CHIRAL SYMMETRY RESTORATION AND THE THERMAL $f_0(500)$ STATE

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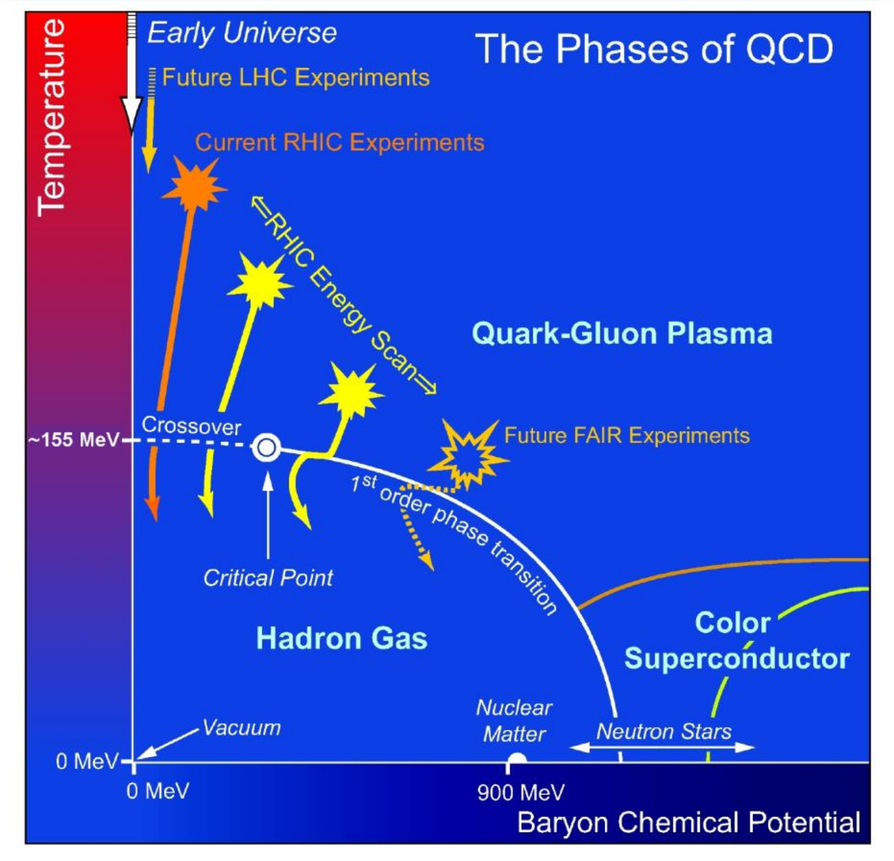
OUTLINE

- Chiral symmetry restoration in QCD
- Quark condensate and scalar susceptibility in the Linear Sigma Model
- Self-Energy and pole position of sigma in the Linear Sigma Model
- Thermal $f_0(500)$ saturation approach
- Topological susceptibility in U(3) ChPT
- Conclusions

QCD PHASE DIAGRAM AND CHIRAL SYMMETRY BREAKING



A. Bazavov, Quark Matter 2017

Ideal chiral restoration $for\ N_f$=2 and m_l=0

CROSSOVER transition for μ_B=0, N_f=2+1 and physical masses

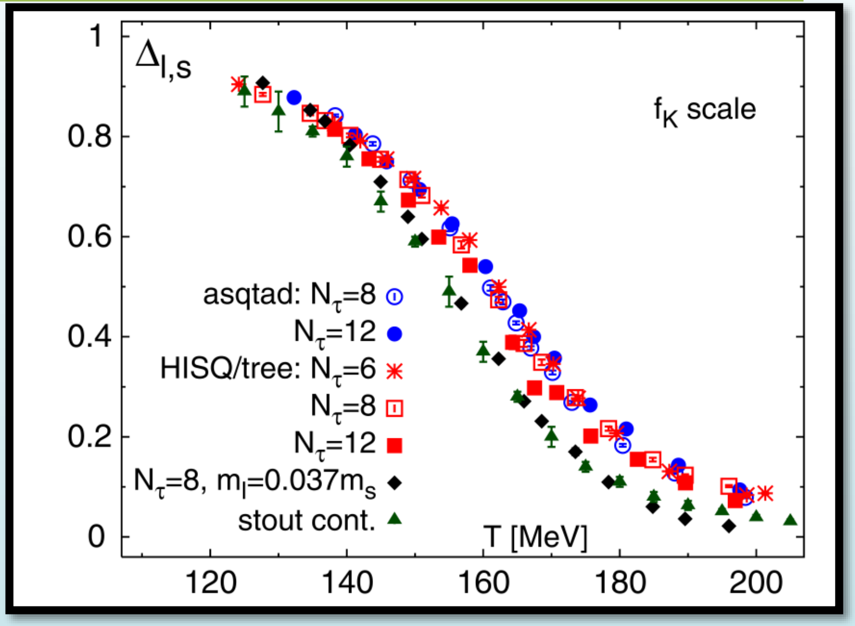
CHIRAL SYMMETRY RESTORATION IN QCD

Free energy density: $z(T) = -\lim_{V\to\infty}(\beta V)^{-1}\log Z$

Quark condensate: $\langle\bar{q}q\rangle = \dfrac{\partial z(T)}{\partial m_q}$

Substrated quark condensate:

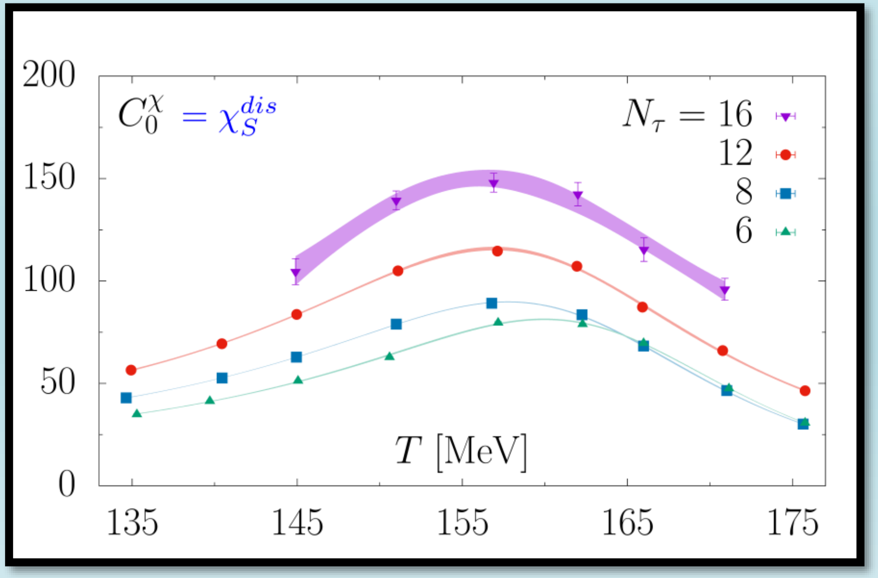
$$\Delta_{s,l} = \frac{\langle\bar{q}q\rangle_T - (2m_q/m_s)\langle\bar{s}s\rangle_T}{\langle\bar{q}q\rangle_0 - (2m_q/m_s)\langle\bar{s}s\rangle_0}$$



A.Bazavov et al (Hot QCD), 2012, 2014, 2018

Scalar susceptibility:

$$\chi_S(T) = -\frac{\partial}{\partial m_q}\langle\bar{q}q\rangle(T)$$
$$= \int_T d^4x\left[\langle\mathcal{T}(\bar{q}q(x)\bar{q}q(0))\rangle - \langle\bar{q}q\rangle^2(T)\right]$$



LINEAR SIGMA MODEL LAGRANGIAN

- LSM lagrangian:

$$\mathcal{L}_{LSM} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \frac{\lambda}{4}\left[\Phi^T\Phi - v_0^2\right]^2 + \underbrace{h\sigma}$$

Breaks chiral symmetry explicitly

- Potential mínima: $\Phi^2 = v^2 \neq 0$
- We choose a vacuum: $\langle \Phi^T \rangle = \left(\vec{0}, v\right)$ → **Spontaneous chiral symmetry breaking**
- Minimum of the potential: $h = \lambda v(v^2 - v_0^2)$
- Shifted field: $\tilde{\sigma} = \sigma - v$

$$M_{0\pi}^2 = \frac{h}{v} = \lambda(v^2 - v_0^2) \qquad M_{0\sigma}^2 = M_{0\pi}^2 + 2\lambda v^2$$

Tree-level pion and sigma masses

QUARK CONDENSATE AND SCALAR SUSCEPTIBILITY IN THE LINEAR SIGMA MODEL

- At T$\neq 0$: $\langle\tilde{\sigma}\rangle(T)=\langle\sigma\rangle(T)-v\neq 0$ → one-particle reducible diagrams enter In the calculation of correlators

- Quark condensate: $\langle\bar{q}q\rangle_l(T)=-\frac{dh}{dm_l}v(T)$

- Scalar susceptibility: $\chi_S(T)=\left(\frac{d^2h}{dm_l^2}\right)v(T)+\left(\frac{dh}{dm_l}\right)^2\int_T dx\ \left\{\langle\mathcal{T}\tilde{\sigma}(x)\,\tilde{\sigma}(0)\rangle-\langle\tilde{\sigma}\rangle^2(T)\right\}$

$$\chi_S(T)=\left(\frac{d^2h}{dm_l^2}\right)v(T)+\left(\frac{dh}{dm_l}\right)^2\Delta_\sigma(k=0;T)$$

$$\Delta_\sigma(k;T)=\frac{1}{k^2+M_{0\sigma}^2+\Sigma(k_0,\vec{k};T)}$$

Euclidean propagator of the $\tilde{\sigma}$ field

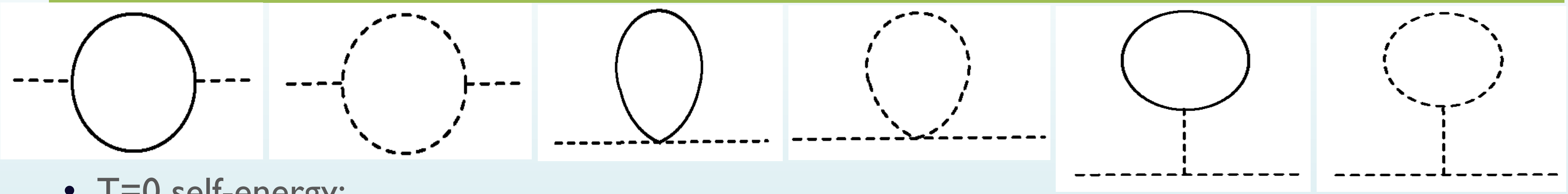
- Near the transition:

$$\chi_S(T)\simeq 4B_0^2v^2\left(\frac{M_{0\sigma}^2}{M_{0\sigma}^2-M_{0\pi}^2}\right)^2\Delta_\sigma(k=0;T)$$

Saturated susceptibility

$$\frac{\chi_S(T)}{\chi_S(0)}\simeq\frac{M_{0\sigma}^2+\Sigma\left(k=0;T=0\right)}{M_{0\sigma}^2+\Sigma\left(k=0;T\right)}$$

