$$
\begin{aligned}
i\mathcal{M}_1 &= (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p) \\
&= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}} \\
&\quad \bar{u}(q)\left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)
\end{aligned}
$$



$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(q-p_1)x_2} \\
&\quad \bar{u}(q)\left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right] u(p)
\end{aligned}
$$



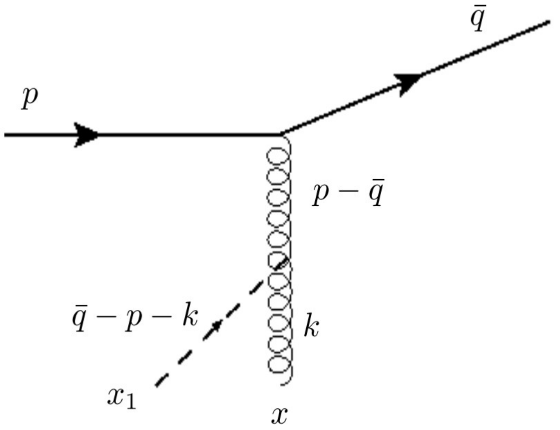
$$
\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\, \theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}
$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\, \frac{\not{p}_1}{2n \cdot p}\, \not{n} = \not{n}$

$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2\, (-i)(i)\, 2\pi\delta(p^+ - q^+) \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+) \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}} \\
&\quad \bar{u}(q)\, \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\, u(p)
\end{aligned}
$$

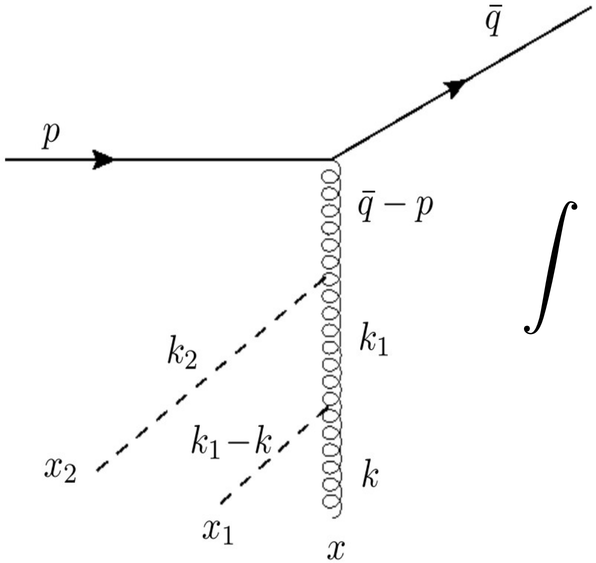
interactions of large and small x modes



$$
\begin{aligned}
i\mathcal{M} \quad = \quad & f_{acd} \int \frac{d^4k}{(2\pi)^4}\, d^4x\, d^4x_1\, e^{i(\bar{q}-p-k)x_1}\, e^{ikx} \\
& \bar{u}(\bar{q})\,(ig\,\gamma^\mu\, t^a)\, u(p)\, A^c_\lambda(x)\ \left[i\, g\, S^d(x_1)\right] \\
& \frac{1}{(p-\bar{q})^2 + i\epsilon} \Big[-g^\mu_\lambda\, n\cdot(p-\bar{q}-k) \\
& + n^\mu \left[p_\lambda - \bar{q}_\lambda\,(1 - \frac{n\cdot k}{n\cdot(p-\bar{q})})\right]\Big]
\end{aligned}
$$

