Sub-eikonal corrections and low-x helicity evolution

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Outline

- Motivation
- Brief review of Operator Product Expansion at high-energy
- Operator Product Expansion at high-energy with sub-eikonal corrections
  - Quark and Gluon propagator with sub-eikonal corrections
- Evolution equation for flavor non-Singlet F_2 structure function at small-x.
- Evolution equation for flavor non-Singlet and Singlet g_1 structure function at small-x.
- Conclusions and Outlook

Based on

G. A. C. JHEP 1901 (2019) 118
G. A. C. in preparation

Motivation

- Unpolarized DIS at low-x_B: dynamics is driven by gluon structure functions
  - gluon structure function grows as $(1/x_B)^\lambda$ with $\lambda > 1$.
  - quark structure function are sub-leading.
- Polarized DIS at low-x_B: polarized gluon structure function grows as $(1/x_B)^\lambda$ with λ close to 0.
  - The polarized quark and gluon structure functions are equally relevant.
- At Electron Ion Collider low-x_B spin TMDs and g_1 structure function are relevant
  - Highly polarized ($\sim 70\%$) electron and nucleon beams
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.

Motivation

- Compare with results obtained in the Leading Log approximation by Bartels-Ermolaev-Ryskin-(1995-1996) and recent work in Saturation formalism obtained by Kovchegov-Pytoniak-Sievert (20016-2017)
- Kovchegov-Pytoniak-Sievert (20016-2017)
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  - Singlet case Agrees with the Large N_c limit of BER for the Ladder Diagrams. They find disagreement for the non ladder diagrams.

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- R. Boussarie, Y. Hatta, F. Yuan (2019)
  - Have generalized BER result to include quark and gluon Orbital Angular Momentum

Spin sum rule

$$\frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g = \frac{1}{2}$$

- BER: quark and gluon helicity
- BHY: Orbital Angular Momentum

Boost of the fields $\quad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad x_* = \sqrt{\frac{s}{2}}x^+ \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

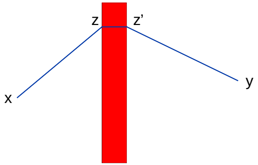
$$\begin{aligned}
A_\bullet(x_\bullet, x_*, x_\perp) &\rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\
A_*(x_\bullet, x_*, x_\perp) &\rightarrow \lambda^{-1} A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\
A_\perp(x_\bullet, x_*, x_\perp) &\rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)
\end{aligned}$$

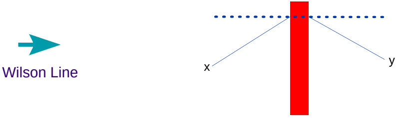
λ is the boost parameter.

$$\langle x| \frac{i}{\hat{P}\!\!\!/ + i\epsilon} |y\rangle \rightarrow \langle x| \frac{i}{\hat{p}\!\!\!/ + \alpha \frac{2}{s} p\!\!\!/_2 \hat{A}_\bullet + i\epsilon} |y\rangle$$

$[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $p\!\!\!/_2 \propto \gamma^+$

Infinite boost: particle does not have time to deviate from straight line





Eikonal interactions give a Wilson lines

$$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$$

$$[x, y] = Pe^{ig \int_0^1 du (x-y)^\mu A_\mu (ux + (1-u)y)} \qquad p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

Quark propagator with eikonal interactions

$$\langle x|\frac{i}{\hat{\not P}+i\epsilon}|y\rangle = \left[\int_0^{+\infty}\frac{đ\alpha}{2\alpha}\theta(x_* - y_*) - \int_{-\infty}^{0}\frac{đ\alpha}{2\alpha}\theta(y_* - x_*)\right] e^{-i\alpha(x_\bullet - y_\bullet)}$$

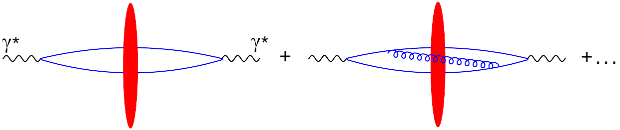
$$\times\frac{1}{\alpha s}\,\langle x_\perp|\,e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\hat{\not p}\not p_2\,[x_*, y_*]\,\hat{\not p}e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle$$

Quark propagator with eikonal interactions

$$\langle x|\frac{i}{\hat{\not P}+i\epsilon}|y\rangle = \left[\int_0^{+\infty}\frac{d\!\!\!{}^-\alpha}{2\alpha}\theta(x_*-y_*) - \int_{-\infty}^0\frac{d\!\!\!{}^-\alpha}{2\alpha}\theta(y_*-x_*)\right] e^{-i\alpha(x_\bullet-y_\bullet)}$$
$$\times\frac{1}{\alpha s}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\hat{\not p}\,\not p_2\,[\infty p_1+z_\perp, -\infty p_1+z_\perp]\,\hat{\not p}e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle$$

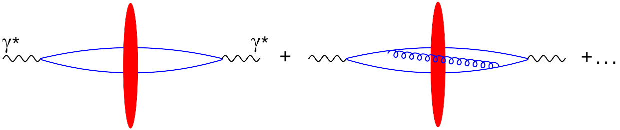
Semi-classical approach: Background field method

Diagrammatic representation of the Operator Product Expansion at High-energy



- The target is highly boosted

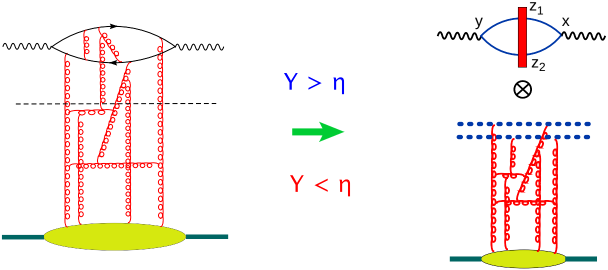
Diagrammatic representation of the Operator Product Expansion at High-energy



