Minkowski space approach to self-energies and scale invariance

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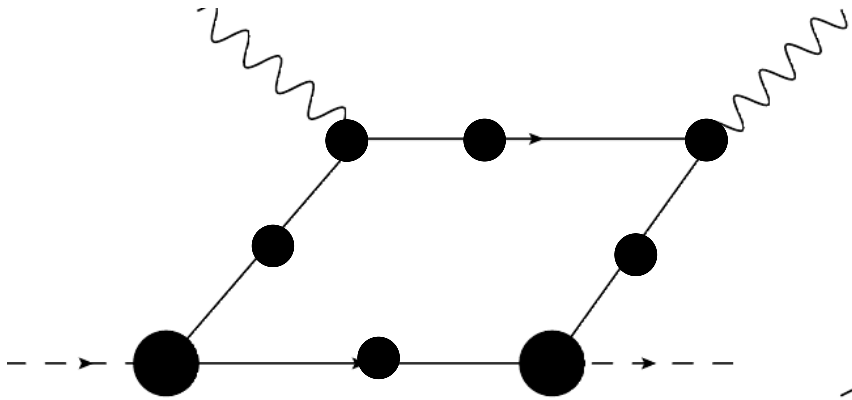
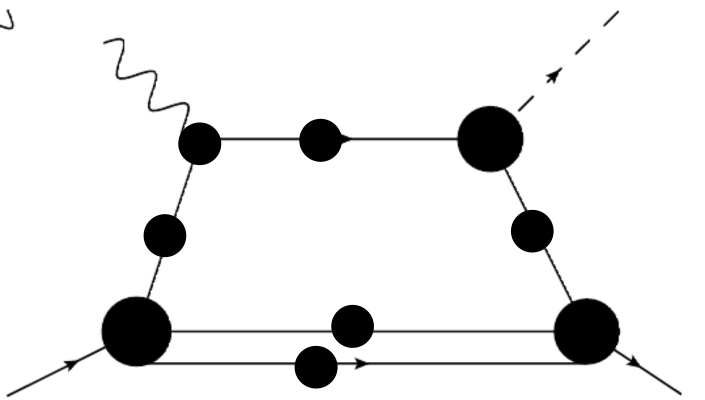
OUTLINE

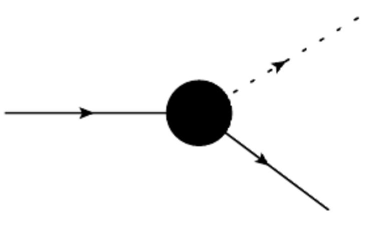
- Motivation
- Schwinger-Dyson equation in Minkowski space - QED-like theory in Rainbow ladder approx.
- Un-Wick rotation of the SD equations & comparison of results
- Miransky scaling and cconsistence with the integral representation
- Scaling properties of the Fermion-boson model and solution of the UV equations
- Conclusion and perspectives

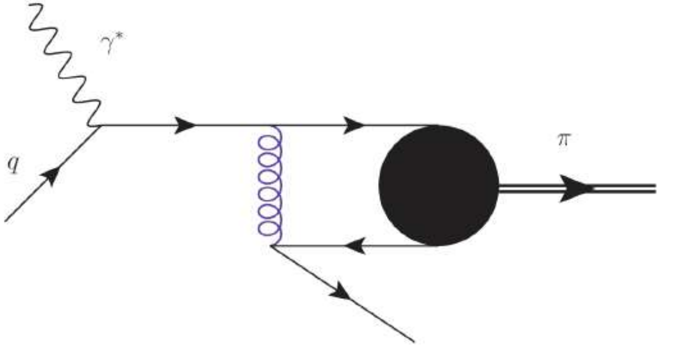
Motivation

Form-factors, DVCS in ERBL – DGLAP regions









Fragmentation function

Dressing quarks

QCD has dynamical chiral symmetry breaking, pions (Goldstone bosons)…

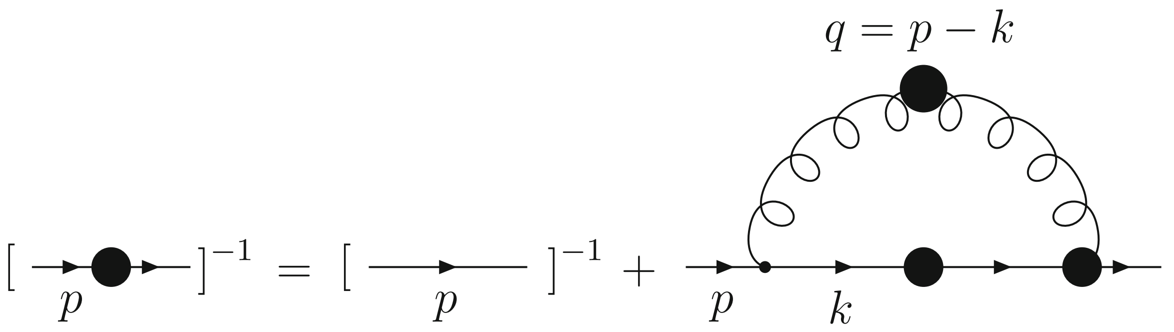


Fig. 2 The Dyson–Schwinger equation for the quark. The solid blobs denote dressed propagators and vertices

Solve Dyson-Schwinger equation in Minkowski space!

