Exploring Pion and Nucleon Structure Through Basis Light Front Quantization

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Palaiseau, September 18, 2019

Overview

Basis Light-Front Quantization (BLFQ)

Vay *et. al*. PRC 81 (2010)

Solve many-body bound state problems in quantum field theories

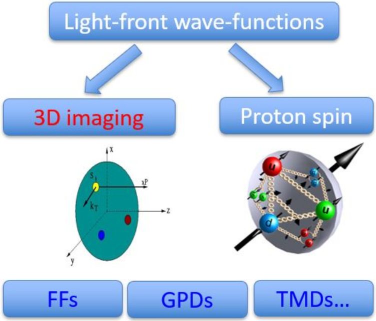
$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

- P^- : light-front Hamiltonian
- $|\beta\rangle$ mass eigenstates
- P^-_β eigenvalue (light-front energy) for eigenstate $|\beta\rangle$
- first-principles / effective Hamiltonian as input
- Evaluate observables

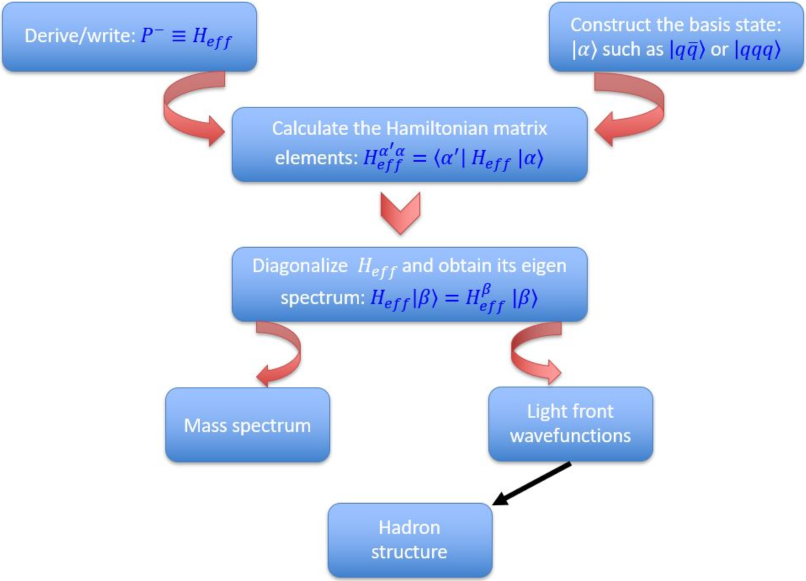
$$O \sim \langle\beta|\hat{O}|\beta\rangle$$

- direct access to wave function of bound states

GOAL



General Procedure for BLFQ



Applications of BLFQ

QCD systems

- Heavy mesons: spectrum, decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, GPDs

—Y Li, G Chen, X Zhao, P Maris, J Vary, L Adhikari, M Li, S. Tang, A El-Hady (2016 - 2019)

- Light mesons: spectrum, decay constant, elastic form factor, radii, distribution amplitude, PDFs and scale evolution

—S Jia, J Vary, J Lan, CM, X. Zhao (2018 - 2019)

QED systems

- Electron: anomalous magnetic moments, GPDs
- positronium wave function, spectroscopy, FFs, GPDs

—Zhao, Wiecki, Li, Honkanen, Chakrabarti, Maris, Vary, Brodsky (2013 - 2018)

Talks on BLFQ: Zhao (16:55), Maris (19/9- 11:00), Vary (19/9- 11:55), Meijian (20/9-11:55)

Example: Light Mesons

S. Jia and J. Vary, PRC (2019)

Light front effective Hamiltonian, H_{eff}:

$$\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF Kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2}_{\text{Transverse}} - \underbrace{\frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right)}_{\text{Longitudinal}} + H_{\text{NJL}}^{\text{eff}}$$

- quark masses: $[m_{u/d},\ m_s] = [337,\ 445]$ MeV
- Confining strength: $[\kappa_{\pi/\rho},\ \kappa_{K/K^*}] = [227,\ 276]$ MeV
- Coupling constants: $[G_{\pi/\rho},\ G_{K/K^*}] = [18.5,\ 13.6]\ \text{GeV}^{-2}$

