The dipole picture and the non-relativistic expansion

Miguel A. Escobedo

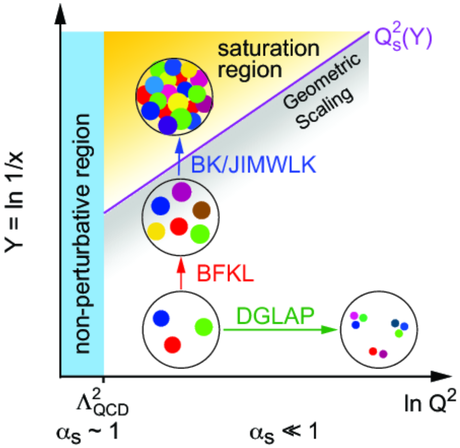
Instituto Galego de Física de Altas Enerxías

18th of September, 2019

Preliminary work. Done in collaboration with Tuomas Lappi (Jyväskylä U).

Plan

Why is exclusive quarkonium production interesting?



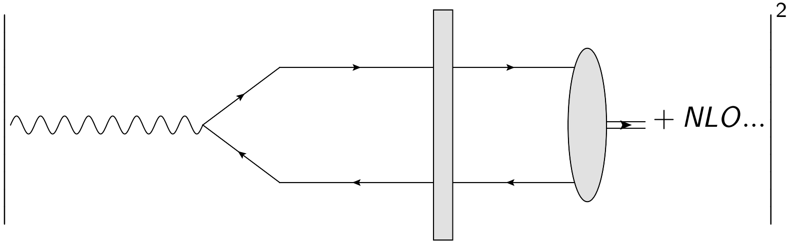
Picture taken from Marquet (2013)

Exclusive quarkonium production is an ideal way to study the gluon distribution at low x in DIS and UPCs

- Exclusive processes depend on the gluon density quadratically.
- Non-perturbative contributions are suppressed with respect to other exclusive processes.

The dipole picture

Application of light cone perturbation theory to exclusive processes



At LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi_{HQ}^*\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2$$

The dipole picture at LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2 r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi_{HQ}^* \Psi_{\gamma^*}\right)_{T,L} \sigma_{q\bar{q}}\right|^2$$

- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.

The dipole picture at LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi_{HQ}^*\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2$$

- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.
- $\sigma_{q\bar{q}}$ is the dipole cross-section. Contains the information about the gluon content of the nucleus and the saturation scale Q_s.

The dipole picture at LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi_{HQ}^*\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2$$

- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.
- $\sigma_{q\bar{q}}$ is the dipole cross-section. Contains the information about the gluon content of the nucleus and the saturation scale Q_s.
- Ψ_{HQ} is the quarkonium wave function.

The dipole picture at LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt} = \frac{1}{16\pi}\left|\int d^2r_\perp \int_0^1 \frac{dz}{4\pi}\left(\Psi_{HQ}^*\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2$$

- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.
- $\sigma_{q\bar{q}}$ is the dipole cross-section. Contains the information about the gluon content of the nucleus and the saturation scale Q_s.
- Ψ_{HQ} is the quarkonium wave function.
- At higher orders the wave functions have to take into account the presence of gluons inside the photon and quarkonium. We also need to take into account $\sigma_{q\bar{q}g}$, $\sigma_{q\bar{q}gg}$ and so on.

The dipole picture at LO

$$\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt}=\frac{1}{16\pi}\left|\int d^2r_\perp\int_0^1\frac{dz}{4\pi}\left(\Psi_{HQ}^*\Psi_{\gamma^*}\right)_{T,L}\sigma_{q\bar{q}}\right|^2$$

- Ψ_{γ^*} is the virtual photon wave function. Its transverse extent is of the order of the inverse of the virtuality Q. Dominated by perturbative physics.
- $\sigma_{q\bar{q}}$ is the dipole cross-section. Contains the information about the gluon content of the nucleus and the saturation scale Q_s.
- Ψ_{HQ} is the quarkonium wave function.
- At higher orders the wave functions have to take into account the presence of gluons inside the photon and quarkonium. We also need to take into account $\sigma_{q\bar{q}g}$, $\sigma_{q\bar{q}gg}$ and so on.
- Our aim is to determine the properties of the nucleus. But in order to do this we need an accurate description of quarkonium wave function.