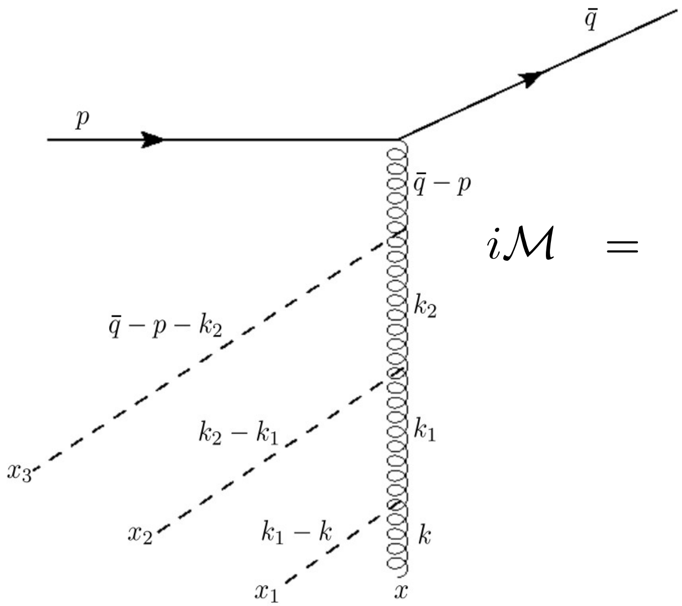
performing k^- integration sets $x_1^+ = x^+$

$$
\begin{aligned}
i\mathcal{M} \quad = \quad & 2 f_{acd} \int d^4x\, e^{i(\bar{q}-p)x} \\
& \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_c(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p)\ \left[i\, g\, S^d(x^+, x_t)\right]
\end{aligned}
$$



$$\int \frac{dk_1^-}{(2\pi)} \frac{e^{ik_1^-(x^+-x_2^+)}}{2(\bar{q}^+ - p^+)\left[k_1^- - \frac{k_{1t}^2 - i\epsilon}{2(\bar{q}^+ - p^+)}\right]} \sim \theta(x^+ - x_2^+)$$

$$\begin{aligned} i\mathcal{M} \quad = \quad & 2\, f_{abc}\, f_{cde} \int d^4x\, dx_2^+\, \theta(x^+ - x_2^+)\, e^{i(\bar{q}^+ - p^+)x^- - i(\bar{q}_t - p_t)\cdot x_t} \\ & \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_e(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p) \\ & \left[i\, g\, S_d(x^+, x_t)\right]\ \left[i\, g\, S_b(x_2^+, x_t)\right] \end{aligned}$$



$$
\begin{aligned}
i\mathcal{M} \;=\; & \frac{2(i)^2}{(\bar{q}-p)^2}\, f^{abc} f^{cde} f^{egf} \int d^4x\, dx_2^+ dx_3^+\, \theta(x^+ - x_2^+)\, \theta(x_2^+ - x_3^+) \\
& \bar{u}(\bar{q})\, (ig\, t^a) \left[n\cdot(p-\bar{q}) \not{A}_f(x) - (p-\bar{q})\cdot A_f(x) \not{n} \right] u(p) \\
& \left[i\, g\, S_g(x^+, x_t) \right] \left[i\, g\, S_d(x_2^+, x_t) \right] \left[i\, g\, S_b(x_3^+, x_t) \right] \\
& e^{i(\bar{q}^+ - p^+) x^- - i(\bar{q}_t - p_t)\cdot x_t}
\end{aligned}
$$

recall

$$
\begin{aligned}
\partial_{x^+} \left[U^\dagger_{AP}(x_t, x^+) \right]^{ab} \;=\; & (if^{bca}) \left[igS_c(x^+, x_t) \right] \\
+\; & (if^{bce})\,(if^{eda}) \int dx_1^+ \theta(x^+ - x_1^+) [[igS_c(x^+, x_t)]\, [igS_d(x_1^+, x_t)] \\
+\; & (if^{bch})\,(if^{gdf})\,(if^{fea}) \int dx_1^+\, dx_2^+\, \theta(x^+ - x_1^+)\, \theta(x_1^+ - x_2^+) \\
& [[igS_c(x^+, x_t)]\, [igS_d(x_1^+, x_t)]\, [[igS_c(x_2^+, x_t)] + \cdots\cdots
\end{aligned}
$$