QED-Like bare vertex, bare photon…

Sauli, Few-Body Systems 39, 45–99 (2006)
(integral representation)

DS Rainbow ladder QED-like theory

In coll. with Dyana Duarte, Emanuel Ydrefors, Wayne de Paula, Shaoyang Ji, Pieter Maris

- Bare massive vector boson (arbitrary gauge)
- Bare vertex
- Pauli-Villars regulators
- Integral representation: Kallén-Lehmann Rep., Nakanishi Int Rep

$$S_f(k) = \frac{1}{\not{k} - \overline{m}_0 + \not{k} A_f(k^2) - B_f(k^2) + i\epsilon}$$

$$\boxed{\begin{aligned} \not{k} A_f(k^2) - B_f(k^2) &= ig^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S(k-q)\gamma_\nu}{q^2 - m_\sigma^2 + i\epsilon} \left[g^{\mu\nu} - \frac{(1-\xi)q^\mu q^\nu}{q^2 - \xi m_\sigma^2 + i\epsilon} \right] \\ &- ig^2 \int \frac{d^4q}{(2\pi)^4} \frac{\gamma_\mu S(k-q)\gamma_\nu}{q^2 - \Lambda^2 + i\epsilon} \left[g^{\mu\nu} - \frac{(1-\xi)q^\mu q^\nu}{q^2 - \xi \Lambda^2 + i\epsilon} \right] \end{aligned}}$$

$\xi = 0$ (Landau Gauge) & $\xi = 1$ (Feynman Gauge)

Main Tool: Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space"
[Nakanishi PR130(1963)1230]

$$S_f\left(k\right) = \frac{1}{\not{k} - \overline{m}_0 + \not{k} A_f\left(k^2\right) - B_f\left(k^2\right) + i\epsilon}$$

$$A_f\left(k^2\right) = \int_0^\infty d\gamma \frac{\rho_A\left(\gamma\right)}{k^2 - \gamma + i\epsilon} \qquad B_f\left(k^2\right) = \int_0^\infty d\gamma \frac{\rho_B\left(\gamma\right)}{k^2 - \gamma + i\epsilon}$$

$$S_f = R\frac{\not{k} + \overline{m}_0}{k^2 - \overline{m}_0^2 + i\epsilon} + \not{k}\int_0^\infty d\gamma \frac{\rho_v\left(\gamma\right)}{k^2 - \gamma + i\epsilon} + \int_0^\infty d\gamma \frac{\rho_s\left(\gamma\right)}{k^2 - \gamma + i\epsilon}$$

physical mass

- Note: Wick-rotation is the exact analytical continuation of the Minkowski space Nakanishi representation (Källen-Lehman):

explore the complex plane!

❖ METHOD TO SOLVE IN MINKOWSKI SPACE

- ✓ **Connection formulas for the $\rho' s$: propagator & self-energy**
- ✓ **Int Rep in the SD eq → Feynman parametrization…**
- ✓ **Uniqueness of the weight functions….**

$$\begin{aligned}\rho_v(\gamma) &= -2\frac{f_A(\gamma)}{d(\gamma)}\left[\gamma\rho_A(\gamma)f_A(\gamma)-\rho_B(\gamma)f_B(\gamma)\right]\\ &+ \frac{\rho_A(\gamma)}{d(\gamma)}\left[\gamma f_A^2(\gamma)-\pi^2\gamma\rho_A^2(\gamma)-f_B^2(\gamma)+\pi^2\rho_B^2(\gamma)\right]\end{aligned}$$

$$d(\gamma)=\left[\gamma f_A^2(\gamma)-\pi^2\gamma\rho_A^2(\gamma)-f_B^2(\gamma)+\pi^2\rho_B^2(\gamma)\right]^2+4\pi^2\left[\gamma\rho_A(\gamma)f_A(\gamma)-\rho_B(\gamma)f_B(\gamma)\right]$$

$$f_A(\gamma) = 1+P.\int_{\gamma_A^{thres}}^{\infty}\frac{\rho_A(\gamma')}{k^2-\gamma'}\gamma' \qquad f_B(\gamma) = \overline{m}_0+P.\int_{\gamma_B^{thres}}^{\infty}\frac{\rho_B(\gamma')}{k^2-\gamma'}d\gamma'$$

$$\rho_A(\gamma) = 0 \text{ for } \gamma < \gamma_A^{thres} = \left(\overline{m}_0 + \sqrt{\xi} m_\sigma\right)^2 \text{ and } \rho_B(\gamma) = 0 \text{ for } \gamma < \gamma_B^{thres} = \left(\overline{m}_0 + m_\sigma\right)^2$$

physical mass (pole mass)

$$0 \le \xi \le 1$$

on-mass-shell renormalization

$$\overline{m}_0^2 f_A^2(\overline{m}_0^2) - f_B^2(\overline{m}_0^2) = 0$$

$$\overline{m}_0 f_A(\overline{m}_0^2) = \overline{m}_0 + \overline{m}_0 P\int_0^\infty d\gamma \frac{\rho_A(\gamma)}{\overline{m}_0^2 - \gamma}$$

$$= f_B(m_0^2) = m_0 + P\int_0^\infty d\gamma \frac{\rho_B(\gamma)}{\overline{m}_0^2 - \gamma}$$

bare mass

in our calculations the pole mass is given

Feynman gauge

$$\rho_A^{\xi=1}\left(k^2\right) = R\,\mathcal{K}_A^{\xi=1}\left(k^2,\overline{m}_0;m_\sigma^2\right) + \int_0^\infty ds\rho_v\left(s\right)\mathcal{K}_A^{\xi=1}\left(k^2,s;m_\sigma^2\right) - \left[m_\sigma \to \Lambda\right]$$

$$\mathcal{K}_A^{\xi=1}\left(k^2,a^2;m_\sigma^2\right) = -\frac{\alpha}{4\pi}\frac{k^2-m_\sigma^2+a^2}{k^4} \times\sqrt{\left(k^2-m_\sigma^2+a^2\right)^2-4k^2a^2}\,\Theta\left[k^2-\left(m_\sigma+\sqrt{a^2}\right)^2\right]$$