- The target is highly boosted
- High-energy OPE for DIS

$$\langle P|\mathrm{T}\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|P\rangle \simeq \int d^2z_1 d^2z_2\; I^{LO}_{\mu\nu}(z_1,z_2;x,y)\langle P|\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}|P\rangle$$
$$+\frac{\alpha_s}{\pi}\int d^2z_1 d^2z_2 d^2z_3\; I^{NLO}_{\mu\nu}(z_1,z_2,z_3;x,y)$$
$$\times\langle P|\Big[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_2}\}-N_c\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}\Big]|P\rangle$$

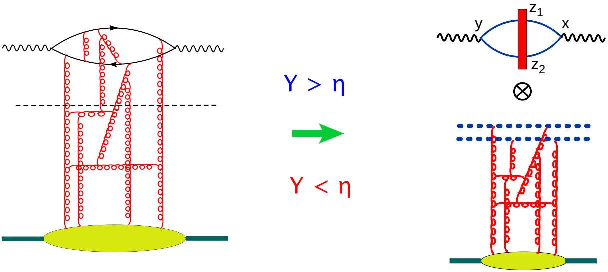
DIS amplitude is factorized in rapidity: η



$|P\rangle$ is the target state.

$$\langle P|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|P\rangle = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle P|{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|P\rangle + \dots$$

High-Energy Operator Product Expansion



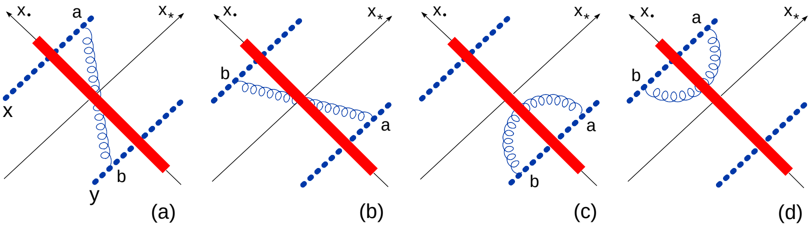
$$\langle P|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|P\rangle = \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle P|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|P\rangle + \ldots$$

- If we use a model to evaluate $\langle P|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|P\rangle$ we can calculate the DIS cross-section.
- If we want to include energy dependence to the DIS cross section, we need to find the evolution of $\langle P|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|P\rangle$ with respect to the rapidity parameter η.

Leading order: BK equation

$$\frac{d}{d\eta}\text{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\text{LO}}\text{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\text{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\text{shockwave}} = \langle K_{\text{LO}}\text{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\text{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$

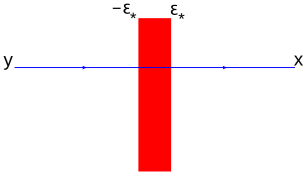
Non linear evolution equation: Balitsky-Kovchegov equation

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\;(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.
- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK eqn
  - background field method: describes recombination process.
- Note: if $x_\perp \;\to\; z_\perp$ or $y_\perp \;\to\; z_\perp$ divergences cancel out.

Shock-wave with finite width



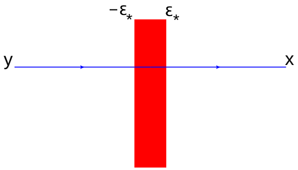
$x_* = \sqrt{\frac{s}{2}} x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}} x^-$

$$A_\bullet(x_\bullet, x_*, x_\perp) \rightarrow \lambda A_\bullet(\lambda^{-1} x_\bullet, \lambda x_*, x_\perp)$$
$$A_*(x_\bullet, x_*, x_\perp) \rightarrow \lambda^{-1} A_*(\lambda^{-1} x_\bullet, \lambda x_*, x_\perp)$$
$$A_\perp(x_\bullet, x_*, x_\perp) \rightarrow A_\perp(\lambda^{-1} x_\bullet, \lambda x_*, x_\perp)$$

λ is the boost parameter

- $p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$
- small α gluons are classical fields large α gluons are quantum fields.
- Longitudinal size classical fields: $\epsilon_* = \frac{\alpha s}{l_\perp^2}$ with $l_\perp$ trans. mom. of classical fileds
- Distance traveled by quantum fields: $z_* = \frac{\alpha s}{k_\perp^2}$ with $k_\perp$ trans. mom. of classical fileds
- We are in the case $l_\perp \sim k_\perp$

Shock-wave with finite width



$x_* = \sqrt{\frac{s}{2}} x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}} x^-$

$$
\begin{aligned}
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A_*(x_\bullet, x_*, x_\perp) &\to \lambda^{-1} A_*(\lambda^{-1} x_\bullet, \lambda x_*, x_\perp) \\
A_\perp(x_\bullet, x_*, x_\perp) &\to A_\perp(\lambda^{-1} x_\bullet, \lambda x_*, x_\perp)
\end{aligned}
$$

λ is the boost parameter

sub-eikonal terms go like $\frac{1}{\lambda}$

$$
\langle x|\frac{i}{\not{P} + i\epsilon}|y\rangle \;\to\; \langle x|\not{P}\,\frac{i}{p^2 + 2\alpha A_\bullet + ig\frac{2}{s}\not{p}_2\gamma^i F_{\bullet i} + \frac{1}{2}F_{ij}\sigma^{ij} + \ldots + i\epsilon}|y\rangle
$$

- Note: $[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}\, p^+$ and $\not{p}_2 \propto \gamma^+$

$$
e^{i\frac{\hat{p}_\perp^2}{\alpha s} z_*}\,\hat{A}_\bullet(z_*)\,e^{-i\frac{\hat{p}_\perp^2}{\alpha s} z_*} \simeq A_\bullet(z_*) - \frac{z_*}{\alpha s}\{p^i, F_{\bullet i}(z_*)\} - \frac{z_*^2}{2\alpha^2 s^2}\{p^j, \{p^i, D_j F_{\bullet i}(z_*)\}\} + \ldots
$$

Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

$$\langle x|\frac{i}{\hat{\not P} + i\epsilon}|y\rangle = \left[\int_0^{+\infty}\frac{d\!\!\!^- \alpha}{2s\alpha^2}\theta(x_* - y_*) - \int_{-\infty}^0 \frac{d\!\!\!^- \alpha}{2s\alpha^2}\theta(y_* - x_*)\right] e^{-i\alpha(x_\bullet - y_\bullet)}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}$$

$$\times\left\{\hat{\not p}\not p_2\,[x_*, y_*]\,\hat{\not p} + \hat{\not p}\not p_2\,\hat{\mathcal{O}}_1(x_*, y_*; p_\perp)\,\hat{\not p} + \frac{1}{2}\hat{\not p}\not p_2\,\hat{\mathcal{O}}_2(x_*, y_*) - \frac{1}{2}\hat{\mathcal{O}}_2(x_*, y_*)\,\not p_2\,\hat{\not p}\right\} e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle$$

$$+O(\lambda^{-2})$$

- Leading-eikonal term
- Sub-eikonal terms

Operators $\hat{\mathcal{O}}_1$ and $\hat{\mathcal{O}}_2$ *measure* the deviation from the straight line.

