Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography and Superconformal Algebra

Fixed $\tau = t + z/c$

with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Tianbo Llu, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur

LIGHT CONE 2019

Stan Brodsky
Mesons and Baryons: Same Regge Slope $M^2 \propto J!$

The leading Regge trajectory: $\Delta$ resonances with maximal $J$ in a given mass range. Also shown is the Regge trajectory for mesons with $J = L+S$.

E. Klempt and B. Ch. Metsch
\( M^2 \) (GeV\(^2\))

\[ \rho - \Delta \] superpartner trajectories

**MESONS** \([q\bar{q}]\)

**BARYONS** \([qqq]\)

\( L_M = L_B + 1 \)

Dosch, de Teramond, sjb

Supersymmetric QCD Spectroscopy
Profound Questions for Hadron Physics

- Color Confinement
- Origin of the QCD Mass Scale
- Meson and Baryon Spectroscopy
- Exotic States: Tetraquarks, Pentaquarks, Gluonium,
- Universal Regge Slopes: n, L, Mesons and Baryons
- Massless Pion: Quark Anti-Quark Bound State
- Hadron Structure and Dynamics: QCD Coupling at all Scales
Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

AdS/QCD
Light-Front Holography
Superconformal Algebra

No parameters except for quark masses!
Evolve in ordinary time

\[ \sigma = ct - z \]

Evolve in light-front time!

\[ \tau = t + \frac{z}{c} \]

Dirac's Amazing Idea:
The "Front Form"

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Instant Form

Front Form

Casual, Boost Invariant!

- Trivial LF Vacuum (up to zero modes)
Bound States in Relativistic Quantum Field Theory: Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi > = M^2 |\psi >$$

Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ P^+, \vec{P}_{\perp} \]

**Dirac: Front Form**

**Measurements of hadron LF wavefunction are at fixed LF time**

**Like a flash photograph**

Fixed \( \tau = t + z/c \)

\[ x_{bj} = x = \frac{k^+}{P^+} \]

**Invariant under boosts! Independent of \( p^\mu \)**
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ P^+, \vec{P}_\perp \]

\[ \psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

Fixed \( \tau = t + z/c \)

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i \]

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_\perp i = 0 \]

Eigenstate of LF Hamiltonian

\[ H_{LF}^{QCD} |\Psi_h \rangle \geq M_h^2 |\Psi_h \rangle \]

\[ |p, J_z \rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_\perp i, \lambda_i) |n; x_i, \vec{k}_\perp i, \lambda_i \rangle \]

Invariant under boosts! Independent of \( P^\mu \)

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS
\[
\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)
\]

\[
q^2 = Q^2 = -q^2
\]

\[
q^+ = 0 \quad \vec{q}_\perp
\]

\[
\gamma^* \quad \text{Fixed } \tau = t + z/c
\]

Form Factors are Overlaps of LFWFs

\[
\psi(x_i, \vec{k}_{\perp i})
\]

struck \quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp

spectators \quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp
Exact LF Formula for Pauli Form Factor

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[ -\frac{1}{q_L} \psi_a^\dagger(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q_R} \psi_a^\dagger(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) \right]$$

$$k'_{\perp i} = k_{\perp i} - x_i q_\perp$$

$$k'_{\perp j} = k_{\perp j} + (1 - x_j) q_\perp$$

$$q_{R,L} = q^x \pm iq^y$$

$${\bf L}_z = +1$$

$${\bf p}, \quad S_z = -1/2$$

$${\bf p} + {\bf q}, \quad S_z = 1/2$$

$${\bf L}_z = 0$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment $\rightarrow$
Nonzero orbital quark angular momentum
Terayev, Okun: $B(0)$ Must vanish because of Equivalence Theorem

LF Proof

Vanishing Anomalous gravitomagnetic moment $B(0)$
Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form.

Boost are dynamical in instant form.
Advantages of the Dirac’s Front Form for Hadron Physics

Poincare’ Invariant

Physics Independent of Observer’s Motion

• Measurements are made at fixed $\tau$
• Causality is automatic
• Structure Functions are squares of LFWFs
• Form Factors are overlap of LFWFs
• LFWFs are frame-independent: no boosts, no pancakes!

Penrose, Terrell, Weisskopf

• Same structure function measured at an e p collider and the proton rest frame
• No dependence of hadron structure on observer’s frame
• LF Holography: Dual to AdS space
• LF Vacuum trivial up to zero modes
• Implications for Cosmological Constant

Roberts, Shrock, Tandy, sjb
Light-Front Wavefunctions underly hadronic observables

\[ \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Transverse density in momentum space

Weak transition form factors

GTMDs

\[ x, \vec{k}_{\perp}, \vec{b}_{\perp} \]

Momentum space \[ \vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp} \]

Position space \[ \vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp} \]

Transverse density in position space

DGLAP, ERBL Evolution
Factorization Theorems

TMDs

\[ x, \vec{k}_{\perp} \]

TMFFs

\[ \vec{k}_{\perp}, \vec{b}_{\perp} \]

GPDs

\[ x, \vec{b}_{\perp} \]

TMSDs

\[ \vec{k}_{\perp} \]

PDFs

\[ x, \vec{k} \]

