Accessing Linearly Polarized Gluon Distribution in J/ψ Production at EIC

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Plan of Talk

Gluon TMDs

Linearly Polarized Gluon Distribution

Probing Linearly Polarized Gluon Distribution in J/ψ Production at EIC

Azimuthal Asymmetry : $\cos 2\phi$

Numerical Results

Conclusions

Gluon TMDs

TMD-PDFs (Transverse Momentum Dependent Parton Distribution Functions): $f(x, k_\perp, Q^2)$ gives the number density of partons , with their intrinsic transverse motion and spin, inside a nucleon, with its spin.

Do not have much information about gluon TMDs, it satisfies positivity bound.

Gluon TMDs are process dependent like the quark TMDs due to presence of gauge link. Each gluon TMD contains two gauge link that makes the process dependence of gluon TMDs more involved than quark TMDs.

The simplest possible gauge link configurations are $++ \; or \; --$ and $-+ \; or \; +-$ where they are described as Weizsacker-Williams (WW) type and Dipole distribution respectively.

Linearly Polarized Gluon TMDs

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

It affects unpolarised cross section and cause azimuthal asymmetries, $\cos 2\phi, \cos 4\phi$.

It's a time-reversal even function and can be WW type or Dipole distribution depending on gauge link.

It can be probed in Drell-Yan process and SIDIS process. Though it has not been extracted from the data yet, but lot of theoretical studies has been done.

Initial state interactions and final state interactions may affect the generalized factorization. Such effects are less complicated in ep compared with pp and pA collsions.

Linearly Polarized Gluon Distribution($h_1^{\perp g}$) in J/ψ Production

$h_1^{\perp g}$ has been probed analytically in J/ψ production at LO process: $\gamma^* + g \to \mathrm{c} + \bar{c}$

A. Mukherjee and S. Rajesh, EPJC 77, 854 (2017)

Contributes at z=1, where ‘z’ is energy fraction of γ^* carried by J/ψ in proton rest frame

Used NRQCD for J/ψ production mechanism.

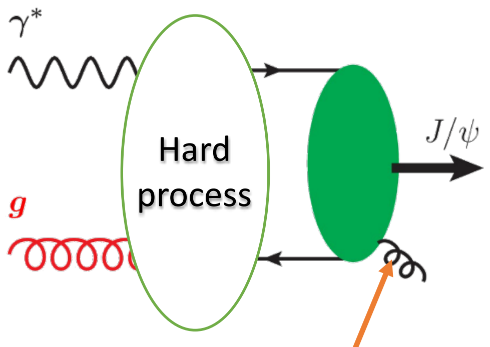
We extend it to the kinematical region z<1. RK and A. Mukherjee; PRD 99 (2019), no. 5, 054012

With the heavy quark pair produced in the hard process: $\gamma^* + g \to \mathrm{c} + \bar{\mathrm{c}} + \mathrm{g}$ assuming the TMD factorization holds.

The heavy quark pair then hadronizes to form J/ψ, a soft process. The hadronization is described in terms of long distance matrix elements (LDMEs), a non-perturbative quantity, can be extracted by fitting experimental data.

J/ψ Production form $c + \bar{c}$

J/ψ is a bound state of $c\bar{c}$



In the rest frame of bound state, the relative momenta of two quarks is small compared to their mass, that allows to nonrelativistic approach called NRQCD.

The heavy $c\bar{c}$ pair can be produced in color singlet (CS) state or in color octate (CO) state.

In CS model, the $c\bar{c}$ produced , in hard process, in CS mode with the same quantum number as of J/ψ.

NRQCD factorization

LDMEs

$$d\sigma^{ab\to J/\psi} = \sum_n d\hat{\sigma}[ab \to c\bar{c}(n)]\langle 0 \mid \mathcal{O}_n^{J/\psi} \mid 0\rangle$$

G. T. Bodwin et al, PRD51 (1995)

J/ψ Production in *ep* collision

We present a cos 2ϕ asymmetry in Process: $e(l) + p(P) \to e(l') + J/\psi(P_h) + X$

J/ψ is produced using CS model.

