Holographic light-front QCD in B meson phenomenology

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QCD on the light cone: from hadrons to heavy ions

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Holographic Schrödinger equation

An important equation in light-front holographic QCD is the holographic Schrödinger equation (HSE) for mesons:

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\mathrm{eff}}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- Derived within a semiclassical approximation of light-front QCD, where quantum loops and quark masses are neglected.
- The holographic variable $\zeta = \sqrt{z\bar{z}}b$ with $\bar{z} \equiv 1-z$ where b is the transverse separation of the quark and antiquark and z is the light-front momentum fraction carried by the quark.
- M is the meson mass.

G. F. de Teramond and S. J. Brodsky, PRL94,201601(2005),PRL96,201601(2006),PRL102,081601(2009)

More details in Ruben Sandapen's review on Thursday

Eigenvalues and eigenfunctions

The confining potential is uniquely determined to have the form:

$$U_{eff}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

Eigenvalues and eigenfunctions

$$M^2 = (4n+2L+2)\kappa^2 + 2\kappa^2(J-1) = 4\kappa^2(n+L+\frac{S}{2})$$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}\exp\left(\frac{-\kappa^2\zeta^2}{2}\right)L_n^L(z^2\zeta^2)$$

- Lightest bound state ($n = L = J = 0$) is massless ($M = 0$)
- $M^2 = 4\kappa^2 L \Rightarrow \kappa = 0.54$ GeV from Regge slope

Light front Wavefunction for ρ, K^* and ϕ

The light front wavefunction is then given as

$$\Psi(\zeta, z, \phi) = e^{iL\phi}\mathcal{X}(z)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

The longitudinal wavefunction $\mathcal{X}(x)$ is obtained from mapping the pion electromagnetic form factors in AdS and in physical spacetime:

$$\mathcal{X}(z) = \sqrt{z(1-z)}$$

For the vector mesons (like ρ, K^* and ϕ), we set $n = 0, L = 0$ to obtain

$$\Psi_{0,0}(z, \zeta) = \frac{\kappa}{\sqrt{\pi}}\sqrt{z(1-z)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]$$

Allowing for small quark masses, the wavefunction becomes

$$\Psi_\lambda(z, \zeta) = \mathcal{N}_\lambda\sqrt{z(1-z)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]\exp\left[-\frac{(1-z)m_q^2 + zm_{\bar{q}}^2}{2\kappa^2 z(1-z)}\right]$$

$m_{u,d} = 0.046$ GeV and $m_s = 0.357$ GeV are fixed from the y-intercepts of the Regge trajectories. Decay constant provides the first test of the wave function

Predictions for leptonic decay width

$$f_V P^+ = \langle 0|\bar{q}(0)\gamma^+ q(0)|V(P,L)\rangle$$

$$f_V = \sqrt{\frac{N_c}{\pi}} \int_0^1 \mathrm{d}z \left[1 + \frac{m_q m_{\bar{q}} - \nabla_r^2}{z(1-z)M_V^2}\right] \Psi_L(r,z)|_{r=0}$$

We can use this decay constant to predict the experimentally measured electronic decay width $\Gamma_{V\to e^+e^-}$ of the vector meson:

$$\Gamma_{V\to e^+e^-} = \frac{4\pi\alpha_{em}^2 C_V^2}{3M_V} f_V^2$$

where $C_\phi = 1/3$ for the $C_\rho = 1/\sqrt{2}$.

Meson	f_V [GeV]	$\Gamma_{e^+e^-}$ [KeV]	$\Gamma_{e^+e^-}$[KeV] (PDG)
ρ	$0.210, 0.211$	$6.355, 6.383$	7.04 ± 0.06
ϕ	$0.191, 0.205$	$0.891, 1.024$	1.251 ± 0.021

Table: Predictions for the electronic decay widths of the ρ and ϕ vector mesons using the holographic wavefunction with $m_{u,d} = 0.046, 0.14$ GeV and $m_s = 0.357, 0.14$ GeV.