G. A. C. JHEP 1901 (2019) 118

Quark propagator with sub-eikonal corrections

$$x_* = \sqrt{\tfrac{s}{2}}\, x^+ \qquad x_\bullet = \sqrt{\tfrac{s}{2}}\, x^- \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$$

$$\hat{\mathcal{O}}_1(x_*, y_*; p_\perp) = \frac{ig}{2\alpha} \int_{y_*}^{x_*} d\frac{2}{s}\omega_* \Big([x_*, \omega_*]\frac{1}{2}\sigma^{ij} F_{ij}[\omega_*, y_*] + \{\hat{p}^i, [x_*, \omega_*]\, \frac{2}{s}\, \omega_*\, F_{i\bullet}(\omega_*)\, [\omega_*, y_*]\}$$
$$+ g\int_{\omega_*}^{x_*} d\frac{2}{s}\, \omega'_*\, \frac{2}{s}(\omega_* - \omega'_*)\, [x_*, \omega'_*] F^i{}_\bullet [\omega'_*, \omega_*]\, F_{i\bullet}\, [\omega_*, y_*] \Big)$$

$$\hat{\mathcal{O}}_2(x_*, y_*; p_\perp) = \frac{ig}{2\alpha} \int_{y_*}^{x_*} d\frac{2}{s}\omega_* \Big[[x_*, \omega_*]\frac{i}{4}\{(i\not{D}_\perp F_{ij}), \gamma^i\gamma^j\}[\omega_*, y_*] + \{\hat{p}^k, [x_*, \omega_*] iF_{kj}\gamma^j [\omega_*, y_*]\}$$
$$+ \int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_* \Big([x_*, \omega'_*] gF^k{}_\bullet [\omega'_*, \omega_*] iF_{kj}\gamma^j [\omega_*, y_*] - [x_*, \omega'_*] iF_{kj}\gamma^j [\omega'_*, \omega_*] gF^k{}_\bullet [\omega_*, y_*] \Big)$$
$$+ \int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_* \Big([x_*, \omega'_*] i\frac{2}{s} F_{\bullet *}[\omega'_*, \omega_*]\gamma^k g F_{k\bullet}[\omega_*, y_*]$$
$$- [x_*, \omega'_*]\gamma^k g F_{k\bullet}[\omega'_*, \omega_*] i\frac{2}{s} F_{\bullet *}[\omega_*, y_*] \Big) + (\hat{\alpha}\not{p}_1 - \hat{\not{p}}_\perp)[x_*, \omega_*]\, i\, \frac{2}{s} F_{\bullet *}[\omega_*, y_*] \Big]$$

G. A. C. JHEP 1901 (2019) 118

Quark propagator with sub-eikonal corrections

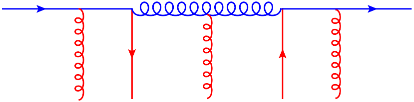
$$x_* = \sqrt{\tfrac{s}{2}}\,x^+ \qquad x_\bullet = \sqrt{\tfrac{s}{2}}\,x^- \qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$$

$$\hat{\mathcal{O}}_1(x_*,y_*;p_\perp) = \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big([x_*,\omega_*]\frac{1}{2}\sigma^{ij}F_{ij}[\omega_*,y_*] + \{\hat{p}^i, [x_*,\omega_*]\,\frac{2}{s}\,\omega_*\,F_{i\bullet}(\omega_*)\,[\omega_*,y_*]\}$$
$$+g\int_{\omega_*}^{x_*} d\frac{2}{s}\,\omega'_*\,\frac{2}{s}(\omega_* - \omega'_*)[x_*,\omega'_*]F^i{}_\bullet[\omega'_*,\omega_*]\,F_{i\bullet}\,[\omega_*,y_*]\Big)$$

$$\hat{\mathcal{O}}_2(x_*,y_*;p_\perp) = \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big[[x_*,\omega_*]\frac{i}{4}\{(i\not{D}_\perp F_{ij}),\gamma^i\gamma^j\}[\omega_*,y_*] + \{\hat{p}^k, [x_*,\omega_*]iF_{kj}\gamma^j[\omega_*,y_*]\}$$
$$+\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\Big([x_*,\omega'_*]F^k{}_\bullet[\omega'_*,\omega_*]igF_{kj}\gamma^j[\omega_*,y_*] - [x_*,\omega'_*]igF_{kj}\gamma^j[\omega'_*,\omega_*]F^k{}_\bullet[\omega_*,y_*]\Big)$$
$$+\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\Big([x_*,\omega'_*]ig\frac{2}{s}F_{\bullet *}[\omega'_*,\omega_*]\gamma^k F_{k\bullet}[\omega_*,y_*]$$
$$-[x_*,\omega'_*]\gamma^k F_{k\bullet}[\omega'_*,\omega_*]ig\frac{2}{s}F_{\bullet *}[\omega_*,y_*]\Big) + (\hat{\alpha}\not{p}_1 - \hat{\not{p}}_\perp)[x_*,\omega_*]\,i\,\frac{2}{s}F_{\bullet *}[\omega_*,y_*]\Big]$$