FFs

\[ \vec{b}_{\perp} \]

Charges

Diffractive DIS from FSI

Sivers, T-odd from lensing

Lorce, Pasquini

Transverse density in momentum space
Light-Front QCD  

**Physical gauge:** $A^+ = 0$

**Exact frame-independent formulation of nonperturbative QCD!**

$$ L^{QCD} \rightarrow H^{QCD}_{LF} $$

$$ H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k_{\perp}^2}{x} \right] \psi_i + H_{LF}^{int} $$

$H_{LF}^{int}$: Matrix in Fock Space

$$ H^{QCD}_{LF} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle $$

$$ |p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle $$

*Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions*

LFWFs: Off-shell in P- and invariant mass
Light-Front Perturbation Theory for pQCD

\[ T = H_I + H_I \frac{1}{\mathcal{M}_{\text{initial}}^2 - \mathcal{M}_{\text{intermediate}}^2} + i\epsilon H_I + \cdots \]

- “History”: Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!

- Wick Theorem applies, but few amplitudes since all \( k^+ > 0 \).

- \( J_z \) Conservation at every vertex \( | \sum_{\text{initial}} S_z - \sum_{\text{final}} S_z | \leq n \) at order \( g^n \)

- Unitarity is explicit

- Loop Integrals are 3-dimensional \( \int_0^1 dx \int d^2 k_\perp \)

- hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions \( \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \)

K. Chiu, Lorcé, sjb
**Light-Front QCD**

\[ \mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD} \]

\[ (H^0_{LF} + H^I_{LF})|\Psi| \geq M^2|\Psi| > \]

\[ \left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\text{eff}} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp) \]

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta) \]

**AdS/QCD:**

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \]

**Fixed** \( \tau = t + z/c \)

\[ \zeta^2 = x(1-x)b^2_\perp \]

**Coupled Fock states**

Eliminate higher Fock states and retarded interactions

**Effective two-particle equation**

Azimuthal Basis \( \zeta, \phi \)

**Single variable Equation**

\( m_q = 0 \)

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD
AdS/QCD

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space
  \[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \]
  \[ x^\mu \rightarrow \lambda x^\mu, \quad z \rightarrow \lambda z, \]
  maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.
  \[ x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z. \]
  
  $x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

\textit{AdS/CFT}
Dilaton-Modified AdS

\[ ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2) \]

- Soft-wall dilaton profile breaks conformal invariance
  \[ e^{\varphi(z)} = e^{+\kappa^2 z^2} \]
- Color Confinement in \( z \)
- Introduces confinement scale \( \kappa \)
- Uses AdS\(_5\) as template for conformal theory
AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)
\]

\[U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)\]

Derived from variation of Action for Dilaton-Modified AdS

Identical to Single-Variable Light-Front Bound State Equation in \(\zeta\)!

\[\zeta = \sqrt{x(1 - x) b_{\perp}^2}\]
\[ \text{Light-Front Holographic Dictionary} \]

\[ \psi(x, \vec{b}_\perp) \quad \leftrightarrow \quad \phi(z) \]

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \leftrightarrow \quad z \]

Fixed \( \tau = t + z/c \)

\[ \psi(x, \zeta) = \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta) \]

\[ (\mu R)^2 = L^2 - (J - 2)^2 \]

**Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion.
\[ e^{\varphi(z)} = e^{+\kappa^2 z^2} \]

\[ \zeta^2 = x(1-x)b^2_\perp. \]

\[ \left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta) \]

**Light-Front Schrödinger Equation**

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \]

**Confinement scale:** \( \kappa \simeq 0.5 \text{ GeV} \)

- de Alfaro, Fubini, Furlan:
- Fubini, Rabinovici:

**Unique Confinement Potential!**

**Conformal Symmetry of the action**

- Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

**AdS/QCD Soft-Wall Model**

- \( e^{\varphi(z)} = e^{+\kappa^2 z^2} \)

**Light-Front Holography**

**Confinement scale:** \( \kappa \simeq 0.5 \text{ GeV} \)

- de Tèramond, Dosch, sjb

**GeV units external to QCD: Only Ratios of Masses Determined**
G. de Teramond, H. G. Dosch, sjb

**Meson Spectrum in Soft Wall Model**

\[ m_\pi = 0 \text{ if } m_q = 0 \]

- Effective potential:
  \[ U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) \]

- LF WE
  \[ \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta) \]

- Normalized eigenfunctions
  \[ \langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1 \]
  \[ \phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n(\kappa^2 \zeta^2) \]

- Eigenvalues
  \[ M^2_{n,J,L} = 4\kappa^2 \left( n + \frac{J + L}{2} \right) \]

\[ \tilde{\zeta}^2 = \overline{b}_x^2 x(1 - x) \]

**Massless pion!**

Pion: Negative term for \( J=0 \) cancels positive terms from LFKE and potential

G. de Teramond, H. G. Dosch, sjb
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

**Same slope in n and L!**

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

- Quark separation increases with $L$
- Pion has zero mass!
- $m_q = 0$
A remarkable empirical feature of the hadronic spectrum is the near equality of the slopes of meson and baryon Regge trajectories. The square of the masses of hadrons composed of light quarks is linearly proportional not only to $L$, the orbital angular momentum, but also to the principal quantum number $n$, the number of radial nodes in the hadronic wavefunction as seen in Fig. 1. The Regge slopes in $n$ and $L$ are equal, as in the meson formula

