

Toward precision jet event shape for future Electron-Ion Collider



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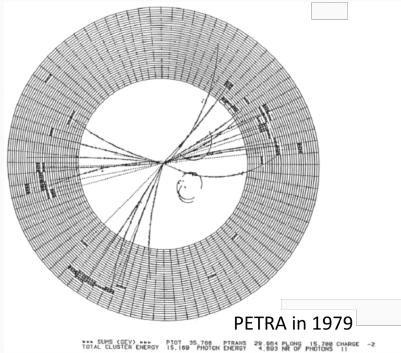
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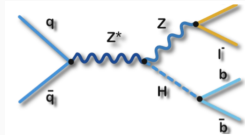
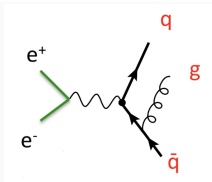
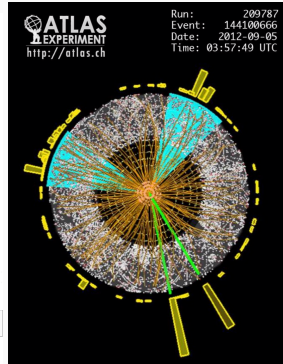
Introduction

Jet production in collider

3-jet event: discovery of gluon



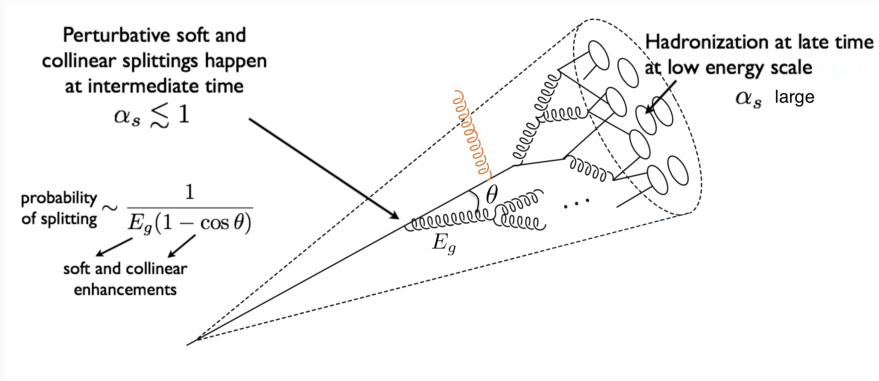
2-jet event: Higgs from LHC



Introduction

Jet production in collider

- Jet?

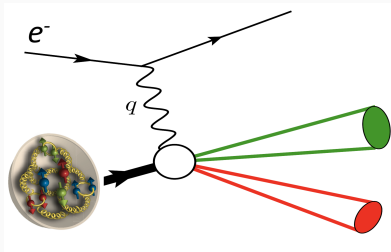


- Jet contains rich information to probe strong dynamics.

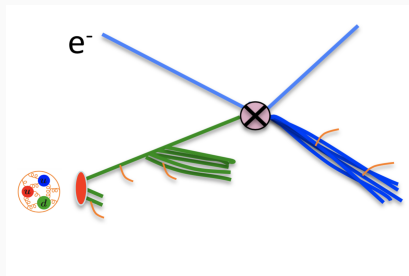
Motivation

Jet production in DIS

$$ep \rightarrow 2 - jet$$



- **Jet:** to investigate short distance phenomenon.
- **Beam:** created from ISR to investigate internal structure of hadron– PDF, GPD, TMD, Spin structure...

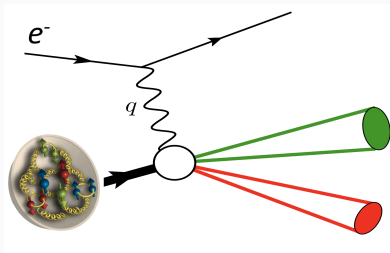


dijet production
in ep collision

Beam encodes internal structure
of incoming proton.

Motivation

Jet production in DIS $ep \rightarrow 2 - jet$



- **Jet:** to investigate **short distance phenomenon**.
- **Beam:** created from ISR to investigate **internal structure of hadron**– PDF, GPD, TMD, Spin structure...

Precision Jet Physics in DIS: a new tool to probe strong dynamics

a new level of precision for jets
at the future
Electron-Ion-Collider (EIC)

- **One of early milestones!**



[GPD at EIC: Talk by Pawel Sznajder]

The jet observables are called **Event Shapes**.