Quarkonium's light cone wave function

- Phenomenological approaches, boosted gaussian, gaussLC... See Kowalski, Motyka and Watt (2006) for a review.
- Obtained by solving the Schrödinger equation using a phenomenological potential. Cepila, Nemchik, Krelina and Pasechnik (2019).
- Vased on light-front holography. Li, Maris and Vary (2017).

Quarkonium's light cone wave function

- Phenomenological approaches, boosted gaussian, gaussLC... See Kowalski, Motyka and Watt (2006) for a review.
- Obtained by solving the Schrödinger equation using a phenomenological potential. Cepila, Nemchik, Krelina and Pasechnik (2019).
- Vased on light-front holography. Li, Maris and Vary (2017).

Non-relativistic assumption

- Widely used in the literature in other contexts, generally combined with EFT approaches (NRQCD, pNRQCD). Inclusive production, spectroscopy, decays.
- Well-defined limit of QCD. Theoretically interesting.
- It has already been used in the dipole model in its simplest form. Ryskin (1993) and Brodsky, Frankfurt, Gunion, Mueller and Strikman (1994).

Energy scales in a non-relativistic quarkonium

- The mass of the heavy quark m is by definition bigger than Λ_{QCD}. Perturbative physics.
- The typical velocity of the heavy quarks around the center of mass v is small. Hence $p \sim \frac{1}{r} \sim mv$. It is estimated that for charmonium $v^2 \sim 0.3$ while for bottomonium $v^2 \sim 0.1$.
- The binding energy is, due to the Virial theorem, of the same order of magnitude as the kinetic energy, mv^2.

Energy scales in a non-relativistic quarkonium

- The mass of the heavy quark m is by definition bigger than Λ_{QCD}. Perturbative physics.
- The typical velocity of the heavy quarks around the center of mass v is small. Hence $p \sim \frac{1}{r} \sim mv$. It is estimated that for charmonium $v^2 \sim 0.3$ while for bottomonium $v^2 \sim 0.1$.
- The binding energy is, due to the Virial theorem, of the same order of magnitude as the kinetic energy, mv^2.

The non-relativistic limit simplifies the treatment because...

- It allows to separate the computation of scale m effects, which are pertubative.
- From the point of view of the scales smaller than m the production of heavy quarks is a local process.

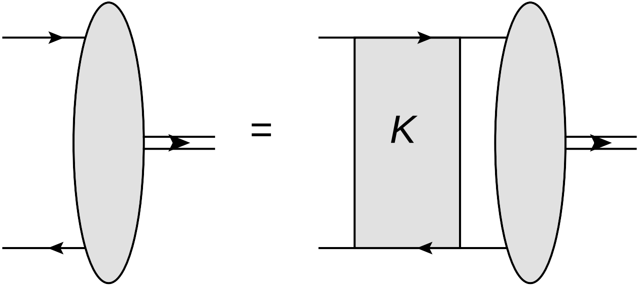
Plan

The non-relativistic limit

Basic assumptions

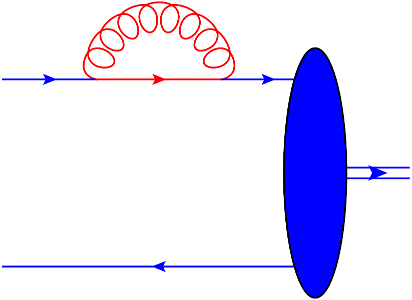
- The leading order light cone wave function can be computed taking into account only non-relativistic quarks and their interaction.
- Relativistic degrees of freedom can appear, but they are a perturbation $\rightarrow$ can appear during small times.
- Non-relativistic quarks (in light-cone perturbation theory) are defined by having a $p_\perp$ much smaller than m and a momentum fraction very close to $\frac{1}{2}$.

The leading order wave function



- Contains only non-relativistic components.
- Fulfils a Bethe-Salpeter equation which can be expressed as a Schrödinger equation.
- Relation between this and the wave function in potential models/NRQCD studied in Bodwin, Kang and Lee (2006).

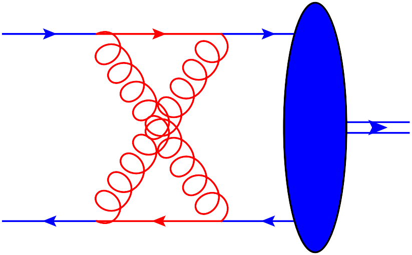
Relativistic corrections to the non-relativistic wave function



Type 1. Contribution of the relativistic degrees of freedom to the wave function renormalization of the non-relativistic quark.
Computed in Mustaki, Pinsky, Shigemitsu and Wilson (1991).