In the kinematic region $z < 1$

The corresponding hard process is : $\gamma^* + g \to \mathrm{c} + \bar{c} + \mathrm{g}$ final state gluon is not detected

$z = P \cdot P_h / P \cdot q$ z is energy fraction of γ^* carried by J/ψ in proton rest frame

$$q = l - l' \qquad Q^2 = s x_B y \qquad s = (l + P)^2 \qquad x_B = \frac{Q^2}{2P \cdot q} \qquad y = P \cdot q / P \cdot l$$

The incoming and outgoing electron forms the leptonic plane. The azimuthal angles are measured wrt this plane.

J/ψ production in *ep* collision

Assuming TMD factorization, the differential cross-section is given by

$$d\sigma = \frac{1}{2s}\frac{d^3l'}{(2\pi)^3 2E_{l'}}\frac{d^3P_h}{(2\pi)^3 2E_{P_h}}\int\frac{d^3p_g}{(2\pi)^3 2E_g}\int dx d^2\boldsymbol{k}_\perp (2\pi)^4\delta(q+k-P_h-p_g)$$
$$\times\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi^{\nu\nu'}(x,\boldsymbol{k}_\perp)\mathcal{M}_{\mu\nu}^{\gamma^*+g\to J/\psi+g}\mathcal{M}_{\mu'\nu'}^{*\gamma^*+g\to J/\psi+g}$$

Lepton tensor $\quad L^{\mu\mu'}(l,q) = e^2(-g^{\mu\mu'}Q^2 + 2(l^\mu l'^{\mu'} + l^{\mu'} l'^\mu))$

Gluon correlator for unpolarized proton

$$\phi_g^{\nu\nu'}(x,\mathbf{k}_\perp) = \frac{1}{2x}\left[-g_\perp^{\nu\nu'} f_1^g(x,\mathbf{k}_\perp^2) + \left(\frac{k_\perp^\nu k_\perp^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'}\frac{\mathbf{k}_\perp^2}{2M_p^2}\right)h_1^{\perp g}(x,\mathbf{k}_\perp^2)\right]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

