Epstein-Glaser's Causal Light-Front Field Theory

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ID tetrad basis $\left\{\tilde{\mathbf{e}}_{(0)}; \cdots ; \tilde{\mathbf{e}}_{(3)}\right\}$: basis of unit orthogonal vectors in Minkowski's space such that: $\left[\eta_{(\mu)(\nu)}\right] = \mathrm{diag}(1; -1; -1; -1)$.
Define NP tetrad basis (basis for light-front dynamics):

$$\mathbf{e}_{(+)} := \frac{1}{\sqrt{2}}\left(\tilde{\mathbf{e}}_{(0)} + \tilde{\mathbf{e}}_{(3)}\right) \quad , \quad \mathbf{e}_{(\perp)} := \tilde{\mathbf{e}}_{(\perp)} \;\; (\perp \equiv 1, 2) \quad , \quad \mathbf{e}_{(-)} := \frac{1}{\sqrt{2}}\left(\tilde{\mathbf{e}}_{(0)} - \tilde{\mathbf{e}}_{(3)}\right) \quad .$$

Properties $(\alpha, \beta = 1, 2)$:

$$\begin{array}{c} \mathrm{e}_{(+)} \cdot \mathrm{e}_{(+)} = 0 = \mathrm{e}_{(-)} \cdot \mathrm{e}_{(-)} \quad , \quad \mathrm{e}_{(+)} \cdot \mathrm{e}_{(-)} = 1 \quad , \\ \mathrm{e}_{(\alpha)} \cdot \mathrm{e}_{(\beta)} = -\delta_{(\alpha)(\beta)} \quad , \quad \mathrm{e}_{(+)} \cdot \mathrm{e}_{(\alpha)} = 0 = \mathrm{e}_{(-)} \cdot \mathrm{e}_{(\alpha)} \end{array} \Rightarrow \left[\eta_{(a)(b)}\right] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad .$$

Invariant NP components of a vector:

$$\mathrm{A} = A^{(a)}\mathrm{e}_{(a)} \;\; ; \;\; A^{(+)} := \mathrm{A} \cdot \mathrm{e}^{(+)} \quad , \quad A^{(\perp)} := \mathrm{A} \cdot \mathrm{e}^{(\perp)} \quad , \quad A^{(-)} := \mathrm{A} \cdot \mathrm{e}^{(-)} \quad .$$

Scalar product:

$$\mathrm{A} \cdot \mathrm{B} = A^{(+)}B^{(-)} + A^{(-)}B^{(+)} - A^{(\perp)}B^{(\perp)} \quad .$$

Choice: $\mathrm{e}_{(+)}$ is the NP-time direction, $x^{(+)}$ is the invariant NP-time.

$S(g)$-operator and T_n distributions

$S(g)$ operator definition:

$$S(g) := 1 + \sum_{n=1}^{+\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n T_n(x_1; \cdots ; x_n) g(x_1) \cdots g(x_n) \quad .$$

Notation: $X := \{x_j \in \mathbb{M} : j = 1, \cdots , n\}$, $T_n \equiv T_n(X), g(X) \equiv g(x_1) \cdots g(x_n)$, $dX \equiv d^4x_1 \cdots d^4x_n$. Then:

$$S(g) = 1 + \sum_{n=1}^{+\infty} \frac{1}{n!} \int dX T_n(X) g(X) \quad .$$

Inverse operator:

$$S(g)^{-1} = 1 + \sum_{n=1}^{+\infty} \frac{1}{n!} \int dX \tilde{T}_n(X) g(X) \quad ; \quad \tilde{T}_n(X) = \sum_{r=1}^{n} (-1)^r \sum_{P_r} T_{n_1}(X_1) \cdots T_{n_r}(X_r) \quad .$$

P_r: partition of X in the r non-empty disjoint subsets $X_1, \cdots , X_r$.
Translation invariance:

$$T_n(x_1; \cdots ; x_n) = T_n(x_1 - x_n; \cdots ; x_{n-1} - x_n; 0) \quad .$$

Causality Condition

Two points $x, y \in \mathbb{M}$ are causally connected if their difference $x - y$ is a time-like vector or a light-like one. For $X, Y \subset \mathbb{M}$ define the relations:

$$X < Y :\Leftrightarrow \ \forall x \in X, y \in Y : \ x^{(+)} < y^{(+)} \ ; \ X \sim Y :\Leftrightarrow \ \forall x \in X, y \in Y : \ (x-y)^2 < 0 \ .$$