G. A. C. JHEP 1901 (2019) 118

Quark propagator in the background of quark fields



$$\langle \mathrm{T}\{\psi(x)\bar\psi(y)\}\rangle_{\psi,\bar\psi}$$

$$= g^2\Big[\int_0^{+\infty}\frac{{\not d}\alpha}{2s^2\alpha^3}\theta(x_*-y_*) - \int_{-\infty}^0\frac{{\not d}\alpha}{2s^2\alpha^3}\theta(y_*-x_*)\Big]e^{-i\alpha(x_\bullet-y_\bullet)}\int_{y_*}^{x_*}dz_*\int_{y_*}^{z_*}dz'_*$$

$$\times\langle x_\perp|e^{-i\frac{\hat p_\perp^2}{\alpha s}x_*}(\alpha\not p_1+\not p_\perp)[x_*,z_*]\gamma_\perp^\mu t^a\psi(z_*)[z_*,z'_*]^{ab}\bar\psi(z'_*)t^b\gamma_\mu^\perp[z'_*,y_*](\alpha\not p_1+\not p_\perp)\,e^{i\frac{\hat p_\perp^2}{\alpha s}y_*}|y_\perp\rangle$$

G. A. C. JHEP 1901 (2019) 118

Gluon propagator in the background of gluon fields

$$i\langle A^a_\mu(x)A^b_\nu(y)\rangle \equiv iG^{ab}_{\mu\nu}(x,y) = \langle x|\frac{1}{\Box^{\mu\nu} - P^\mu P^\nu + \frac{1}{w}p_2^\mu p_2^\nu}|y\rangle^{ab}$$

with $\Box^{\mu\nu} = P^2 g^{\mu\nu} + 2i\,g\,F^{\mu\nu}$

$$\begin{aligned}
&\langle A^a_\mu(x)A^b_\nu(y)\rangle_A \\
&= \left[-\int_0^{+\infty}\frac{\text{đ}\,\alpha}{2\alpha}\theta(x_* - y_*) + \int_{-\infty}^0\frac{\text{đ}\,\alpha}{2\alpha}\theta(y_* - x_*)\right]e^{-i\alpha(x_\bullet - y_\bullet)} \\
&\times\langle x_\perp|e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\left(\delta^\xi_\mu - \frac{p_{2\mu}}{p_*}p^\xi\right)\mathcal{O}_\alpha(x_*,y_*)\left(g_{\xi\nu} - p_\xi\frac{p_{2\nu}}{p_*}\right)e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle^{ab} + i\langle x|\frac{p_{2\mu}p_{2\nu}}{p_*^2}|y\rangle^{ab} \\
&+ \left[-\int_0^{+\infty}\frac{\text{đ}\,\alpha}{2\alpha}\theta(x_* - y_*) + \int_{-\infty}^0\frac{\text{đ}\,\alpha}{2\alpha}\theta(y_* - x_*)\right]e^{-i\alpha(x_\bullet - y_\bullet)}\langle x_\perp|e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*} \\
&\times\left[\mathfrak{G}^{ab}_{1\mu\nu}(x_*,y_*;p_\perp) + \mathfrak{G}^{ab}_{2\mu\nu}(x_*,y_*;p_\perp) + \mathfrak{G}^{ab}_{3\mu\nu}(x_*,y_*;p_\perp) + \mathfrak{G}^{ab}_{4\mu\nu}(x_*,y_*;p_\perp)\right] \\
&\times e^{i\frac{\hat{p}_\perp^2}{\alpha s}y_*}|y_\perp\rangle + O(\lambda^{-2}),
\end{aligned}$$

G. A. C. JHEP 1901 (2019) 118

Gluon propagator in the background of gluon fields

G. A. C. JHEP 1901 (2019) 118

$$\mathcal{O}_\alpha(x_*,y_*) \equiv \quad [x_*,y_*] + \frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_* \Big(\{p^i, [x_*,\omega_*]\,\frac{2}{s}\,\omega_*\, F_{i\bullet}(\omega_*)\,[\omega_*,y_*]\} + g\int_{\omega_*}^{x_*} d\frac{2}{s}\,\omega'_*\,\frac{2}{s}(\omega_*-\omega'_*)[x_*,\omega'_*]F^i{}_\bullet[\omega'_*,\omega_*]\;F_{i\bullet}\,[\omega_*,y_*]\Big).$$

$$\mathfrak{G}^{ab}_{1\mu\nu}(x_*,y_*;p_\perp) = -\frac{g\,p_{2\mu}p_{2\nu}}{s^2\alpha^3}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big[4p^i[x_*,\omega_*]F_{ij}[\omega_*,y_*]p^j + ig\int_{\omega'_*}^{x_*} d\frac{2}{s}\omega'_*\,\frac{2}{s}(\omega'_*-\omega_*)[x_*,\omega'_*]iD^iF_{i\bullet}[\omega'_*,\omega_*]iD^jF_{j\bullet}[\omega_*,y_*]\Big]^{ab},$$

$$\mathfrak{G}^{ab}_{2\mu\nu}(x_*,y_*;p_\perp) = -\frac{g}{\alpha}\delta^i_\mu\delta^j_\nu\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\big([x_*,\omega_*]F_{ij}[\omega_*,y_*]\big)^{ab},$$

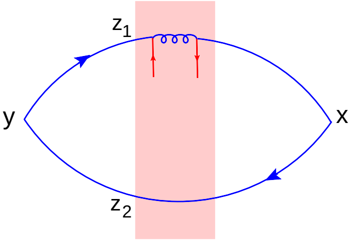
$$\mathfrak{G}^{ab}_{3\mu\nu}(x_*,y_*;p_\perp) = \frac{g}{\alpha^2 s}\Big(\delta^j_\mu p_{2\nu} + \delta^j_\nu p_{2\mu}\Big)\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\,\big([x_*,\omega_*]iD^iF_{ij}[\omega_*,y_*]\big)^{ab},$$

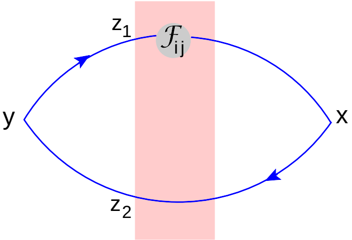
$$\mathfrak{G}^{ab}_{4\mu\nu}(x_*,y_*;p_\perp) = -\frac{2g^2}{\alpha^2 s}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\Big(\delta^j_\nu p_{2\mu}[x_*\omega'_*]F^i{}_\bullet[\omega'_*,\omega_*]F_{ij}[\omega_*,y_*] + \delta^j_\mu p_{2\nu}[x_*\omega'_*]F_{ij}[\omega'_*,\omega_*]F^i{}_\bullet[\omega_*,y_*]\Big)^{ab}.$$