Feynman diagrams

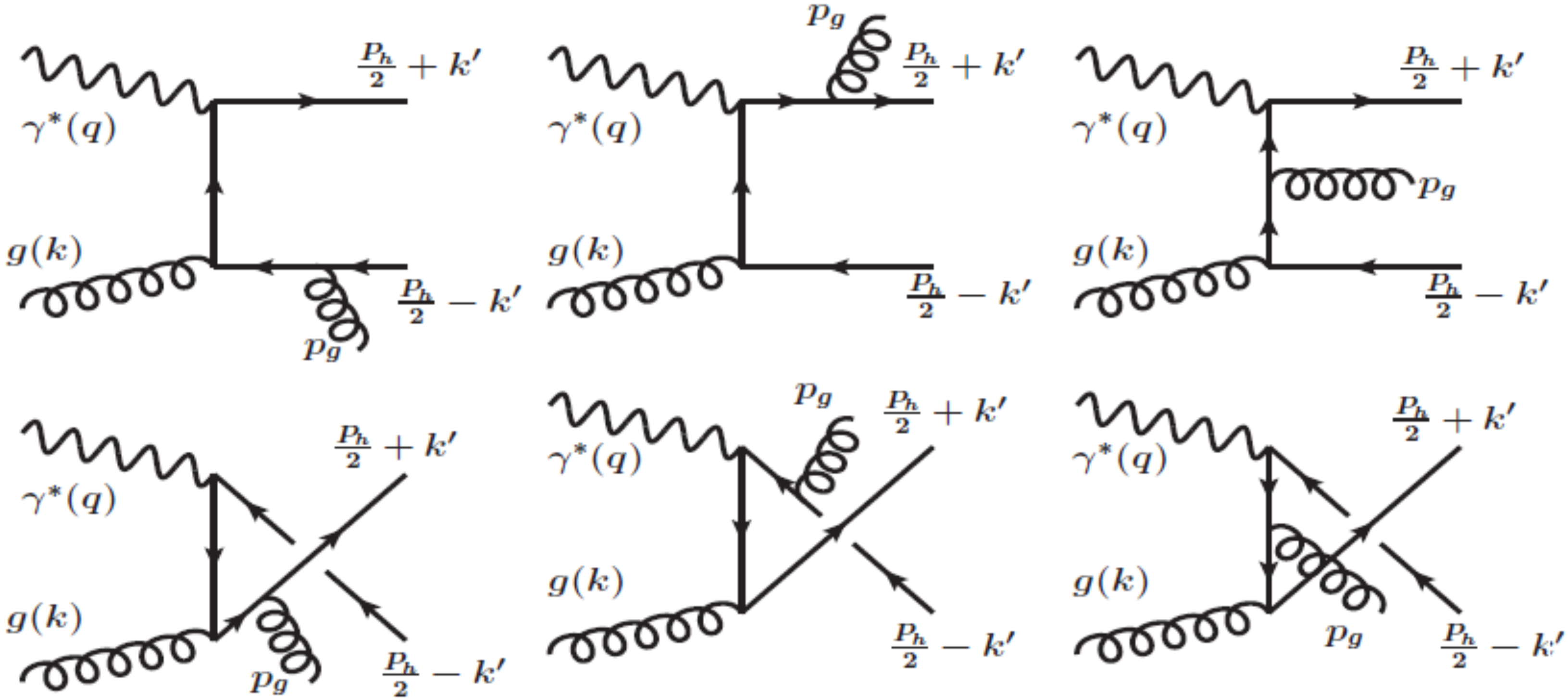


FIG. 1. Feynman diagrams for $\gamma^* + g \to J/\psi + g$ process

Amplitude calculation

The amplitude can be written as

$$\mathcal{M}\left(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1)}](P_h) + g\right) = \sum_{L_z S_z} \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \Psi_{LL_z}(\boldsymbol{k}') \langle LL_z; SS_z | JJ_z \rangle$$

$$\times \mathrm{Tr}[O(q,k,P_h,k')\mathcal{P}_{SS_z}(P_h,k')],$$

D. Boer and C. Pisano, PRD 86,094007 (2012)

The operator $O(q,k,P_h,k')$ is calculated from the Feynman diagrams

$$O(q,k,P_h,k') = \sum_{m=1}^{6} \mathcal{C}_m O_m(q,k,P_h,k').$$

The spin projection operator that projects the spin triplet and spin singlet states

$$\mathcal{P}_{SS_z}(P_h,k') = \sum_{s_1 s_2} \langle \frac{1}{2}s_1; \frac{1}{2}s_2 | SS_z \rangle v(\frac{P_h}{2} - k', s_1) \bar{u}(\frac{P_h}{2} + k', s_2)$$

$$= \frac{1}{4M^{3/2}}(-\not{P}_h + 2\not{k}' + M)\Pi_{SS_z}(\not{P}_h + 2\not{k}' + M) + \mathcal{O}(k'^2)$$

Amplitude calculation

Since, $k' \ll P_h$, amplitude expanded in Taylor series about $k' = 0$

First term in the expansion gives S wave state.
We calculate $^3S_1$ state as it contributes in CS model.

The final expression of amplitude

$$\mathcal{M}[^3S_1^{(1)}](P_h, k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\delta_{ab}}{\sqrt{N_c}} \mathrm{Tr}\left[\sum_{m=1}^{3} O_m(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right]$$

Where,

$$\sum_{m=1}^{3} O_m(0) = g_s^2 (ee_c) \varepsilon_{\lambda_g}^{\rho *}(p_g) \left[\frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\mu(-\not{P}_h - 2\not{p}_g + M)\gamma_\rho}{(\hat{s} - M^2)(\hat{u} - M^2 + q^2)} + \frac{\gamma_\rho(\not{P}_h + 2\not{p}_g + M)\gamma_\nu(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{s} - M^2)(\hat{t} - M^2)} + \frac{\gamma_\nu(\not{P}_h - 2\not{q} + M)\gamma_\rho(-\not{P}_h + 2\not{k} + M)\gamma_\mu}{(\hat{t} - M^2)(\hat{u} - M^2 + q^2)} \right].$$