Event Shapes: Jet observables

Several event shapes: Thrust (τ), Angularity (τ_a), Jet Broadening...

Event Shapes: Jet observables

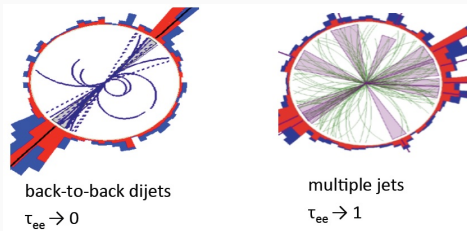
Several event shapes: **Thrust** (τ), **Angularity** (τ_a), Jet Broadening...

◀ Thrust (τ) ▶

One of the most precisely measured observables!

$$\tau = \frac{2}{Q} \sum_{i \in \mathcal{X}} |\mathbf{p}_{\perp}^i| e^{-|\eta_i|}$$

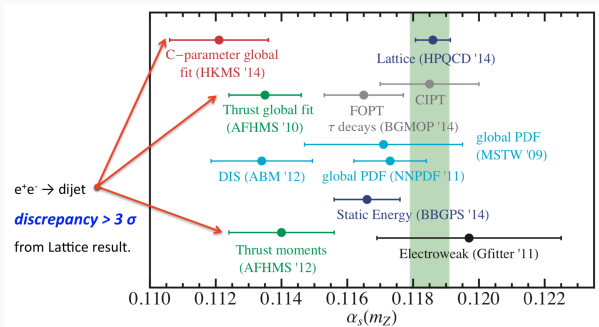
Thrust event shape characterised the geometry of the collision!



- Expt.: LEP, CMS and ATLAS, HERA by the ZEUS and H1

Event Shapes: Determination of strong coupling

► Puzzle in strong coupling constant



- Tension between $e^+e^- \rightarrow 2 - jet$ result and Lattice result!
- **Need a new test from an independent experiment!**

DIS: $ep \rightarrow \text{dijet}$!!
Angularity event shapes

future Electron-Ion-Collider

Event Shapes: Angularity

◀ Angularity (τ) ▶

A more general event shapes!

$$\tau_a = \frac{2}{Q} \sum_{i \in \mathcal{X}} |\mathbf{p}_{\perp}^i| e^{-|\eta_i|(1-a)}$$

Angularity depends on the continuous parameter a .

$a \rightarrow 0$: Thrust

$a \rightarrow 1$: Jet Broadening

Event Shapes: Angularity

◀ Angularity (τ) ▶

A more general event shapes!

$$\tau_a = \frac{2}{Q} \sum_{i \in \mathcal{X}} |\mathbf{p}_{\perp}^i| e^{-|\eta_i|(1-a)}$$

Angularity depends on the continuous parameter a .

$a \rightarrow 0$: Thrust

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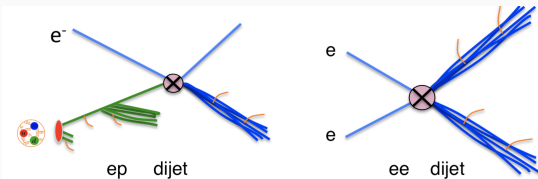
- In terms of the four-vectors q_B along the incident proton beam direction and q_J along the direction of the final state jet we wish to measure

- 1-jettiness axis Choice:

$$q_B = xP$$

$$q_J = \text{Jet axis}$$

Breit frame!



Event Shapes: Angularity

Theoretical Prediction:

Accuracy in the theoretical prediction to the event shapes!

Tool:

Soft-collinear effective theory (SCET)

a systematic way to achieve high precision in high-energy scattering

SCET Predictions so far...

- Thrust :

e^+e^- [Becher, Schwartz'08; Stewart, Tackmann, Waalewijn'10]

pp-collision [Stewart, Tackmann, PRL'10, '11; PRD'13],

DIS [D. Kang, C. Lee, I. Stewart'13]

...

- Angularity:

e^+e^- [Hornig, Lee, Ovanessian'09; Bell, Lee'19]

DIS [??]

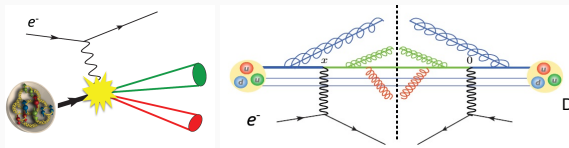
Could be important for future EIC!

We do...