Causality Axiom:

1.- Let $g_1, g_2 \in \mathcal{S}(\mathbb{R}^4)$ with causally connected supports, $\mathrm{supp}(g_1) < \mathrm{supp}(g_2)$. Then:

$$S(g_1 + g_2) = S(g_2) S(g_1) \quad .$$

2.- Let $g_1, g_2 \in \mathcal{S}(\mathbb{R}^4)$ two switching functions such that $\mathrm{supp}(g_1) \sim \mathrm{supp}(g_2)$. Then:

$$S(g_1 + g_2) = S(g_2) S(g_1) = S(g_1) S(g_2) \quad . \ \square$$

Immediate consequence: For $X_1 < X_2$ or $X_1 \sim X_2$ we can perform the following decomposition:

$$T_n(X) = T_m(X_2) T_{n-m}(X_1) \quad ; \quad \tilde{T}_n(X) = \tilde{T}_m(X_1) \tilde{T}_{n-m}(X_2) \quad .$$

For $X_1 \sim X_2$ the order of the decomposition is arbitrary.

Inductive Construction Idea

Suppose we know $T_m, \tilde{T}_m$ $(m = 1, \cdots, n-1)$. We want to find the next order T_n distribution. Define:

$$A'_n(X) := \sum_{P_2} \tilde{T}_{n_1}(X_1) T_{n-n_1}(X_2 \cup \{x_n\}) \quad , \quad R'_n(X) := \sum_{P_2} T_{n-n_1}(X_2 \cup \{x_n\}) \tilde{T}_{n_1}(X_1) \quad ,$$

P_2: partition of $X \setminus \{x_n\}$ in disjoint X_1, X_2; $X_1 \neq \emptyset$.

Advanced and **retarded** distributions of order n:

$$A_n(X) := \sum_{P_2^{(0)}} \tilde{T}_{n_1}(X_1) T_{n-n_1}(X_2 \cup \{x_n\}) = A'_n(X) + T_n(X) \quad ,$$

$$R_n(X) := \sum_{P_2^{(0)}} T_{n-n_1}(X_2 \cup \{x_n\}) \tilde{T}_{n_1}(X_1) = R'_n(X) + T_n(X) \quad .$$

$P_2^{(0)}$: now in P_2 it is allowed $X_1 = \emptyset$. We see that the T_n distribution can be found as:

$$T_n(X) = A_n(X) - A'_n(X) = R_n(X) - R'_n(X) \quad .$$

Causal distribution:

$$D_n(X) := R'_n(X) - A'_n(X) = R_n(X) - A_n(X) \quad .$$

Terminology:

$$\tilde{V}^{\pm}(x) := \left\{ y \in \mathbb{M} \,\middle|\, (y-x)^2 \geq 0; y^{(+)} \geq / \leq x^{(+)} \right\} \quad = \overline{V^{\pm}}(x) \cup x^{(-)} - \text{axis} \quad .$$

Sets of n points:

$$\Gamma_n^{\pm}(x) := \left\{ (x_1; \cdots ; x_n) \in \mathbb{M}^n \,\middle|\, \forall j \in \{1, \cdots , n\} \;:\; x_j \in \tilde{V}^{\pm}(x) \right\} \quad .$$

Theorem 1: For $n \geq 3$:

$$\text{supp}\left(D_n(x_1; \cdots ; x_n) \right) \subseteq \Gamma_n^{+}(x_n) \cup \Gamma_n^{-}(x_n) \quad .$$

Moreover, $D_n(X)$ can be non null only if $X \in \Gamma_n^{+}(x_n)$ or $X \in \Gamma_n^{-}(x_n)$. □

Theorem 2: Let $D_n(X)$ be a causal distribution with causal support, which therefore can be splitted into a retarded and advanced distributions $R_n(X)$ and $A_n(X)$, with supports in $\Gamma_n^{+}(x_n)$ and $\Gamma_n^{-}(x_n)$, respectively. The distribution $T_n(X)$ constructed with them satisfies the causality conditions. □