Gluon propagator in the background of quark fields

$$
\begin{aligned}
\langle A^a_\mu(x)A^b_\nu(y)\rangle_{\psi,\bar\psi} &= \Big[-\int_0^{+\infty} {\textstyle d\!\!\!/}\alpha\,\theta(x_*-y_*) + \int_{-\infty}^0 {\textstyle d\!\!\!/}\alpha\,\theta(y_*-x_*)\Big] e^{-i\alpha(x_\bullet - y_\bullet)} \\
&\times \frac{g^2}{2\alpha^2 s^2}\int_{y_*}^{x_*} dz_{1*}\int_{y_*}^{z_{1*}} dz_{2*}\Bigg[\langle x_\perp|\, e^{-i\frac{\hat p_\perp^2}{\alpha s}x_*}\Big(g^\xi_{\perp\mu} - \frac{p_{2\mu}}{p_*}p^\xi_\perp\Big) \\
&\times \bar\psi(z_{1*})\gamma^\perp_\xi \not p_1\,[z_{1*},x_*]t^a[x_*,y_*]t^b[y_*,z_{2*}]\gamma^\sigma_\perp\psi(z_{2*})\Big(g^\perp_{\sigma\nu} - p^\perp_\sigma\frac{p_{2\nu}}{p_*}\Big)e^{i\frac{\hat p_\perp^2}{\alpha s}y_*}|y_\perp\rangle \\
&+ \langle y_\perp|\, e^{-i\frac{\hat p_\perp^2}{\alpha s}y_*}\Big(g^\xi_{\perp\nu} - \frac{p_{2\nu}}{p_*}p^\xi_\perp\Big)\bar\psi(z_{2*})\gamma^\perp_\xi \not p_1[z_{2*},y_*]t^b[y_*,x_*]t^a[x_*,z_{1*}]\gamma^\sigma_\perp\psi(z_{1*}) \\
&\times\Big(g^\perp_{\sigma\mu} - p^\perp_\sigma\frac{p_{2\mu}}{p_*}\Big)e^{i\frac{\hat p_\perp^2}{\alpha s}x_*}|x_\perp\rangle\Bigg] + O(\lambda^{-2})
\end{aligned}
$$

G. A. C. JHEP 1901 (2019) 118





Let $|P\rangle$ be proton or nuclear target

$$\langle P|J^\mu(x)J^\nu(y)|P\rangle \;\to\; \langle J^\mu(x)J^\nu(y)\rangle_{A\,\psi\,\bar\psi} = \mathrm{Tr}\{\gamma^\mu\langle x|\frac{1}{\not{P}+i\epsilon}|y\rangle\,\gamma^\nu\langle y|\frac{1}{\not{P}+i\epsilon}|y\rangle\}$$

Quark propagator with sub-eikonal corrections: $\Rightarrow$ New evolution equations

$$
\begin{aligned}
\langle P|\mathrm{T}\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|P\rangle \simeq \int d^2z_1 d^2z_2 \Bigg[& I^{eikonal}_{\mu\nu}(z_{1\perp}, z_{2\perp}; x, y)\langle P|\mathrm{Tr}\{U_{z_1}U^\dagger_{z_2}\}|P\rangle \\
& + I^{sub\,eik}_{1\,\mu\nu}(z_1, z_2; x, y)\langle P|\Big(Q_1(z_{1\perp})\mathrm{Tr}\{U_{z_1}U^\dagger_{z_2}\} - \frac{1}{N_c}\mathrm{Tr}\{\tilde{Q}_1(z_{1\perp})U^\dagger_{z_2}\}\Big) + h.c|P\rangle \\
& + I^{sub\,eik}_{5\,\mu\nu}(z_{1\perp}, z_2; x, y)\langle P|\Big(Q_5(z_{1\perp})\mathrm{Tr}\{U_{z_1}U^\dagger_{z_2}\} - \frac{1}{N_c}\mathrm{Tr}\{\tilde{Q}_5(z_{1\perp})U^\dagger_{z_2}\Big) + h.c|P\rangle \\
& + 2\,C_F\, I^{sub\,eik}_{\mathcal{F}\,\mu\nu}(z_{1\perp}, z_2; x, y)\langle P|\mathrm{Tr}\{\mathcal{F}_{z_1}U^\dagger_{z_2}\} + h.c|P\rangle \Bigg]
\end{aligned}
$$

OPE in Wilson lines with sub-eikonal corrections

$\not{p}_1 = \sqrt{\frac{s}{2}}\gamma^- \qquad x_* = \sqrt{\frac{s}{2}}x^+$

$$Q_1(x_\perp) = g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z'_*} dz'_*\, \bar\psi(z_*, x_\perp)\, i\not{p}_1 [z_*, z'_*]_x \psi(z'_*)$$

$$Q_5(x_\perp) = g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z'_*} dz'_*\, \bar\psi(z_*, x_\perp)\, \gamma^5\not{p}_1 [z_*, z'_*]_x \psi(z'_*)$$

$$\tilde{Q}_1(x_\perp) = g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* [\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar\psi(z'_*, x_\perp)\, i\not{p}_1\}[z'_*, -p_1\infty]$$

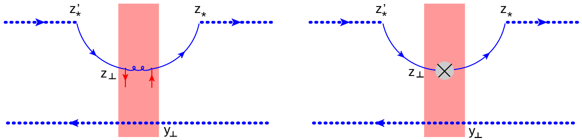
$$\tilde{Q}_5(x_\perp) = g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* [\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar\psi(z'_*, x_\perp)\, \gamma^5\not{p}_1\}[z'_*, -p_1\infty]$$

$$\mathcal{F}(x_\perp) = \frac{ig}{2C_F}\,\frac{s}{2}\int_{-\infty}^{+\infty} dz_*\, [\infty, z_*]_x\, \epsilon^{ij} F_{ij}(z_*, x_\perp)[z_*, -\infty]_x$$