SELF-ENERGY AND POLE POSITION AT T=0



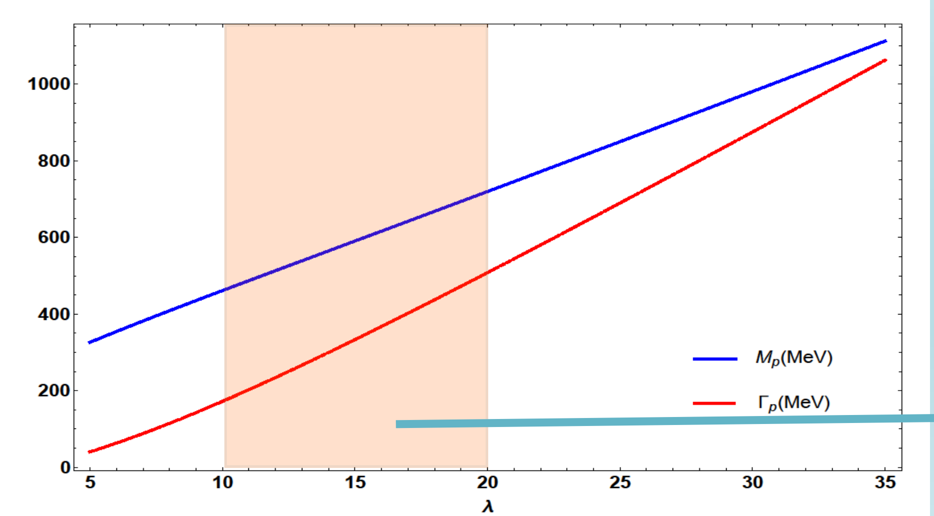
- T=0 self-energy:

$$\Sigma(s,T=0) = \frac{3\lambda}{16\pi^2}(M_\sigma^2 - M_\pi^2)\left[\sigma_\pi(s)\log\left(\frac{\sigma_\pi(s)+1}{\sigma_\pi(s)-1}\right) + 3\,\sigma_\sigma(s)\log\left(\frac{\sigma_\sigma(s)+1}{\sigma_\sigma(s)-1}\right) + \log\left(\frac{M_\pi^2}{M_\sigma^2}\right) - \frac{13}{3}\right]$$

- Perturbatively, the pole of the propagator is at $s_p = M_\sigma^2 + \Sigma(k^2 = M_\sigma^2) \rightarrow s_p = (M_p - i\Gamma_p/2)^2$

Breit-Wigner resonance

- The mass and the width of the pole cannot agree for a given λ with their experimental values:



M_π (MeV)	M_p (MeV)	Γ_p (MeV)	λ
0	450.0	172.5	8.4
0	775.1	550.0	20.0
140	450.0	159.2	9.6
140	750.1	550.0	21.2

$\lambda \sim 10 - 20$

Reference range where the deviations from the PDG value are not large.

SELF-ENERGY AT FINITE TEMPERATURE

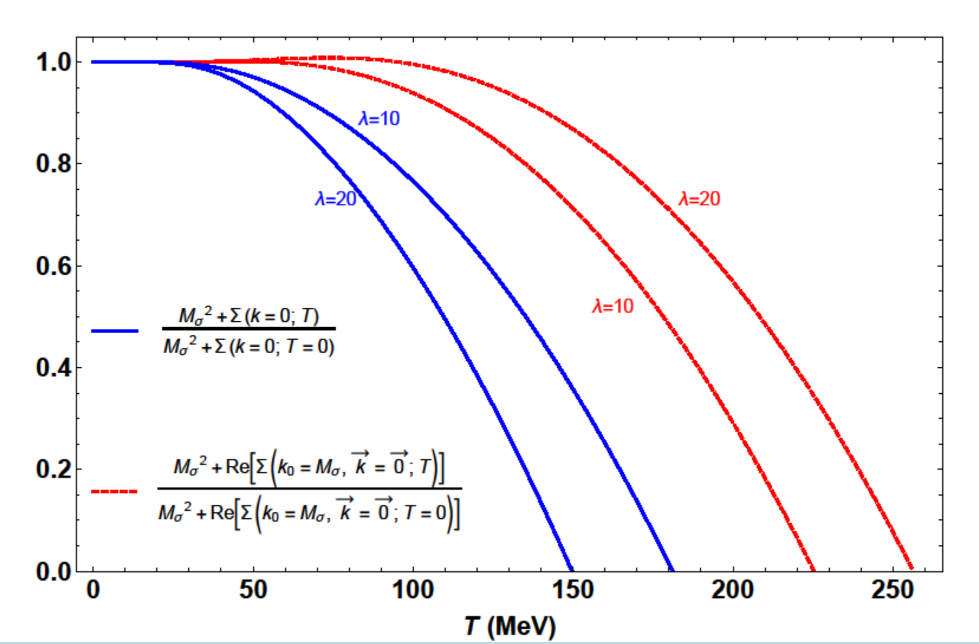
- The $k \to 0^+$ limit for the self-energy is

$$\Sigma(k=0;T) = \frac{\lambda}{16\pi^2}(M_\sigma^2 - M_\pi^2)\left[11 + 3\log\left(\frac{M_\pi^2}{M_\sigma^2}\right)\right]$$

