Exploring Pion and Nucleon Structure Through Basis Light Front Quantization

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Overview

1. Basis Light-Front Quantization (BLFQ)
2. Application to light mesons
3. Application to nucleon
4. Conclusions & Outlook
Basis Light-Front Quantization (BLFQ)

Vay et. al. PRC 81 (2010)

Solve many-body bound state problems in quantum field theories

\[ P^- |\beta⟩ = P^-_\beta |\beta⟩ \]

- \( P^- \) : light-front Hamiltonian
- \( |\beta⟩ \) : mass eigenstates
- \( P^-_\beta \) : eigenvalue (light-front energy) for eigenstate \( |\beta⟩ \)

GOAL

- direct access to wave function of bound states

3D imaging

Proton spin

FFs  GPDs  TMDs...
General Procedure for BLFQ

Derive/write: $P^- \equiv H_{eff}$

Construct the basis state: $|\alpha\rangle$ such as $|q\bar{q}\rangle$ or $|qqq\rangle$

Calculate the Hamiltonian matrix elements: $H_{eff}^{\alpha'_\alpha} = \langle \alpha' | H_{eff} | \alpha \rangle$

Diagonalize $H_{eff}$ and obtain its eigen spectrum: $H_{eff} |\beta\rangle = H_{eff}^{\beta} |\beta\rangle$

Mass spectrum

Light front wavefunctions

Hadron structure
Applications of BLFQ

**QCD systems**

- **Heavy mesons**: spectrum, decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, GPDs
  

- **Light mesons**: spectrum, decay constant, elastic form factor, radii, distribution amplitude, PDFs and scale evolution
  

**QED systems**

- **Electron**: anomalous magnetic moments, GPDs
  

- **positronium** wave function, spectroscopy, FFs, GPDs


**Talks on BLFQ**

Zhao (16:55), Maris (19/9- 11:00), Vary (19/9- 11:55), Meijian ( 20/9-11:55)
Example: Light Mesons

S. Jia and J. Vary, PRC (2019)

Light front effective Hamiltonian, $H_{\text{eff}}$:

$$
\begin{align*}
\vec{k}^2 + m_q^2 &\times \frac{\vec{k}^2 + m_{\bar{q}}^2}{1 - x} \\
\text{LF Kinetic energy} &+ \kappa^4 x(1 - x) \vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1 - x) \partial_x) + H_{\text{NJL}}^{\text{Longitudinal}}
\end{align*}
$$

- Quark masses: $[m_u/d, m_s] = [337, 445]$ MeV
- Confining strength: $[\kappa_{\pi/\rho}, \kappa_{K/K^*}] = [227, 276]$ MeV
- Coupling constants: $[G_{\pi/\rho}, G_{K/K^*}] = [18.5, 13.6]$ GeV$^{-2}$

<table>
<thead>
<tr>
<th>Mass</th>
<th>BLFQ-NJL</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\pi^+}$</td>
<td>139.57 MeV</td>
<td>139.57 MeV</td>
</tr>
<tr>
<td>$m_{\rho^+}$</td>
<td>775.23 ± 0.04 MeV</td>
<td>775.26 ± 0.25 MeV</td>
</tr>
<tr>
<td>$m_{K^+}$</td>
<td>493.68 MeV</td>
<td>493.68 ± 0.02 MeV</td>
</tr>
<tr>
<td>$m_{K^{*+}}$</td>
<td>891.82 ± 0.06 MeV</td>
<td>891.76 ± 0.25 MeV</td>
</tr>
<tr>
<td>$m_{K_{0}^{*+}}$</td>
<td>858.35 MeV</td>
<td>824 ± 30 MeV</td>
</tr>
</tbody>
</table>
Elastic Form Factor

S. Jia and J. Vary, PRC (2019)

The elastic form factors of the pseudoscalar states:

\[ F_P(Q^2) = I_{0,0}(Q^2). \]

\[ I_{m_J, m'_J}(Q^2) = \langle \psi(P', m'_J) \mid \frac{J^+(0)}{2P^+} \mid \psi(P, m_J) \rangle \]

with \( q = P' - P \) and \( Q^2 = -q^2 \).

