Holographic light-front QCD in B meson phenomenology

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QCD on the light cone: from hadrons to heavy ions

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An important equation in light-front holographic QCD is the holographic Schrödinger equation (HSE) for mesons:

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)
\]

- Derived within a semiclassical approximation of light-front QCD, where quantum loops and quark masses are neglected.
- The holographic variable \( \zeta = \sqrt{z\bar{z}}b \) with \( \bar{z} \equiv 1 - z \) where \( b \) is the transverse separation of the quark and antiquark and \( z \) is the light-front momentum fraction carried by the quark.
- \( M \) is the meson mass.

More details in Ruben Sandapen’s review on Thursday
The confining potential is uniquely determined to have the form:

\[ U_{\text{eff}}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \]

Eigenvalues and eigenfunctions

\[ M^2 = (4n + 2L + 2)\kappa^2 + 2\kappa^2(J - 1) = 4\kappa^2(n + L + \frac{S}{2}) \]

\[ \phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp \left( -\frac{\kappa^2 \zeta^2}{2} \right) L_n(z^2 \zeta^2) \]

- Lightest bound state \((n = L = J = 0)\) is massless \((M = 0)\)
- \(M^2 = 4\kappa^2 L \Rightarrow \kappa = 0.54\) GeV from Regge slope
The light front wavefunction is then given as

$$\Psi(\zeta, z, \phi) = e^{iL\phi} \chi(z) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

The longitudinal wavefunction $\chi(x)$ is obtained from mapping the pion electromagnetic form factors in AdS and in physical spacetime:

$$\chi(z) = \sqrt{z(1 - z)}$$

For the vector mesons (like $\rho$, $K^*$ and $\phi$), we set $n = 0$, $L = 0$ to obtain

$$\Psi_{0,0}(z, \zeta) = \frac{\kappa}{\sqrt{\pi}} \sqrt{z(1 - z)} \exp \left[ -\frac{\kappa^2 \zeta^2}{2} \right]$$

Allowing for small quark masses, the wavefunction becomes

$$\Psi_\lambda(z, \zeta) = N_\lambda \sqrt{z(1 - z)} \exp \left[ -\frac{\kappa^2 \zeta^2}{2} \right] \exp \left[ -\frac{(1 - z)m_q^2 + zm_{\bar{q}}^2}{2\kappa^2 z(1 - z)} \right]$$

$m_{u,d} = 0.046$ GeV and $m_s = 0.357$ GeV are fixed from the y-intercepts of the Regge trajectories. Decay constant provides the first test of the wave function.
Predictions for leptonic decay width

\[ f_V P^+ = \langle 0 | \bar{q}(0) \gamma^+ q(0) | V(P, L) \rangle \]

\[ f_V = \sqrt{\frac{N_c}{\pi}} \int_0^1 \text{d}z \left[ 1 + \frac{m_q m_{\bar{q}} - \nabla_r^2}{z(1 - z) M_V^2} \right] \Psi_L(r, z) \big|_{r=0} \]

We can use this decay constant to predict the experimentally measured electronic decay width \( \Gamma_{V \to e^+e^-} \) of the vector meson:

\[ \Gamma_{V \to e^+e^-} = \frac{4\pi \alpha_{em}^2 C_V^2}{3 M_V} f_V^2 \]

where \( C_\phi = 1/3 \) for the \( C_\rho = 1/\sqrt{2} \).

<table>
<thead>
<tr>
<th>Meson</th>
<th>( f_V ) [GeV]</th>
<th>( \Gamma_{e^+e^-} ) [KeV]</th>
<th>( \Gamma_{e^+e^-} ) [KeV] (PDG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.210, 0.211</td>
<td>6.355, 6.383</td>
<td>7.04 ± 0.06</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.191, 0.205</td>
<td>0.891, 1.024</td>
<td>1.251 ± 0.021</td>
</tr>
</tbody>
</table>