K^* decay constants

f_{K^*} and "transverse decay constant" $f^{\perp}_{K^*}$ defined as:

$$\langle 0|\bar{q}[\gamma^\mu,\gamma^\nu]s|K^*(P,\epsilon)\rangle = 2f^{\perp}_{K^*}(\epsilon^\mu P^\nu - \epsilon^\nu P^\mu)$$

$$f^{\perp}_{K^*}(\mu) = \sqrt{\frac{N_c}{2\pi}}\int_0^1 \mathrm{d}z(zm_{\bar{q}} + (1-z)m_s)\int \mathrm{d}b\,\mu J_1(\mu b)\frac{\Psi_T(\zeta,z)}{z(1-z)}$$

Approach	Scale μ	$m_{\bar{q}}$[MeV]	m_s[MeV]	f_{K^*}[MeV]	$f^{\perp}_{K^*}(\mu)$[MeV]	$f^{\perp}_{K^*}/f_{K^*}(\mu)$
AdS/QCD	~ 1 GeV	140	280	200	118	0.59
AdS/QCD	~ 1 GeV	195	300	200	132	0.66
AdS/QCD	~ 1 GeV	250	320	200	142	0.71
Experiment				205 ± 6		
Lattice	2 GeV					0.780 ± 0.008
Lattice	2 GeV					0.74 ± 0.02

Comparison between AdS/QCD predictions for the decay constant of the K^* meson with experiment (obtained from $\Gamma(\tau^- \to K^{*-}\nu_\tau)$), and the ratio of couplings with lattice data.

Light cone distribution amplitudes

Light cone coordinates: $x^\mu = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp$ any combinations of x_1 and x_2.
At equal light-front time $x^+ = 0$ and in the light-front gauge $A^+ = 0$,

$$\langle 0|\bar{q}(0)\gamma^\mu q(x^-)|\rho(P,\epsilon)\rangle = f_\rho M_\rho \frac{\epsilon\cdot x}{P^+x^-} P^\mu \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} \phi_\rho^\parallel(u,\mu)$$
$$+ f_\rho M_\rho \left(\epsilon^\mu - P^\mu \frac{\epsilon\cdot x}{P^+x^-}\right) \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} g_\rho^{\perp(v)}(u,\mu)$$

$$\langle 0|\bar{q}(0)[\gamma^\mu,\gamma^\nu] q(x^-)|\rho(P,\epsilon)\rangle = 2f_\rho^\perp(\epsilon^\mu P^\nu - \epsilon^\nu P^\mu) \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} \phi_\rho^\perp(u,\mu)$$

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma^5 q(x^-)|\rho(P,\epsilon)\rangle = -\frac{1}{4}\epsilon^\mu_{\nu\rho\sigma}\epsilon^\nu P^\rho x^\sigma f_\rho M_\rho \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} g_\rho^{\perp(a)}(u,\mu)$$

Vector meson's polarization vectors ϵ are chosen as

$$\epsilon_L = \left(\frac{P^+}{M_\rho}, -\frac{M_\rho}{P^+}, 0_\perp\right) \quad \text{and} \quad \epsilon_{T(\pm)} = \frac{1}{\sqrt{2}}(0,0,1,\pm i)$$

Light cone DAs in terms of LFWF

R. Sandapen, MA, PRD87.054013(2013)

$$\phi_\rho^{\parallel}(z,\mu) = \frac{N_c}{\pi f_\rho M_\rho} \int \mathrm{d}r\, \mu J_1(\mu r)[M_\rho^2 z(1-z) + m_f^2 - \nabla_r^2] \frac{\Psi_L(r,z)}{z(1-z)} ,$$

$$\phi_\rho^{\perp}(z,\mu) = \frac{N_c m_f}{\pi f_\rho^{\perp}} \int \mathrm{d}r\, \mu J_1(\mu r) \frac{\Psi_T(r,z)}{z(1-z)} ,$$