Asymmetry calculation

Three amplitudes and conjugates, with i= 1,2,3

$$\mathcal{M}_i[^3S_1^{(1)}](P_h,k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\delta_{ab}}{\sqrt{N_c}} \mathrm{Tr}\left[O_i(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}\right],$$

Cross section will have the contributions of the form

$$M_i M_j = L^{\mu\mu'}(l,q)\Phi^{\nu\nu'}(x,\boldsymbol{k}_\perp)\mathcal{M}_{i\mu\nu}^{\gamma^*+g\to J/\psi+g}\mathcal{M}_{j\mu'\nu'}^{*\gamma^*+g\to J/\psi+g}$$

The final expression of the differential cross section

$$\frac{d\sigma}{dy dx_B dz d^2\mathbf{P}_{h\perp}} = \frac{1}{256\pi^4}\frac{1}{x_B^2 s^3 y^2 z(1-z)}\int d^2\mathbf{k}_\perp |\ M\ |^2$$

Asymmetry calculation

We are interested in small-x, we neglected the terms with higher power of x_B, also kept terms up to $(k_\perp^2/M_p^2)$.

The leading terms to the asymmetry come from

$$P_{h\perp} = P_{hT}$$

$$\frac{d\sigma}{dy\,dx_B\,dz\,d^2\mathbf{P}_{hT}} = \frac{1}{256\pi^4}\frac{1}{x_B^2 s^3 y^2 z(1-z)}\int k_\perp dk_\perp$$

$$\{(A_0 + A_1 cos\phi_h) f_1^g(x, \mathbf{k}_\perp^2)\} + \frac{k_\perp^2}{M_p^2}\{(B_0 + B_1 cos\phi_h + B_2 cos2\phi_h) h_1^{\perp g}(x, \mathbf{k}_\perp^2)\}$$

The cos $2\phi_h$ asymmetry:

$$\langle cos(2\phi_h)\rangle = \frac{\int d\phi_h cos(2\phi_h) d\sigma}{\int d\phi_h d\sigma}$$

ϕ_h is the azimuthal angle of J/ψ production plane with lepton plane.

Parametrization of TMDs

Linearly polarized gluon distribution satisfy the positivity bound. Saturating this, we obtain the upper bound of the asymmetry

$$\frac{\mathbf{k}_\perp^2}{2M_p^2}\left|h_1^{\perp g}(x,\mathbf{k}_\perp^2)\right| \le f_1^g(x,\mathbf{k}_\perp^2)$$

$$f_1^g(x,\mathbf{k}_\perp^2) = f_1^g(x,\mu)\frac{1}{\pi\langle k_\perp^2\rangle}e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Gaussian parametrization

Boer and Pisano (2012)

We took $r = 1/3,\ \langle k_\perp^2\rangle = 0.25 GeV^2$

$$h_1^{\perp g}(x,\mathbf{k}_\perp^2) = \frac{M_p^2 f_1^g(x,Q^2)}{\pi\langle k_\perp^2\rangle^2}\frac{2(1-r)}{r}e^{1-\frac{k_\perp^2}{r\langle k_\perp^2\rangle}}$$

Mclerran-Venugopalan (MV) model for WW gluon distribution in small-x.

Regularized version:

$$\frac{\mathbf{k}_\perp^2}{2M_p^2}\frac{h_1^{\perp g}(x,\mathbf{k}_\perp^2)}{f_1^g(x,\mathbf{k}_\perp^2)} = \frac{\int dr \frac{J_2(k_\perp r)}{r\log(\frac{1}{r^2\Lambda_{QCD}^2})}\left(1-e^{-\frac{r^2}{4}Q_{sg0}^2\log(\frac{1}{r^2\Lambda_{QCD}^2})}\right)}{\int dr \frac{J_0(k_\perp r)}{r}\left(1-e^{-\frac{r^2}{4}Q_{sg0}^2\log(\frac{1}{r^2\Lambda_{QCD}^2})}\right)}$$

McLerran and Venugopalan, PRD (1994)

Bacchetta, Boer, Pisano, Taels (2018)

$$Q_{sg0}^2 = (N_c/C_F)\times Q_{s0}^2 \qquad Q_{s0}^2 = 0.35\ \mathrm{GeV}^2 \text{ at } x = 0.01 \text{ and } \Lambda_{QCD} = 0.2\ \mathrm{GeV}$$

Where Q_{sg} is saturation scale. The ratio is less than 1 for all transverse momentum.