$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

but even more surprising, they are observed to be equal for both the meson and baryon trajectories, as shown in Fig. 2. The mean value for all of the slopes is $\pi = \rho = 0.523$ GeV. See Fig. 3.
Structure of the Vacuum in Light-Front Dynamics

- Results easily extended to light quarks masses (Ex: $K$-mesons)
  
  

- First order perturbation in the quark masses
  
  
  
  
  
  
  $M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_0^2}{1-x} \right| X \right\rangle$

- Holographic LFWF with quark masses
  
  
  
  
  
  
  $\left[ S. J. Brodsky and GdT, arXiv:0802.0514 [hep-ph]\right]$

- Ex: Description of diffractive vector meson production at HERA
  
  
  
  
  
  
  $\left[ J. R. Forshaw and R. Sandapen, PRL 109, 081601 (2012)\right]$

- For the $K^\ast M^2_{n,L,S} = M^2_{K^\ast} + 4\sqrt{n} + J + L^2$

- Effective quark masses from reduction of higher Fock states as functionals of the valence state:
  
  
  
  
  
  
  $m_u = m_d = 46$ MeV, $m_s = 357$ MeV

- Effective mass from $m(p^2)$

- from LF Higgs mechanism

\[ \text{Effective mass from } m(p^2) \]
Prediction from AdS/QCD: Meson LFWF

Note coupling

\[ k^2_\perp, x \]

\[ \psi_M(x, k^2_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k^2_\perp}{2\kappa^2 x(1-x)}} \]

\[ f_\pi = \sqrt{\frac{P_{q\bar{q}}}{8}} \frac{\sqrt{3}}{\kappa} = 92.4 \text{ MeV} \]

Same as DSE! C. D. Roberts et al.

Provides Connection of Confinement to Hadron Structure
Holographic Mapping of AdS Modes to QCD LFWFs

• Integrate Soper formula over angles:

\[
F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),
\]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

• Transversality variable

\[
\zeta = \sqrt{x(1-x)b^2_\perp}
\]

• Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[
\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),
\]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!

Identical to Polchinski-Strassler Convolution of AdS Amplitudes
$Q^2 F_π(Q^2)$

$Q^2 \text{ GeV}^2$
Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

\[ F_{\pi}(s) = (1 - \gamma) \left( 1 - \frac{s}{M_{\rho}^2} \right) + \gamma \left( 1 - \frac{s}{M_{\rho'}^2} \right) \left( 1 - \frac{s}{M_{\rho''}^2} \right) \]

\[ M_{\rho_n} = 4\kappa^2 (1/2 + n) \]

\[ \gamma = 0.17 \]

Prescription for Timelike poles:

\[ \frac{1}{s - M^2 + i\sqrt{s}\Gamma} \]

14\% four-quark probability

G. de Teramond & sjb
J. R. Forshaw, R. Sandapen

\( \gamma^* p \rightarrow \rho^0 p' \)

\[
\psi_M(x, k) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k^2}{2\kappa^2 x(1-x)}}
\]
• **Light Front Wavefunctions:** \( \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \)

off-shell in \( P^- \) and invariant mass \( M_{q\bar{q}}^2 \)

\[
\text{Fixed } \tau = t + z/c
\]

\[
\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)
\]

"**Hadronization at the Amplitude Level**"

**Boost-invariant LFWF connects confined quarks and gluons to hadrons**
Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

PHYSICAL REVIEW LETTERS 120, 182001 (2018)
Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks
QCD Lagrangian

\[ L_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} \bar{\Psi}_f \gamma^5 \gamma^\mu \gamma_5 \gamma^\mu \Psi_f \]

\[ i D^\mu = i \partial^\mu - g A^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g [A^\mu, A^\nu] \]

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean! Only Ratios of Masses Determined

- de Alfaro, Fubini, Furlan: Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Unique confinement potential!
\[ G\psi(\tau) = i \frac{\partial}{\partial \tau} \psi(\tau) \]

\[ G = uH + vD + wK \]

\[ G = H_{\tau} = \frac{1}{2} \left( -\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4} x^2 \right) \]

Retains conformal invariance of action despite mass scale!

\[ 4uw - v^2 = \kappa^4 = [M]^4 \]

Identical to LF Hamiltonian with unique potential and dilaton!