Investigation of **angularity in DIS** in the frame-work of **soft-collinear effective theory (SCET)** tools and to push the frontier of the **precision jet physics in the future Electron-Ion Collider (EIC)**

DIS factorization and Beam function at NNLL

DIS process: $e P \rightarrow 2\text{Jets}$



$$\text{SCET facto. } \sigma_{eP} = \text{Hard} \times \text{Beam} \otimes \text{Jet} \otimes \text{Soft}$$

- Beam is produced due to the initial state radiation (ISR) and can be used to study internal structure of proton!

$$\text{Beam func. } B(\tau_a, x, \mu) = \text{pdf} \otimes \left[\delta_{qj} \delta(\tau_a) + \tilde{\mathcal{I}}_{qj}^{(1)} + \mathcal{O}(\alpha_s^2) + \dots \right]$$

NP LO NLO $NNLO$

- Angularity Jet and soft function is presented at NNLL accuracy [Bell, Horning, Lee'2018]

Angularity beam function at NLO is presented for the first time!

Fixed order Beam function at NLO

$$\text{Beam func. } B(\tau_a, x, \mu) = \text{pdf} \otimes \left[\delta_{qj} \delta(\tau_a) + \tilde{\mathcal{I}}_{qj}^{(1)} + \mathcal{O}(\alpha_s^2) + \dots \right]$$

NP *LO* *NLO* *NNLO*

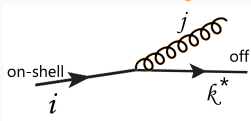
Angularity beam function at NLO is presented for the first time!

$$\tilde{\mathcal{I}}_{qj}^{(1)} = \frac{\alpha_s}{4\pi} \left[\left(j_B \kappa_B \frac{\Gamma_0}{2} L_B^2 + \gamma_0^B L_B \right) 1 + 4C_{qj} P_{qj}(z) L_B + c_1^{qj}(z) \right]$$

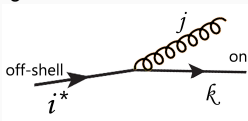
$$L_B(\tau_a) = \log \left[\frac{Q}{\mu_B} (\tau_a e^{-\gamma_E})^{1/j_B} \right]$$

Beam Func. and Fragmentation Func.

Beam at NLO: $i \rightarrow k^* j$



Fragmentation at NLO: $i^* \rightarrow k j$



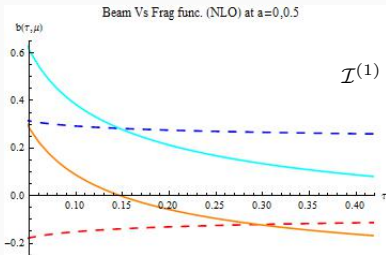
Crossing Symmetry!

Splitting Function:

[M.Ritzmann,W.J.Waalewijn,PRD90(2014)]

$$P_{i \rightarrow k^* j}(2p_i \cdot p_j, x) \equiv (-1)^{\Delta_f} P_{k^* \rightarrow i j}(-2p_i \cdot p_j, 1/x)$$

- Change comes only from the two-particle phase-space and effectively change in sign of the $\log(x)$ term in the matching co-efficient $\mathcal{I}^{(1)}$.



$$\mathcal{I}^{(1)} \sim \dots - \frac{\alpha_s C_F}{2\pi} \frac{2(1-a)}{2-a} \frac{1+x^2}{1-x} \log x$$

Beam match. Coeffi.: $a = 0$

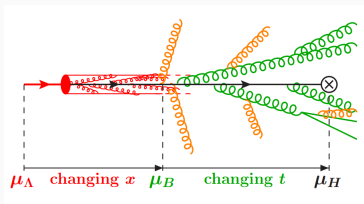
Beam match. Coeffi.: $a = 0.5$

FF match. Coeffi.: $a = 0$

FF match. Coeffi.: $a = 0.5$

- Difference decreases with the increase of angularity parameter a .