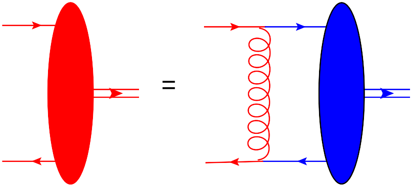
Color code

- Relativistic particles and gluons with virtuality of order m^2.
- Non-relativistic particles and softer gluons.



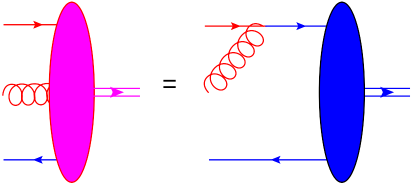
Type 2. Contributions that can be encoded as a redefinition of the potential between non-relativistic quarks.
In our case, it will not appear explicitly in our equations. However, it modifies the value of the non-relativistic wave-function at the origin.

Relativistic quark-antiquark pair component



- Can be computed in perturbation theory. $\rightarrow$ Proportional to $\alpha_s(m)$.
- Point like interaction from the point of view of non-relativistic quarks. $\rightarrow$ Proportional to the leading order non-relativistic wave function at the origin.

Relativistic quark, hard gluon and non-relativistic antiquark



- Proportional to $g(m)$.
- Cross-check. One can recover the wave-function renormalization by computing the square of this contribution.

Mathematical structure

$$\int dz f(z)\Psi^n_{HQ}(z,\mathbf{x}_\perp) = \sum_{m,k}\int dz f(z) C^k_{n\leftarrow m}(z,\mathbf{x}_\perp)\left(\frac{\nabla}{m}\right)^k \int \frac{d\lambda}{4\pi}\phi^m(\lambda,\mathbf{0})$$

- In this formula it is assumed that $x_\perp \sim \frac{1}{m}$ or smaller. The momentum fraction of a non-relativistic quark is $\lambda + \frac{1}{2}$ where $\lambda \ll 1$.
- ϕ represents the non-relativistic part. n and m label de components of the Fock space. For example, $C^k_{q\bar{q}\leftarrow q\bar{q}}(z,\mathbf{x}_\perp)$ means how the $q\bar{q}$ component of the full wave function depends on the same component of the non-relativistic wave function.
- The terms in the rhs scale as v^k. Note that if $mv^2 \gg \Lambda_{QCD}$ then $v \sim \alpha_s(mv)$.
- $C^k_{n\leftarrow m}(z,\mathbf{x}_\perp)$ can be computed as and expansion in $\alpha_s(m)$.

$$16\pi\frac{d\sigma_{T,L}^{\gamma^*+N\to HQ+N}}{dt}=$$

$$\left|\sum_{n,m,k}\left(\left(\frac{\nabla}{M}\right)^k\int\frac{d\lambda}{4\pi}\phi^m(\lambda,\mathbf{0})\right)\int d^2r_\perp\int_0^1\frac{dz}{4\pi}\left(\left(C^k_{n\leftarrow m}(z,\mathbf{r}_\perp)\right)^*\Psi_{\gamma^*}\right)^n_{T,L}\sigma_n\right|^2$$

Power counting

- The first correction from terms with $k\neq 0$ will enter at NNLO in α_s.
- At NLO we only need to take into account $C^0_{q\bar{q}\leftarrow q\bar{q}}$ and $C^0_{q\bar{q}g\leftarrow q\bar{q}}$.
- In this power counting we did not consider the difference between $\alpha_s(m)$ and $\alpha_s(mv)$.

Results: $q\bar{q}$ longitudinal polarization

$$
\begin{aligned}
\int d\theta_r C^0_{q\bar{q}\leftarrow q\bar{q}}(z,\mathbf{r}_\perp;\lambda_1,\lambda_2;\lambda_1',\lambda_2')_{long} &= 8\pi^2\delta(z-\frac{1}{2})(1+\delta Z)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} \\
&+\frac{4g^2C_F z(1-z)}{\left(z-\frac{1}{2}\right)^2}\delta_{\lambda_1,\lambda_1'}\delta_{\lambda_2,\lambda_2'}\Bigg\{K_0(\tau) \\
&+\left(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z\right)\left[\frac{\left(z-\frac{1}{2}\right)^2-\frac{1}{2}}{z(1-z)}\left(K_0(\tau)\right.\right. \\
&\left.\left.-\frac{\tau}{2}K_1(\tau)\right)-\frac{\left(z-\frac{1}{2}\right)^2}{2z(1-z)}\tau K_1(\tau)\right]\Bigg\}
\end{aligned}
$$

Longitudinal polarization. We do the transverse angle integration to get a more compact expression. $\tau = 2m\left(z-\frac{1}{2}\right)r_\perp$.