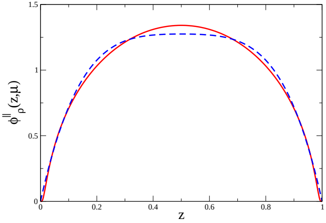
$$g_\rho^{\perp(v)}(z,\mu) = \frac{N_c}{2\pi f_\rho M_\rho} \int \mathrm{d}r\, \mu J_1(\mu r) \left[m_f^2 - (z^2 + (1-z)^2)\nabla_r^2\right] \frac{\Psi_T(r,z)}{z^2(1-z)^2}$$

$$\frac{\mathrm{d}g_\rho^{\perp(a)}}{\mathrm{d}z}(z,\mu) = \frac{\sqrt{2} N_c}{\pi f_\rho M_\rho} \int \mathrm{d}r\, \mu J_1(\mu r)(1-2z)[m_f^2 - \nabla_r^2] \frac{\Psi_T(r,z)}{z^2(1-z)^2} .$$

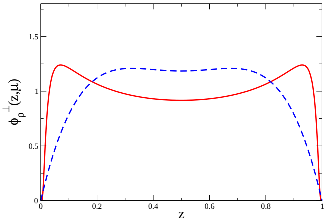
Distribution amplitudes are normalized:

$$\int_0^1 \mathrm{d}u\, \phi_\rho^{\perp,\parallel}(u,\mu) = \int_0^1 \mathrm{d}u\, g_\rho^{\perp(a,v)}(u,\mu) = 1$$

AdS/QCD DAs for ρ:comparison to Sum Rules



(d) Twist-2 DA for the longitudinally polarized ρ meson



(e) Twist-2 DA for the transversely polarized ρ meson

Figure: Twist-2 DAs for the ρ meson. Solid Red: AdS/QCD DA at $\mu \sim 1$ GeV; Dashed Blue: Sum Rules DA at $\mu = 2$ GeV.

DAs for K^*:comparison to Sum Rules

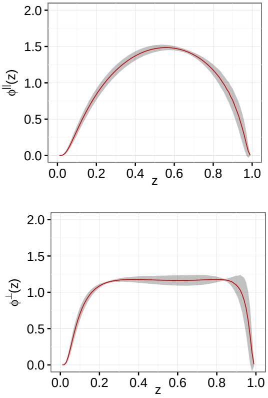


Figure: Twist-2 DAs predicted by AdS/QCD (graphs on the left) and SR (graphs on the right). The uncertainty band is due to the variation of the quark masses for AdS/QCD and the error bar on Gegenbauer coefficients for SR.

Isospin asymmetry in $B \to K^*\gamma$

Isospin asymmetry defined as:

$$\Delta_{0-} = \frac{\Gamma(\bar{B}^\circ \to \bar{K}^*\gamma) - \Gamma(B^- \to K^{*-}\gamma)}{\Gamma(\bar{B}^\circ \to \bar{K}^*\gamma) + \Gamma(B^- \to K^{*-}\gamma)} \; .$$

Branching ratio	BABAR	BELLE	CLEO	PDG
$\mathcal{B}(B^0 \to K^{*0}\gamma) \times 10^6$	$44.7 \pm 1.0 \pm 1.6$	$45.5^{+7.2}_{-6.8} \pm 3.4$	$40.1 \pm 2.1 \pm 1.7$	43.3 ± 1.5
$\mathcal{B}(B^+ \to K^{*+}\gamma) \times 10^6$	$42.2 \pm 1.4 \pm 1.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	42.1 ± 1.8
Δ_{0-}	$6.6 \pm 2.1 \pm 2.2$	$1.2 \pm 4.4 \pm 2.6$		5.2 ± 2.6

Isospin calculation

Based on original work by Kagan and Neubert: Phys.Lett. B539, 227 (2002)

$$F_{\perp}(\mu_h) = \int_0^1 dz \, \frac{\phi_{K^*}^{\perp}(z,\mu_h)}{3(1-z)}$$

$$G_{\perp}(s_c,\mu_h) = \int_0^1 dz \, \frac{\phi_{K^*}^{\perp}(z,\mu_h)}{3(1-z)} G(s_c,\bar{z})$$

$$X_{\perp}(\mu_h) = \int_0^1 dz \, \phi_{K^*}^{\perp}(z,\mu_h) \left(\frac{1+\bar{z}}{3\bar{z}^2} \right)$$

and

$$H_{\perp}(s_c,\mu_h) = \int_0^1 dz \, \left(g_{K^*}^{\perp(v)}(z,\mu_h) - \frac{1}{4} \frac{\mathrm{d}g_{K^*}^{\perp(a)}}{dz}(z,\mu_h) \right) G(s_c,\bar{z})$$