Sample of diagrams for the quark operator Q_1 or Q_5

a) b)

Sample of diagrams for the quark operator $\mathrm{Tr}\{U_y^\dagger \tilde{Q}_{1\omega}\}$ or $\mathrm{Tr}\{U_y^\dagger \tilde{Q}_{1\omega}\}$



a) b)

Evolution equation flavor non-singlet case

Unpolarized case

$$\langle Q_{1\,\omega}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\,\frac{1}{(\omega-\zeta)^2_\perp}\left(\frac{1}{2}Q_{1\,\zeta}\mathrm{Tr}\{U_\zeta U^\dagger_\omega\} - \frac{1}{2N_c}\mathrm{Tr}\{U^\dagger_\omega \tilde{Q}_{1\,\zeta}\}\right)$$

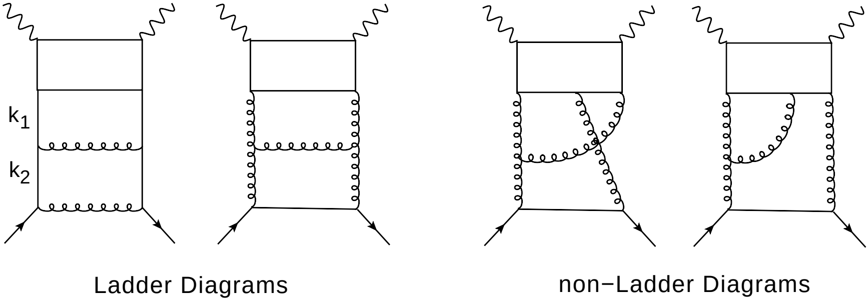
$$\langle \mathrm{Tr}\{U^\dagger_z\tilde{Q}_{1\,\omega}\}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\left[\frac{1}{(\omega-\zeta)^2_\perp}\left(\frac{1}{2}\mathrm{Tr}\{U^\dagger_z U_\zeta\}Q_{1\,\zeta} - \frac{1}{2N_c}\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{1\,\zeta}\}\right)\right.$$
$$\left. + \frac{(\omega-z)^2_\perp}{(\omega-\zeta)^2_\perp(\zeta-z)^2_\perp}\left(\mathrm{Tr}\{U_\zeta U^\dagger_z\}\mathrm{Tr}\{U^\dagger_\zeta\tilde{Q}_{1\,\omega}\} - (\frac{1}{N_c}+C_F)\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{1\,\omega}\}\right)\right]$$

Polarized case

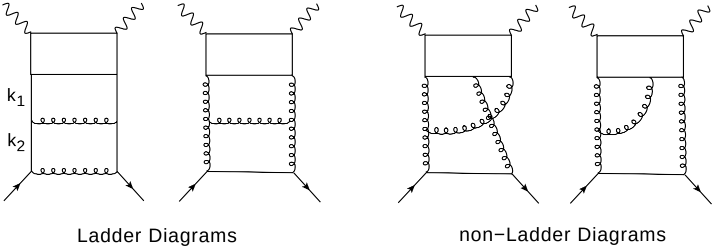
$$\langle Q_{5\,\omega}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\,\frac{1}{(\omega-\zeta)^2_\perp}\left(\frac{1}{2}\mathrm{Tr}\{U^\dagger_\omega U_\zeta\}\,Q_{5\,\zeta} - \frac{1}{2N_c}\mathrm{Tr}\{U^\dagger_\omega \tilde{Q}_{5\,\zeta}\}\right)$$

$$\langle \mathrm{Tr}\{U^\dagger_z\tilde{Q}_{5\,\omega}\}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\left[\frac{1}{(\omega-\zeta)^2_\perp}\left(\frac{1}{2}\mathrm{Tr}\{U^\dagger_z U_\zeta\}Q_{5\,\zeta} - \frac{1}{2N_c}\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{5\,\zeta}\}\right)\right.$$
$$\left. + \frac{(\omega-z)^2_\perp}{(\omega-\zeta)^2_\perp(\zeta-z)^2_\perp}\left(\mathrm{Tr}\{U_\zeta U^\dagger_z\}\mathrm{Tr}\{U^\dagger_\zeta\tilde{Q}_{5\,\omega}\} - (\frac{1}{N_c}+C_F)\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{5\,\omega}\}\right)\right]$$

Infrared Evolution Equation



- Frolov, Gorshkov, Gribov, Lipatov (1966): QED
- Kirschner, Lipatov (1983)
- Bartels, Ermolaev, Ryskin-(1995-1996)



$k^\mu = \alpha p_1^\mu + \beta p_2^\mu + k_\perp \qquad \alpha = \sqrt{\frac{2}{s}} k^+$ and $\beta = \sqrt{\frac{2}{s}} k^-$

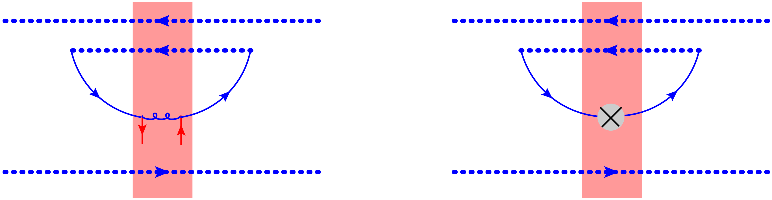
$1 \gg \alpha_1 \gg \alpha_2$ and $1 \gg \beta_2 \gg \beta_1$

$$\frac{k_{1\perp}^2}{\alpha_1 s} \ll \frac{k_{2\perp}^2}{\alpha_2 s} \quad \Rightarrow \quad \alpha_1 x_{1\perp}^2 \gg \alpha_2 x_{2\perp}^2$$

$$\alpha_s \int_0^{\alpha_1} \frac{d\alpha}{\alpha} \int \frac{dx_{2\perp}^2}{x_{2\perp}^2} \to \alpha_s \ln^2 \frac{\alpha_1}{\alpha_0}$$

