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Tetraquark properties at large N_c

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Multiquark states in QCD

Hadrons are color-singlet bound states of quarks and gluons.

Mesons: $\overline{q}q$.

Baryons: qqq.

Are there other types of structure for bound states (exotics)?

Tetraquarks: $\overline{q}\overline{q}qq$.

Pentaquarks: $\overline{q}qqqq$.

Hexaquarks: $qqqqqq$ or $\overline{q}\overline{q}\overline{q}qqq$.

Possibility considered long ago by many authors (Jaffe, 1977).

We concentrate in the following on tetraquark states.

QCD at large N_c

Framework: $SU(N_c)$ gauge theory, with quarks in the fundamental representation, considered in the limit $N_c \to \infty$ with $g \sim 1/N_c^{1/2}$.
('t Hooft, 1974.)

In this limit, QCD catches the main properties of confinement, while being simplified with respect to secondary complications, like inelasticity and screening effects. $1/N_c$ plays the role of a perturbative parameter.

Properties of the theory analyzed by Witten (1979).

The spectrum is saturated by an infinite number of free stable mesons; $M = O(N_c^0)$.

- Three-meson interaction $\sim\ N_c^{-1/2}$.
- Four-meson interaction $\sim\ N_c^{-1}$.

Meson decay widths: $\Gamma(M) = O(N_c^{-1})$.

$\Longrightarrow$ Mesons are stable at large-N_c. One of the main properties of confinement.

Can we have similar predictions with tetraquarks?

$$T(x) = (\overline{q}\overline{q}qq)(x) \qquad \text{(color singlet)},$$

$$j(x) = (\overline{q}q)(x) \qquad \text{(color singlet)}.$$

$$\langle T(x)T^{\dagger}(0)\rangle \underset{N_c\to\infty}{=} \sum \langle j(x)j^{\dagger}(0)\rangle\langle j(x)j^{\dagger}(0)\rangle.$$

Equivalent to the propagation of two free mesons. (Coleman, 1980.)

No tetraquark poles can appear at leading order.

For a long time, this fact has been considered as a theoretical proof of the non-existence of tetraquarks as elementary stable particles, surviving in the large-N_c limit, like the ordinary mesons.

However, tetraquark poles may appear at subleading orders. If they have narrow widths, they might be observable.

Weinberg (2013).

Knecht and Peris (2013), Cohen and Lebed (2014), Maiani, Polosa, Riquer (2016), Lucha, Melikhov, H.S. (2017).

Widths of order $1/N_c^2$ expected.

Line of approach

Study of exotic tetraquark properties, through the analysis of meson-meson scattering amplitudes.
Exotics: contain four different quark flavors.

Four-point correlation functions of color-singlet quark bilinears,

$$j_{ab} = \overline{q}_a q_b,$$

having coupling with a meson M_{ab}:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}; \qquad f_M \sim N_c^{1/2}.$$

Spin and parity ignored; not relevant for the qualitative aspects.

Consider all possible channels where a tetraquark may be present.

To be sure that a QCD diagram may contain a tetraquark contribution, through a pole term, one has to check that it receives a four-quark contribution in its s-channel singularities, plus additional gluon singularities that do not modify the N_c-behavior of the diagram.

If the tetraquark contains quarks and antiquarks with masses $m_j,\ j = a, b, c, d$, then the diagram should have a four-particle cut starting at $s = (m_a + m_b + m_c + m_d)^2$.
Its existence is checked with the use of the Landau equations.

Diagrams that do not have s-channel singularities, or have only two-particle singularities (quark-antiquark), cannot contribute to the formation of tetraquarks at their N_c-leading order. They should not be taken into account for the N_c-behavior analysis of the tetraquark properties.

Exotic tetraquarks

Four distinct quark flavors, denoted 1,2,3,4, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{34} = \overline{q}_3 q_4, \quad j_{14} = \overline{q}_1 q_4, \quad j_{32} = \overline{q}_3 q_2.$$

The following scattering processes are considered:

$$M_{12} + M_{34} \ \rightarrow \ M_{12} + M_{34}; \qquad \text{Direct channel I};$$

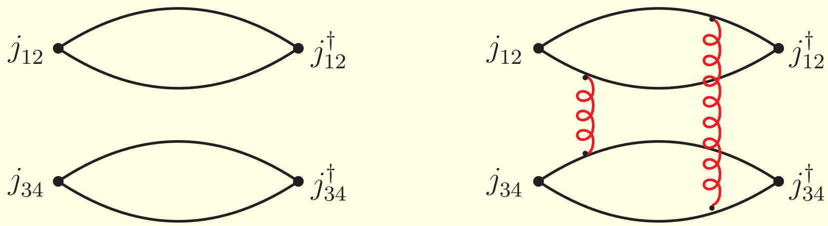
$$M_{14} + M_{32} \ \rightarrow \ M_{14} + M_{32}; \qquad \text{Direct channel II};$$

$$M_{12} + M_{34} \ \rightarrow \ M_{14} + M_{32}; \qquad \text{Recombination channel}.$$

‘Direct’ 4-point functions

$$\Gamma_I^{(\mathrm{dir})} = \langle j_{12} j_{34} j_{34}^\dagger j_{12}^\dagger \rangle \ , \qquad \Gamma_{II}^{(\mathrm{dir})} = \langle j_{14} j_{32} j_{32}^\dagger j_{14}^\dagger \rangle \ .$$

Leading and subleading diagrams for $\Gamma_I^{(dir)}$:



$O(N_c^2)$ (a) $\qquad$ $O(N_c^0)$ (b)

Similar diagrams for $\Gamma_{II}^{(dir)}$.