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta) \]

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \]
dAFF: New Time Variable

\begin{equation}
\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left( \frac{2tw + v}{\sqrt{4uw - v^2}} \right),
\end{equation}

- Identify with difference of LF time \( \Delta x^+/P^+ \) between constituents
- Finite range
- Measure in Double-Parton Processes

Retains conformal invariance of action despite mass scale!
Remarkable Features of 
Light-Front Schrödinger Equation

- Relativistic, frame-independent
- QCD scale appears - unique LF potential
- Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter
- Zero-mass pion for zero mass quarks!
- Regge slope same for n and L -- not usual HO
- Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry
- Phenomenology: LFWFs, Form factors, electroproduction
- Extension to heavy quarks

\[ U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \]
Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale (De Alfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- PoincarèInvariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-plet: Meson-Baryon-Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests
- Systematically improvable: Basis LF Quantization (BLFQ)
\{\psi, \psi^+\} = 1 \quad B = \frac{1}{2} [\psi^+, \psi] = \frac{1}{2} \sigma_3

\psi = \frac{1}{2} (\sigma_1 - i \sigma_2), \quad \psi^+ = \frac{1}{2} (\sigma_1 + i \sigma_2)

Q = \psi^+ [-\partial_x + \frac{f}{x}], \quad Q^+ = \psi [\partial_x + \frac{f}{x}], \quad S = \psi^+ x, \quad S^+ = \psi x

\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K

\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD

generates conformal algebra

\[ [H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK \]

\[ Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K} \]
Consider \( R_w = Q + wS \);
\( w \): dimensions of mass squared

\[
G = \{ R_w, R_w^+ \} = 2H + 2w^2K + 2wfI - 2wB \quad 2B = \sigma_3
\]

Retains Conformal Invariance of Action

**Baryon Equation** \( Q \approx \sqrt{H}, \quad S \approx \sqrt{K} \)

Fubini and Rabinovici

**New Extended Hamiltonian** \( G \) is diagonal:

\[
G_{11} = \left( -\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)
\]

\[
G_{22} = \left( -\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)
\]

Identify \( f - \frac{1}{2} = L_B \), \( w = \kappa^2 \)

\( \lambda = \kappa^2 \)

Eigenvalue of \( G \):
\[
M^2(n, L) = 4\kappa^2(n + L_B + 1)
\]
Baryon Equation

\[
\left( - \partial^2_\zeta + \kappa^4 \zeta^2 + 2\kappa^2 (L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi^+_J = M^2 \psi^+_J
\]

\[
\left( - \partial^2_\zeta + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi^-_J = M^2 \psi^-_J
\]

\[
M^2(n, L_B) = 4\kappa^2 (n + L_B + 1)
\]

Meson Equation

\[
\left( - \partial^2_\zeta + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J
\]

\[
M^2(n, L_M) = 4\kappa^2 (n + L_M)
\]

**S=0, P=+**

\[
\lambda = \kappa^2
\]

\[
S=0, I=1 \text{ Meson is superpartner of } S=1/2, I=1 \text{ Baryon}
\]

**Same \( \kappa \)!**
• Nucleon LF modes

\[ \psi_+ (\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2/2} L^{L+1} (\kappa^2 \zeta^2) \]

\[ \psi_- (\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2/2} L^{L+2} (\kappa^2 \zeta^2) \]

• Normalization

\[ \int d\zeta \psi_+^2 (\zeta) = \int d\zeta \psi_-^2 (\zeta) = 1 \]

• Eigenvalues

\[ \int_0^\infty d\zeta \int_0^1 dx \psi_+^2 (\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2 (\zeta^2, x) = \frac{1}{2} \]

Quark Chiral Symmetry of Eigenstate!

Nucleon: Equal Probability for L=0,1

\[ J^z = + \frac{1}{2} : \quad \frac{1}{\sqrt{2}} [ | S^z_q = + \frac{1}{2}, L^z = 0 > + | S^z_q = - \frac{1}{2}, L^z = + 1 > ] \]

Nucleon spin carried by quark orbital angular momentum
Figure 2: Orbital and radial baryon excitation spectrum. Positive-parity spin-$\frac{1}{2}$ nucleons (a) and spectrum gap between the negative-parity spin-$\frac{3}{2}$ and the positive-parity spin-$\frac{1}{2}$ nucleons families (b). Minus parity $N$ (c) and plus and minus parity $\Delta$ families (d), for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas). The predictions for the daughter trajectories for $n = 1$, $n = 2$, $\cdots$ are also shown in this figure. Only confirmed PDG states are shown. The lowest state $N(1440)$ and the $N(1710)$ are well accounted for as the first and second radial excited states of the proton. The newly identified state, the $N(1900)$ is depicted here as the first radial excitation of the $N(1720)$. The model is successful in explaining the parity degeneracy observed in the light baryon spectrum, such as the $L = 2$, $N(1680) - N(1720)$ pair in Fig. 2A. In Fig. 2B we compare the positive parity spin-$\frac{1}{2}$ parent nucleon trajectory with the negative parity spin-$\frac{3}{2}$ daughter nucleon trajectory.
Superconformal Quantum Mechanics

Light-Front Holography

\[ M^2(n, L_B) = 4\kappa^2(n + L_B + 1)N^{7-}_2 \]

Same slope

\[ M^2(n, L_M) = 4\kappa^2(n + L_M) \]

Universal slopes in \( n, L \)

Meson-Baryon Mass Degeneracy for \( L_M=L_B+1 \)

\[ \frac{M^2_{\text{meson}}}{M^2_{\text{nucleon}}} = \frac{n + L_M}{n + L_B + 1} \]
$M^2 \ (\text{GeV}^2)$