Kinematical region for asymmetry

We use a framework based on generalized parton model with the inclusion of intrinsic transverse momentum.

We consider the region where $P_{h\perp} < \mathrm{M}$, M is mass of J/ψ.

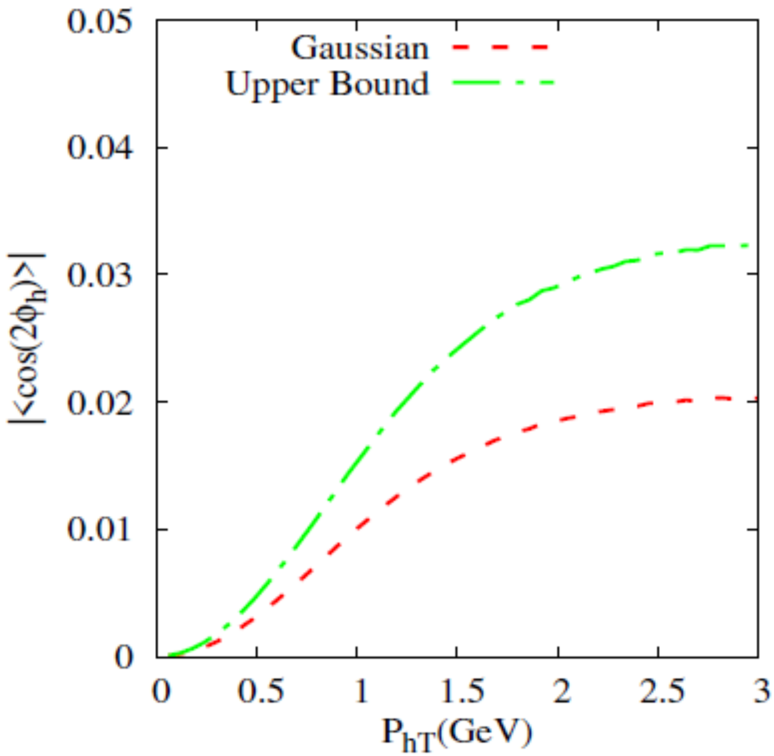
This upper limit on $P_{h\perp}$, reduces the fragmentation contributions from heavy quark.