Feynman gauge

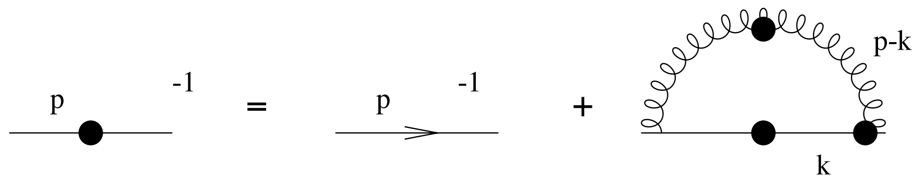
$$\begin{aligned}\rho_B^{\xi=1}\left(k^2\right) &= R\overline{m}_0 \mathcal{K}_B^{\xi=1}\left(k^2, \overline{m}_0; m_\sigma^2\right) \\ &\quad + \int_0^\infty ds \rho_s\left(s\right) \mathcal{K}_B^{\xi=1}\left(k^2, s; m_\sigma^2\right) \\ &\quad - \left[m_\sigma \to \Lambda\right]\end{aligned}$$

$$\begin{aligned}\mathcal{K}_B^{\xi=1}\left(k^2, a^2; m_\sigma^2\right) &= -\frac{\alpha}{4\pi}\frac{4}{k^2}\sqrt{\left(k^2 - m_\sigma^2 + a^2\right)^2 - 4k^2 a^2} \\ &\quad \times \Theta\left[k^2 - \left(\sqrt{a^2} + \sqrt{m_\sigma^2}\right)^2\right]\end{aligned}$$

Dyson-Schwinger equation in Rainbow ladder truncation

from Euclidean to Minkowski: Un-Wick rotating

in coll. with Dyana Duarte, Emanuel Ydrefors, Wayne de Paula, Shaoyang Ji, Pieter Maris



$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

QED-like, Feynman Gauge, bare vertices, massive vector boson, Pauli-Villars regulator

Wick-rotated SD equation (Euclidean momentum)

$$\begin{aligned}
B(p^2) &= m_0 - \frac{2\,g^2}{(2\pi)^3}\int_0^\infty k^3\,dk\,\frac{4\,B(k^2)}{k^2\,A^2(k^2) - B^2(k^2)} \\
&\quad \times \int_0^\pi \sin^2\theta\,d\theta\,\frac{\Lambda^2 - \mu^2}{(q^2-\mu^2)(q^2-\Lambda^2)}\,, \\
A(p^2) &= 1 - \frac{2\,g^2}{(2\pi)^3}\int_0^\infty k^3\,dk\,\frac{A(k^2)}{k^2\,A^2(k^2) - B^2(k^2)} \\
&\quad \times \int_0^\pi \sin^2\theta\,d\theta\,\frac{2\,k\cos\theta}{p}\,\frac{\Lambda^2 - \mu^2}{(q^2-\mu^2)(q^2-\Lambda^2)}\,.
\end{aligned}$$

$\boxed{\textbf{Un-Wick rotation: } p \to \mathrm{e}^{-i\delta}\, p\,, \quad k \to \mathrm{e}^{-i\delta}\, k\,, \quad dk \to \mathrm{e}^{-i\delta}\, dk}$

$$p_{\mathrm{E}}^2 \to -p_{\mathrm{E}}^2 = p^2\,, \quad k_{\mathrm{E}}^2 \to -k_{\mathrm{E}}^2 = k^2 \quad k_{\mathrm{E}}^3\, dk_{\mathrm{E}} \to k_{\mathrm{E}}^3\, dk_{\mathrm{E}} = k^3\, dk$$

$$\boxed{\theta = \pi/2 - \delta}$$

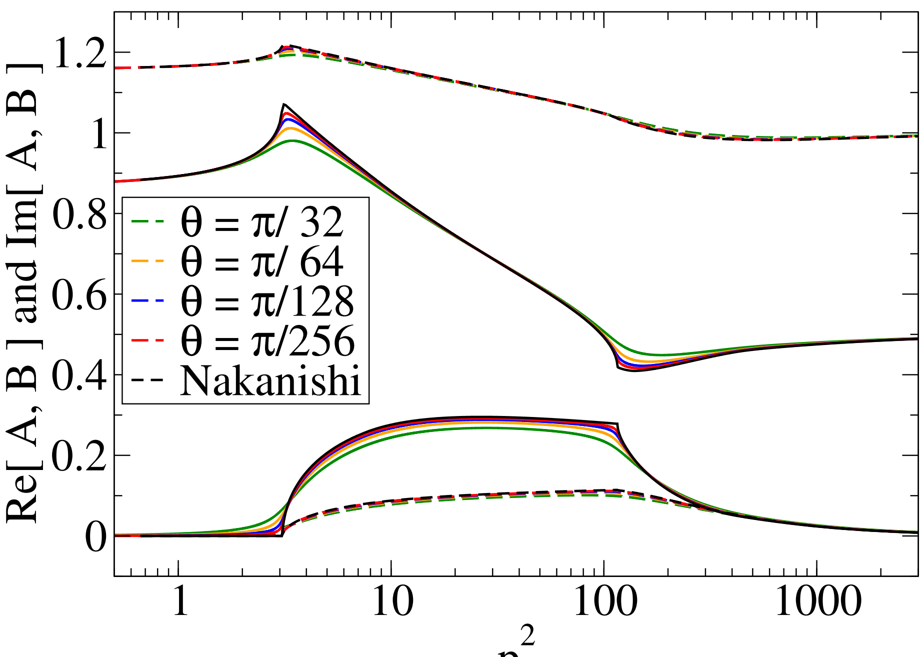
Euclidean $\theta = \pi/2$ $\Rightarrow$ **Minkowski** $\theta = 0$