Mass	BLFQ-NJL	PDG
m_{π^+}	139.57 MeV	139.57 MeV
m_{ρ^+}	775.23 ± 0.04 MeV	775.26 ± 0.25 MeV
m_{K^+}	493.68 MeV	493.68 ± 0.02 MeV
$m_{K^{*+}}$	891.82 ± 0.06 MeV	891.76 ± 0.25 MeV
$m_{K_0^{*+}}$	858.35 MeV	824 ± 30 MeV

Elastic Form Factor

S. Jia and J. Vary, PRC (2019)

Drell & Yan (PRL, 70); West (PRL, 70)

- The elastic form factors of the pseudoscalar states:

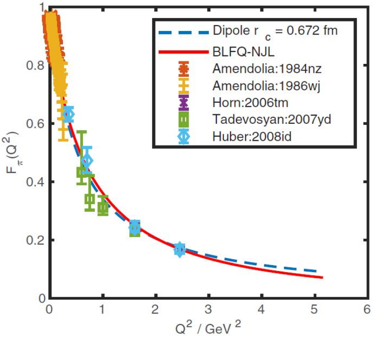
$$F_{\mathrm{P}}(Q^2) = I_{0,0}(Q^2).$$

$$I_{m_J,m_J'}(Q^2) = \left\langle \Psi(P', m_J') \middle| \frac{J^+(0)}{2P^+} \middle| \Psi(P, m_J) \right\rangle$$

with $q = P' - P$ and $Q^2 = -q^2$.

- The charge radius:

$$\langle r_c^2 \rangle = -6 \lim_{Q^2 \to 0} \frac{d}{dQ^2} F_{\mathrm{P}}(Q^2).$$



Pion charge radius: $\sqrt{\langle r_c^2 \rangle}|_{\pi^+}$

BLFQ: 0.68 ± 0.05 fm

PDG: 0.672 ± 0.008 fm

Parton Distribution Amplitude

in preparation, CM *et al*.

Lepage & Brodsky, PRD (1980)

- Distribution amplitude

$$\phi_{\mathrm{P}}(x;\mu) = \frac{2\sqrt{2N_c}}{f_P}\frac{1}{4\pi\sqrt{x(1-x)}} \times \int \frac{d^2k_\perp}{(2\pi)^2}\frac{1}{\sqrt{2}}\left[\psi_{\uparrow\downarrow}(x,k_\perp) - \psi_{\downarrow\uparrow}(x,k_\perp)\right]$$

- Fitting function for pion PDA at $\mu_0 = 442$ MeV

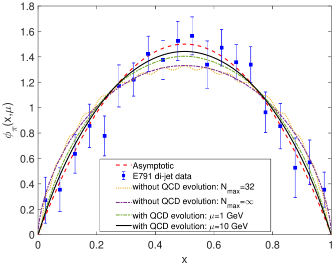
$$\phi(x) = \frac{x^a(1-x)^b}{B(a+1,b+1)},$$

extrapolation: $a = b = 0.60$

Decay constant:

BLFQ: 142.9 MeV

PDG: 130.2 ± 1.7 MeV



- DA evolution: Gegenbauer basis   Ruiz, *et. al*. PRD 66, (2002)
- Our evolved DA ≈ Asymptotic DA

$\pi\to\gamma(\gamma^*)$ Transition Form Factor

in preparation, CM *et al*.