Resummation from evolution



-Stewart et. al. PRD81(2010)

Evolution Equation for beam function

$$\mu \frac{d}{d\mu} B(\nu, \mu) = \gamma_G(\mu) B(\nu, \mu) ; \quad \text{similar to } J, S, H$$

$$\text{Solution : } B(\nu, \mu) = B(\nu, \mu_B) e^{K_B(\mu_B, \mu) + j_B \eta_B(\mu_B, \mu) L_B} ,$$

- Jet and beam functions are defined by same collinear operator: $\gamma_J(\mu) = \gamma_B(\mu)$

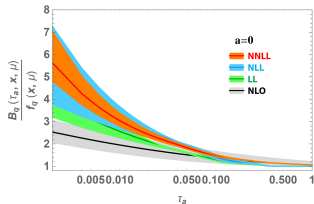
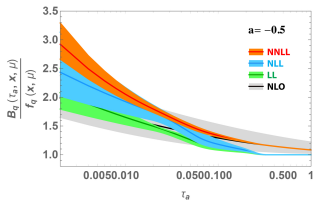
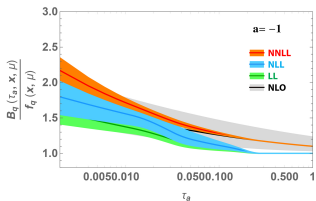
$$K_B(\mu_B, \mu) = L_B \sum_{k=1}^{\infty} (\alpha_s L_B)^k + \sum_{k=1}^{\infty} (\alpha_s L_B)^k + \dots$$

LL *NLL*

$$L_B = \ln(\mu/\mu_B)$$

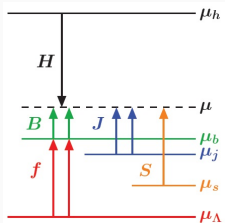
- Resummation of large logs provides better perturbative convergence.

Beam function result at NNLL



Differential Cross-Section

$$\sigma_{eP} = \text{Hard} \times \text{Beam} \otimes \text{Jet} \otimes \text{Soft}$$



Evolved form

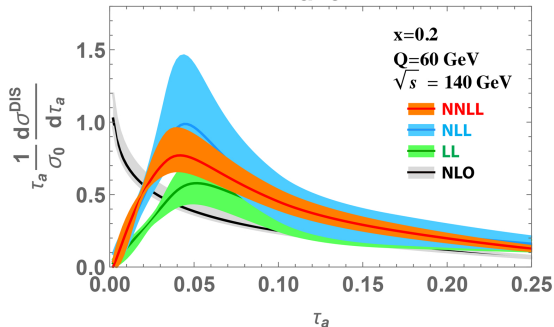
$$\begin{aligned} \sigma(x, Q^2, \tau_a, \mu) &= \sigma_0(x, Q^2) \left(\frac{Q}{\mu_H} \right)^{\eta_H(\mu, \mu_H)} e^{\kappa(\mu_H, \mu_J, \mu_B, \mu_S, \mu)} \\ &\times \left(\left(\frac{Q}{\mu_J} \right)^{2-a} \tau_a e^{-\gamma_E} \right)^{\eta_J(\mu, \mu_J)} \left(\left(\frac{Q}{\mu_B} \right)^{2-a} \tau_a e^{-\gamma_E} \right)^{\eta_B(\mu, \mu_B)} \left(\frac{Q^2}{\mu_S} \tau_a e^{-\gamma_E} \right)^{2\eta_S(\mu, \mu_S)} \\ &\times \tilde{j}_q \left(\partial_\Omega + \log \left(\frac{Q^{2-a}}{\mu_J^{2-a}} \tau_a e^{-\gamma_E} \right), \mu_J \right) \tilde{s} \left(\frac{1}{Q_R} \left(\partial_\Omega + \log \left(\frac{Q}{\mu_S} \tau_a e^{-\gamma_E} \right) \right), \mu_S \right) \\ &\times \left[H_q(y, Q^2, \mu_H) \tilde{b}_q \left(\partial_\Omega + \log \left(\frac{Q^{2-a}}{\mu_B^{2-a}} \tau_a e^{-\gamma_E} \right), x, \mu_B \right) \right. \\ &\quad \left. + H_{\bar{q}}(y, Q^2, \mu_H) \tilde{b}_{\bar{q}} \left(\partial_\Omega + \log \left(\frac{Q^{2-a}}{\mu_B^{2-a}} \tau_a e^{-\gamma_E} \right), x, \mu_B \right) \right] \frac{1}{\tau_a \Gamma(\Omega)} \end{aligned} \quad 16$$

Cross-section at NNLL

Prediction to EIC: NNLL resummation accuracy for angularity in $ep \rightarrow 2 - jets$