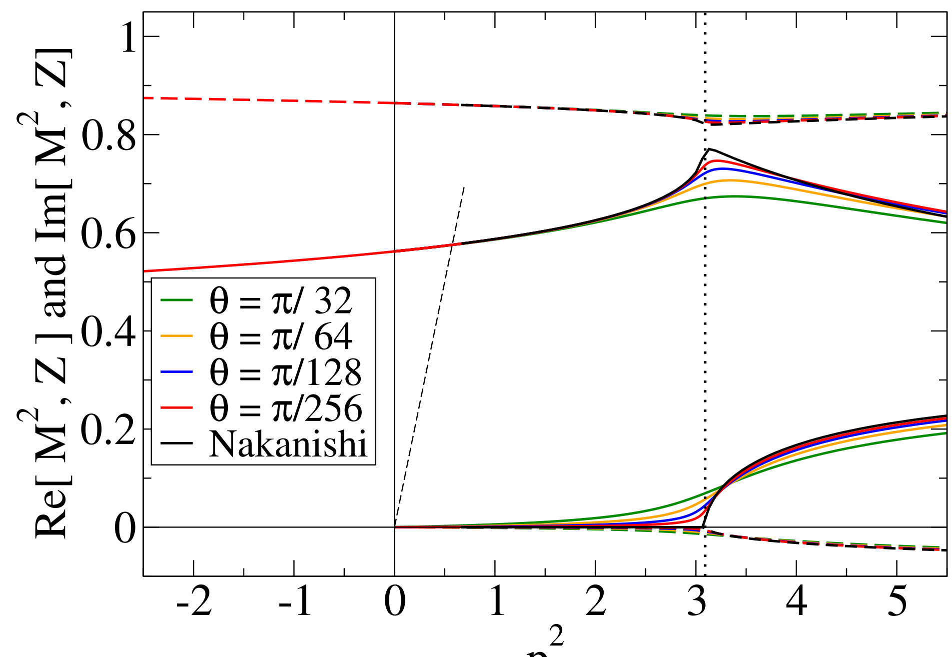
Spectral representation of the self-energy

$$B(p^2) = m_0 + \int_0^\infty ds \frac{\rho_B(s)}{p^2 - s + i\varepsilon} \quad \text{with} \quad \rho_B(s) = -\mathrm{Im}\,[B(s)/\pi]$$

$$A(p^2) = 1 + \int_0^\infty ds \frac{\rho_A(s)}{p^2 - s + i\varepsilon} \quad \text{with} \quad \rho_A(s) = -\mathrm{Im}\,[A(s)/\pi]$$

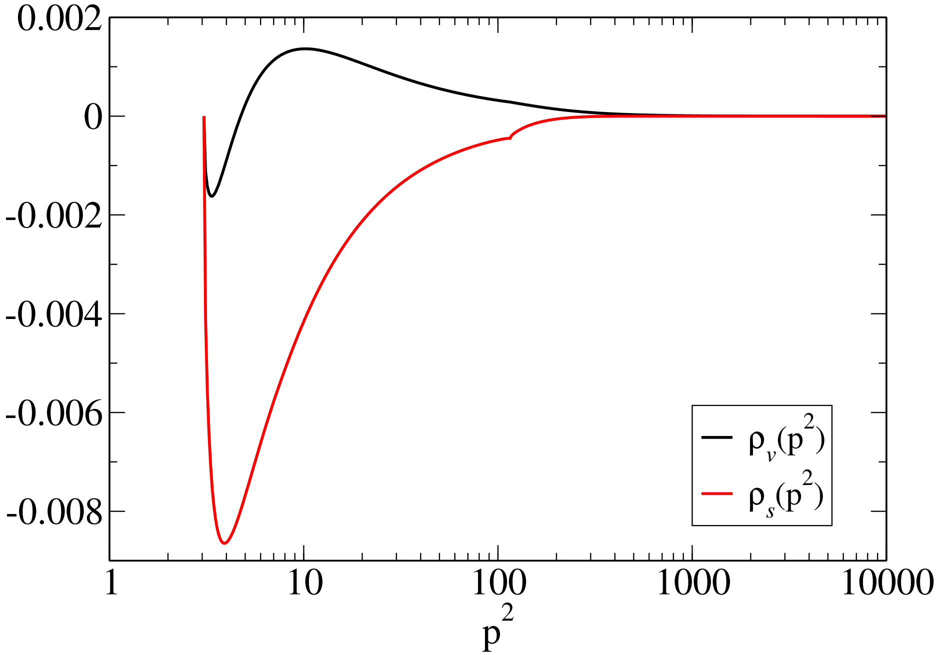
Parameters: $m_\sigma = 1, \Lambda = 10, \alpha = 0.5$ and $m_0 = 0.5$ $\Rightarrow$ $\overline{m}_0 = 0.7586$