The charge radius:

\[ \langle r_c^2 \rangle = -6 \lim_{Q^2 \to 0} \frac{d}{dQ^2} F_P(Q^2). \]

Pion charge radius: \( \sqrt{\langle r_c^2 \rangle} \mid_{\pi^+} \)

BLFQ: \( 0.68 \pm 0.05 \text{ fm} \)

PDG: \( 0.672 \pm 0.008 \text{ fm} \)
Parton Distribution Amplitude

Lepage & Brodsky, PRD (1980)

- **Distribution amplitude**

\[ \phi_P(x; \mu) = \frac{2\sqrt{2N_c}}{f_P} \frac{1}{4\pi \sqrt{x(1-x)}} \times \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{\sqrt{2}} \left[ \psi_{\uparrow \downarrow}(x, k_\perp) - \psi_{\downarrow \uparrow}(x, k_\perp) \right] \]

- **Fitting function for pion PDA at**
  \( \mu_0 = 442 \text{ MeV} \)

\[ \phi(x) = \frac{x^a(1-x)^b}{B(a+1, b+1)} , \]

extrapolation: \( a = b = 0.60 \)

**Decay constant:**

- **BLFQ:** 142.9 MeV
- **PDG:** 130.2 ± 1.7 MeV

**DA evolution:** Gegenbauer basis


Our evolved DA \( \approx \) Asymptotic DA
\[ \pi \rightarrow \gamma (\gamma^*) \] Transition Form Factor

Lepage & Brodsky, PRD (1980)

- \[ \pi \rightarrow \gamma^* \gamma \] TFF:
  \[ F_{\pi \gamma} (Q^2) = \frac{\sqrt{2}}{3} f_P \int_0^1 dx \frac{\phi_\pi(x, xQ)}{Q^2x} \]

- TFF data prefer QCD evolution of DA

- \[ \pi \rightarrow \gamma^* \gamma^* \] TFF: \[ F_{\pi \gamma^*} (Q^2) \]
  \[ \approx \frac{\sqrt{2}}{3} f_P \int_0^1 dx \frac{\phi_\pi(x)}{Q_1^2x + Q_2^2(1 - x)} \]

Light Meson PDFs

\[ f(x) = x^a (1 - x)^b / B(a + 1, b + 1), \]

\[ a = b = 0.5961 \] for pion, while \[ a = 0.6337 \] and \[ b = 0.8546 \] for kaon.

- LF wavefunctions ➤ eigenvectors of effective Hamiltonian.
- PDFs evolution ➤ based on the NNLO DGLAP equations.

<table>
<thead>
<tr>
<th>Order</th>
<th>Initial scale of pion</th>
<th>Initial scale of kaon</th>
<th>E-0615 $\chi^2/(d.o.f.)$</th>
<th>NA-003 $\chi^2/(d.o.f.)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>0.120 ± 0.012 GeV$^2$</td>
<td>0.133 ± 0.013 GeV$^2$</td>
<td>6.71</td>
<td>0.88</td>
</tr>
<tr>
<td>NLO</td>
<td>0.205 ± 0.020 GeV$^2$</td>
<td>0.210 ± 0.021 GeV$^2$</td>
<td>4.67</td>
<td>0.56</td>
</tr>
<tr>
<td>NNLO</td>
<td>0.240 ± 0.024 GeV$^2$</td>
<td>0.246 ± 0.024 GeV$^2$</td>
<td>3.64</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Moments of Pion PDF

Moments of the valence quark PDF

\[ \langle x^n \rangle = \int_0^1 dx \ x^n f_\pi(x, \mu^2), \ n = 1, 2, 3, 4. \]