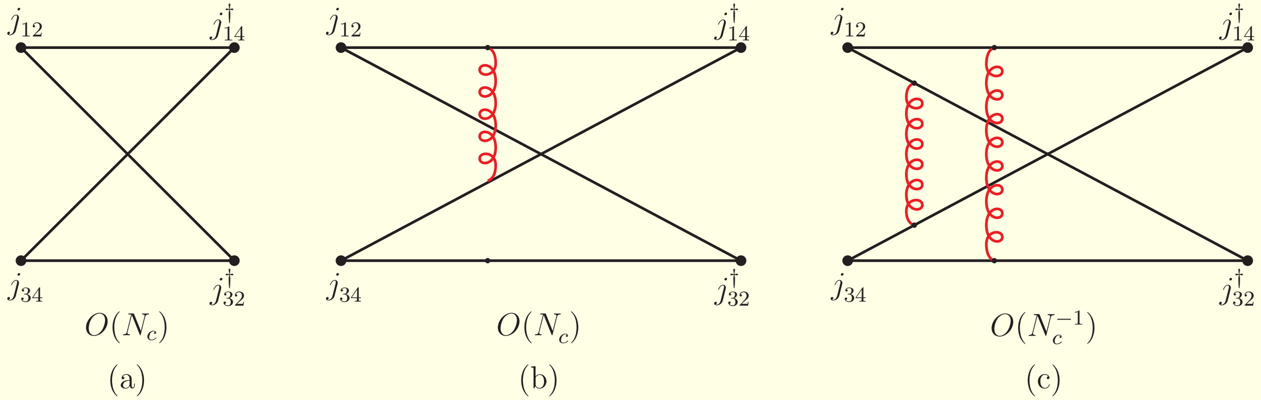
Only diagram (b) may receive contributions from tetraquark states.

$$\Gamma_{I,T}^{(\mathrm{dir})} = O(N_c^0), \qquad \Gamma_{II,T}^{(\mathrm{dir})} = O(N_c^0).$$

'Recombination' 4-point function

$$\Gamma^{(\mathrm{recomb})} = \langle j_{12} j_{34} j_{32}^{\dagger} j_{14}^{\dagger} \rangle \, .$$

Leading and subleading diagrams:



Only diagram (c) may receive contributions from tetraquark states.

$$\Gamma_T^{(\mathrm{recomb})} = O(N_c^{-1}).$$

We have found that the direct and recombination amplitudes have different behaviors in N_c:

$$\Gamma_{I,T}^{(\mathrm{dir})} = O(N_c^0), \qquad \Gamma_{II,T}^{(\mathrm{dir})} = O(N_c^0), \qquad \Gamma_T^{(\mathrm{recomb})} = O(N_c^{-1}).$$

The solution requires the contribution of two different tetraquarks, T_A and T_B, each having different couplings to the meson pairs.

One finds for the tetraquark – two-meson transition amplitudes:

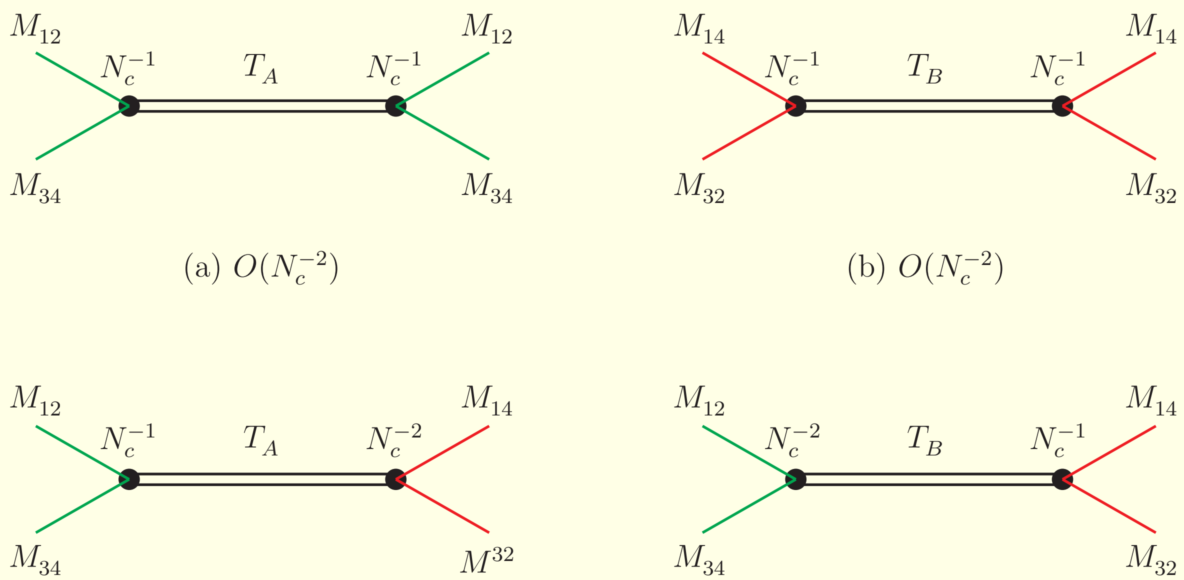
$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}),$$