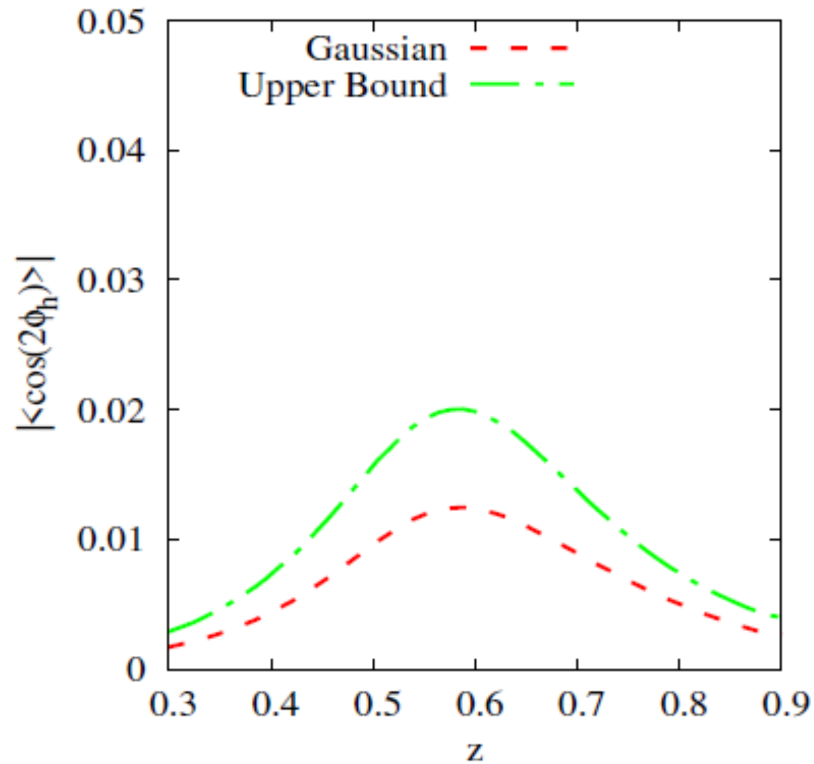
We impose cutoff on z : $0.3 < z < 0.9$

Cutoff $z < 0.9$ made outgoing gluon hard and no contributions from virtual diagrams

Cutoff $0.3 < z$ to eliminate fragmentation of hard gluon to J/ψ.

Cos(2ϕ) Asymmetry at EIC





Z is fraction of momentum taken by J/ψ from photon

$\sqrt{s} = 45\ GeV$

$x_B = 0.01$

$0.3 < z < 0.9$

$0 < P_{hT} < 3\ GeV$

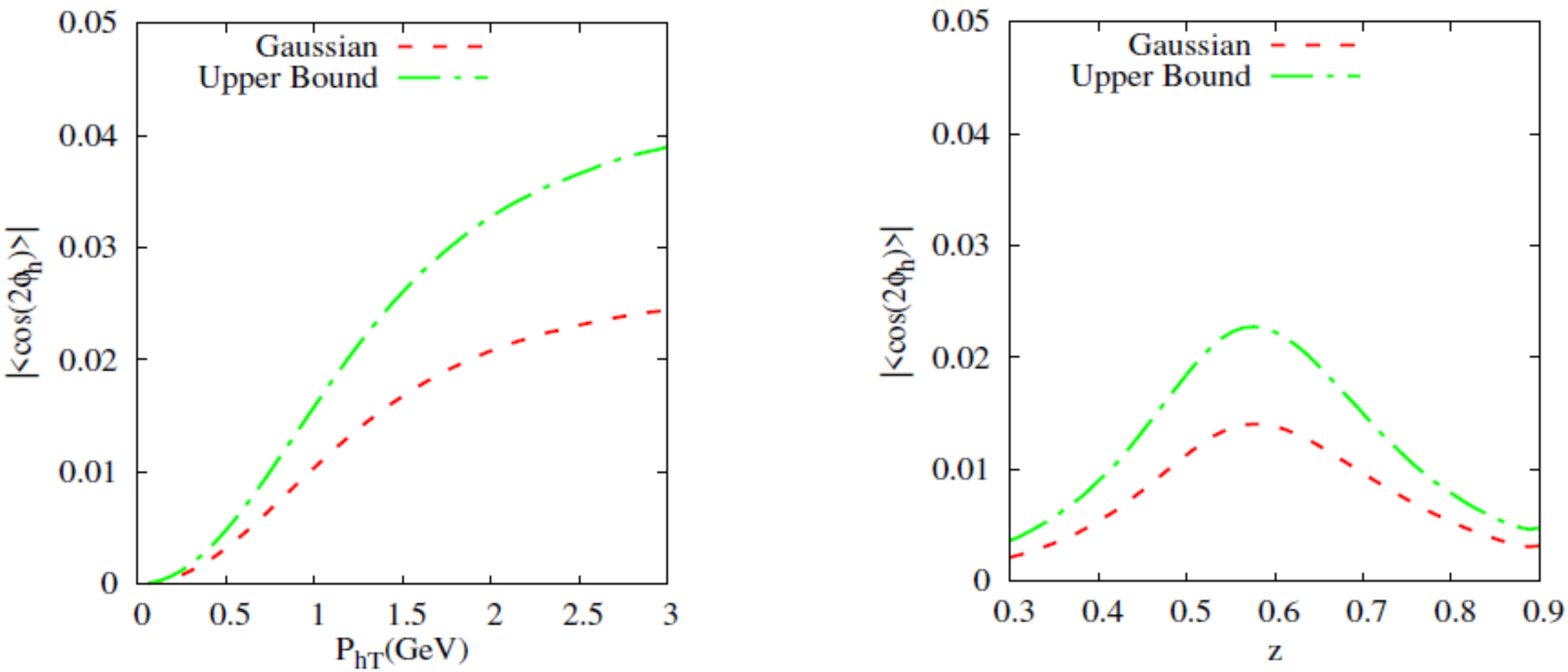
$0.05 < y < 0.4$

We have used MSTW2008 pdfs, TMD evolution is not used.