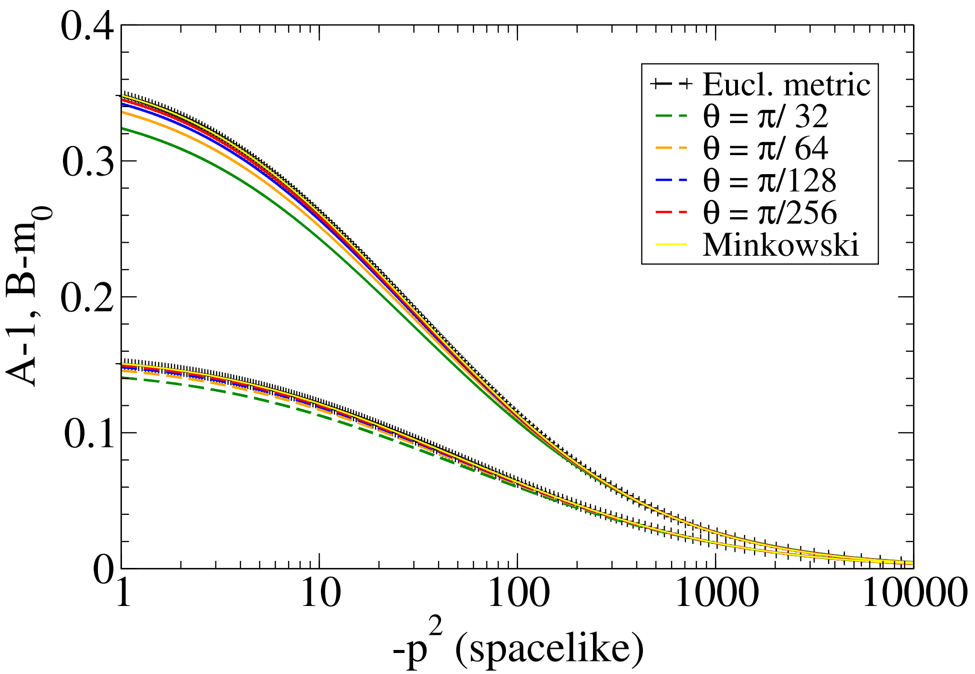


$$M^2(p^2) = \frac{B^2(p^2)}{A^2(p^2)} \qquad Z(p^2) = \frac{1}{A(p^2)}$$

Propagator spectral densities



Space like self-energies



No violation of positivity (opposite sign in our definitions)

- **$\Lambda \rightarrow$ infinite ...**

Scale invariance & breaking

- **scale invariance in UV→ strong coupling is broken**

- *Spontaneous Chiral symmetry breaking & Miransky scaling, N.Cim. A90(1985)149*
  Kaplan,Lee,Son PRD80 (2009)12005

- *Relativistic bound states within Bethe-Salpeter approach:*

Fermions coupled to scalar, vector etc fields: coupling constant is dimensionless (QCD) - instabilities above critical value associated with log-periodic solutions… Efimov physics!

Fermion-fermion: $\alpha_c = \frac{\pi}{4}$ (vector exchange) Mangin-Brinet, Carbonell, Karmanov, PRD64 (2001) 027701 & 125005; Dorkin, Beyer, Semikh, Kaptari, Few Body Syst. 42 (2008) 1

Fermion-boson: Alvarenga Nogueira***,*** Gherardi, TF, Salmè, Colasante, Pace
PRD100 (2019)016021

Scale invariance & breaking in DS equation

UV limit with $m_\sigma = m_0 = 0,\ A \to 1\ \ \&\ \ B \to 0$

$$B\left(p^2\right) \;=\; \frac{4\alpha}{\pi^2}\int_0^\infty dk\,\frac{k^3 B\left(k^2\right)}{k^2 A^2\left(k^2\right)+B^2\left(k^2\right)}\int_{-1}^{1}\frac{dx\sqrt{1-x^2}}{k^2+p^2-2k\,p\,x}$$

invariant under a scale transformation: $p \to \lambda\, p\ \ \&\ \ k \to \lambda\, k$

solution homogeneous function $B\left(k^2\right) = k^\eta\ \&\ \eta < 0$

$$\boxed{(2\alpha)^{-1} \;=\; ((2+\eta)\pi)^{-1} - (\eta\pi)^{-1}}$$

$$\eta \;=\; -1 \pm \sqrt{1-\frac{4\alpha}{\pi}} \qquad\qquad \alpha_c = \frac{\pi}{4}$$

Miransky scaling, N.Cim. A90(1985)149 $\alpha > \pi/4$

unstable solution and necessity of a cut-off - log-periodic solutions (analogous to the Landau "fall-to-center" with -1/r² potential in QM)

Consistence of the Wick-rotated solution and integral representation

$$\rho_B\left(k^2\right) = -\frac{\alpha}{4\pi}\int_0^{k^2} ds \frac{4}{k^2}\sqrt{\left(k^2+s\right)^2 - 4s\,k^2}\rho_s\left(s\right)$$

$$= -\frac{\alpha}{\pi}\int_0^{k^2} ds \frac{k^2 - s}{s\,k^2}\rho_B\left(s\right) \ ,$$

Scale invariance → $\rho_B\left(k^2\right) \to k^{2\eta'}$

$$1 = -\frac{\alpha}{\pi}\left[\int_0^1 dy\, y^{\eta'-1} - \int_0^1 dy\, y^{\eta'}\right] = -\frac{\alpha}{\pi}\frac{1}{\eta'\left(\eta'+1\right)}$$