Sample of diagrams

Diagrams for the operator $\mathrm{Tr}\{U_\omega U_z^\dagger\}Q_{1\omega}$ or $\mathrm{Tr}\{U_\omega U_z^\dagger\}Q_{5\omega}$



+ BK type of diagrams

Evolution equation flavor non-singlet case in double log approx

Unpolarized case

$$\left[\mathrm{Tr}\{U_\omega U_z^\dagger\}Q_{1\omega}\right]^{\alpha_1,\beta_1} \simeq \quad \frac{\alpha_s}{\pi}\, C_F\, \ln^2\frac{\alpha_1}{\alpha_2}\left[\mathrm{Tr}\{U_\omega U_z^\dagger\}Q_{1\omega}\right]^{\alpha_2,\beta_2}$$

$$[\mathrm{Tr}\{U_z^\dagger \tilde{Q}_{1\,\omega}\}]^{\alpha_1,\beta_1} \simeq \frac{3\alpha_s}{2\pi}\ln^2\frac{\alpha_1}{\alpha_2}\left(\mathrm{Tr}\{U_z^\dagger U_\omega\}Q_{1\,\omega} - \frac{1}{N_c}\mathrm{Tr}\{U_z^\dagger \tilde{Q}_{1\,\omega}\}\right)^{\alpha_2,\beta_2}$$

Polarized case

$$\left[\mathrm{Tr}\{U_\omega U_z^\dagger\}Q_{5\omega}\right]^{\alpha_1,\beta_1} \simeq \quad \frac{\alpha_s}{\pi}\, C_F\, \ln^2\frac{\alpha_1}{\alpha_0}\left[\mathrm{Tr}\{U_\omega U_z^\dagger\}Q_{5\omega}\right]^{\alpha_2,\beta_2}$$

$$[\mathrm{Tr}\{U_z^\dagger \tilde{Q}_{5\,\omega}\}]^{\alpha_1,\beta_1} \simeq \frac{3\alpha_s}{2\pi}\ln^2\frac{\alpha_1}{\alpha_2}\left(\mathrm{Tr}\{U_z^\dagger U_\omega\}Q_{5\,\omega} - \frac{1}{N_c}\mathrm{Tr}\{U_z^\dagger \tilde{Q}_{5\,\omega}\}\right)^{\alpha_2,\beta_2}$$

Evolution equation flavor Singlet case for g_1 structure function

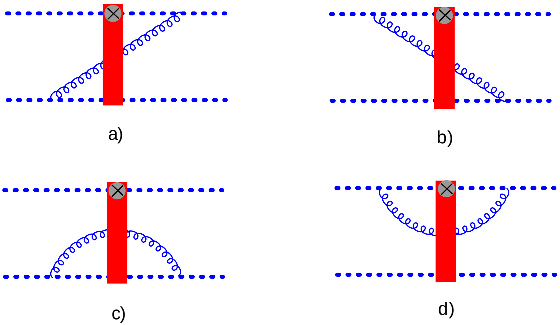
$$\langle Q_{5\,\omega}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\,\frac{1}{(\omega-\zeta)_\perp^2}\Big(\frac{1}{2}\mathrm{Tr}\{U_\omega^\dagger U_\zeta\}\,Q_{5\,\zeta} - \frac{1}{2N_c}\mathrm{Tr}\{U_\omega^\dagger \tilde{Q}_{5\,\zeta}\} + 2C_F\,\mathrm{Tr}\{U_\omega^\dagger\,\mathcal{F}(\zeta_\perp)\}\Big)$$

$$\langle \mathrm{Tr}\{U_z^\dagger \tilde{Q}_{5\,\omega}\}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\Big[\frac{1}{(\omega-\zeta)_\perp^2}\Big(\frac{1}{2}\mathrm{Tr}\{U_z^\dagger U_\zeta\}Q_{5\,\zeta} - \frac{1}{2N_c}\mathrm{Tr}\{U_z^\dagger \tilde{Q}_{5\,\zeta}\}$$
$$+2C_F\,\mathrm{Tr}\{U_z^\dagger\,\mathcal{F}(\zeta)\}\Big) + \frac{(\omega-z)_\perp^2}{(\omega-\zeta)_\perp^2(\zeta-z)_\perp^2}\Big(\mathrm{Tr}\{U_\zeta U_z^\dagger\}\mathrm{Tr}\{U_\zeta^\dagger \tilde{Q}_{5\,\omega}\} - \big(\frac{1}{N_c}+C_F\big)\mathrm{Tr}\{U_z^\dagger \tilde{Q}_{5\,\omega}\}\Big)\Big]$$

From these evolution equation we can again extract the evolution equation valid in the double Log approximation.

Evolution equation case: Gluon operator

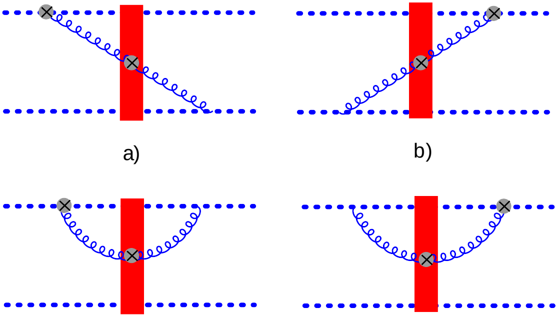
BK-type diagrams



$$\langle \mathrm{Tr}\{\mathcal{F}_\omega\, U_z^\dagger\}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\,\frac{(\omega-z)_\perp^2}{(\omega-\zeta)_\perp^2(\zeta-z)_\perp^2}$$
$$\times\Big[\mathrm{Tr}\{U_\zeta U_z^\dagger\}\mathrm{Tr}\{U_\zeta^\dagger \mathcal{F}_\omega\} - \big(\frac{1}{N_c}+C_F\big)\mathrm{Tr}\{U_z^\dagger \mathcal{F}_\omega\}\Big]$$

Diagrams with $\mathcal{F}_{ij}$ quantum

Sample of diagrams



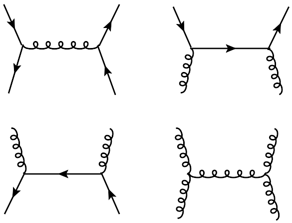
a) b) c) d)