Causal distribution of order n: $D_n(X) = \sum_k d_n^k(X) \; :C_k(\psi^A):$.
Define $d \in \mathcal{S}(\mathbb{R}^{4n-4})$ as: $d(X) := d_n^k(x_1 - x_n; \cdots ; x_{n-1} - x_n; 0)$; $\text{supp}(d) \subseteq \Gamma^+_{n-1}(0) \cup \Gamma^-_{n-1}(0)$, which is split as:

$$d = r - a \quad ; \quad \text{supp}(r) \subseteq \Gamma^+_{n-1}(0) \quad , \quad \text{supp}(a) \subseteq \Gamma^-_{n-1}(0) \quad .$$

First proposal:

$$r(X) = \chi(X) d(X) \quad \text{with} \quad \chi(X) := \prod_{j=1}^{n-1} \Theta\left(x_j^{(+)} - x_n^{(+)}\right) \quad .$$

$\{\text{Discontinuity surface of } \chi(X)\} \cap \text{supp}(d) = X^{(-)}-$axis $\Rightarrow$ the behaviour of $d(X)$ in the neighbourhood of the $X^{(-)}-$axis is essential for the splitting procedure.

Definition 1: Let $d \in \mathcal{S}'(\mathbb{R}^m)$, ρ be a positive-definite continuous function. If the limit

$$\lim_{s \to 0^+} \rho(s) s^{3m/4} d\left(sX^{(+)}; sX^{(\perp)}; X^{(-)}\right) = d_-(X) \neq 0$$

exists in $\mathcal{S}'(\mathbb{R}^m)$, then d_- is the quasi-asymptote of d at the $X^{(-)}-$axis, with regard to the function ρ. □

Definition 2: If the quasi-asymptote of d at the $X^{(-)}-$axis is obtained for $\rho(s) = s^{\omega_-}$, then ω_- is the singular order of d at the $X^{(-)}-$axis. □

Idea for the splitting: Substitute the discontinuous Heaviside's functions by a continuous function which goes to it in some limit. Define the continuous non-decreasing function:

$$\chi(t) := \begin{cases} 0 & ; \quad t \leq 0 \\ < 1 & ; \quad 0 < t < 1 \\ 1 & ; \quad t \geq 1 \end{cases} \quad .$$

The retarded and advanced distributions will be given by the limits:

$$r(X) = \lim_{s\to 0} \chi\left(\frac{X^{(+)}}{s}\right) d(X) \quad , \quad a(X) = -\lim_{s\to 0} \chi\left(-\frac{X^{(+)}}{s}\right) d(X) \quad .$$

Using Cauchy's criterion, the above limit exists if $\omega_- < 0$, and it could not exist for $\omega_- \geq 0$.

Negative singular order ($\omega_- < 0$):

$$r(X) = \lim_{s\to 0} \chi\left(\frac{X^{(+)}}{s}\right) d(X) \equiv \Theta\left(X^{(+)}\right) d(X) \quad .$$

Definition 3: Let $\hat{d} \in \mathcal{S}'(\mathbb{R}^m)$, and let ρ be a positive-definite continuous function. If the limit

$$\lim_{s\to 0^+} \rho(s) \left(\hat{d}\left(\frac{P_{(+)}}{s}; \frac{P_{(\perp)}}{s}; P_{(-)} \right); \check{\varphi}(P) \right) = (\hat{d}_-; \check{\varphi})$$

exists $\forall \check{\varphi} \in \mathcal{S}(\mathbb{R}^m)$, then $\hat{d}_-$ is the quasi-asymptote of $\hat{d}$ at $P_{(+)}, P_{(\perp)} \to +\infty$. □

Negative singular order ($\omega_- < 0$):

$$\hat{r}(P) = \frac{i}{2\pi} \mathrm{sgn}\left(p_{1(+)}\right) \int_{-\infty}^{+\infty} \frac{\hat{d}\left(\left(tp_{1(+)}; \boldsymbol{p}_1\right); p_2; \cdots; p_{n-1}\right)}{1 - t + \mathrm{sgn}\left(p_{1(+)}\right) i0^+} dt \quad .$$

Scalar field: Commutation distribution: $\hat{D}(p) = \frac{i}{2\pi}\text{sgn}\left(p_{(-)}\right)\delta(p^2 - m^2)$, with $\omega_- = -2$.