Lepage & Brodsky, PRD (1980)

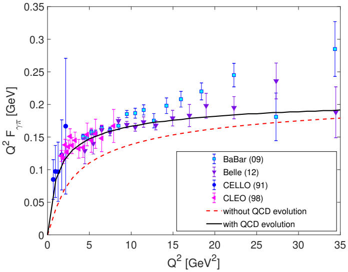
- $\pi\to\gamma^*\gamma$ TFF:

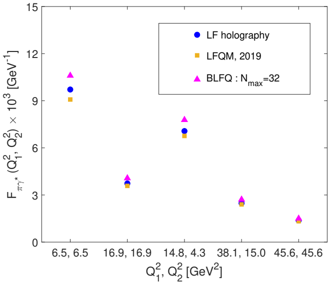
$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}}{3} f_P \int_0^1 \mathrm{d}x \frac{\phi_\pi(x, xQ)}{Q^2 x}$$

- TFF data prefer QCD evolution of DA
- $\pi\to\gamma^*\gamma^*$ TFF: $F_{\pi\gamma^*}(Q^2)$

$$\approx \frac{\sqrt{2}}{3} f_P \int_0^1 \mathrm{d}x \frac{\phi_\pi(x)}{Q_1^2 x + Q_2^2(1-x)}$$

LFQM: Choi, Ryu, Ji, PRD 99 (2019)



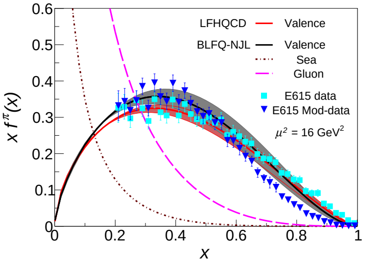


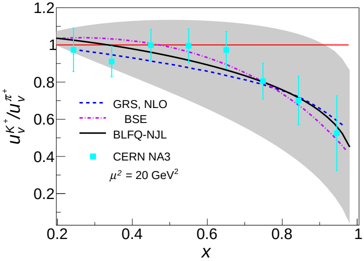
Light Meson PDFs

Lan, CM, Jia, Zhao, Vary: PRL 122 (2019)

$$f(x) = x^a(1-x)^b / B(a+1, b+1),$$

$a = b = 0.5961$ for pion, while $a = 0.6337$ and $b = 0.8546$ for kaon





- LF wavefunctions ► eigenvectors of effective Hamiltonian.
  PDFs evolution ► based on the NNLO DGLAP equations.

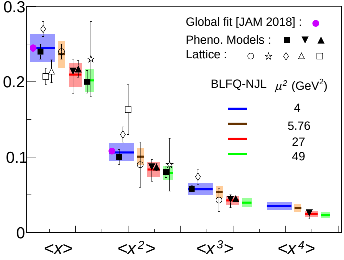
Order	Initial scale of pion	Initial scale of kaon	E-0615 χ^2/(d.o.f.)	NA-003 χ^2/(d.o.f.)
LO	0.120 ± 0.012 GeV^2	0.133 ± 0.013 GeV^2	6.71	0.88
NLO	0.205 ± 0.020 GeV^2	0.210 ± 0.021 GeV^2	4.67	0.56
NNLO	0.240 ± 0.024 GeV^2	0.246 ± 0.024 GeV^2	3.64	0.50

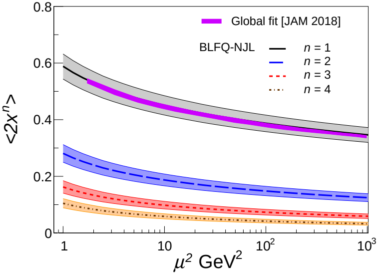
Moments of Pion PDF

Lan, CM, Jia, Zhao, Vary: arXiv: 1907.01509

Moments of the valence quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^\pi(x, \mu^2),\ \ n = 1, 2, 3, 4.$$





- The robustness of our results motivates the application of analogous effective Hamiltonians to the baryons.

Effective Light-front Hamiltonian for Nucleon

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

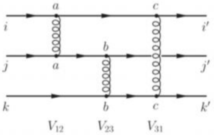
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{trans} \sim \kappa_T^4 r^2$$ -- Brodsky, Teramond arXiv: 1203.4025

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{OGE} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$ Color factor : $C_F = -\frac{2}{3}$

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + \cdots\cdots$$



Three active-quark approach

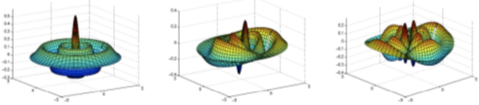
Basis Construction

- Fock's space expansion:

$$|\,N\,\rangle_{\text{baryon}} = a|\,qqq\,\rangle + b|\,qqqg\,\rangle + c|\,qqqq\bar{q}\,\rangle + \cdots .$$