The robustness of our results motivates the application of analogous effective Hamiltonians to the baryons.
Effective Light-front Hamiltonian for Nucleon

\[ P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE} \]

\[ H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+} \]
\[ H_{trans} \sim \kappa_T^4 r^2 \quad \text{--- Brodsky, Teramond arXiv: 1203.4025} \]
\[ H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial x_i \left( x_i x_j \partial x_j \right) \quad \text{--- Y Li, X Zhao, P Maris, J Vary, PLB 758(2016)} \]
\[ H_{OGE} = -\frac{C_F 4\pi \alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j) \quad \text{Color factor: } C_F = -\frac{2}{3} \]

\[ |P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq \, q\bar{q}\rangle + \ldots \]

Three active-quark approach
Basis Construction

- Fock’s space expansion:
  \[ | N \rangle_{\text{baryon}} = a | qqq \rangle + b | qqg \rangle + c | qqqq \bar{q} \rangle + \cdots. \]

- For each Fock particle:
  - **Transverse**: 2D harmonic oscillator basis: \( \Phi_{n,m}(\vec{p}_\perp) \)
    labeled by radial (angular) quantum number \( n \) (\( m \)); scale parameter \( b \)
    
    e.g., \( n=4 \)

    \[ \begin{align*}
    &m=0 & &m=1 & &m=2 \\
    &\text{e.g., } n=4 & & & &
    \end{align*} \]

  - **Longitudinal**: plane-wave basis, labeled by \( k \)
  - **Helicity**: labeled by \( \lambda \)

- For the first Fock sector:
  \[ | qqq \rangle = | n_{q1}, m_{q1}, k_{q1}, \lambda_{q1} \rangle \otimes | n_{q2}, m_{q2}, k_{q2}, \lambda_{q2} \rangle \otimes | n_{q3}, m_{q3}, k_{q3}, \lambda_{q3} \rangle. \]
Basis Truncation Scheme

- **Symmetries of Hamiltonian:**
  - Net fermion number: \( \sum_i n_i^f = N^f \)
  - Total angular momentum projection: \( \sum_i (m_i + s_i) = J_z \)
  - Longitudinal momentum: \( \sum_i k_i = K \)

- **Further truncation:**
  - Fock sector truncation
  - Discretization in longitudinal direction
  - “\( N_{\text{max}} \)” truncation in transverse directions
    \( \sum_i [2n_i + |m_i| + 1] \leq N_{\text{max}} \)
  - UV cutoff \( \Lambda \sim b \sqrt{N_{\text{max}}} \); IR cutoff \( \lambda \sim b / \sqrt{N_{\text{max}}} \)
Nucleon Form Factors

EM form factors in light-front

\[ \langle P'; \uparrow | \frac{J^+(0)}{2P^+} | P; \uparrow \rangle = F_1(q^2) \]

\[ \langle P'; \uparrow | \frac{J^+(0)}{2P^+} | P; \downarrow \rangle = -(q^1 - iq^2) \frac{F_2(q^2)}{2M} \]

Drell & Yan (PRL, 70); West (PRL, 70)
Nucleon Form Factors

- Only valence quarks contributions
- Missing meson-cloud effects
- $|qqq\bar{q}\rangle$ has a significant effect on Pauli FF: 30% in proton; 40% in neutron

[Sufian et al. PRD 95 (2017)]

Magnetic moments ($\mu_N$):

Proton: 2.51 (Exp. : 2.79)
Neutron: -1.45 (Exp. : -1.91)
Nucleon Sach’s Form Factors

In preparation, CM, Siqi Xu, et. al.
Axial Form Factor

In preparation, CM, Siqi Xu, et. al.

\[ \langle N(p) | A^\mu | N(p') \rangle = \bar{u}(p') \left[ \gamma^\mu G_A(t) + \frac{(p' - p)^\mu}{2m} G_p(t) \right] \gamma_5 u(p) \]

- Axial vector current:
  \[ A^\mu = \bar{q} \gamma^\mu \gamma_5 q \]

- Axial form factor can be measured by ordinary muon capture (OMC)
  \[ \mu^- (l) + p(r) \rightarrow \nu_\mu (l') + n(r') \]