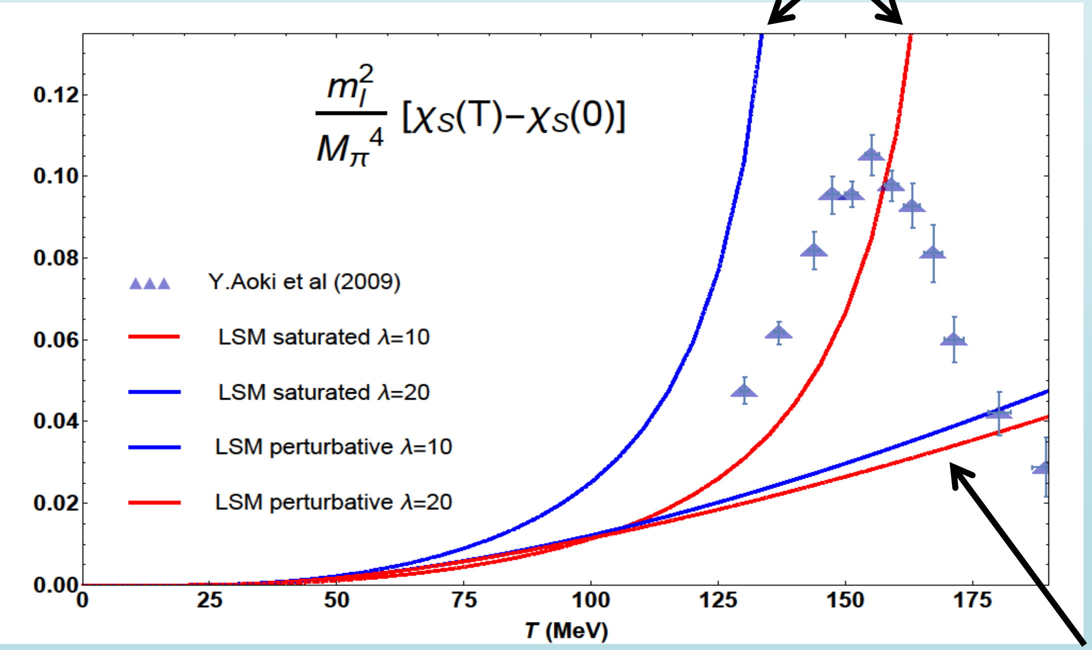
$$+3\lambda\left\{\frac{3M_\pi^2 - 2M_\sigma^2}{M_\sigma^2}\left[g_1(M_\pi,T) + g_1(M_\sigma,T)\right] - \left(M_\sigma^2 - M_\pi^2\right)\left[g_2(M_\pi,T) + 3g_2(M_\sigma,T)\right]\right\}$$

Thermal part of the bubble diagrams

Thermal part of the tadpole diagrams



Saturated



Perturbative

The qualitative behaviour is the same

$\Sigma(s = s_p)$ → vanishes at a higher temperature

Saturated susceptibility → diverges around T_c

Chiral symmetry restoration tendency

THERMAL f_0(500) SATURATION APPROACH

- Within UChPT approach, the $f_0(500)$ state emerges as a second Riemann sheet pole of the scattering amplitude.
- We define the unitarized scalar susceptibility:

$$\chi_S^U(T) = A\frac{M_\pi^4}{4m_l^2}\frac{M_S^2(0)}{M_S^2(T)}$$

with

$$M_S^2(T) = \mathrm{Re}(s_{pole}(T)) \sim \mathrm{Re}\Sigma_{f_0}$$

- For that, we assume

There are no decay channels open at k=0 → $\mathrm{Im}\Sigma_{f_0}(k=0)=0$

Taking into account the uncertainty range of this approach → $\mathrm{Re}\Sigma_{f_0}(s=0) \sim \mathrm{Re}\Sigma_{f_0}(s=s_{pole})$

S. Ferreres-Solé, A. Gómez Nicola and A. Vioque-Rodríguez (2019). arXiv:1811.07304

THERMAL f_0(500) SATURATION APPROACH

- To guarantee the key qualitative features of a crossover behaviour:

1. Thermal unitarity.

ChPT amplitude is not unitary → Unitarization

Different unitarization methods:

$$t_K(s;T) = \frac{t_2(s)}{1-\sigma_T(s,T)t_2(s)} \qquad t_{Umod}(s;T) = \frac{t_2^2(s)}{t_2(s)-t_{4J}(s,T)} \qquad t_{IAM}(s;T) = \frac{t_2(s)^2}{t_2(s)-t_4(s,T)}$$

2. Analyticity.

$t_K(s;T)$ → is not analytic → is not posible to define properly the second Riemann sheet

3. T=0 pole prediction.

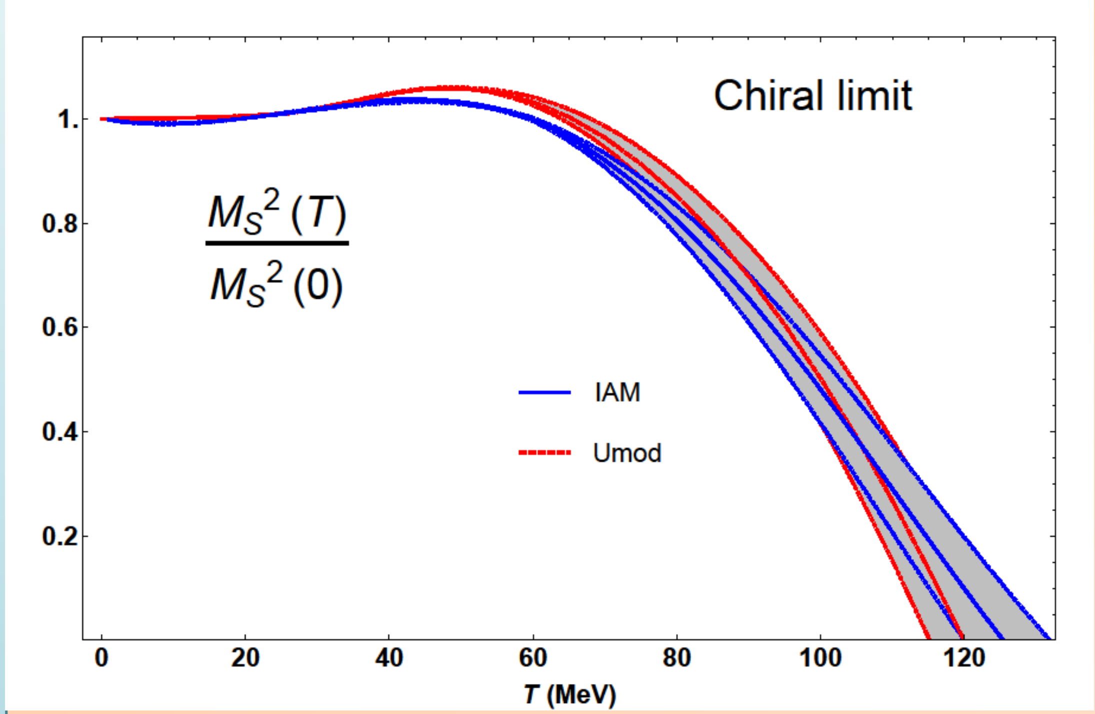
LECs used → fitted with the full IAM.