- For each Fock particle:
  - Transverse: 2D harmonic oscillator basis: $\Phi^{b}_{n,m}(\vec{p}_\perp)$
    labeled by radial (angular) quantum number n (m); scale parameter b

    e.g., n=4



  - Longitudinal: plane-wave basis, labeled by k
  - Helicity: labeled by λ

- For the first Fock sector:

$$|\,qqq\,\rangle = |\,\mathbf{n_{q_1}}, \mathbf{m_{q_1}}, \mathbf{k_{q_1}}, \lambda_{\mathbf{q_1}}\,\rangle \otimes |\,\mathbf{n_{q_2}}, \mathbf{m_{q_2}}, \mathbf{k_{q_2}}, \lambda_{\mathbf{q_2}}\,\rangle \otimes |\,\mathbf{n_{q_3}}, \mathbf{m_{q_3}}, \mathbf{k_{q_3}}, \lambda_{\mathbf{q_3}}\,\rangle.$$

Basis Truncation Scheme

- Symmetries of Hamiltonian:
  - Net fermion number: $\sum_i n_i^f = N^f$
  - Total angular momentum projection: $\sum_i (m_i + s_i) = J_z$
  - Longitudinal momentum: $\sum_i k_i = K$

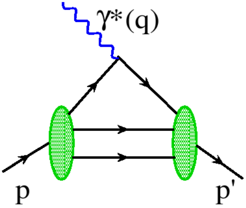
- Further truncation:
  - Fock sector truncation
  - Discretization in longitudinal direction $k_i = \begin{cases} 1, 2, 3.... & \text{bosons} \\ 0.5, 1.5, 2.5 ... & \text{fermions} \end{cases}$
  - "N_{max}" truncation in transverse directions $\sum_i [2n_i + |m_i| + 1] \le N_{max}$

  UV cutoff $\Lambda \sim b\sqrt{N_{\max}}$; IR cutoff $\lambda \sim b/\sqrt{N_{\max}}$

Nucleon Form Factors

In preparation, CM, Siqi Xu, *et. al.*

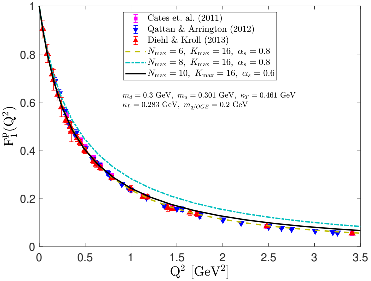
EM form factors in light-front

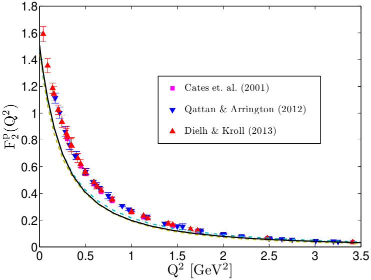


$$\langle P';\uparrow|\frac{J^+(0)}{2P^+}|P;\uparrow\rangle = F_1(q^2)$$

$$\langle P';\uparrow|\frac{J^+(0)}{2P^+}|P;\downarrow\rangle = -(q^1 - iq^2)\frac{F_2(q^2)}{2M}$$

Drell & Yan (PRL, 70); West (PRL, 70)





Nucleon Form Factors

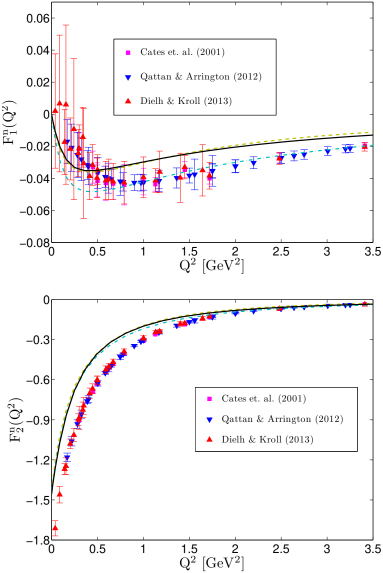
In preparation, CM, Siqi Xu, *et. al.*

- Only valence quarks contributions
- Missing meson-cloud effects
- $|qqqq\bar{q}\rangle$ has a significant effect on Pauli FF: 30% in proton; 40% in neutron