Numerical results

R. Sandapen, MA, PRD88.014042(2013)

Integral	SR	AdS/QCD
$X_\perp$	∞	26.9
$F_\perp$	1.14	1.38
$G_\perp$	$2.55 + 0.43i$	$2.89 + 0.30i$
$H_\perp$	$2.48 + 0.50i$	$2.12 + 0.21i$

Branching ratio for $B \to K^*\gamma$: 44.3×10^{-6} from AdS/QCD compared with 45.9×10^{-6} from Sum Rules
$\Delta_{0-} = 3.3\%$ from AdS/QCD

$B \to \rho$ transition form factors

Form factors are defined as:

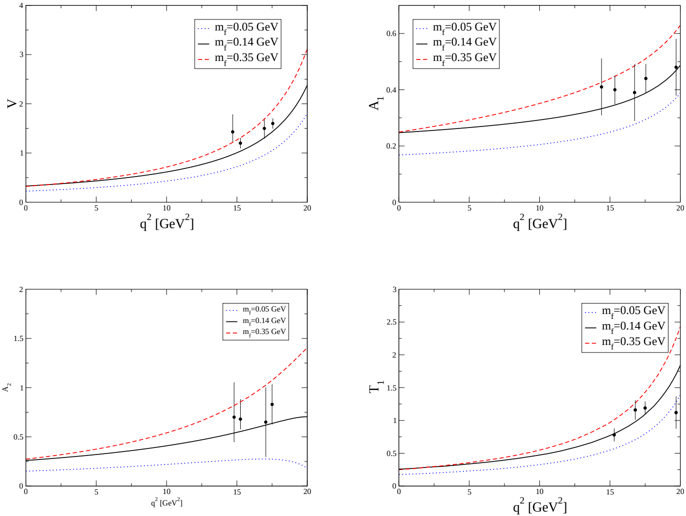
$$
\begin{aligned}
\langle \rho(k,\varepsilon)|\bar{q}\gamma^\mu(1-\gamma^5)b|B(p)\rangle &= \frac{2iV(q^2)}{m_B+m_\rho}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma - 2m_\rho A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu \\
&- (m_B+m_\rho)A_1(q^2)\left(\varepsilon^{\mu *} - \frac{\varepsilon^*\cdot q q^\mu}{q^2}\right) \\
&+ A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_\rho}\left[(p+k)^\mu - \frac{m_B^2-m_\rho^2}{q^2}q^\mu\right]
\end{aligned}
$$

$$
\begin{aligned}
q_\nu\langle \rho(k,\varepsilon)|\bar{d}\sigma^{\mu\nu}(1-\gamma^5)b|B(p)\rangle &= 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho k_\sigma \\
&- iT_2(q^2)[(\varepsilon^*\cdot q)(p+k)_\mu - \varepsilon^*_\mu(m_B^2-m_\rho^2)] \\
&- iT_3(q^2)(\varepsilon^*\cdot q)\left[\frac{q^2}{m_B^2-m_\rho^2}(p+k)_\mu - q_\mu\right]
\end{aligned}
$$

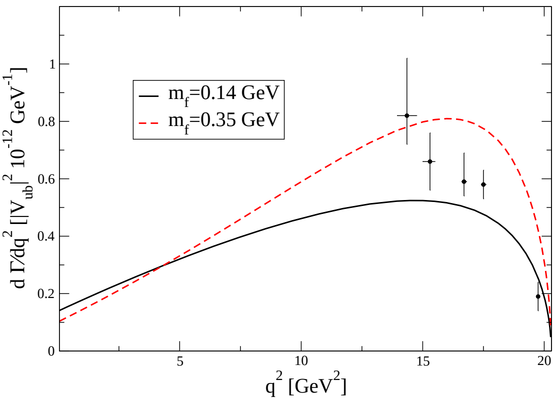
AdS/QCD prediction for $B \to \rho$ transition form factors

R. Campbell, S. Lord, R. Sandapen, MA, PRD88.074031(2013)

Using light-cone sum rules with holographic DAs



Differential decay rate for semileptonic $B \to \rho\ell\nu$



(a) Differential decay rate for the semileptonic $B \to \rho\ell\bar{\nu}$ decay. The lattice data points are from UKQCD Collaboration.