$$\eta' = \frac{-1 \pm \sqrt{1 - \frac{4\alpha}{\pi}}}{2} = \frac{\eta}{2}$$

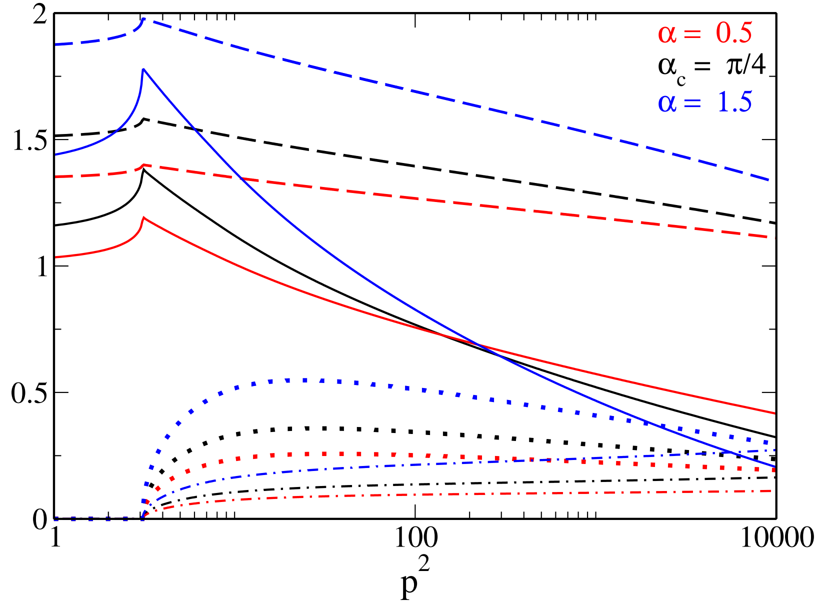
$$B\left(k^2\right) = \int_0^\infty d\gamma \frac{\rho_B\left(\gamma\right)}{k^2 - \gamma + i\epsilon} \sim k^\eta$$

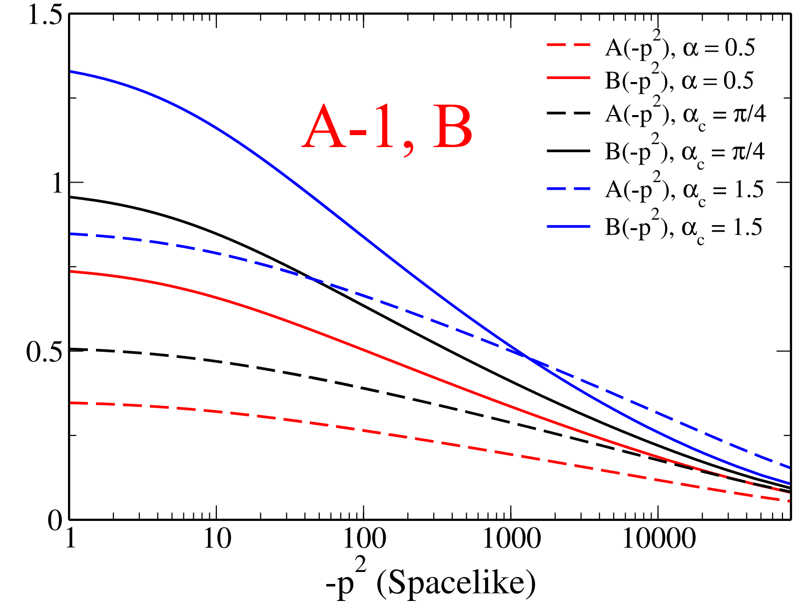
Scale invariance & breaking in DS equation

Table I: Values of the bare mass m_0 and propagator residue R for different values of Λ and α, for $m_\sigma = 1$ and $\overline{m}_0 = 0.7586$.

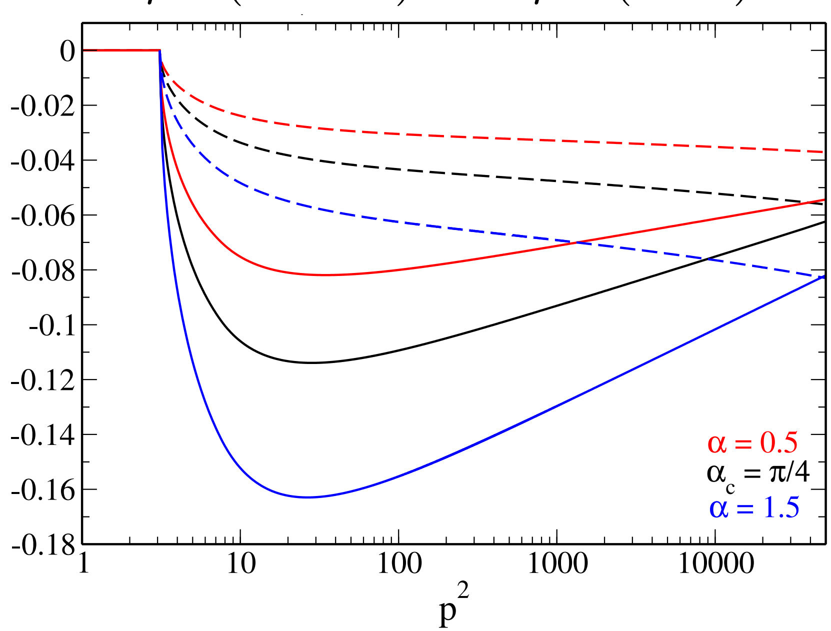
Λ	α	R	m_0
	0.5	0.8835	0.5001
10^1	$\alpha_c = \pi/4$	0.8329	0.4067
	1.5	0.7372	0.2564
	0.5	0.7808	0.3382
10^2	$\alpha_c = \pi/4$	0.7067	0.2371
	1.5	0.5873	0.1127
	0.5	0.7137	0.2307
10^3	$\alpha_c = \pi/4$	0.6320	0.1358
	1.5	0.5106	3.2572×10^{-2}

Re$[A(p^2)]$ (dashed), Re$[B(p^2)]$ (solid), Im$[A(p^2)]$ (dot-dashed) and Im$[B(p^2)]$ (dotted)