Sufian *et. al.* PRD 95 (2017)

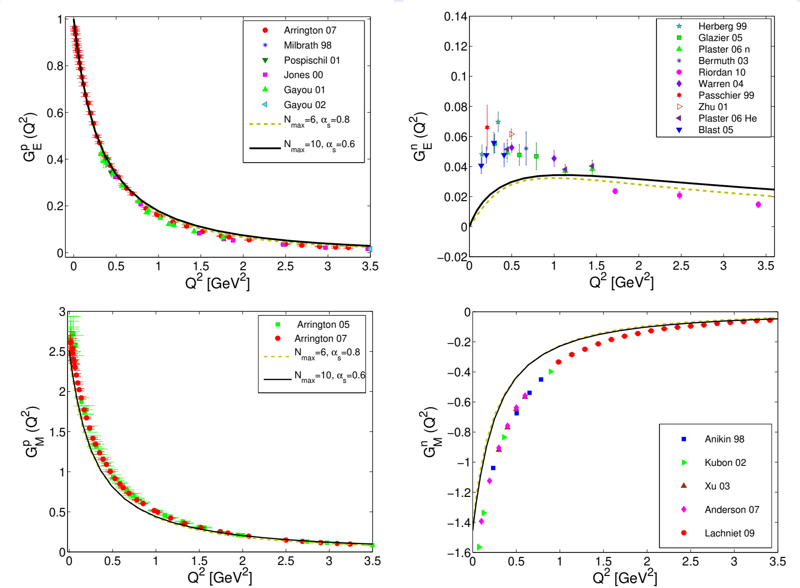
Magnetic moments (μ_N):

Proton: 2.51 (Exp. : 2.79)
Neutron: -1.45 (Exp. : -1.91)



Nucleon Sach's Form Factors

In preparation, CM, Siqi Xu, *et. al.*



Axial Form Factor

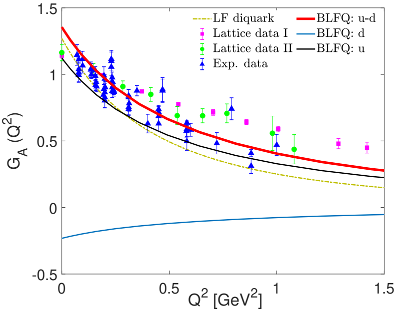
In preparation, CM, Siqi Xu, *et. al.*

$$\langle N(p)|A^\mu|N(p')\rangle = \bar{u}(p')\left[\gamma^\mu G_A(t) + \frac{(p'-p)^\mu}{2m} G_p(t)\right]\gamma_5 u(p)$$

- Axial vector current:
  $A^\mu = \bar{q}\gamma^\mu\gamma_5 q$
- Axial form factor can be measured by ordinary muon capture (OMC)

  $$\mu^-(l) + p(r) \to \nu_\mu(l') + n(r')$$

- Provide information on spin-isospin distributions



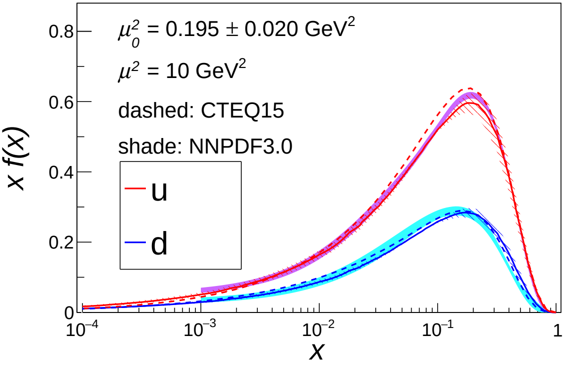
$$G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$