$\rho - \Delta$ superpartner trajectories

$M^2 (\text{GeV}^2)$

$\rho - \Delta$ superpartner trajectories

$\rho, \omega \quad a_2, f_2 \quad \rho_3, \omega_3 \quad a_4, f_4$

$\rho, \omega \quad a_2, f_2 \quad \rho_3, \omega_3 \quad a_4, f_4$

$L_M = L_B + 1$

Dosch, de Teramond, sjb

Dosch, de Teramond, sjb
\( \lambda = \kappa^2 \)

\[ m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV} \]

\[ \kappa = \sqrt{\lambda} = 0.523 \pm 0.024 \]

From \( \alpha_{g1}(Q^2) \)

Deur

**Universal Mass Scale**

Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons:

Supersymmetric feature of hadron physics
Figure 1:
The supersymmetric quadruplet \( \{ M, B^+, B^-, T \} \). Open circles represent quarks, full circles antiquarks. The tetraquark has the same mass as its baryon partner in the multiplet. Notice that the LF angular momentum of the negative-chirality component wave function of a baryon \( B \) is one unit higher than that of the positive-chirality (leading-twist) component \( B^+ \).

According to this analysis, the lowest-lying light-quark tetraquark is a partner of the \( b_1(1235) \) and the nucleon; it has quantum numbers \( I, J^P = 0^+, 0^+ \). The partners of the \( a_2(1320) \) and the \( f_0(1270) \) have the quantum numbers \( I = 0, J^P = 1^+ \). Candidates for these states are the \( f_0(980) \) and \( a_1(1260) \), respectively.

2.4 Inclusion of quark masses and comparison with experiment

We have argued in \[11\] that the natural way to include light quark masses in the hadron mass spectrum is to leave the LF potential unchanged as a first approximation and add the additional term of the invariant mass \( m^2 = \sum_{i=1}^n m_i^2 x_i^2 \) to the LF kinetic energy. The resulting LF wave function is then modified by the factor \( e^{-m^2/2m} \), thus providing a relativistically invariant form for the hadronic wave functions. The effect of the nonzero quark masses for the squared hadron masses is then given by the expectation value of \( m^2 \) evaluated using the modified wave functions. This prescription leads to

It is interesting to note that in Ref. \[20\] mesons, baryons and tetraquarks are also hadronic states within the same multiplet.
\[ R_\lambda^\dagger \bar{q} \rightarrow (qq) \quad S = 1 \]
\[ \bar{3}_C \rightarrow \bar{3}_C \]

\[ f_2(1270) \]
\[ L = 1, S = 1 \]
\[ J^{PC} = 2^{++} \]

\[ \Delta^+(1232) \]
\[ L = 0 \]
\[ J^P = \frac{3^+}{2} \]

\[ a_1(1260) \]
\[ S = 0 \]
\[ L = 0 \]

**Superconformal Algebra 4-Plet**

**Vector (0+) Scalar ([ ]) Diquarks**

**Meson**

**Baryon**

\[ R_\lambda^\dagger \quad q \rightarrow [\bar{q}\bar{q}] \]
\[ 3_C \rightarrow 3_C \]
<table>
<thead>
<tr>
<th>Meson</th>
<th>Baryon</th>
<th>Tetraquark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q$-cont</td>
<td>$J^P$</td>
</tr>
<tr>
<td>$\bar{q}q$</td>
<td>0$^{--}$</td>
<td>$\pi$(140)</td>
</tr>
<tr>
<td>$\bar{q}q$</td>
<td>1$^{-+}$</td>
<td>$b_1$(1235)</td>
</tr>
<tr>
<td>$\bar{q}q$</td>
<td>2$^{-+}$</td>
<td>$\pi_2$(1670)</td>
</tr>
<tr>
<td>$\bar{q}q$</td>
<td>3$^{--}$</td>
<td>$\rho_3$(1690), $\omega_3$(1670)</td>
</tr>
<tr>
<td>$\bar{q}q$</td>
<td>4$^{-+}$</td>
<td>$a_4(2040)$, $f_4(2050)$</td>
</tr>
<tr>
<td>$\bar{q}s$</td>
<td>0$^{-(+)}$</td>
<td>$K$(495)</td>
</tr>
<tr>
<td>$\bar{q}s$</td>
<td>1$^{-(+)}$</td>
<td>$\bar{K}_1$(1270)</td>
</tr>
<tr>
<td>$\bar{s}q$</td>
<td>0$^{-(+)}$</td>
<td>$K$(495)</td>
</tr>
<tr>
<td>$\bar{s}q$</td>
<td>1$^{-(+)}$</td>
<td>$K_1$(1270)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>1$^{-+}$</td>
<td>$K^*$(890)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>2$^{-(+)}$</td>
<td>$K^*_2$(1430)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>3$^{-+}$</td>
<td>$K^*_3$(1780)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>4$^{-(+)}$</td>
<td>$K^*_4$(2045)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>0$^{-+}$</td>
<td>$\eta$(550)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>1$^{-+}$</td>
<td>$\eta_1$(1170)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>2$^{-+}$</td>
<td>$\eta_2$(1645)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>1$^{-(+)}$</td>
<td>$\Phi$(1020)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>2$^{-(+)}$</td>
<td>$\Phi_2$(1525)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>3$^{-(+)}$</td>
<td>$\Phi_3$(1850)</td>
</tr>
<tr>
<td>$\bar{s}s$</td>
<td>4$^{-(+)}$</td>
<td>$\eta_2(1645)$</td>
</tr>
</tbody>
</table>