$$\hat{D}^{ret}(p) = -(2\pi)^{-2}\text{sgn}\left(p_{(+)}\right)\int\limits_{-\infty}^{+\infty}\frac{\text{sgn}\left(p_{(-)}\right)\delta\left(2tp_{(+)}p_{(-)} - p_{(\perp)}^2 - m^2\right)}{1 - t + \text{sgn}\left(p_{(+)}\right) i0^+}dt$$

$$= -(2\pi)^{-2}\frac{\text{sgn}\left(p_{(+)}p_{(-)}\right)}{\left|2p_{(+)}p_{(-)}\right|}\int\limits_{-\infty}^{+\infty}\frac{\delta\left(t - \dfrac{\omega_p^2}{2p_{(+)}p_{(-)}}\right)}{1 - t + \text{sgn}\left(p_{(+)}\right) i0^+}dt$$

$$= -(2\pi)^{-2}\frac{1}{p^2 - m^2 + \text{sgn}\left(p_{(-)}\right) i0^+} \quad .$$

Feynman's propagator, $\hat{D}^F(p) := \hat{D}^{ret}(p) - \hat{D}^{(-)}(p) = -(2\pi)^{-2}\dfrac{1}{p^2 - m^2 + i0^+}$.

Fermion field: Comm. dist.: $\hat{S}(p) = \frac{i}{2\pi}(\not{p} + m)\text{sgn}\left(p_{(-)}\right)\delta(p^2 - m^2)$ has $\omega_- = -1$.

$$\hat{S}^{ret}(p) = -(2\pi)^{-2}\left(\frac{\not{p} + m}{p^2 - m^2 + \text{sgn}\left(p_{(-)}\right) i0^+} - \frac{\gamma^{(+)}}{2p_{(-)}}\right) \quad .$$

Feynman's propagator: $\hat{S}^F(p) := \hat{S}^{(-)}(p) - \hat{S}^{ret}(p) = (2\pi)^{-2}\left(\frac{\not{p} + m}{p^2 - m^2 + i0^+} - \frac{\gamma^{(+)}}{2p_{(-)}}\right)$.

Radiation field in NP-gauge: Commutation distribution:
$\hat{D}_{(a)(b)}(k) = \frac{i}{2\pi}\text{sgn}\left(k_{(-)}\right)\delta(k^2)\left(\eta_{(a)(b)} - \frac{k_{(a)}\eta_{(b)} + \eta_{(a)}k_{(b)}}{k_{(-)}}\right)$, with $\omega_- = -2, -1$.

$$\hat{D}^{ret}_{(a)(b)}(k) = -\frac{(2\pi)^{-2}}{k^2 + \text{sgn}\left(k_{(-)}\right) i0^+}\left\{\eta_{(a)(b)} - \frac{k_{(a)}\eta_{(b)} + \eta_{(a)}k_{(b)}}{k_{(-)}} + \frac{k^2}{k^2_{(-)}}\eta_{(a)}\eta_{(b)}\right\} \quad .$$

Feynman's prop.: $\hat{D}^F_{(a)(b)}(k) = -\frac{(2\pi)^{-2}}{k^2 + i0^+}\left\{\eta_{(a)(b)} - \frac{k_{(a)}\eta_{(b)} + \eta_{(a)}k_{(b)}}{k_{(-)}} + \frac{k^2}{k^2_{(-)}}\eta_{(a)}\eta_{(b)}\right\}$.

Instantaneous terms of Feynman's propagators are a consequence of the splitting of the causal distribution.

Splitting of the Causal Distribution: Configuration Space II

Non-negative singular order ($\omega_- \geq 0$): The splitting is trivial for test functions:

$$\varphi(X) = \left(X^{(+,\perp)}\right)^{\lfloor \omega_- \rfloor + 1} \varphi_0(X) \;\Leftrightarrow\; D^{b'}_{(+,\perp)} \varphi\left(0; 0^{(\perp)}; X^{(-)}\right) = 0 \text{ for } b' \leq \lfloor \omega_- \rfloor \; .$$