Unpolarized PDFs

In preparation, CM, Siqi Xu, *et. al.*

Unpolarized PDF is defined as ($\Gamma \equiv \gamma^+$):

$$\Phi^{\Gamma(\nu)}(x) = \frac{1}{2}\int \frac{dz^-}{2(2\pi)} e^{ip^+z^-/2} \langle P; S|\bar{\psi}^{(\nu)}(0)\Gamma\psi^{(\nu)}(z^-)|P; S\rangle\Big|_{z^+=z_T=0}.$$

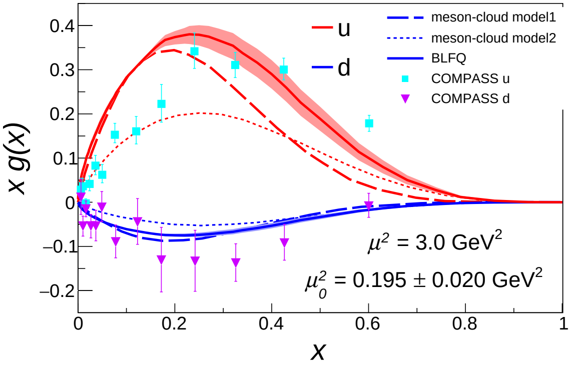


Helicity PDFs

In preparation, CM, Siqi Xu, *et. al.*

Helicity PDF is defined as ($\Gamma \equiv \gamma^+\gamma_5$):

$$\Phi^{\Gamma(\nu)}(x) = \frac{1}{2}\int \frac{dz^-}{2(2\pi)} e^{ip^+z^-/2} \langle P;S|\bar{\psi}^{(\nu)}(0)\Gamma\psi^{(\nu)}(z^-)|P;S\rangle\Big|_{z^+=z_T=0}.$$



model1 & model2: meson-cloud models

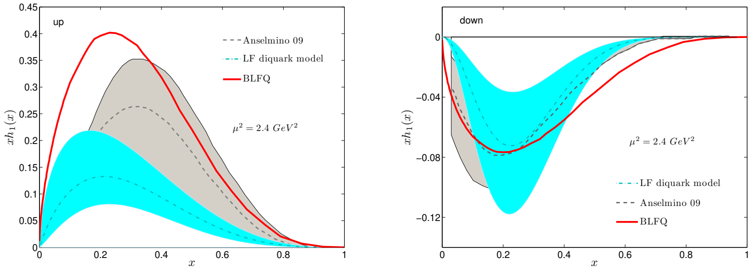
Kofler & Pasquini, PRD 95 (2017)

Transversity PDFs

In preparation, CM, Siqi Xu, *et. al.*

Transversity PDF is defined as ($\Gamma \equiv i\sigma^{j+}\gamma_5$):

$$\Phi^{\Gamma(\nu)}(x) = \frac{1}{2}\int \frac{dz^-}{2(2\pi)} e^{ip^+z^-/2} \langle P;S|\bar{\psi}^{(\nu)}(0)\Gamma\psi^{(\nu)}(z^-)|P;S\rangle\Big|_{z^+=z_T=0}.$$



Qualitative behavior of BLFQ results are in more or less agreement with other calculations.

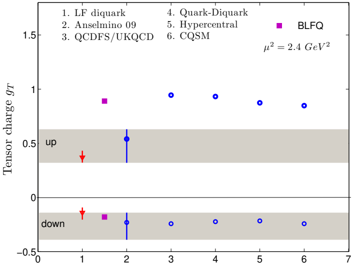
Anselmino, *et al.* Nucl.Phys.Proc.Suppl. 191 (2009); Maji & Chakrabarti, PRD 94 (2016)

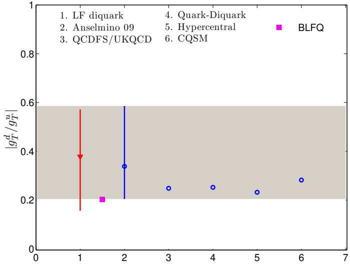
Tensor Charge

In preparation, CM, Siqi Xu, *et. al.*

The tensor charge is defined as

$$g_T^q(\mu^2) = \int_0^1 dx \; h_1^q(x, \mu^2).$$





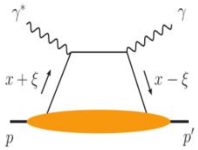
BLFQ results are consistent with other calculations.