New Organization of the Hadron Spectrum

M. Nielsen, sjb
\[
\frac{M_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)
\]

- **Universal quark light-front kinetic energy**
  \[
  \Delta M_{LFKE}^2 = \kappa^2 (1 + 2n + L)
  \]
- **Universal quark light-front potential energy**
  \[
  \Delta M_{LFPE}^2 = \kappa^2 (1 + 2n + L)
  \]
- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**
  \[
  \Delta M_{spin}^2 = 2\kappa^2 (L + 2S + B - 1)
  \]

**Equal: Virial Theorem**

**Universal Hadronic Decomposition**
Using $SU(6)$ flavor symmetry and normalization to static quantities
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

Harmonic Oscillator Confinement
Normalized to anomalous moment

$$F_2^p(Q^2)$$

$$\kappa = 0.49 \text{ GeV}$$

G. de Teramond, sjb
Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

**Universality of Generalized Parton Distributions in Light-Front Holographic QCD**

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

PHYSICAL REVIEW LETTERS 120, 182001 (2018)
Supersymmetry across the light and heavy-light spectrum
Supersymmetry across the light and heavy-light spectrum

Heavy charm quark mass does not break supersymmetry
Superpartners for states with one c quark

<table>
<thead>
<tr>
<th>q-cont</th>
<th>$J^P(C)$</th>
<th>Name</th>
<th>Meson</th>
<th>Baryon</th>
<th>Tetraquark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>q-cont</td>
<td>$J^P$</td>
<td>Name</td>
</tr>
<tr>
<td>$\bar{q}c$</td>
<td>0$^-$</td>
<td>$D(1870)$</td>
<td>$\bar{q}c$</td>
<td>1$^+$</td>
<td>$D_1(2420)$</td>
</tr>
<tr>
<td>$\bar{q}c$</td>
<td>2$^-$</td>
<td>$D_J(2600)$</td>
<td>$\bar{c}q$</td>
<td>0$^-$</td>
<td>$\bar{D}(1870)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{c}q$</td>
<td>1$^+$</td>
<td>$\bar{D}_1(2420)$</td>
</tr>
<tr>
<td>$\bar{q}c$</td>
<td>1$^-$</td>
<td>$D^*(2010)$</td>
<td>$\bar{c}q$</td>
<td>2$^+$</td>
<td>$D_2^*(2460)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\bar{c}q$</td>
<td>3$^-$</td>
</tr>
<tr>
<td>$\bar{s}c$</td>
<td>0$^-$</td>
<td>$D_s(1968)$</td>
<td>$\bar{s}c$</td>
<td>1$^+$</td>
<td>$D_{s1}(2460)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{s}c$</td>
<td>2$^-$</td>
<td>$D_{s2}(\sim 2860)$?</td>
</tr>
<tr>
<td>$\bar{s}c$</td>
<td>1$^-$</td>
<td>$D_s^*(2110)$</td>
<td>$\bar{s}c$</td>
<td>2$^+$</td>
<td>$D_{s2}^*(2573)$</td>
</tr>
<tr>
<td>$\bar{c}s$</td>
<td>1$^+$</td>
<td>$D_{s1}(\sim 2700)$?</td>
<td>$\bar{c}s$</td>
<td>2$^+$</td>
<td>$D_{s2}^*(\sim 2750)$?</td>
</tr>
<tr>
<td>$\bar{c}s$</td>
<td>2$^+$</td>
<td>$D_{s2}^*(\sim 2750)$?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

M. Nielsen, sjb

predictions

beautiful agreement!
Supersymmetry across the light and heavy-light spectrum

Heavy bottom quark mass does not break supersymmetry
Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector
  [H. G. Dosch, GdT, S. J. Brodsky, PRD 92, 074010 (2015), PRD 95, 034016 (2017)]

- Extension to the double-heavy hadronic sector
  [M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

- Extension to the isoscalar hadronic sector
Supersymmetry in QCD

- A hidden symmetry of Color SU(3)_c in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

de Téramond, Dosch, Lorcé, sjb
Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

Meson

\[ \phi_M, \ L_B + 1 \]

Baryon

\[ \psi_{B+}, \ L_B \]

Baryon

\[ \psi_{B-}, \ L_B + 1 \]

Tetraquark

\[ \phi_T, \ L_B \]

Proton: \( lu[ud] > \) Quark + Scalar Diquark

Equal Weight: \( L=0, L=1 \)
\[ \sigma(e^+e^- \rightarrow MT) \propto \frac{1}{s^{N-1}} \quad N = 6 \]

Use counting rules to identify composite structure
Underlying Principles

- Polncarè Invariance: Independent of the observer’s Lorentz frame: Quantization at Fixed Light-Front Time $\tau$

- Causality: Information within causal horizon: Light-Front

- Light-Front Holography: $AdS_5 = LF (3+1)$

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b^2_\perp x(1-x)$$

- Introduce mass scale $\kappa$ while retaining the Conformal Invariance of the Action (dAFF)