For a general test function, define the projection operator W as:

$$(W\varphi)(X) := \varphi(X) - w(X) \sum_{b=0}^{\lfloor \omega_- \rfloor} \frac{\left(X^{(+,\perp)}\right)^b}{b!} D^{b}_{(+,\perp)} \varphi\left(0; 0^{(\perp)}; X^{(-)}\right) \quad ,$$

with $w \in \mathcal{S}\left(\mathbb{R}^m\right)$ an auxiliary function such that:

$$w\left(0; 0^{(\perp)}; X^{(-)}\right) = 1 \quad , \quad D^{c}_{(+,\perp)} w\left(0; 0^{(\perp)}; X^{(-)}\right) = 0 \;\text{ for }\; 1 \leq c \leq \lfloor \omega_- \rfloor \quad .$$

With these considerations, the retarded distribution is defined in the following way:

$$(r; \varphi) := \left(\lim_{s \to 0} \chi\left(\frac{X^{(+)}}{s}\right) d(X); (W\varphi)(X)\right) \equiv (d; \Theta W \varphi) \quad .$$

Non-negative singular order ($\omega_- \geq 0$):

$$\hat{r}(P) = (2\pi)^{-\frac{m}{2}} \int dQ \hat{\Theta}(Q) \Big(\hat{d}(P-Q)$$
$$-(2\pi)^{-\frac{m}{2}} \sum_{b=0}^{\lfloor \omega_- \rfloor} \frac{P^b_{(+,\perp)}}{b!} \int dP' D^b_{P'(+,\perp)} \hat{d}(P'-Q) \check{w}\left(P'_{(+)}; P'_{(\perp)}; P'_{(-)} - P_{(-)}\right)^* \Big) \quad .$$

$r = d$ on $\Gamma^+_{n-1}(0) \setminus X^{(-)}$−axis, even if we add to r terms which are non-zero on the $X^{(-)}$−axis. Those additional terms can be chosen in such a way that the dependence with w vanishes: Retarded distribution with normalization line $K = \left(K_{(+)}; K_{(\perp)}; P_{(-)}\right)$:

$$\hat{r}_K(P) = \frac{i}{2\pi} \int \frac{dq}{q + i0^+} \Big\{ \hat{d}\left(\left(p_{1(+)} - q; \boldsymbol{p}_1\right); p_2; \cdots; p_{n-1}\right)$$
$$- \sum_{b=0}^{\lfloor \omega_- \rfloor} \frac{\left(P_{(+,\perp)} - K_{(+,\perp)}\right)^b}{b!} D^b_{K(+,\perp)} \hat{d}\left(\left(k_{1(+)} - q; \boldsymbol{k}_1\right); k_2; \cdots; k_{n-1}\right) \Big\} \quad ,$$

which satisfies the normalization condition:

$$D^b_{(+,\perp)} \hat{r}_K(K) = 0 \quad ; \quad b \leq \lfloor \omega_- \rfloor \quad .$$

Central splitting solution: $K = \left(0; 0_{(\perp)}; P_{(-)}\right)$.

$(r; a)$ and $(\tilde{r}; \tilde{a})$: two solutions of the splitting problem.

$$d = r - a = \tilde{r} - \tilde{a} \Rightarrow r - \tilde{r} = a - \tilde{a} .$$

Support of l.h.s.: $\Gamma^{+}_{n-1}(0)$, support of r.h.s.: $\Gamma^{-}_{n-1}(0)$, hence the equality can hold only if the support of the above quantities is the intersection of those two sets, that is, the $X^{(-)}$-axis. That means that r and $\tilde{r}$ can only differ by *normalization terms* of the form:

$$r(X) - \tilde{r}(X) = \sum_{a=0}^{\lfloor \omega^{r}_{-} \rfloor} C_a \left(X^{(-)}\right) D^{a}_{(+,\perp)} \delta\left(X^{(+,\perp)}\right) \quad ,$$

with $C_a\left(X^{(-)}\right)$ some distributions of the variables $X^{(-)}$. Those normalization terms only appear for $\omega^{r}_{-} \geq 0$, while the retarded solution for $\omega^{r}_{-} < 0$ is unique.
In momentum space, the indefinite terms are:

$$\sum_{a=0}^{\lfloor \omega^{r}_{-} \rfloor} \hat{C}_a \left(P_{(-)}\right) P^{a}_{(+,\perp)} \quad .$$

This normalization terms (the distributions $\hat{C}_a$) must be fixed by other physical conditions besides causality.