Table: Predictions for the electronic decay widths of the \( \rho \) and \( \phi \) vector mesons using the holographic wavefunction with \( m_{u,d} = 0.046, 0.14 \) GeV and \( m_s = 0.357, 0.14 \) GeV.
$f_{K^*}$ and "transverse decay constant" $f_{K^*}^\perp$ defined as:

$$
\langle 0| \bar{q}[\gamma^\mu, \gamma^\nu] s|K^*(P, \epsilon)\rangle = 2f_{K^*}^\perp(\epsilon^\mu P^\nu - \epsilon^\nu P^\mu)
$$

$$
f_{K^*}^\perp(\mu) = \sqrt{\frac{N_c}{2\pi}} \int_0^1 dz (zm_\bar{q} + (1 - z)m_s) \int db \, \mu J_1(\mu b) \frac{\Psi_T(\zeta, z)}{z(1 - z)}
$$

<table>
<thead>
<tr>
<th>Approach</th>
<th>Scale $\mu$</th>
<th>$m_\bar{q}$[MeV]</th>
<th>$m_s$[MeV]</th>
<th>$f_{K^*}$[MeV]</th>
<th>$f_{K^*}^\perp(\mu)$[MeV]</th>
<th>$f_{K^<em>}^\perp/f_{K^</em>}(\mu)$</th>
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<tbody>
<tr>
<td>AdS/QCD</td>
<td>$\sim 1$ GeV</td>
<td>140</td>
<td>280</td>
<td>200</td>
<td>118</td>
<td>0.59</td>
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<tr>
<td>AdS/QCD</td>
<td>$\sim 1$ GeV</td>
<td>195</td>
<td>300</td>
<td>200</td>
<td>132</td>
<td>0.66</td>
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<tr>
<td>AdS/QCD</td>
<td>$\sim 1$ GeV</td>
<td>250</td>
<td>320</td>
<td>200</td>
<td>142</td>
<td>0.71</td>
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<td>Experiment</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lattice</td>
<td>2 GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$0.780 \pm 0.008$</td>
</tr>
<tr>
<td>Lattice</td>
<td>2 GeV</td>
<td></td>
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<td></td>
<td>$0.74 \pm 0.02$</td>
</tr>
</tbody>
</table>

Comparison between AdS/QCD predictions for the decay constant of the $K^*$ meson with experiment (obtained from $\Gamma(\tau^- \rightarrow K^*-\nu_\tau)$), and the ratio of couplings with lattice data.
Light cone distribution amplitudes

Light cone coordinates: $x^\mu = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp$ any combinations of $x_1$ and $x_2$.

At equal light-front time $x^+ = 0$ and in the light-front gauge $A^+ = 0$,

$$
\langle 0 | \bar{q}(0) \gamma^\mu q(x^-) | \rho(P, \epsilon) \rangle = f_\rho M_\rho \frac{\epsilon \cdot x}{P^+ x^-} P^\mu \int_0^1 du \ e^{-iuP^+ x^-} \phi_\parallel(u, \mu) \\
+ f_\rho M_\rho \left( \epsilon^\mu - P^\mu \frac{\epsilon \cdot x}{P^+ x^-} \right) \int_0^1 du \ e^{-iuP^+ x^-} g_\parallel(\nu)(u, \mu)
$$

$$
\langle 0 | \bar{q}(0) [\gamma^\mu, \gamma^\nu] q(x^-) | \rho(P, \epsilon) \rangle = 2f_\rho \left( \epsilon^\mu P^\nu - \epsilon^\nu P^\mu \right) \int_0^1 du \ e^{-iuP^+ x^-} \phi_\parallel(u, \mu)
$$

$$
\langle 0 | \bar{q}(0) \gamma^\mu \gamma^5 q(x^-) | \rho(P, \epsilon) \rangle = -\frac{1}{4} \epsilon_\nu \epsilon_{\rho \sigma} P^\rho x^\sigma f_\rho M_\rho \int_0^1 du \ e^{-iuP^+ x^-} g_\parallel(a)(u, \mu)
$$