- Unique Dilaton in $AdS_5$: $e^{+\kappa^2 z^2}$

- Unique color-confining LF Potential $U(\zeta^2) = \kappa^4 \zeta^2$

- Superconformal Algebra: Mass Degenerate 4-Plet:

  Meson $q\bar{q} \leftrightarrow$ Baryon $q[qq] \leftrightarrow$ Tetraquark $[qq][\bar{q}\bar{q}]$

Stan Brodsky

Supersymmetric Features of Hadron Physics from Superconformal Algebra and Light-Front Holography

19 Sept 2019
Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

\[ e^\phi(z) = e^{\kappa^2 z^2} \]

\[ S = -\frac{1}{4} \int d^4 x \, dz \, \sqrt{g} \, e^{\varphi(z)} \frac{1}{g_5^2} G^2 \]

Flow equation

\[ \frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0) \]

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

Coupling measured at momentum scale $Q$

\[ \alpha_s^{\text{AdS}}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{\text{AdS}}(\zeta) \]

Solution

\[ \alpha_s^{\text{AdS}}(Q^2) = \alpha_s^{\text{AdS}}(0) \, e^{-Q^2/4\kappa^2}. \]

where the coupling $\alpha_s^{\text{AdS}}$ incorporates the non-conformal dynamics of confinement
Bjorken sum rule defines effective charge

\[\int_0^1 dx [g_{1ep}^e(x, Q^2) - g_{1en}^e(x, Q^2)] \equiv \frac{g_{a}}{6} [1 - \frac{\alpha g_1(Q^2)}{\pi}]\]

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large \(Q^2\)**
- **Computable at large \(Q^2\) in any \(pQCD\) scheme**
- **Universal \(\beta_0, \beta_1\)**
AdS/QCD dilaton captures the higher twist corrections to effective charges for \( Q < 1 \) GeV

\[
e^\varphi = e^{+\kappa^2 z^2}
\]

Deur, de Teramond, sjb
\[ m_\rho = \sqrt{2\kappa} \]
\[ m_p = 2\kappa \]

Nonperturbative QCD (Quark Confinement)

\[ \alpha_{g_1}(Q^2) = \frac{Q^2}{\pi} e^{-\frac{Q^2}{4\kappa^2}} \]

Transition scale \( Q_0 \)

\[ Q_0 = 0.87 \pm 0.08 \text{ GeV} \]

Perturbative QCD (Asymptotic Freedom)

\[ \lambda \equiv \kappa^2 \]

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:
\[ \kappa = 0.513 \pm 0.007 \text{ GeV} \]

5-Loop \( \beta \) Prediction:
\[ \Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV} \]

Experiment:
\[ \Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV} \]

Reverse Dimensional Transmutation!

MS scheme
Underlying Principles

- **Poincaré Invariance:** Independent of the observer’s Lorentz frame: Quantization at Fixed Light-Front Time $\tau$
- **Causality:** Information within causal horizon: Light-Front
- **Light-Front Holography:** $\text{AdS}_5 = \text{LF (3+1)}$
  \[ z \leftrightarrow \zeta \quad \text{where} \quad \zeta^2 = b^2_{\perp} x (1 - x) \]
- **Introduce Mass Scale $\kappa$ while retaining the Conformal Invariance of the Action ($\text{dAFF}$)**
- **Unique Dilaton in $\text{AdS}_5$:** $e^{+\kappa^2 z^2}$
- **Unique color-confining LF Potential** $U(\zeta^2) = \kappa^4 \zeta^2$
- **Superconformal Algebra:** Mass Degenerate 4-Plet:

  \[
  \text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]
  \]
What is PMC?

**Principle of Maximum Conformality**

- Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$
- Choose $\mu_R^{\text{init}}$; arbitrary initial renormalization scale
- Identify $\{\beta_i^R\}$ - terms using $n_f$ - terms through the PMC – BLM correspondence principle
- Order-by-order
- Shift scale of $\alpha_s$ to $\mu_R^{\text{PMC}}$ to eliminate $\{\beta_i^R\}$ - terms
- Conformal Series
- Result is independent of $\mu_R^{\text{init}}$ and scheme at fixed order

**Xing-Gang Wu, Matin Mojaza, Leonardo di Giustino, SJF**

- PMC–BLM – one
- $R_\delta$–scheme – two

**nf dependence of pQCD series does not uniquely identify the $\beta$ terms**
Renormalization scale depends on the thrust
Not constant!

\[ T = \frac{\max \sum_{\vec{n}} \left| \vec{p}_i \cdot \vec{n} \right|}{\sum_{i} \left| \vec{p}_i \right|}, \]

\[ e^+e^- \rightarrow Z \rightarrow q\bar{q}g + \ldots \]

T. Gehrmann, N. H`afliger, P. F. Monni
S.-Q. Wang, L. Di Giustino, X.-G. Wu, sjb
The mean value of event shapes have also been extensively measured in the two-jet region. Since the calculation of the mean value involves an integration over the full phase space, it is crucial to ensure that the scale setting is appropriate. The conventional scale setting may be needed for the PMC predictions near the two-jet and multi-jet regions, outside of the region of $0 \leq r_{\mu} \leq 1$. The PMC scales of thrust differential distributions using the conformal coefficients are very different from those of the conventional scale setting, the perturbative series shows a slow convergence and the estimation of the magnitude of the conformal coefficients is very different from those of the thrust distributions. These cases are similar to those of the thrust differential distributions using the conformal coefficients. This is reasonable, since we have shown in Fig. (1) that the PMC scales of thrust differential distributions using the conformal coefficients are very different from those of the thrust distributions.