In CPT the one-point distribution is defined as (linear in e):

$$T_1(x) = -i \ :j_{(a)}(x): \ A^{(a)}(x) \quad ; \quad :j_\mu(x): = ie \ :\varphi^\dagger(x)\overleftrightarrow{\partial}_\mu\varphi(x): \quad .$$

To go to second order we need:

$$A_2'(x_1;x_2) = \tilde{T}_1(x_1)T_1(x_2) = -T_1(x_1)T_1(x_2)\ , \ R_2'(x_1;x_2) = -T_1(x_2)T_1(x_1)\ .$$

Causal distribution of second order ($y \equiv x_1 - x_2$):

$$\begin{aligned} D_2(x_1;x_2) = e^2 \Big\{ \Big[&:\varphi^\dagger(x_1)\partial_\mu\varphi(x_1)\varphi^\dagger(x_2)\partial_\nu\varphi(x_2): - :\varphi^\dagger(x_1)\partial_\mu\varphi(x_1)\partial_\nu\varphi^\dagger(x_2)\varphi(x_2): \\ &- :\partial_\mu\varphi^\dagger(x_1)\varphi(x_1)\varphi^\dagger(x_2)\partial_\nu\varphi(x_2): + :\partial_\mu\varphi^\dagger(x_1)\varphi(x_1)\partial_\nu\varphi^\dagger(x_2)\varphi(x_2): \Big] iD^{\mu\nu}(y) \quad (1) \\ &+ \Big[- :\varphi(x_1)\varphi^\dagger(x_2): \ i\partial_\mu\partial_\nu D(y) + :\partial_\mu\varphi(x_1)\varphi^\dagger(x_2): \ i\partial_\nu D(y) \\ &- :\varphi(x_1)\partial_\nu\varphi^\dagger(x_2): \ i\partial_\mu D(y) + :\partial_\mu\varphi(x_1)\partial_\nu\varphi^\dagger(x_2): \ iD(y) \\ &- :\varphi^\dagger(x_1)\varphi(x_2): \ i\partial_\mu\partial_\nu D(y) + :\partial_\mu\varphi^\dagger(x_1)\varphi(x_2): \ i\partial_\nu D(y) \\ &- :\varphi^\dagger(x_1)\partial_\nu\varphi(x_2): \ i\partial_\mu D(y) + :\partial_\mu\varphi^\dagger(x_1)\partial_\nu\varphi(x_2): \ iD(y) \Big] :A^\mu(x_1)A^\nu(x_2): + \cdots \Big\} \ . \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2) \end{aligned}$$

Moeller's scattering: $D_2^M(x_1;x_2) = -iD_{(a)(b)}(y)\ :j^{(a)}(x_1)j^{(b)}(x_2):$.

$$T_2^M(x_1;x_2) = R_2^M(x_1;x_2) - {R'}_2^M(x_1;x_2) = -iD^F_{\mu\nu}(y)\ :j^{\mu}(x_1)j^{\nu}(x_2):$$

Second order S_2 operator in the adiabatic limit ($g \to 1$) with normalization term allowed for $\omega^r_- = 0$:

$$S_2^M = -\frac{i}{2}(2\pi)^{-2}\int d^4k d^4x_1 d^4x_2 e^{-iky}\left(\hat{D}^F_{\mu\nu}(k) + \hat{C}\left(k_{(-)}\right)\right)\ :j^{\mu}(x_1)j^{\nu}(x_2): \quad .$$

Choosing (locality condition):

$$\hat{C}\left(k_{(-)}\right) = (2\pi)^{-2}\frac{\eta_\mu\eta_\nu}{k^2_{(-)}}$$

$$\Rightarrow\ \hat{D}^F_{\mu\nu}(k) + \hat{C}\left(k_{(-)}\right) = -(2\pi)^{-2}\frac{1}{k^2+i0^+}\left(g_{\mu\nu} - \frac{k_\mu\eta_\nu + \eta_\mu k_\nu}{k_{(-)}}\right) .$$