Particle production in high energy collisions

pQCD and collinear factorization at high p_t

breaks down at low p_t (small x)

CGC at low p_t

breaks down at large x (high p_t)

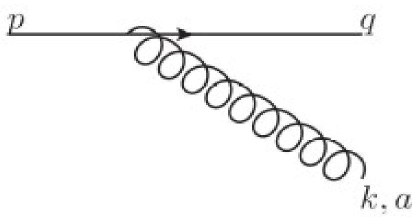
need a unified formalism:

CGC at low x (low p_t)

leading twist pQCD (DGLAP) at large x (high p_t)

1-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)



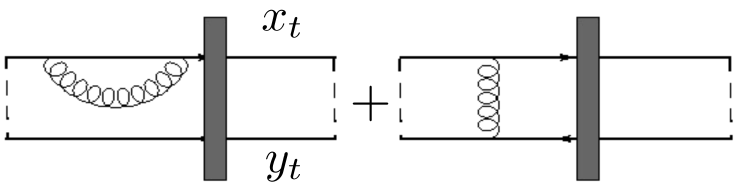
$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^\mu_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$

coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2} e^{ik_t\cdot(x_t-z_t)} 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

x_t, z_t are transverse coordinates of the quark and gluon

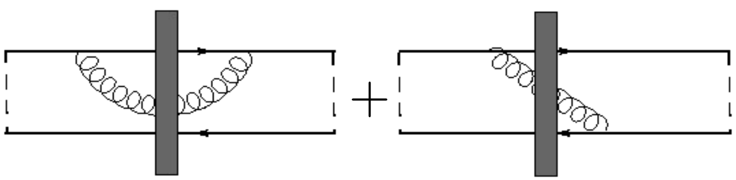
virtual corrections:



$$+ \qquad + \qquad \longrightarrow Tr\,V(x_t)\,V^\dagger(y_t)$$

a dipole

real corrections:



$$+ \qquad + \qquad \longrightarrow Tr\,V(x_t)\,V^\dagger(z_t)\,Tr\,V(z_t)\,V^\dagger(y_t)$$

$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

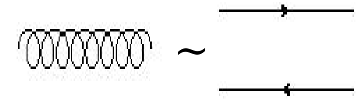
the S matrix

$$S(x_t,y_t) \equiv \frac{1}{N_c}Tr\,V(x_t)\,V^\dagger(y_t)$$

1-loop correction: BK eq.

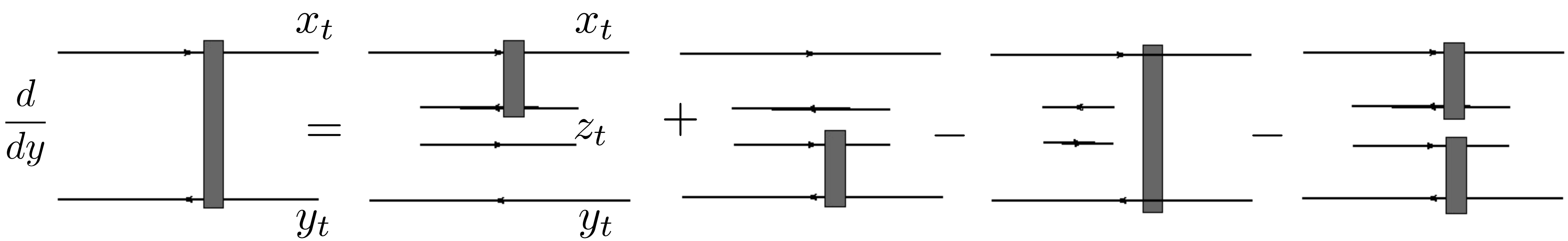
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy} T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t) T(z_t, y_t) \right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t \, dy}}{A^{1/3} \frac{d\sigma^{pp}}{d^2 p_t \, dy}}$$

suppression of p_t spectra

nuclear shadowing

…………………