Vector meson’s polarization vectors $\epsilon$ are chosen as

$$
\epsilon_L = \left( \frac{P^+}{M_\rho}, -\frac{M_\rho}{P^+}, 0_\perp \right) \quad \text{and} \quad \epsilon_T(\pm) = \frac{1}{\sqrt{2}} (0, 0, 1, \pm i)
$$
Light cone DAs in terms of LFWF

R. Sandapen, MA, PRD87.054013(2013)

\[ \phi^\parallel_\rho(z, \mu) = \frac{N_c}{\pi f_\rho M_\rho} \int dr \mu J_1(\mu r)[M_\rho^2 z(1 - z) + m_f^2 - \nabla_r^2] \frac{\Psi_L(r, z)}{z(1 - z)}, \]

\[ \phi^\perp_\rho(z, \mu) = \frac{N_c m_f}{\pi f_\rho} \int dr \mu J_1(\mu r) \frac{\Psi_T(r, z)}{z(1 - z)}, \]

\[ g^\perp_\rho(\nu)(z, \mu) = \frac{N_c}{2\pi f_\rho M_\rho} \int dr \mu J_1(\mu r) [m_f^2 - (\nu^2 + (1 - z)^2)\nabla_r^2] \frac{\Psi_T(r, z)}{z^2(1 - z)^2} \]

\[ \frac{dg^\perp_\rho(a)}{dz}(z, \mu) = \frac{\sqrt{2}N_c}{\pi f_\rho M_\rho} \int dr \mu J_1(\mu r)(1 - 2z)[m_f^2 - \nabla_r^2] \frac{\Psi_T(r, z)}{z^2(1 - z)^2}. \]

Distribution amplitudes are normalized:

\[ \int_0^1 \text{d}u \phi^\perp,\parallel_\rho(u, \mu) = \int_0^1 \text{d}u g^\perp_\rho(\nu,a)(u, \mu) = 1 \]
AdS/QCD DAs for $\rho$: comparison to Sum Rules

Figure: Twist-2 DAs for the $\rho$ meson. Solid Red: AdS/QCD DA at $\mu \sim 1$ GeV; Dashed Blue: Sum Rules DA at $\mu = 2$ GeV.

(d) Twist-2 DA for the longitudinally polarized $\rho$ meson

(e) Twist-2 DA for the transversely polarized $\rho$ meson
DAs for $K^*$: comparison to Sum Rules

Figure: Twist-2 DAs predicted by AdS/QCD (graphs on the left) and SR (graphs on the right). The uncertainty band is due to the variation of the quark masses for AdS/QCD and the error bar on Gegenbauer coefficients for SR.
Isospin asymmetry in $B \to K^*\gamma$

Isospin asymmetry defined as:

$$\Delta_{0^-} = \frac{\Gamma(\bar{B}^0 \to \bar{K}^*\gamma) - \Gamma(B^- \to K^*\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^*\gamma) + \Gamma(B^- \to K^*\gamma)}.$$

<table>
<thead>
<tr>
<th>Branching ratio</th>
<th>BABAR</th>
<th>BELLE</th>
<th>CLEO</th>
<th>PDG</th>
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</thead>
<tbody>
<tr>
<td>$B(B^0 \to K^{*0}\gamma) \times 10^6$</td>
<td>44.7 ± 1.0 ± 1.6</td>
<td>45.5$^{+7.2}_{-6.8}$ ± 3.4</td>
<td>40.1 ± 2.1 ± 1.7</td>
<td>43.3 ± 1.5</td>
</tr>
<tr>
<td>$B(B^+ \to K^{*+}\gamma) \times 10^6$</td>
<td>42.2 ± 1.4 ± 1.6</td>
<td>42.5 ± 3.1 ± 2.4</td>
<td>37.6$^{+8.9}_{-8.3}$ ± 2.8</td>
<td>42.1 ± 1.8</td>
</tr>
<tr>
<td>$\Delta_{0^-}$</td>
<td>6.6 ± 2.1 ± 2.2</td>
<td>1.2 ± 4.4 ± 2.6</td>
<td></td>
<td>5.2 ± 2.6</td>
</tr>
</tbody>
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Isospin calculation