Application: SQED_4

We identify the chosen term $\hat{C}\left(k_{(-)}\right)$ as the instantaneous term of the interaction Lagrangian density in the usual approach, which has arose in CPT as a normalization term by imposing a locality condition:

$$S_2^M = \cdots - \frac{i}{2}\int d^4x_1 \;\; \boldsymbol{:} j^{(+)}(x_1)\frac{1}{\partial^2_{(-)}} j^{(+)}(x_1)\boldsymbol{:} \qquad .$$

For the initial and final defined momenta states

$$b^\dagger(\boldsymbol{q}_1)b^\dagger(\boldsymbol{p}_1)\Omega \quad \text{and} \quad b^\dagger(\boldsymbol{q}_2)b^\dagger(\boldsymbol{p}_2)\Omega$$

the contribution of the remaining non-local terms vanish, so that the result is equivalent to use simply:

$$-(2\pi)^{-2}\frac{g_{\mu\nu}}{k^2 + i0^+} \qquad .$$

This proves the equivalence with instant dynamics for Moeller's scattering of scalar particles.

Compton's scattering: In D_2^C appear the numerical distributions ($\alpha, \beta = 1, 2$):

$$\omega_-[D] = -2 \quad ; \quad \omega_-\left[\partial_{(-)} D\right] = -2 \quad , \quad \omega_-\left[\partial_{(\alpha)} D\right] = -1 \quad ;$$
$$\omega_-\left[\partial_{(-)}^2 D\right] = -2 \quad , \quad \omega_-\left[\partial_{(-)}\partial_{(\alpha)} D\right] = -1 \quad , \quad \omega_-\left[\partial_{(\alpha)}\partial_{(\beta)} D\right] = 0 \quad .$$

Performing the splitting and writing the normalization terms:

$$\begin{aligned}
T_2^C(x_1; x_2) = ie^2 \; &: A^{(a)}(x_1) A^{(b)}(x_2) : \\
\times \Big\{ &- \Big(:\varphi(x_1)\varphi^\dagger(x_2): + :\varphi^\dagger(x_1)\varphi(x_2): \Big) \partial_{(a)}\partial_{(b)} D^F(y) \\
&+ \Big(:\partial_{(a)}\varphi(x_1)\varphi^\dagger(x_2): + :\partial_{(a)}\varphi^\dagger(x_1)\varphi(x_2): \Big) \partial_{(b)} D^F(y) \\
&- \Big(:\varphi(x_1)\partial_{(b)}\varphi^\dagger(x_2): + :\varphi^\dagger(x_1)\partial_{(b)}\varphi(x_2): \Big) \partial_{(a)} D^F(y) \\
&+ \Big(:\partial_{(a)}\varphi(x_1)\partial_{(b)}\varphi^\dagger(x_2): + :\partial_{(a)}\varphi^\dagger(x_1)\partial_{(b)}\varphi(x_2): \Big) D^F(y) \Big\} \\
- ie^2 \; &: A^{(\perp)}(x_1) A^{(\perp)}(x_2) : \Big\{ C\left(y^{(-)}\right) :\varphi(x_1)\varphi^\dagger(x_2): \\
&+ C'\left(y^{(-)}\right) :\varphi^\dagger(x_1)\varphi(x_2): \Big\} \delta\left(y^{(+,\perp)}\right) \quad .
\end{aligned}$$

Application: $SQED_4$

We use physical conditions to fix the normalization terms:

(A) Charge conjugation invariance:

$$(\mathbf{U}_\varphi \otimes \mathbf{U}_A)\, T_n(x_1; \cdots; x_n)\, (\mathbf{U}_\varphi \otimes \mathbf{U}_A)^\dagger = T_n(x_1; \cdots; x_n) \quad ,$$

with: $\mathbf{U}_\varphi \varphi(x) \mathbf{U}_\varphi^\dagger = \varphi^\dagger(x)$, $\mathbf{U}_A A^\mu(x) \mathbf{U}_A^\dagger = -A^\mu(x)$. Then: $C\left(y^{(-)}\right) = C'\left(y^{(-)}\right)$.