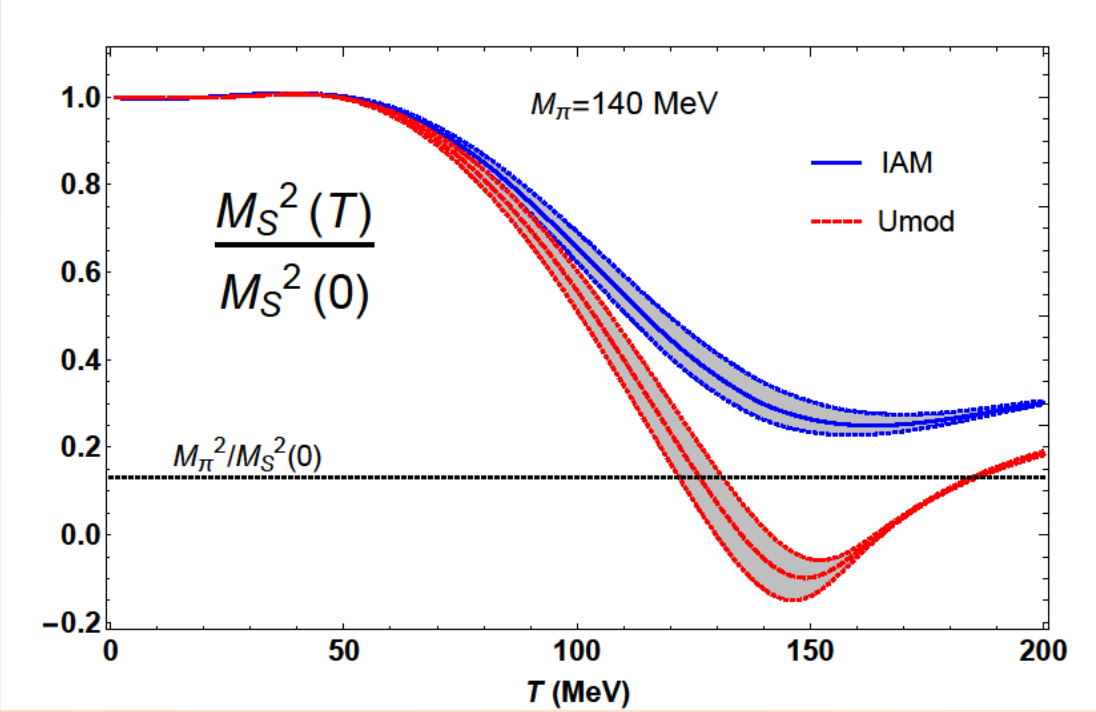
Including that, we recover the IAM amplitude at T=0.

UNCERTAINTIES OF THE APPROACH

- **Three types of uncertainties:**

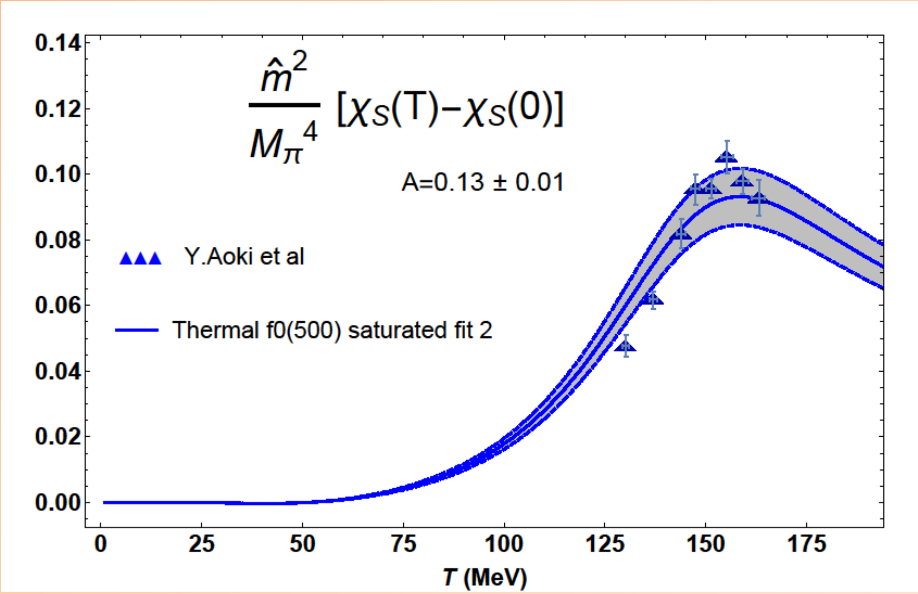
The unitarization method:





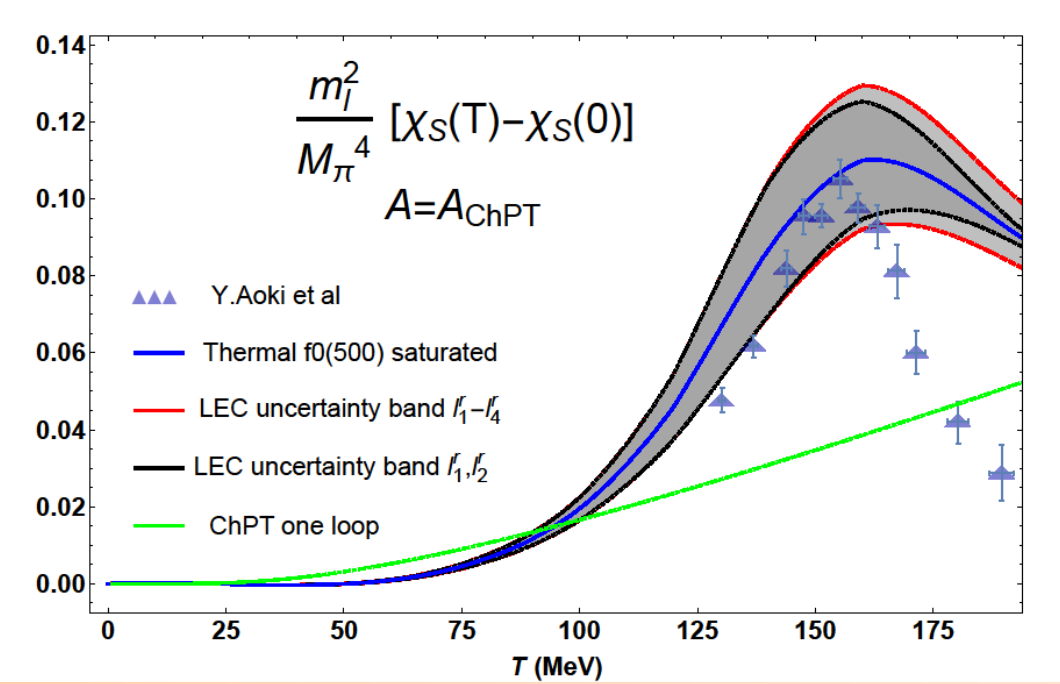
UNCERTAINTIES OF THE APPROACH

The normalization factor A.



$$A_{\text{ChPT}} = \frac{\chi_S^{\text{ChPT}}}{B_0^2}$$

The numerical uncertainties in the LEC.