\begin{align*}
F_{\perp}(\mu_h) &= \int_0^1 dz \frac{\phi_{K^*}(z, \mu_h)}{3(1 - z)} \\
G_{\perp}(s_c, \mu_h) &= \int_0^1 dz \frac{\phi_{K^*}(z, \mu_h)}{3(1 - z)} G(s_c, \bar{z}) \\
X_{\perp}(\mu_h) &= \int_0^1 dz \phi_{K^*}(z, \mu_h) \left( \frac{1 + \bar{z}}{3\bar{z}^2} \right) \\
H_{\perp}(s_c, \mu_h) &= \int_0^1 dz \left( g_{K^*}^{(v)}(z, \mu_h) - \frac{1}{4} \frac{dg_{K^*}^{(a)}}{dz}(z, \mu_h) \right) G(s_c, \bar{z})
\end{align*}
Numerical results

R. Sandapen, MA, PRD88.014042(2013)

<table>
<thead>
<tr>
<th>Integral</th>
<th>SR</th>
<th>AdS/QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_\perp$</td>
<td>$\infty$</td>
<td>26.9</td>
</tr>
<tr>
<td>$F_\perp$</td>
<td>1.14</td>
<td>1.38</td>
</tr>
<tr>
<td>$G_\perp$</td>
<td>$2.55 + 0.43i$</td>
<td>$2.89 + 0.30i$</td>
</tr>
<tr>
<td>$H_\perp$</td>
<td>$2.48 + 0.50i$</td>
<td>$2.12 + 0.21i$</td>
</tr>
</tbody>
</table>

Branching ratio for $B \to K^*\gamma$: $44.3 \times 10^{-6}$ from AdS/QCD compared with $45.9 \times 10^{-6}$ from Sum Rules

$\Delta_{0-} = 3.3\%$ from AdS/QCD
Form factors are defined as:

\[ \langle \rho(k, \varepsilon) | \bar{q} \gamma^\mu (1 - \gamma^5) b | B(p) \rangle = \frac{2iV(q^2)}{m_B + m_\rho} \epsilon^{\mu \nu \rho \sigma} \varepsilon^*_\nu k_\rho p_\sigma - 2m_\rho A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q^\mu \]

\[ - (m_B + m_\rho) A_1(q^2) \left( \frac{\varepsilon^* - \frac{\varepsilon^* \cdot q q^\mu}{q^2}}{q^2} \right) \]

\[ + A_2(q^2) \frac{\varepsilon^* \cdot q}{m_B + m_\rho} \left[ (p + k)^\mu - \frac{m_B^2 - m_\rho^2}{q^2} q^\mu \right] \]

\[ q_\nu \langle \rho(k, \varepsilon) | \bar{d} \sigma^{\mu \nu} (1 - \gamma^5) b | B(p) \rangle = 2T_1(q^2) \epsilon^{\mu \nu \rho \sigma} \varepsilon^*_\nu p_\rho k_\sigma \]

\[ - iT_2(q^2) [(\varepsilon^* \cdot q)(p + k)^\mu - \varepsilon^*_\mu (m_B^2 - m_\rho^2)] \]

\[ - iT_3(q^2)(\varepsilon^* \cdot q) \left[ \frac{q^2}{m_B^2 - m_\rho^2} (p + k)_\mu - q_\mu \right] \]
AdS/QCD prediction for $B \rightarrow \rho$ transition form factors

R. Campbell, S. Lord, R. Sandapen, MA, PRD88.074031(2013)
Using light-cone sum rules with holographic DAs
(a) Differential decay rate for the semileptonic $B \rightarrow \rho \ell \bar{\nu}$ decay. The lattice data points are from UKQCD Collaboration.
Numerical predictions