Results: $q\bar{q}$ transverse polarization

$$\int d\theta_r C^0_{q\bar{q}\leftarrow q\bar{q}}(z,\mathbf{r}_\perp;\lambda_1,\lambda_2;\lambda_1',\lambda_2')_{trans} = 8\pi^2\delta(z-\frac{1}{2})(1+\delta Z)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}$$
$$+\frac{4g^2C_Fz(1-z)\delta_{\lambda_1,\lambda_1'}\delta_{\lambda_2,\lambda_2'}}{\left(z-\frac{1}{2}\right)^2}\left[K_0(\tau)\right.$$
$$\left.-\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{2z(1-z)}\left(K_0(\tau)-\frac{\tau}{2}K_1(\tau)\right)\right]$$

Transverse polarization. We do the transverse angle integration to get a more compact expression. $\tau = 2m\left(z-\frac{1}{2}\right)r_\perp$.

Results: $q\bar{q}g$

$$\int \frac{d\lambda}{4\pi} \left\{\Psi^{q\bar{q}g}_{HQ}\right\}^{i}_{\lambda_{RQ},\lambda_G,\lambda_{\bar{Q}}} (x, \mathbf{l}_\perp; \lambda, \mathbf{r}_\perp = \mathbf{0}) =$$

$$-\frac{ig\left(1-\frac{x}{2}\right) x l^i_\perp \epsilon^{*\,A,j}_\perp(\lambda_G)}{2\pi\sqrt{2x(1-x)}\, l_\perp} \left(\frac{mx}{2\pi l_\perp \mu^2}\right)^{\frac{D-4}{2}} K_{\frac{D-2}{2}}(mx l_\perp) \sum_{\lambda_Q} \bar{u}(\hat{p}_{RQ}, \lambda_{RQ}) T^A$$

$$\times \left[\delta^{ij} - \frac{x}{4\left(1-\frac{x}{2}\right)}[\gamma^i_\perp, \gamma^j_\perp]\right] \not{n} u(mv, \lambda_Q) \int \frac{d\lambda}{4\pi} \phi^i_{q\bar{q}}(\lambda, \mathbf{0})$$

$$+ \frac{g\sqrt{x(1-x)}x}{8\sqrt{2}\pi(1-x)} \left(\frac{mx}{2\pi l_\perp \mu^2}\right)^{\frac{D-4}{2}} K_{\frac{D-4}{2}}(mx l_\perp)$$

$$\times \sum_{\lambda_Q} \bar{u}(\hat{p}_{RQ}, \lambda_{RQ}) \not{\epsilon}^*_\perp(\lambda_G) \not{n} u(mv, \lambda_Q) \int \frac{d\lambda}{4\pi} \phi^i_{q\bar{q}}(\lambda, \mathbf{0})$$

$l_\perp$ is the distance between the gluon and the relativistic quark. x is the momentum fraction of the gluon with respect to the non-relativistic quark.

Plan

Light-cone distribution amplitude

- The light-cone distribution amplitude is a function that is defined in collinear factorization.
- In light-cone gauge, it is equal to the light-cone wave function at $r_\perp = 0$.
- It must fulfil the Efremov-Radyushkin-Brodsky-Lepage (ERBL)[1] equation.

[1] Lepage and Brodsky (1980), Chernyak and Zhitnitsky (1984)

Light-cone distribution amplitude

The limit $r_\perp \to 0$ introduces ultraviolet divergences, they can be regulated in dimensional regularization.

$$\lim_{r_\perp \to 0} f(z, \mathbf{r}_\perp) = \int \frac{d^{D-2}p_\perp}{(2\pi)^{D-2}} f(z, \mathbf{p}_\perp)$$