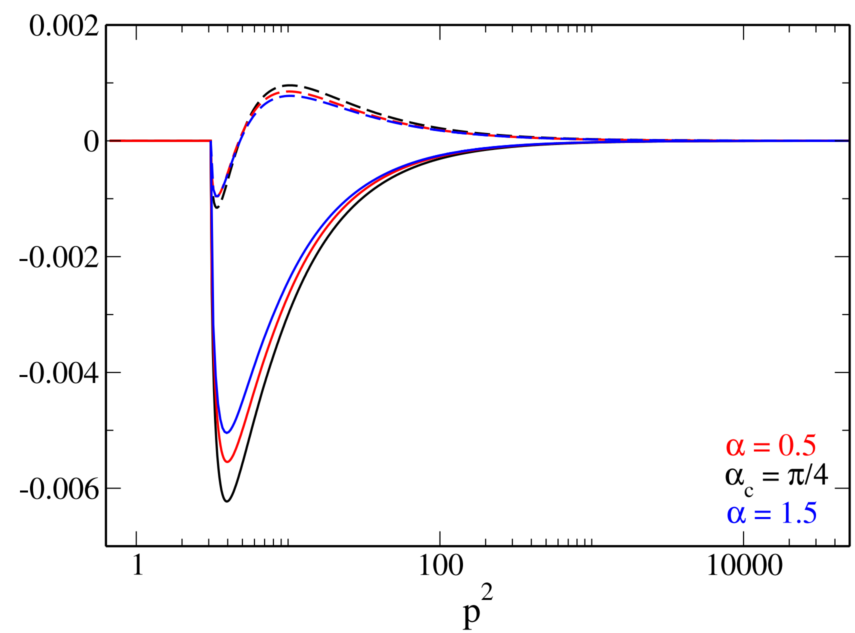




ρ_A (dashed) and ρ_B (solid)



ρ_v (dashed) and ρ_s (solid)



Scale invariance: Fermion-boson Bethe-Salpeter equation ½⁺

Alvarenga Nogueira et al. in preparation

$$\Phi^{\pi}(k,p,J_z) = \Big[O_1(k)\ \phi_1(k,p) + O_2(k)\ \phi_2(k,p)\Big]\ U(p,J_z)$$

$$O_1(k) = \mathbb{I}\ , \qquad O_2(k) = \frac{\not{k}}{M}\ , \qquad (\not{p} - M)\ U(p,J_z) = 0$$

Ladder BSE in Euclidean space – vector exchange

$$k_4 = K\cos\varphi \quad \text{and} \quad k = K\sin\varphi \qquad 0 < \varphi < \pi$$

$$-5 < \mathrm{Real}[\eta] < -4$$

Maximum value of the couplings product

$$\alpha_c = 1.18691\ldots \qquad \alpha^V = \frac{\lambda_F^v\ \lambda_S^v}{8\pi}$$

$$\phi_1(k_4,k) = K^{\eta+1} \quad \text{and} \quad \phi_2(k_4,k) = 0 \qquad \eta = -4.08918\ldots$$

$$\phi_1(k_4,k) = 0 \quad \text{and} \quad \phi_2(k_4,k) = K^{\eta} \qquad \eta = -4.91082\ldots.$$

Alvarenga Nogueira et al. in preparation

$$\psi_i(\xi, \gamma; \kappa^2) = iM \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} \phi_i(k,p) \sim \gamma^{1+\frac{\eta_i}{2}}$$

Solution of the Ladder BS equation in Minkowski space via Nakanishi integral representation [PRD100 (2019)016021]

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_0^{\infty} d\gamma' \frac{g(\gamma', z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