BaBar collaboration has measured partial branching fractions in $q^2$ bins: PRD83, 032007 (2011)

\[
\Delta B_{\text{low}} = \int_0^8 \frac{dB}{dq^2} dq^2 = (0.564 \pm 0.166) \times 10^{-4}
\]

\[
\Delta B_{\text{mid}} = \int_8^{16} \frac{dB}{dq^2} dq^2 = (0.912 \pm 0.147) \times 10^{-4}
\]

\[
\Delta B_{\text{high}} = \int_{16}^{20.3} \frac{dB}{dq^2} dq^2 = (0.268 \pm 0.062) \times 10^{-4}
\]

\[
R_{\text{low}} = \frac{\Delta B_{\text{low}}}{\Delta B_{\text{mid}}} = 0.618 \pm 0.207
\]

\[
R_{\text{high}} = \frac{\Delta B_{\text{high}}}{\Delta B_{\text{mid}}} = 0.294 \pm 0.083
\]

Our predictions for $m_f = 0.14, 0.35$:

\[
R_{\text{low}} = 0.580, 0.424
\]

\[
R_{\text{high}} = 0.427, 0.503
\]
AdS/QCD prediction for $B \rightarrow K^*$ transition form factors

R. Campbell, S. Lord, R. Sandapen, MA, PRD89.074021(2014)
Isospin asymmetry in dileptonic $B \to K^* \mu^+ \mu^-$

S. Lord, R. Sandapen, MA, PRD90.074010(2014)

(b) Isospin asymmetry in $B \to K^* \mu^+ \mu^-$ decay vs dileptonic invariant mass. The data points are from LHCb. The dashed red curve is the prediction of the QCD sum rules.
Differential decay rate for $B \rightarrow K^* \nu \bar{\nu}$


(c) The AdS/QCD (Solid line) and SR (Dashed line) predictions for the differential Branching Ratio for $B \rightarrow K^* \nu \bar{\nu}$. The shaded band represents the uncertainty coming from the form factors.
(d) The AdS/QCD (red line) and SR (blue line) predictions for the differential Branching Ratio $B_s \rightarrow \phi \mu^+ \mu^-$.
AdS/QCD LFWF is used to obtain $\rho$ and $K^*$ DAs.
DAs are essential ingredients for the calculation of the $B \rightarrow \rho, K^*$ transition form factors via LCSR.
We are in the process of calculating the form factors directly from LFWF.
We have looked into the proper LFWF for pseudoscalar mesons and working to predict $B \rightarrow \pi, K$ transition form factors. (F. Chishtie, R. Sandapen, MA PRD95, 074008(2017); C. Mondal, R. Sandapen, MA, PRD98, 034010(2018))
Conformal invariance and the dAFF mechanism

3 mechanisms to break conformal symmetry

1. Spontaneous
2. Explicit
3. de Alfaro, Fubini, Furlan (dAFF)


In conformal QM, changing the evolution parameter allows the introduction of a mass scale in the Hamiltonian while preserving the conformal invariance of the underlying action.

4. dAFF/LF mapping ⇒ Semiclassical QCD LF Hamiltonian

\[
H_{\text{scQCD}}^{\text{LF}} = \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right)
\]

with \( U(\zeta) = \kappa^4 \zeta^2 \)
A remarkable feature in light-front holography is that the form of the confinement potential is uniquely determined to be that of a harmonic oscillator, i.e. \( U_{\text{eff}} = \kappa^4 \zeta^2 \) where \( \kappa \) is the fundamental of the model. S. J. Brodsky, G. F. De Tramond, and H. G. Dosch, Phys. Lett. B729, 3 (2014), 1302.4105.

\( \zeta \rightarrow z \) (the fifth dimension of anti-de Sitter (AdS) space), the HSE also described the propagation of weakly-coupled spin-\( J \) modes in a modified AdS space with the confining QCD potential then determined by the form of the dilaton field, \( \varphi(z) \), which modifies the pure AdS geometry.

\[
U_{\text{eff}}(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)
\]

To recover this harmonic potential, the dilaton field has to be quadratic, i.e. \( \varphi(z) = \kappa^2 z^2 \)

\[
U_{\text{eff}}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)
\]

where \( J = L + S \).
Diffractive vector meson production

- $ep \rightarrow epV$ or $\gamma^* p \rightarrow pV$
- Sensitivity to non-perturbative physics
- Can be used to fine tune the vector meson wavefunction