Numerical predictions

BaBar collaboration has measured partial branching fractions in q^2 bins:PRD83, 032007 (2011)

$$\Delta B_{\rm low} = \int_0^8 \frac{\mathrm{d}B}{\mathrm{d}q^2}\mathrm{d}q^2 = (0.564 \pm 0.166) \times 10^{-4}$$

$$\Delta B_{\rm mid} = \int_8^{16} \frac{\mathrm{d}B}{\mathrm{d}q^2}\mathrm{d}q^2 = (0.912 \pm 0.147) \times 10^{-4}$$

$$\Delta B_{\rm high} = \int_{16}^{20.3} \frac{\mathrm{d}B}{\mathrm{d}q^2}\mathrm{d}q^2 = (0.268 \pm 0.062) \times 10^{-4}$$

$$R_{\rm low} = \frac{\Delta B_{\rm low}}{\Delta B_{\rm mid}} = 0.618 \pm 0.207$$

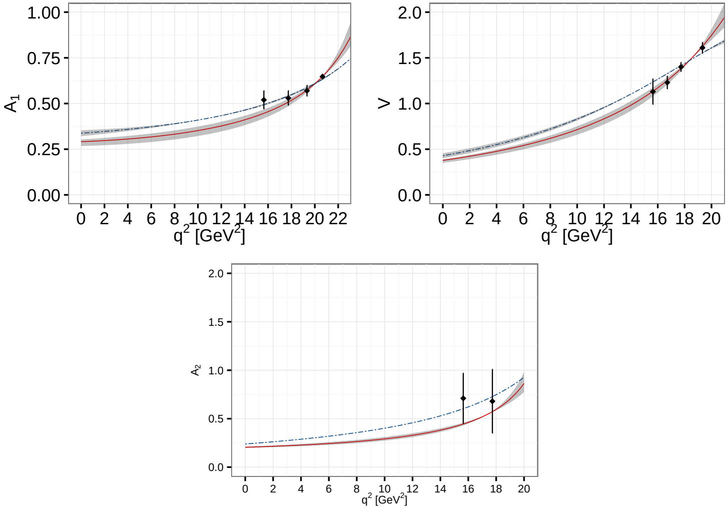
$$R_{\rm high} = \frac{\Delta B_{\rm high}}{\Delta B_{\rm mid}} = 0.294 \pm 0.083$$

Our predictions for $m_f = 0.14, 0.35$:

$$R_{\rm low} = 0.580, 0.424$$
$$R_{\rm high} = 0.427, 0.503$$

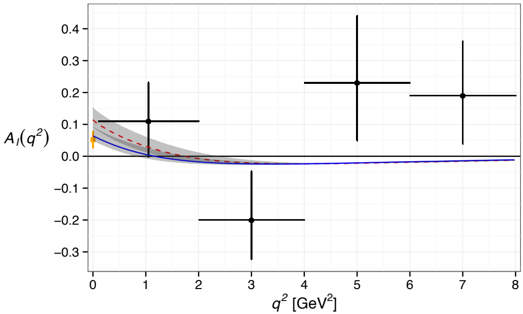
AdS/QCD prediction for $B \to K^*$ transition form factors

R. Campbell, S. Lord, R. Sandapen, MA, PRD89.074021(2014)



Isospin asymmetry in dileptonic $B \to K^* \mu^+ \mu^-$

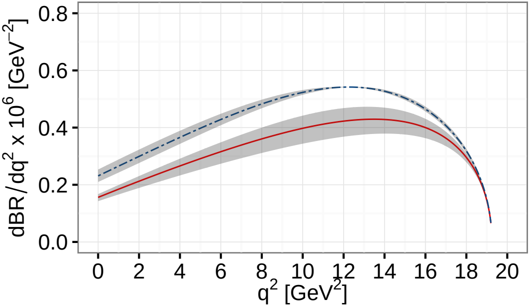
S. Lord, R. Sandapen, MA, PRD90.074010(2014)



(b) Isospin asymmetry in $B \to K^* \mu^+ \mu^-$ decay vs dileptonic invariant mass. The data points are from LHCb. The dashed red curve is the prediction of the QCD sum rules.