Upper bound of the asymmetry is obtained by saturating the positivity bound.

RK and A. Mukherjee; PRD 99 (2019), no. 5, 054012

Cos(2ϕ) Asymmetry at EIC



Asymmetry increases with increase of center of mass energy for fixed x_B.

Asymmetry is coming negative, consistent with the LO.
Plots show the magnitude of asymmetry.

$\sqrt{s} = 150\ GeV$

$x_B = 0.01$

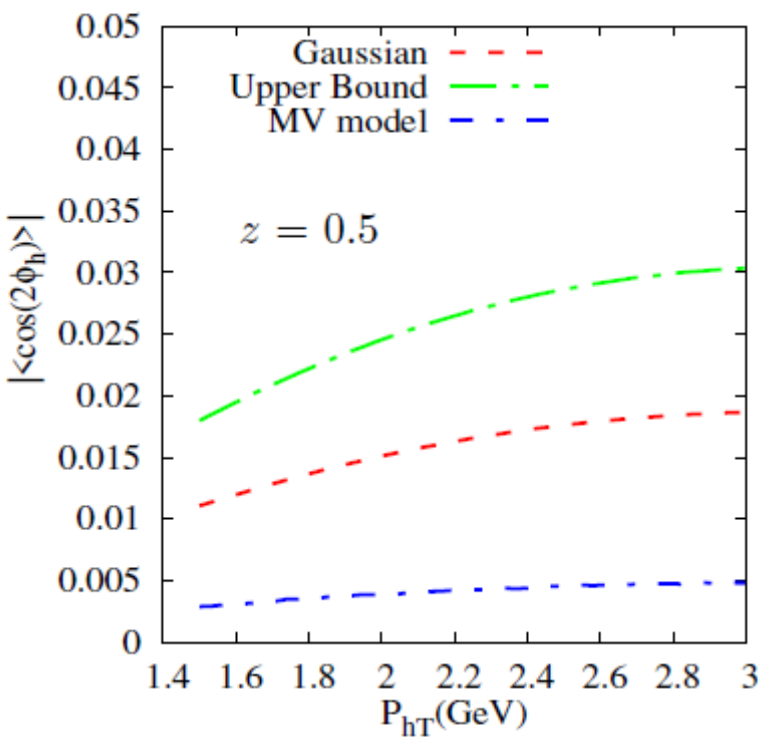
$0.3 < z < 0.9$

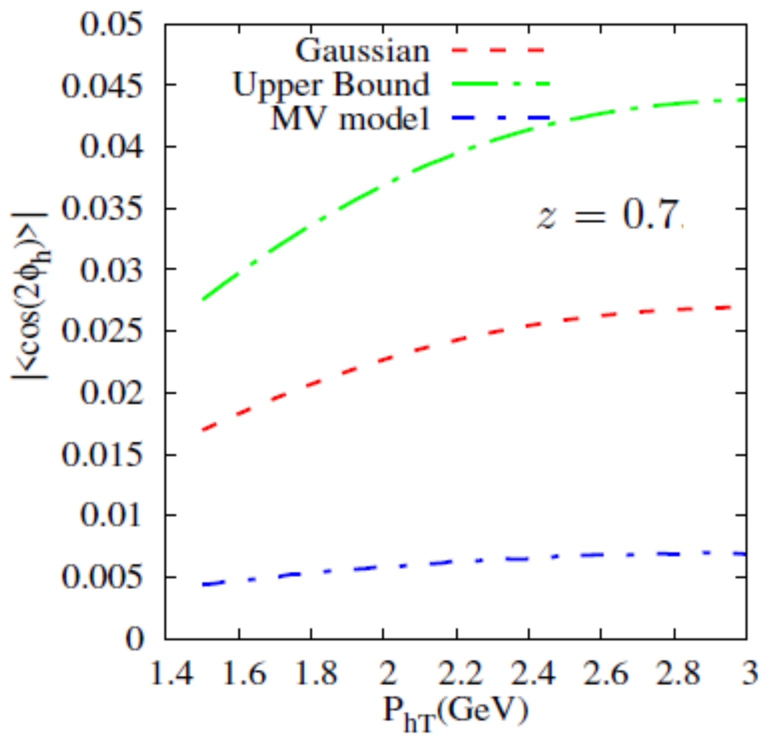
$0 < P_{hT} < 3\ GeV$

$0.005 < y < 0.04$

RK and A. Mukherjee; PRD 99 (2019), no. 5, 054012

Cos(2ϕ) Asymmetry at EIC





$61 GeV < \sqrt{s} < 181 GeV$

$58 GeV < \sqrt{s} < 182 GeV$

$Q^2 = 9\ GeV^2$ $\quad x = 0.01$

MV model is giving smaller asymmetry than the Gaussian for fixed value of x and z

RK and A. Mukherjee; PRD 99 (2019), no. 5, 054012

Conclusions

- A small but sizable azimuthal asymmetry (cos 2ϕ) is calculated in electroproduction of J/ψ at EIC, which probes the linearly polarized WW gluon distributions.
- Asymmetry calculated in kinematical region $z < 1$ and for $small - x$.
- Asymmetry calculated using CS model.
- The asymmetry found in Gaussian model is more as compared in case of MV model.