For the longitudinal polarization

$$\begin{aligned}
D^3(z) &= 4\pi(1+\delta Z)\delta\left(z-\frac{1}{2}\right)\int \frac{d\lambda}{4\pi}\phi^3_{q\bar{q}}(\lambda, \mathbf{0}) \\
&- \frac{2g^2 C_F z(1-z)}{\pi\left(z-\frac{1}{2}\right)^2}\left[\left(\frac{1}{D-4}+\frac{1}{2}\log\left(\frac{m^2\left(z-\frac{1}{2}\right)^2}{\pi\mu^2}\right)+\frac{\gamma_E}{2}\right)\right. \\
&\times\left(1+(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)\frac{(D-2)\left(z-\frac{1}{2}\right)^2-1}{2z(1-z)}\right) \\
&\left.+\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{4z(1-z)}\left(2(D-2)\left(z-\frac{1}{2}\right)^2-1\right)\right]\int \frac{d\lambda}{4\pi}\phi^3_{q\bar{q}}(\lambda, \mathbf{0})
\end{aligned}$$

Light-cone distribution amplitude

For the transverse polarization

$$D^i(z) = 4\pi(1+\delta Z)\delta\left(z-\frac{1}{2}\right)\int\frac{d\lambda}{4\pi}\phi^3_{q\bar{q}}(\lambda,\mathbf{0})$$
$$-\frac{2g^2C_Fz(1-z)}{\pi\left(z-\frac{1}{2}\right)^2}\Bigg[\left(\frac{1}{D-4}+\frac{1}{2}\log\left(\frac{m^2\left(z-\frac{1}{2}\right)^2}{\pi\mu^2}\right)+\frac{\gamma_E}{2}\right)$$
$$\left(1-\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{2z(1-z)}\right)$$
$$-\frac{(\theta(z-\frac{1}{2})(1-z)+\theta(\frac{1}{2}-z)z)}{4z(1-z)}\Bigg]\int\frac{d\lambda}{4\pi}\phi^i_{q\bar{q}}(\lambda,\mathbf{0})$$

The ERBL equation

We have checked that it is fulfilled that

$$\frac{\partial D^i(z)}{\partial \log \mu^2} = \frac{\alpha_s C_F}{2\pi} \int_0^1 dz' K_{L,T}(z,z') D^i(z')$$

where

$$K_L(z,z') = \theta(z-z')\frac{1-z}{1-z'}\left(1+\left[\frac{1}{z-z'}\right]_+\right) + \theta(z'-z)\frac{z}{z'}\left(1+\left[\frac{1}{z'-z}\right]_+\right) + \frac{3}{2}\delta(z-z')$$

and

$$K_T(z,z') = \theta(z-z')\frac{1-z}{1-z'}\left[\frac{1}{z-z'}\right]_+ + \theta(z'-z)\frac{z}{z'}\left[\frac{1}{z'-z}\right]_+ + \frac{3}{2}\delta(z-z')$$

Quarkonium decay into leptons

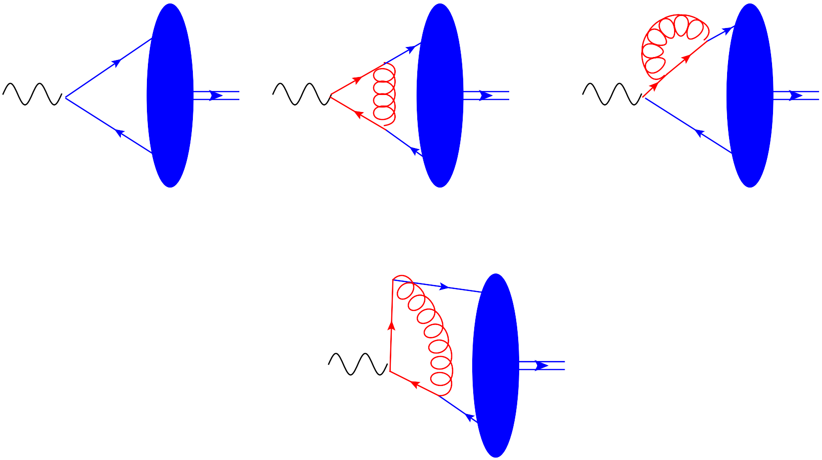
This is a quantity that can be computed knowing the light-cone wave function. It has also been computed at one loop using NRQCD and related approaches[2] (in dimensional regularization). Therefore, we know

$$\int_0^1 dz \sum_n \Psi^n_{HQ}(z, \mathbf{0}_\perp) = \left(1 - \frac{2\alpha_s C_F}{\pi}\right) \int d\lambda \phi(\lambda, \mathbf{0})$$

However, we are not using dimensional regularization.