Giovanni Salmè talk on Monday afternoon

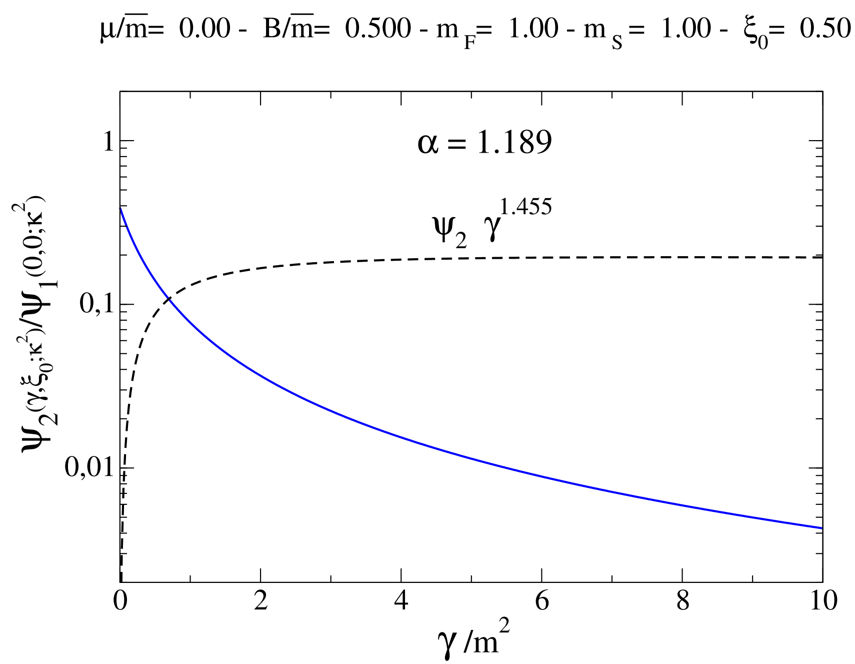


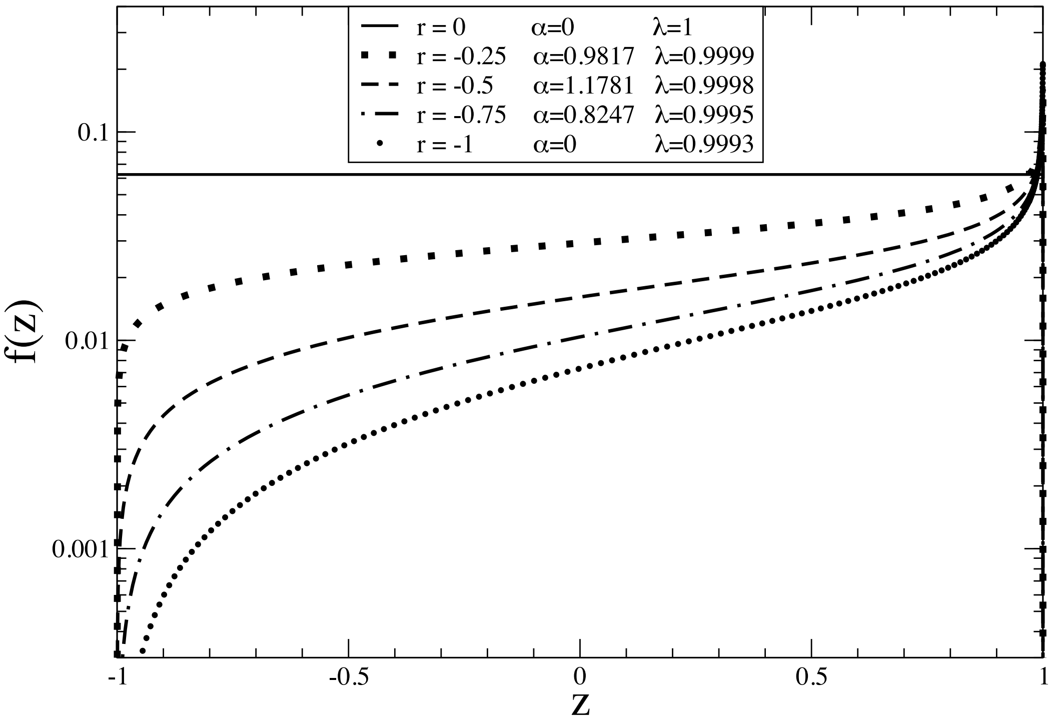
Figure 5.4. The light-front wave function $\psi_2(\gamma, z_0 = 0)$ obtained from the solution of the original equation (5.8) as a function of γ (solid blue curve) and its product with the asymptotic limit found in the high momentum limit (dashed black curve).

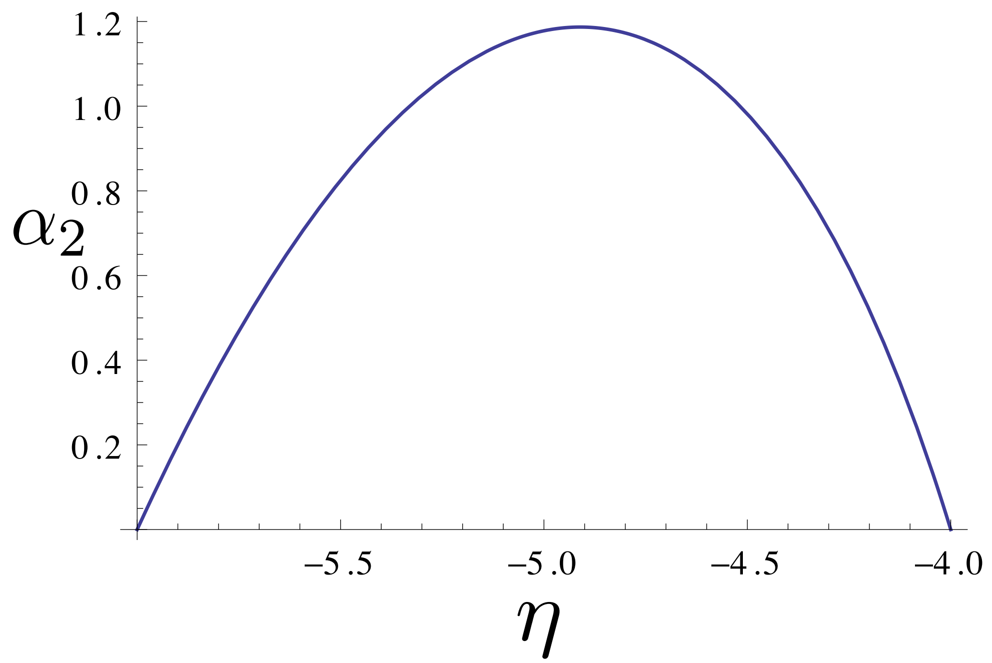
Solution of the integral equations for the Nakanishi weight functions in UV limit

$$g_2(\gamma, z) = \gamma^r f_2(z)$$

$$r = 2 + \frac{\eta}{2} \text{ with the constraint that } -1 < r < 0.$$

$$\lambda f(z) = \frac{1+|r|}{2+4|r|} \int_{-1}^{1} dz' f(z')$$
$$\times \left\{ \left[\frac{1+z}{1+z'}\right]^{|r|} \theta(z'-z) + \left[\frac{1-z}{1-z'}\right]^{|r|} \left[1 + \frac{4|r|}{(1-z')}\right] \theta(z-z') \right\}$$





Conclusions and Perspectives

- **Integral Representation to solve Dyson-Schwinger in diferente gauges;**
- **Un-Wick rotation: BSE and SD - promissing tool allied to Integral Representations;**
- **Consistence of the scale invariance analysis in Euclidean and self-energies NIR;**
- **Cosnsitence of the scale invariance analysis and BS solution for fermion-boson problem;**
- **Self-energies, quark-gluon vertex, ingredients from LQCD ….**
- **Confinement – How to include with Int. Representation?**
- **Apply to the study the structure: pion, kaon, D, B, rho…, and the nucleon**
- **Form-Factors, PDFs, TMDs, FRAGMENTATION FUNCTIONS...**

THANK YOU!

LIA/CNRS - SUBATOMIC PHYSICS: FROM THEORY TO APPLICATIONS

IPNO (U.Van Kolck, Jaume Carbonell)…. + Brazilian Institutions …