$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}).$$

Total widths:

$$\Gamma(T_A) = O(N_c^{-2}), \qquad \Gamma(T_B) = O(N_c^{-2}).$$

The meson-meson scattering amplitudes at the tetraquark poles (leading contributions):



(a) $O(N_c^{-2})$ (b) $O(N_c^{-2})$

(c) $O(N_c^{-3})$ (d) $O(N_c^{-3})$

One may also extract, from the previous results and the color structure of the intermediate states in the Feynman diagrams, the flavor structure of each of the tetraquarks T_A and T_B. In particular, the intermediate states are characterized by color-exchange between the quarks.

$$T_A \sim (\overline{q}_1 q_4)(\overline{q}_3 q_2), \qquad T_B \sim (\overline{q}_1 q_2)(\overline{q}_3 q_4).$$

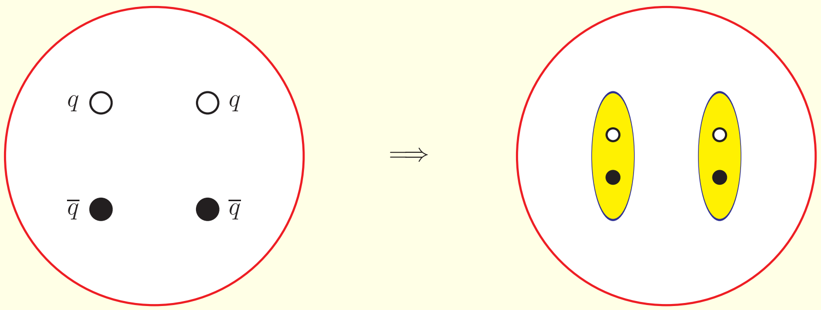
Mixings of order $1/N_c$ between the two configurations are possible.

Favors a color singlet-singlet structure of the tetraquarks.

Implications

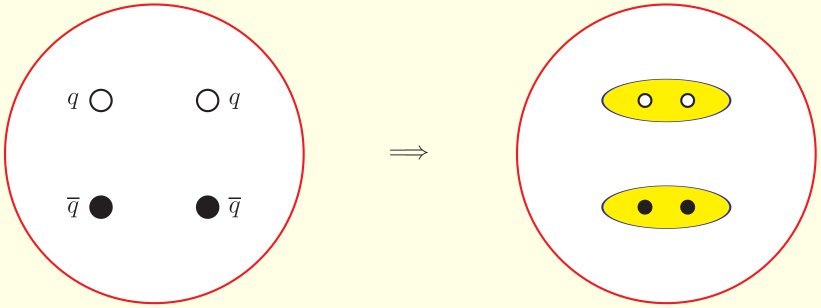
The fact that we have two different tetraquarks, each having a structure made of two color-singlet clusters, raises several questions.

Once the color-singlet clusters are formed inside the four-body system, their mutual interaction can no longer be confining.



The resulting system is no longer compact. Similar to a molecular state.

The diquark model provides a mechanism to produce compact multiquark states (Jaffe and Wilczek (2003), Shuryak and Zahed (2004), Maiani, Piccinini, Polosa, and Riquer (2004-)).



The binding of the diquark and antidiquark clusters is expected to be realized by means of confining forces, hence the appearance of a compact tetraquark.

However, only one type of diquark is expected to be formed, in its color-antisymmetric representation, where the interaction forces are attractive, decaying with equal weights into the two different two-meson channels.

Conclusion

The necessity of having two different tetraquarks, each decaying into a preferred two-meson channel, to accomodate the N_c-counting constraints, does not fit the diquark formation scheme, at least in the sector of fully exotic case (four different quark flavors).

To avoid the dominance of quark-antiquark forces over the diquark forces, hidden dynamical mechanisms have been proposed (Brodsky, Hwang, and Lebed (2014), Maiani, Polosa, and Riquer (2018)).

Large-N_c analysis of molecular-type tetraquarks is more involved. Since effective meson-meson interactions are, in general, of order $1/N_c$, the formation of bound states or resonances necessitates the summation of chains of diagrams with different N_c behaviors (Peláez (2016)).