[2] Barbieri, Gatto, Kogerler and Kunszt (1975)

Diagrams



Remark: The last diagram vanishes in the longitudinal polarization case.

Result

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, \mathbf{0}_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right) \int d\lambda \phi(\lambda, \mathbf{0})$$

Result

$$\int_0^1 dz \Psi^{q\bar{q}}_{HQ}(z, \mathbf{0}_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right) \int d\lambda \phi(\lambda, \mathbf{0})$$

- We are using a cut-off x_0 to regulate the integration in z and DR to regulate the transverse component.
- Power like divergences do not appear in dimensional regularization (DM) but they can appear in our case.
- The divergence comes from the region in which $p_\perp \sim m x_0 \ll m$ and $\left|z - \frac{1}{2}\right| \sim \frac{x_0}{2} \ll 1$. Coulomb singularity.

Result

$$\int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, \mathbf{0}_\perp) = \left(1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{x_0} - 2\right)\right) \int d\lambda \phi(\lambda, \mathbf{0})$$

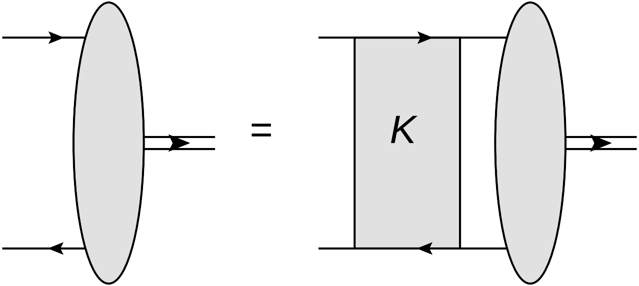
- We are using a cut-off x_0 to regulate the integration in z and DR to regulate the transverse component.
- Power like divergences do not appear in dimensional regularization (DM) but they can appear in our case.
- The divergence comes from the region in which $p_\perp \sim m x_0 \ll m$ and $\left|z - \frac{1}{2}\right| \sim \frac{x_0}{2} \ll 1$. Coulomb singularity.

We need that...

$$\frac{d}{dx_0} \int_0^1 dz \Psi_{HQ}^{q\bar{q}}(z, \mathbf{0}_\perp) = -\frac{\alpha_s C_F}{\pi x_0^2} \int d\lambda \phi(\lambda, \mathbf{0}) + \frac{d}{dx_0} \int d\lambda \phi(\lambda, \mathbf{0}) = 0$$

Coulomb singularity

We want to check if the condition is fulfilled.



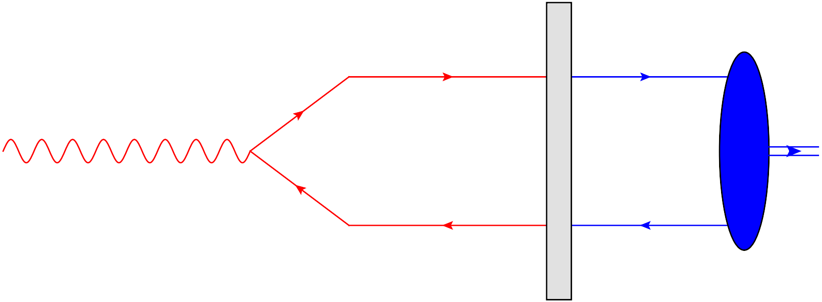
- We look at the ultraviolet behaviour, we can substitute the kernel by a Coulomb exchange.
- We focus on the case in which $\left|z - \frac{1}{2}\right| \sim \frac{x_0}{2}$ and $m \gg p_\perp \sim m x_0 \gg m v$.
- Indeed, the dependence of $\int d\lambda \phi$ on x_0 makes decay width finite.

Plan

Exclusive quarkonium production in the $Q \gg m$ limit at NLO

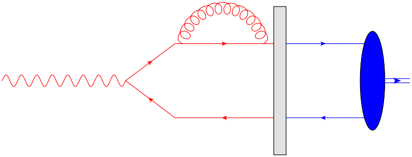
- Our final goal is to compute this process in the general case $Q \sim m$.
- For this we need the NLO photon wave function with massive quarks. This is being investigated at the moment, see talks by Beuf and Paatelainen in LC2019.
- At the moment, we check that all divergences cancel in the $Q \gg m$ limit. We focus on the simpler, longitudinal polarization case.
- We get consistent results compatible with B-JIMWLK evolution.