Diagrams with $\mathcal{F}_{ij}$ quantum

Polarized case

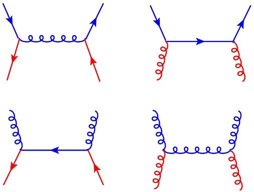
$$\langle \mathrm{Tr}\{\mathcal{F}_\omega\, U_z^\dagger\}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2\zeta\Bigg\{\frac{N_f}{2}\Bigg[-\Big(\frac{(\omega-\zeta,\zeta-z)}{(\omega-\zeta)_\perp^2(z-\zeta)_\perp^2}+\frac{1}{(\omega-\zeta)_\perp^2}\Big)$$
$$\times\Big(\mathrm{Tr}\{U_z^\dagger\tilde{Q}_{5\,\zeta}\}\mathrm{Tr}\{U_\zeta^\dagger U_\omega\} - \frac{1}{N_c}\mathrm{Tr}\{U_\omega U_z^\dagger U_\zeta\tilde{Q}_{5\zeta}^\dagger\} - \frac{1}{N_c}\mathrm{Tr}\{U_z^\dagger U_\omega\tilde{Q}_{5\zeta}^\dagger U_\zeta\} + \frac{1}{N_c^2}\mathrm{Tr}\{U_z^\dagger U_\omega\}Q_{5\,\zeta}^\dagger$$
$$+\mathrm{Tr}\{U_\omega\tilde{Q}_{5\,\zeta}^\dagger\}\mathrm{Tr}\{U_z^\dagger U_\zeta\} - \frac{1}{N_c}\mathrm{Tr}\{U_\omega U_z^\dagger\tilde{Q}_{5\zeta}U_\zeta^\dagger\} - \frac{1}{N_c}\mathrm{Tr}\{U_z^\dagger U_\omega U_\zeta^\dagger\tilde{Q}_{5\zeta}\} + \frac{1}{N_c^2}\mathrm{Tr}\{U_z^\dagger U_\omega\}Q_{5\,\zeta}\Big)\Bigg]$$
$$+2C_F\Bigg[\frac{(\omega-\zeta,\zeta-z)}{(z-\zeta)_\perp^2(\zeta-\omega)_\perp^2}+\frac{1}{(\omega-\zeta)_\perp^2}+4\pi^2\int d\!\!{}^-{}^2q_1\frac{e^{i(q_1,z-\zeta)}-e^{i(q_1,\omega-\zeta)}}{q_{1\perp}^2}\,\delta^{(2)}(\zeta-\omega)\Bigg]$$
$$\times\Big(\mathrm{Tr}\{U_\omega U_\zeta^\dagger\}\mathrm{Tr}\{U_z^\dagger\,\mathcal{F}(\zeta_\perp)\} + \mathrm{Tr}\{U_z^\dagger U_\zeta\}\mathrm{Tr}\{U_\omega\,\mathcal{F}^\dagger(\zeta_\perp)\}\Big)\Bigg\}$$

BER formalism: splitting functions ΔP_{ij}



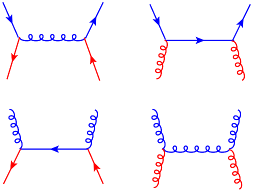
$$\begin{pmatrix} P_{q\leftarrow q} & P_{g\leftarrow q} \\ P_{q\leftarrow g} & P_{g\leftarrow g} \end{pmatrix} \quad \longleftrightarrow \quad \begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

BER formalism

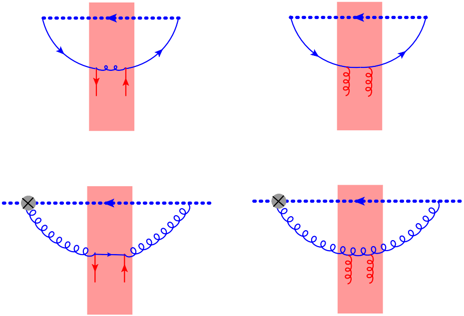


Ladder Diagrams in Singlet case: BER vs. Shock-wave formalism

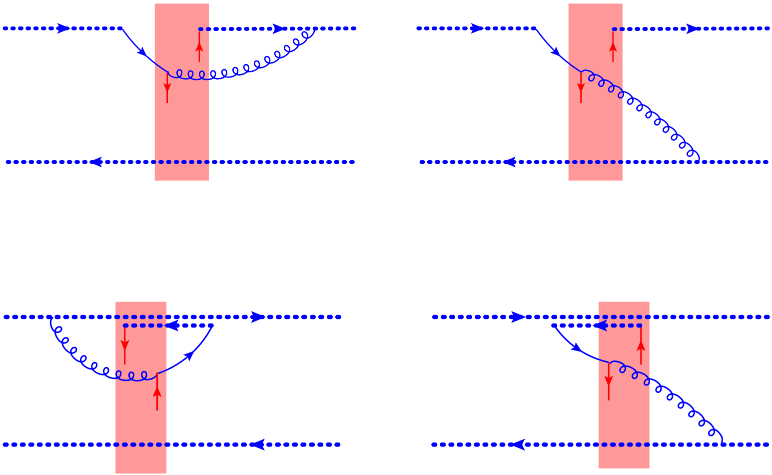
BER formalism



Shock-wave formalism



Sample of quark-to-gluon diagrams



Calculated but not presented (ask in private if interested).

Conclusions

- High-energy Quark and Gluon propagator with sub-eikonal corrections has been presented.
- BK-type of evolution equation for Unpolarized flavor-non-singlet F_2 structure function has been presented.
- BK-type of evolution equations for Polarized flavor-non-singlet g_1 structure function has been presented and agrees with BER result in the double-log approximation.
- BK-type of evolution equations for Polarized flavor-singlet g_1 structure function has been presented.

Outlook

- Include the quark-to-gluon type of diagrams.
- Refine the linearization procedure.
- Solve the coupled evolution equations in the Double Log Approximation for the flavor-singlet case and compare with the non-ladder BER diagrams.
- Orbital Angular Momentum and spin-TMDs at small-x.