(B) Residual gauge invariance: Imposing that T_2^C is invariant under $A'^{(a)}(x) = A^{(a)}(x) + \partial^{(a)}\Lambda\left(x^{(+)}; x^{(\perp)}\right)$, which guarantees $A'^{(+)} = A^{(+)} = 0$:

$$\begin{aligned}
&- \left[:\varphi^\dagger(x_2)\varphi(x_1): + :\varphi^\dagger(x_1)\varphi(x_2): \right] \left(\partial_{(b)}\delta(y) - \partial_{(b)}\left[C\left(y^{(-)}\right)\delta\left(y^{(+,\perp)}\right)\right]\right) \\
&+ \left[:\varphi^\dagger(x_2)\partial_{(b)}\varphi(x_1): + :\partial_{(b)}\varphi^\dagger(x_1)\varphi(x_2): \right] C\left(y^{(-)}\right)\delta\left(y^{(+,\perp)}\right) \\
&- \left[:\partial_{(b)}\varphi^\dagger(x_2)\varphi(x_1): + :\varphi^\dagger(x_1)\partial_{(b)}\varphi(x_2): \right] \delta(y) = 0 \;\Rightarrow\; C\left(y^{(-)}\right) = \delta\left(y^{(-)}\right) \;.
\end{aligned}$$

The contribution of this normalization terms to S_2 in the adiabatic limit is:

$$\frac{1}{2!}\int d^4x_1 d^4x_2\, T_2^C(x_1; x_2) = \cdots - ie^2 \int d^4x_1 \;:A^{(\perp)}(x_1)A^{(\perp)}(x_1): \;:\varphi^\dagger(x_1)\varphi(x_1): \quad .$$

Application: $\overline{\psi}\Gamma\psi\varphi$ Model

φ: Neutral spin zero field; ψ: fermion field. One-point distribution:

$$T_1(x) = -ig \ :\overline{\psi}(x)\Gamma\psi(x): \ \varphi(x) \quad .$$

$\Gamma = 1$ for φ scalar, $\Gamma = \gamma^{(5)} := i\gamma^{(0)}\gamma^{(1)}\gamma^{(2)}\gamma^{(3)}$ for φ pseudoscalar.

Boson-fermion scattering:

$$D_2^{(BF)}(x_1; x_2) = ig^2 \left(:\overline{\psi}(x_1)\Gamma S(y)\Gamma\psi(x_2): - :\overline{\psi}(x_2)\Gamma S(-y)\Gamma\psi(x_1): \right) :\varphi(x_1)\varphi(x_2): \quad .$$

Singular order: $\omega_- = -1$. Performing the splitting and writing the normalization terms:

$$T_2^{(BF)}(x_1; x_2) = \Big\{ :\overline{\psi}(x_1) \left(-ig^2\Gamma S^F(y)\Gamma + C\left(y^{(-)}\right)\delta\left(y^{(+,\perp)}\right)\right)\psi(x_2): - :\overline{\psi}(x_2) \left(-ig^2\Gamma S^F(-y)\Gamma + C'\left(y^{(-)}\right)\delta\left(y^{(+,\perp)}\right)\right)\psi(x_1): \Big\} :\varphi(x_1)\varphi(x_2): \quad .$$

Choosing $\hat{C}\left(p_{(-)}\right) = -i(2\pi)^{-2}g^2\epsilon\dfrac{\gamma^{(+)}}{2p_{(-)}} = -\hat{C}'\left(p_{(-)}\right)$ ($\epsilon = \pm 1$ for $\Gamma = 1, \gamma^{(5)}$) the instantaneous part in $\hat{S}^F(y)$ is cancelled (locality condition).
Contribution of normalization terms to S_2 in the adiabatic limit:

$$\frac{1}{2!}\int d^4x_1 d^4x_2\, T_2^{(BF)}(x_1; x_2) = \cdots + \int d^4x_1 \ :\varphi(x_1)\varphi(x_1): \ :\overline{\psi}(x_1)\epsilon g^2\frac{\gamma^{(+)}}{2\partial_{(-)}}\psi(x_1): \quad .$$

Conclusions

- When the causality axiom is referred to the $x^{(+)}$ coordinate, causality theorems in LF dynamics allows the retarded and advanced distributions to be non-null on the entire $x^{(-)}-$axis. Then the normalization terms are defined in this region.
- Instantaneous terms in the propagators arise in the splitting procedure of the commutation distributions.
- In the applications, the normalization distributions can be chosen in such a way that locality is preserved. They reproduce the non-local terms in the Lagrangian density in the usual approach.
- Residual gauge invariance implies the arising of the second order vertex of SQED as a normalization term.

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