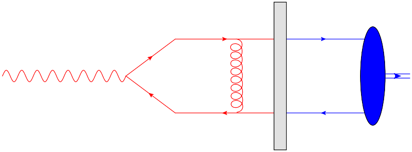
Tree level



Dependence with x_0 hidden in two terms. $\sigma_{q\bar{q}}$, which fulfils B-JIMWLK evolution, and the non-relativistic wave function, which depends on x_0 in the way we described when discussing the decay into leptons.

One loop corrections to photon wave function



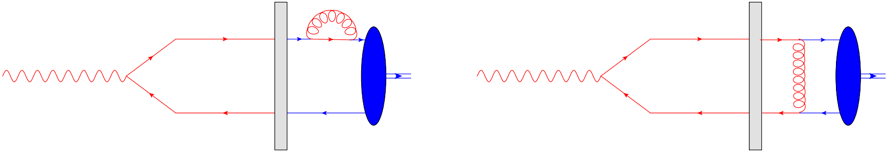


$$\Psi_\gamma(z, r_\perp)|_{NLO} = \Psi_\gamma(z, r_\perp)|_{LO}\,(1 + \delta Z_\gamma(z, r_\perp))$$

Recently computed in Beuf (2017). In our case we need the value at $z = \frac{1}{2}$.

$$\delta Z_\gamma(\frac{1}{2}, r_\perp) = -\frac{2C_F\alpha_s}{\pi}\left(\left(\log x_0 + \frac{3}{4}\right)\left(\frac{1}{D-4} - \frac{\gamma_E}{2} - \frac{1}{2}\log(\pi\mu^2 r_\perp^2)\right)\right.$$
$$\left. + \frac{\pi^2}{24} - \frac{3}{4}\right)$$

One loop corrections to quarkonium wave function



Dependence with μ

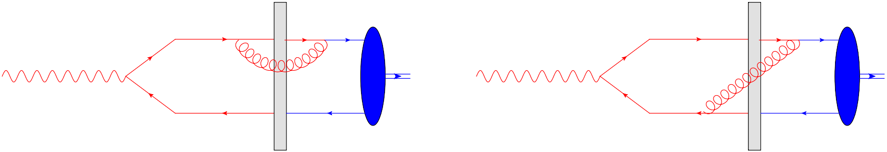
Comes only from the wave function renormalization and fulfils that $\frac{d\delta Z}{d\mu} = \frac{d\delta Z_\gamma\left(\frac{1}{2}, r_\perp\right)}{d\mu}$

Dependence with x_0

Can be divided into two pieces:

- One which cancels the x_0 dependence of the non-relativistic wave function. Coulomb singularity.
- One whose derivative is proportional to $\frac{d\delta Z_\gamma\left(\frac{1}{2}, r_\perp\right)}{dx_0}$.

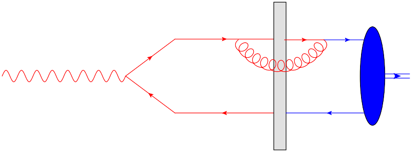
Contribution of the $q\bar{q}g$ Fock state

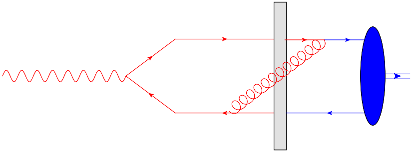


Dependence with μ

Note that in the ultraviolet $\sigma_{q\bar{q}g} \to \sigma_{q\bar{q}}$. It has a divergence that cancels that of the wave functions of the photon and quarkonium.

Contribution of the $q\bar{q}g$ Fock state





Dependence with x_0

Can be divided in two terms:

- One proportional to $(\sigma_{q\bar{q}g} - \sigma_{q\bar{q}})$ which cancels the B-JIMWLK evolution of the target.
- One proportional to $\sigma_{q\bar{q}}$ which cancels the divergences of the wave functions of the photon and quarkonium, except for the piece related with the Coulomb singularity

Plan

- We have computed the NLO corrections to the quarkonium wave function in the non-relativistic limit.
- We have checked that the light-cone distribution amplitude obtained in this framework fulfils ERBL equation.
- We recovered known results for the decay of quarkonium into leptons. To our knowledge, first computation in light-cone gauge.
- We have checked that when the wave function is applied to compute exclusive quarkonium production all divergences will cancel.
- Once the photon wave function with massive quarks is known we are ready for phenomenological applications.