- We plan to see the effect of the CO mechanism on the asymmetry in future work.

Thank You…

Backup Slides

$$
\begin{aligned}
A_0 =&64\pi M^4\Big\{\Big\{M^2(z-1)\left(M^2+{P_{h\perp}}^2-sx_By(z-2)z\right)\Big(M^2(z-1)(2z-3)\\
&+{P_{h\perp}}^2(2z(4z-9)+9)-sy(z-1)\Big(x(z+1)(2z-3)\\
&+x_Bz\left(3(y(y+6)-6)z^2-6(y(y+4)-3)z+4y+3\right)+2x_By+x_B\Big)\Big)\\
&+M^4z\Big(-2M^2(z-1)^2(3z-5)-4{P_{h\perp}}^2(z-1)(z(6z-13)+8)\\
&+sy\Big(x(3z-10)(z-1)^2+x_Bz\Big((50-31y(y+2))z^3\\
&+(y(57y+118)-80)z^2-y(43y+106)z\\
&+y(9y+46)+78z-56\Big)+2x_B((y-4)y+10)\Big)\Big)\Big\}/s\\
&+2xyz^2\Big(M^4(z-1)^3+M^2\Big({P_{h\perp}}^2(z-1)(8(z-1)z+3)\\
&+sy\Big(x_B\left(z\left((1-6y(y+2))z^2+2y(y+4)z-2y+z+1\right)-1\right)\\
&-x(z-1)^3\Big)\Big)+{P_{h\perp}}^4(z(z(11z-23)+13)-3)\\
&+{P_{h\perp}}^2sy\Big(x(z(-2(z-5)z-9)+3)+x_B\Big(z\Big(-3(y(5y+16)-6)z^2\\
&+2(y(y+14)-5)z-6y+7\Big)-3\Big)\Big)\\
&+s^2xx_By^2z(z((y-2)y(2z-1)+2(8z-7))+4)\Big)\Big\}/\Big\{x^2y^3(z-1)^2z^3\\
&\times\left(M^2+sy(x_B-x)\right)^2\left(M^2+{P_{h\perp}}^2-sx_By(z-2)z\right)^2\Big\}
\end{aligned}
$$

$$
\begin{aligned}
B_2 =&\Big\{128\pi M^4{P_{h\perp}}^2sx_B(y-1)(z(5z-4)+2)\Big\}/\Big\{xy(z-1)^2\\
&\times\left(M^2+sy(x_B-x)\right)^2\left(M^2+{P_{h\perp}}^2-sx_By(z-2)z\right)^2\Big\}
\end{aligned}
$$