- Provide information on spin-isospin distributions

\[ G_A(Q^2) = G_u(Q^2) - G_d(Q^2) \]
Unpolarized PDF is defined as \((\Gamma \equiv \gamma^+)\):

\[
\Phi_{\Gamma}(x) = \frac{1}{2} \int \frac{dz^-}{2(2\pi)} e^{i p^+ z^- / 2} \langle P; S | \bar{\psi}^{(\nu)}(0) \Gamma \psi^{(\nu)}(z^-) | P; S \rangle \bigg|_{z^+ = z_T = 0}.
\]

\[
\mu^2 = 0.195 \pm 0.020 \text{ GeV}^2
\]
\[
\mu^2 = 10 \text{ GeV}^2
\]

- dashed: CTEQ15
- shade: NNPDF3.0

\(x f(x)\)
Helicity PDFs

Helicity PDF is defined as \( \Gamma \equiv \gamma^+ \gamma_5 \):

\[
\Phi_{\Gamma}^{(\nu)}(x) = \frac{1}{2} \int \frac{dz^-}{2(2\pi)} e^{ip^+z^-/2} \langle P; S| \bar{\psi}^{(\nu)}(0) \Gamma \psi^{(\nu)}(z^-)|P; S \rangle \bigg|_{z^+ = z_T = 0}
\]

\[\mu^2 = 3.0 \text{ GeV}^2\]
\[\mu_0^2 = 0.195 \pm 0.020 \text{ GeV}^2\]

model1 & model2: meson-cloud models

Kofler & Pasquini, PRD 95 (2017)
Transversity PDFs

In preparation, CM, Siqi Xu, et. al.

Transversity PDF is defined as \( \Gamma \equiv i \sigma^j \gamma_5 \):

\[
\Phi_{\Gamma}(\nu)(x) = \frac{1}{2} \int \frac{dz^-}{2(2\pi)} e^{ip^+z^-/2} \langle P; S| \bar{\psi}^{(\nu)}(0) \Gamma \psi^{(\nu)}(z^-)|P; S\rangle \bigg|_{z^+=z_T=0}.
\]

Qualitative behavior of BLFQ results are in more or less agreement with other calculations.

Tensor Charge

The tensor charge is defined as

$$g_T^q(\mu^2) = \int_0^1 dx \ h_1^q(x, \mu^2).$$

BLFQ results are consistent with other calculations.

Lattice QCD: $g_T^u = 0.3(2)$ and $g_T^d = -0.7(2)$ at $\mu^2 = 2$ GeV$^2$

Lin et. al. PRL, 120 (2018)
GPDs

- **Deeply Virtual Compton Scattering (DVCS)/vector meson productions experiment:**

- **GPDs** appear in **DVCS** processes.
  - GPDs are functions of three variables:
    - Longitudinal momentum fraction $x = \frac{k^+}{P^+}$
    - Longitudinal momentum transfer $\rightarrow$ skewness $\xi = \frac{\Delta^+}{P^+} = 0$
    - Square of total mom transfer $t = \Delta^2 = (P' - P)^2$

where the $b_\perp$ is transverse position of parton.
Proton unpolarized GPDs in BLFQ

Encode information about three dimensional spatial structure, as well as the spin and orbital angular momentum of the constituents.

As momentum transfer (t) increases, peaks of distributions shift to larger x; qualitative behavior of BLFQ results are consistent is with LF quark-diquark model results.

\[ t = \Delta^2, \quad x = \frac{k^+}{p^+}, \quad \zeta = \frac{\Delta^+}{p^+} = 0 \]
Conclusions & Outlook

- We discussed the structure of pion and proton from the eigenstates of light front effective Hamiltonian.
- Consider the leading Fock sectors (|q\bar{q}\rangle for pion, |qqq\rangle for proton).
- BLFQ provides a good description of the experimental data & Global fits for various observables.

Outlook:

- Other distribution functions: spin asymmetries, GTMD, DPD...
- Study meson-cloud effects and gluon content by including higher Fock sectors.
- Mechanical properties, spin structure of proton, mass decomposition.

Thank You