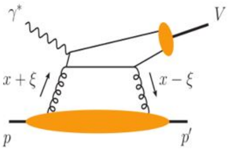
Lattice QCD: $g_T^u = 0.3(2)$ and $g_T^d = -0.7(2)$ at $\mu^2 = 2$ GeV^2

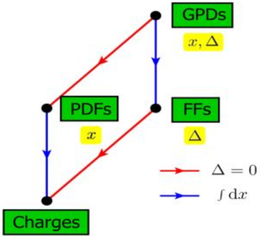
Lin *et. al*. PRL, 120 (2018)

GPDs

- ***Deeply Virtual Compton Scattering (DVCS)/*** vector meson productions ***experiment:***

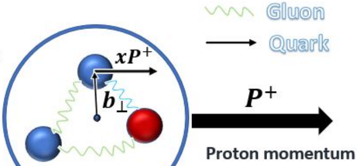






- **GPDs** appear in **DVCS** processes.

  - GPDs are functions of three variables :
    - ***Longitudinal momentum fraction*** $x = \frac{k^+}{P^+}$
    - ***Longitudinal momentum transfer --> skewness*** $\xi = \frac{\Delta^+}{P^+} = 0$
    - ***Square of total mom transfer*** $t = \Delta^2 = (P' - P)^2$



$$\Delta_\perp \overset{FT}{\longleftrightarrow} b_\perp$$

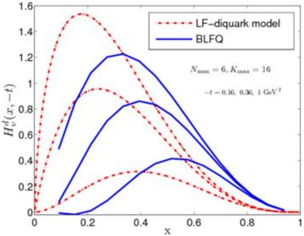
where the $b_\perp$ is transverse position of parton

Proton unpolarized GPDs in BLFQ

In preparation, CM, Siqi Xu, *et. al.*

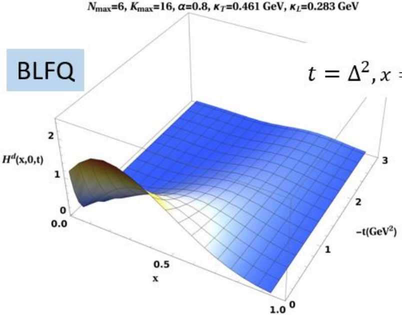
Encode information about three dimensional spatial structure, as well as the spin and orbital angular momentum of the constituents

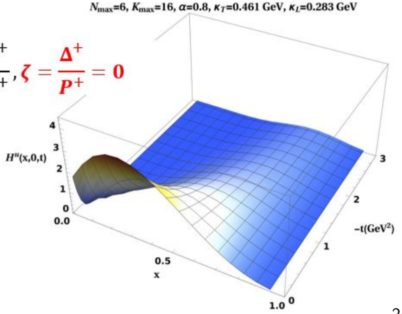
As momentum transfer (t) increases, peaks of distributions shift to larger x; qualitative behavior of BLFQ results are consistent is with LF quark-diquark model results



BLFQ

$$t = \Delta^2, x = \frac{k^+}{P^+}, \zeta = \frac{\Delta^+}{P^+} = 0$$





Conclusions & Outlook

- We discussed the structure of pion and proton from the eigenstates of light front effective Hamiltonian
- Consider the leading Fock sectors ($|q\bar{q}\rangle$ for pion, $|qqq\rangle$ for proton).
- BLFQ provides a good description of the experimental data & Global fits for various observables.

Outlook:

- Other distribution functions : spin asymmetries ,GTMD, DPD. . .
- Study meson-cloud effects and gluon content by including higher Fock sectors.
- Mechanical properties, spin structure of proton, mass decomposition.

Thank You