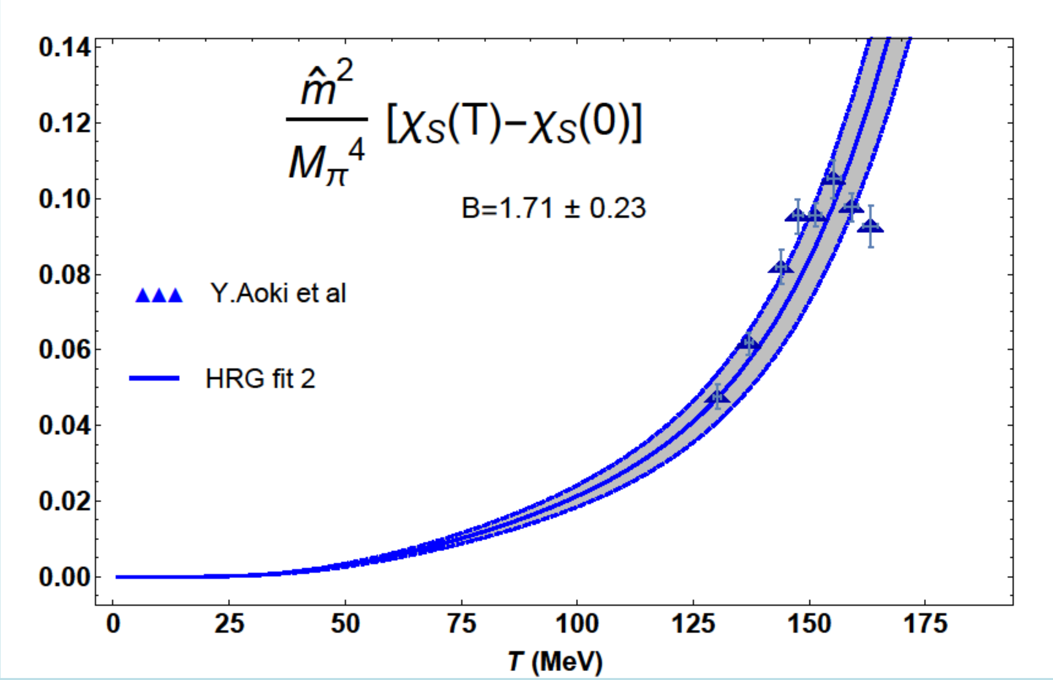


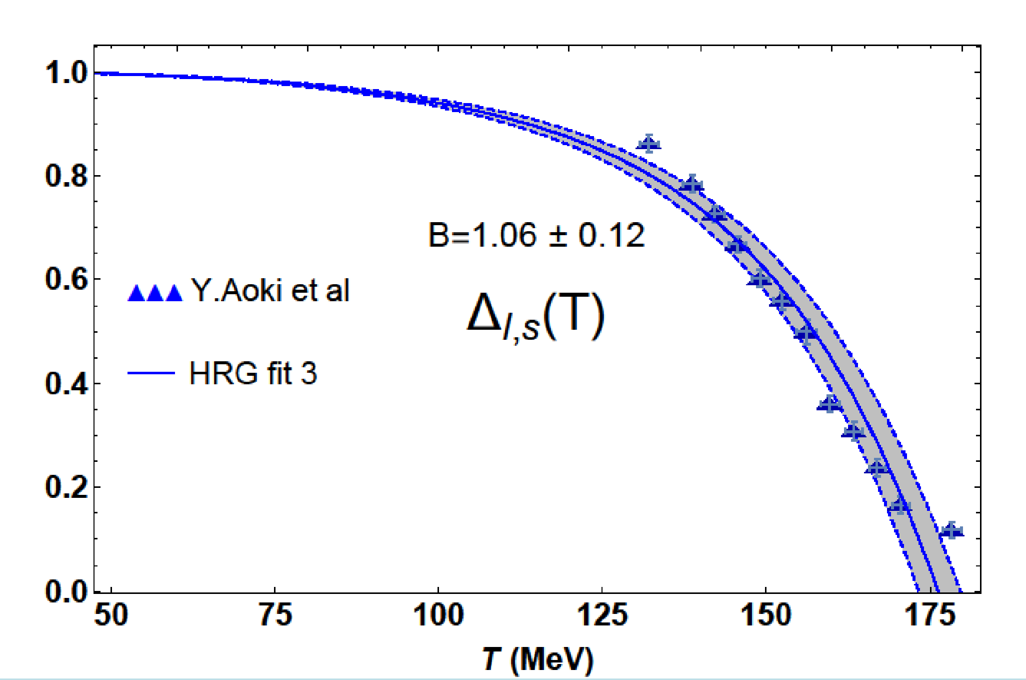
$l_{1,2}^r$ → ππ scattering vertices

$l_{3,4}^r$ → renormalization of M_π and F_π

FITS TO LATTICE DATA AND HADRON RESONANCE GAS

Free energy density: $z \to Bz$ HRG Jankowski et al 2013





Fit	A	B	χ^2/dof	T_{max}(MeV)
Thermal f_0 fit 1	0.13± 0.02	—	6.25	155
Thermal f_0 fit 2	0.13± 0.01	—	4.93	165
HRG fit 1	—	1.90± 0.02	1.33	155
HRG fit 2	—	1.71± 0.23	10.30	165
HRG fit 3	—	1.06± 0.12	3.77	155

TOPOLOGICAL SUSCEPTIBILITY IN U(3) CHPT AT T=0

- Energy vacuum: $\epsilon_{\text{vac}}(\theta) = \epsilon_{\text{vac}}(0) + \frac{1}{2}\chi_{\text{top}}\theta^2 + \frac{1}{24}c_4\theta^4 + \dots$

$m_u = m_d$ $\left[\chi_{\text{top}}^{\text{latt}}\right]^{1/4} = 73(9)$

$\chi_{top}^{1/4}$ [MeV]	$U(3)$	$SU(2)$	$SU(3)$
LO	74(3)	75(3)	75(3)
NLO	74(3)	78(3)	83(2)
NNLO	81(2)		

$$\chi_{top}^{U(3),LO} = \Sigma \frac{M_0^2 \bar{m}}{M_0^2 + 6B_0\bar{m}}$$

$$\bar{m} = \left[\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right]^{-1}$$

$$c_4^{U(3),LO} = -\Sigma \frac{\hat{m}^4}{\bar{m}^{[3]}}$$

$$\bar{m}^{[3]} = \left[\frac{1}{m_u^3} + \frac{1}{m_d^3} + \frac{1}{m_s^3}\right]^{-1}$$

$z = m_u/m_d$

$$\chi_{top}^{U(3),LO} = \frac{F^2 M_0^2}{6 + \frac{(1+z)^2}{z}\frac{M_0^2}{M_{\pi^0}^2} + \frac{(1+z)M_0^2}{(1+z)M_{K^0}^2 - M_{\pi^0}^2}}$$

$z = 0.485$ → $\left[\chi_{\text{top}}^{U(3),LO,IB}\right]^{1/4} = 72 \text{ MeV}$

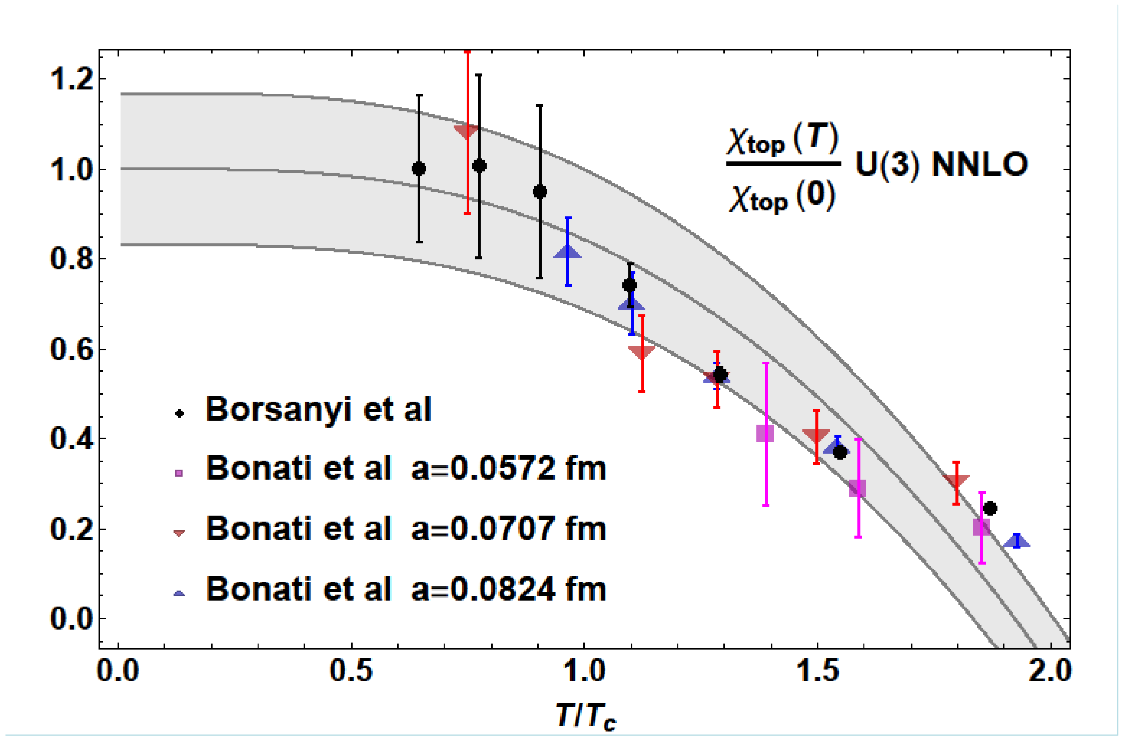
A.G.Nicola, J.Ruiz De Elvira and A.Vioque-Rodríguez (2019). arXiv:1907.11734