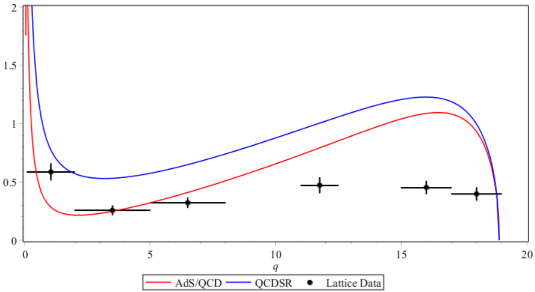
Differential decay rate for $B \to K^* \nu \bar{\nu}$

A. Leger, Z. McIntyre, A. Morrison, R. Sandapen, MA, PRD98.053002(2018)



(c) The AdS/QCD (Solid line) and SR (Dashed line) predictions for the differential Branching Ratio for $B \to K^* \nu \bar{\nu}$. The shaded band represents the uncertainty coming from the form factors.

Differential decay rate for $B_s \to \phi\mu^+\mu^-$ (Preliminary)



(d) The AdS/QCD (red line) and SR (blue line) predictions for the differential Branching Ratio $B_s \to \phi\mu^+\mu^-$.

Summary and outlook

- AdS/QCD LFWF is used to obtain ρ and K^* DAs.
- DAs are essential ingredients for the calculation of the $B \to \rho, K^*$ transition form factors via LCSR.
- We are in the process of calculating the form factors directly from LFWF.
- We have looked into the proper LFWF for pseudoscalar mesons and working to predict $B \to \pi, K$ transition form factors. (F. Chishtie, R. Sandapen, MA PRD95,074008(2017); C. Mondal, R. Sandapen, MA, PRD98, 034010(2018))

Conformal invariance and the dAFF mechanism

3 mechanisms to break conformal symmetry

1. Spontaneous
2. Explicit
3. de Alfaro, Fubini, Furlan (dAFF)✓ V. Alfaro, S. Fubini and G. Furlan. Nuovo. Cim. A34 (1976) 569

   in conformal QM, changing the evolution parameter allows the introduction of a mass scale in the Hamiltonian while preserving the conformal invariance of the underlying action.

4. dAFF/LF mapping ⇒ Semiclassical QCD LF Hamiltonian

$$H^{\mathrm{LF}}_{\mathrm{scQCD}} = \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right)$$

with $\boxed{U(\zeta) = \kappa^4 \zeta^2}$

Confining potential $U_{\rm eff}$

A remarkable feature in light-front holography is that the form of the confinement potential is uniquely determined to be that of a harmonic oscillator, i.e. $U_{\rm eff} = \kappa^4\zeta^2$ where κ is the fundamental of the model.
S. J. Brodsky, G. F. De Tramond, and H. G. Dosch, Phys. Lett. B729, 3 (2014), 1302.4105.
$\zeta \to z$ (the fifth dimension of anti-de Sitter (AdS) space), the HSE also described the propagation of weakly-coupled spin-J modes in a modified AdS space with the confining QCD potential then determined by the form of the dilaton field, $\varphi(z)$, which modifies the pure AdS geometry.

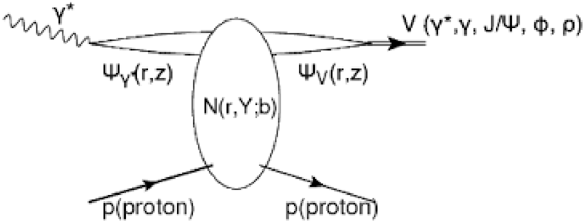
$$U_{\rm eff}(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

To recover this harmonic potential, the dilaton field has to be quadratic, i.e. $\varphi(z) = \kappa^2 z^2$

$$U_{\rm eff}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

where $J = L + S$.

Diffractive vector meson production



- $ep \to epV$ or $\gamma^* p \to pV$
- Sensitivity to non-perturbative physics
- Can be used to fine tune the vector meson wavefunction

N. Sharma, R. Sandapen, MA, PhysRevD.94.074018(2016)