The electron-positron colliders have collected large numbers of experimental data for the thrust mean value and are in agreement with the theoretical predictions with high precision. However, the pQCD predictions near the two-jet and multi-jet regions are plagued by significant uncertainty, and substantial deviation from perturbative pQCD calculations may be needed for the PMC predictions.

**Differential distributions using the conformal coefficients**

$\sigma_{h} / d(1-T)$

**PMC scale**

**Conventional scale**

ALEPH
- DELPH
- OPAL
- L3
- SLD

**Principle of Maximum Conformality (PMC)**
In this paper, we will apply the Principle of Maximum Conformality (PMC) to make comprehensive analyses for two classic event shapes: the thrust, and for the C-parameter it is 0 and 1 for the Abelian limit to the Gell-Mann-Low method. We use the RunDec program to precisely calculate the perturbative coefficients at the next-to-leading order (NLO). The perturbative series prediction, and the NNLO calculation do not overlap higher-order terms by varying mental data. The conventional predictions are plagued by conventional scale-setting method even up to NNLO QCD case of conventional scale setting, Fig. shows that the C-parameter distribution is in excellent agreement with the ALEPH experiment. After using the PMC, the relative magnitude at NLO is 0.0 ± 0.0, where the estimate of unknown higher-order terms is natural for a convergent perturbative series. This estimate of the unknown higher-order terms is not depend on the choice of the renormalization scheme, and its error band is the squared averages of the errors for each respective and corresponding error bands are obtained by the ALEPH experiment.

\[ \sqrt{s} = M_Z \]
Determine QCD running coupling from measurement of the thrust \( T \) and \( C \)-distribution at one energy!

\[ e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}g + \cdots \quad \alpha_s(Q^2) \text{ in } \overline{\text{MS}} \text{ scheme} \]

**A new way to measure** \( \alpha_s(Q^2) \)

\( \Delta \) Thrust

■ C–parameter

S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB
Features of BLM/PMC

- Predictions are scheme-independent at every order
- Matches conformal series
- No $n!$ Renormalon growth of pQCD series
- New scale appears at each order; $n_F$ determined at each order - matches virtuality of quark loops
- Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)
- Rigorous: Satisfies all Renormalization Group Principles
- Realistic Estimate of Higher-Order Terms
- Reduces to standard QED scale $N_C \rightarrow 0$
- GUT: Must use the same scale setting procedure for QED, QCD
- Eliminates unnecessary theory error
- Maximal sensitivity to new physics
- Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)
- PMC Reduces to BLM at NLO: Example: BFKL intercept (Fadin, Kim, Lipatov, Pivovarov, sjb)
\[ q^+ = 0 \quad q^2_{\perp} = Q^2 = -q^2 \]

Illustrates the LF time sequence

One-Step / Two-Step Interference

Study Double Virtual Compton Scattering $\gamma^* A \rightarrow \gamma^* A$

Cannot reduce to matrix element of local operator! No Sum Rules!

Liuti, Schmidt sjb
• Unlike shadowing, anti-shadowing from Reggeon exchange is flavor specific;
• Each quark and anti-quark will have distinctly different constructive interference patterns.
• The flavor dependence of antishadowing explains why anti-shadowing is different for electron (neutral electromagnetic current) vs. neutrino (charged weak current) DIS reactions.
• Test of the explanation of antishadowing: Bjorken-scaling leading-twist charge exchange DDIS reaction $\gamma^* p \rightarrow nX^+$ with a rapidity gap due to $I=1$ Reggeon exchange.

The finite path length due to the on-shell propagation of $V^0$ between $N_1$ and $N_2$ contributes a finite distance $(\Delta z)^2$ between the two virtual photons in the DVCS amplitude.

The usual “handbag” diagram where the two $J^\mu(x)$ and $J^\nu(0)$ currents acting on an uninterrupted quark propagator are replaced by a local operator $T^{\mu\nu}(0)$ as $Q^2 \rightarrow \infty$, is inapplicable in deeply virtual Compton scattering from a nucleus since the currents act on different nucleons.

$$\Delta z^2 \text{ does not vanish as } \frac{1}{Q^2}.$$
Invariance Principles of Quantum Field Theory

- **Poincaré Invariance**: Physical predictions must be independent of the observer’s Lorentz frame: Front Form

- **Causality**: Information within causal horizon: Front Form

- **Gauge Invariance**: Physical predictions of gauge theories must be independent of the choice of gauge

- **Scheme-Independence**: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)

- **Mass-Scale Invariance**: Conformal Invariance of the Action (DAFF)
Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography and Superconformal Algebra

Fixed \( \tau = t + z/c \)

with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Tianbo Llu, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur

Ecole Polytechnique, Palaiseau, France

September 19, 2019