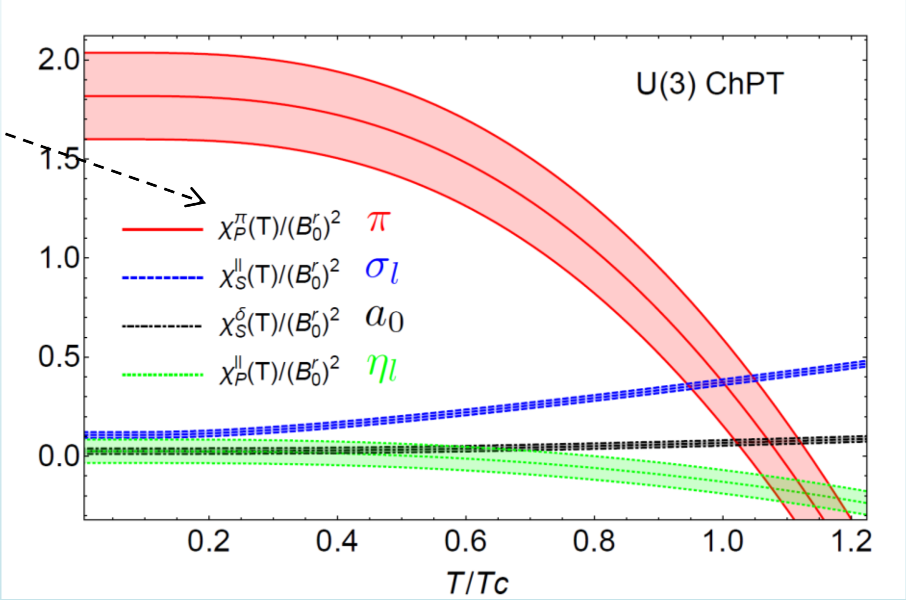
TOPOLOGICAL SUSCEPTIBILITY IN U(3) CHPT AT FINITE TEMPERATURE

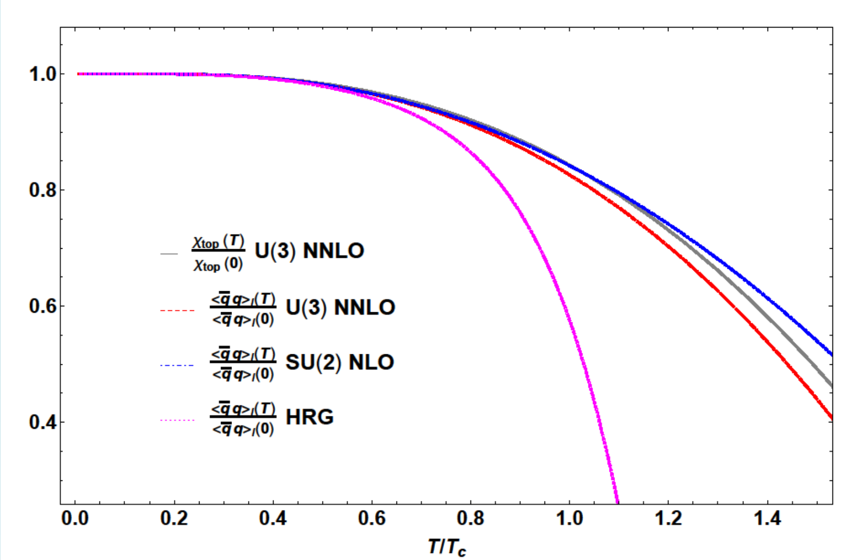
Temperature dependence:

$$\chi^{\pi}_{P} = \frac{\langle \bar{q}q \rangle_l}{m_q}$$

$$\chi_{top} = -\frac{1}{4}\left[m_q\langle \bar{q}q \rangle_l + m_q^2 \chi_P^{ll}\right]$$







CONCLUSIONS

LSM and thermal saturation approach

- The scalar susceptibility in the LSM can be related to the propagator of the lightest scalar state at zero momentum.
- Using the IAM, we reproduce the crossover peak and most of the lattice data fall into the uncertainty band.

Topological susceptibility in U(3) ChPT

- The η' meson and mixing angle corrections to the topological susceptibility are comparable to the kaon and η ones.
- T-dependence dominated by the quark condensate, although the second term of χ_{top} is relevant near T_c.