EIC $\sqrt{s} = 140 GeV, 0.01 \leq y \leq 0.95$

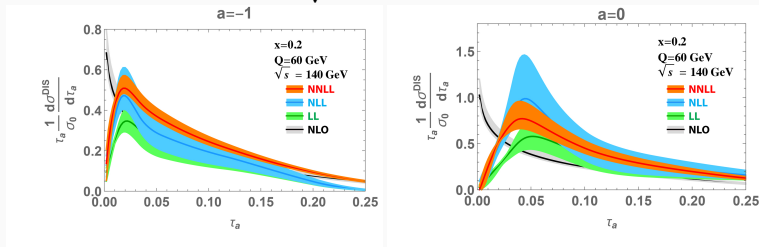
$a=0$



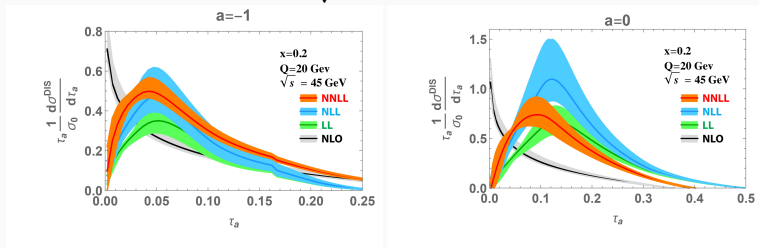
- Low angularity region: Log singularity at NLO
- Tail region: Resummation gives convergence from LL to NNLL.
- Far-tail region: Need full QCD.

Cross-section dependency on a

EIC $\sqrt{s} = 140$ GeV



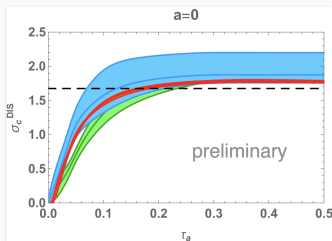
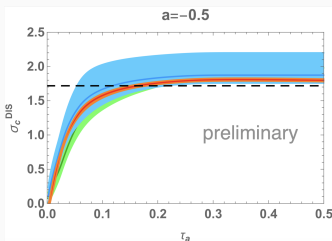
EIC $\sqrt{s} = 45$ GeV



- Peak shifts towards right for larger value of angularity parameter a .

Cumulant

$$\sigma_c(x, Q^2, \tau_a) = \frac{1}{\sigma_0} \int_0^{\tau_a} d\tau'_a \frac{d\sigma}{dx dQ^2 d\tau'_a}.$$



- Deviation of NNLL prediction from the total cross-section (at $\mathcal{O}(\alpha_s)$) at large angularity is due to the non singular term that not considered.
- Non-singular contribution decreases with the decreasing angularity parameter a .

Summary & Future direction

- ▶ We define DIS angularity event shape, τ_a^{DIS} , for the axis aligned to the jet axis and present angularity beam function at NNLL accuracy.
- ▶ We present precision prediction to differential cross-section and cumulant for the future EIC kinematics which could be early prediction to future Electron-Ion-Collider.

[In collaboration with: Daekyoung Kang, IMP, Fudan University]

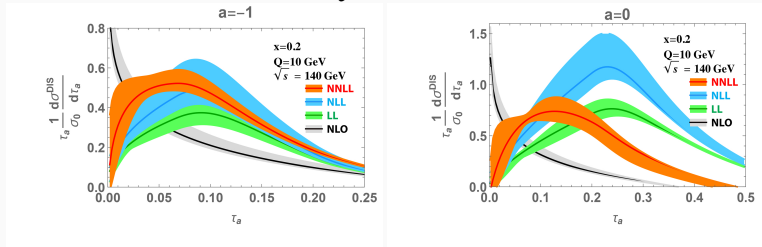
Towards the precision measurement for electron proton scattering
(DIS)

“And miles to go before I sleep”

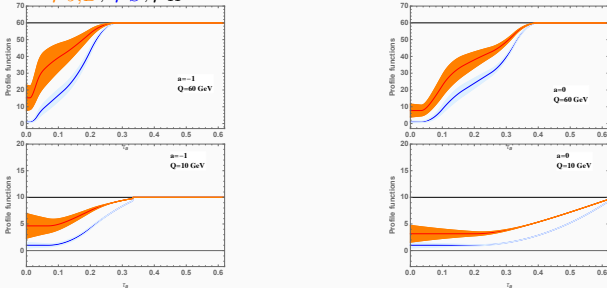
Thank you!

Cross-section dependency on a

Low $Q = 10$ GeV



Profile function: $\mu_{J,B}$, μ_S , μ_H



- Choice of other scale profile functions may improve tail region for low Q .

Profile Function

