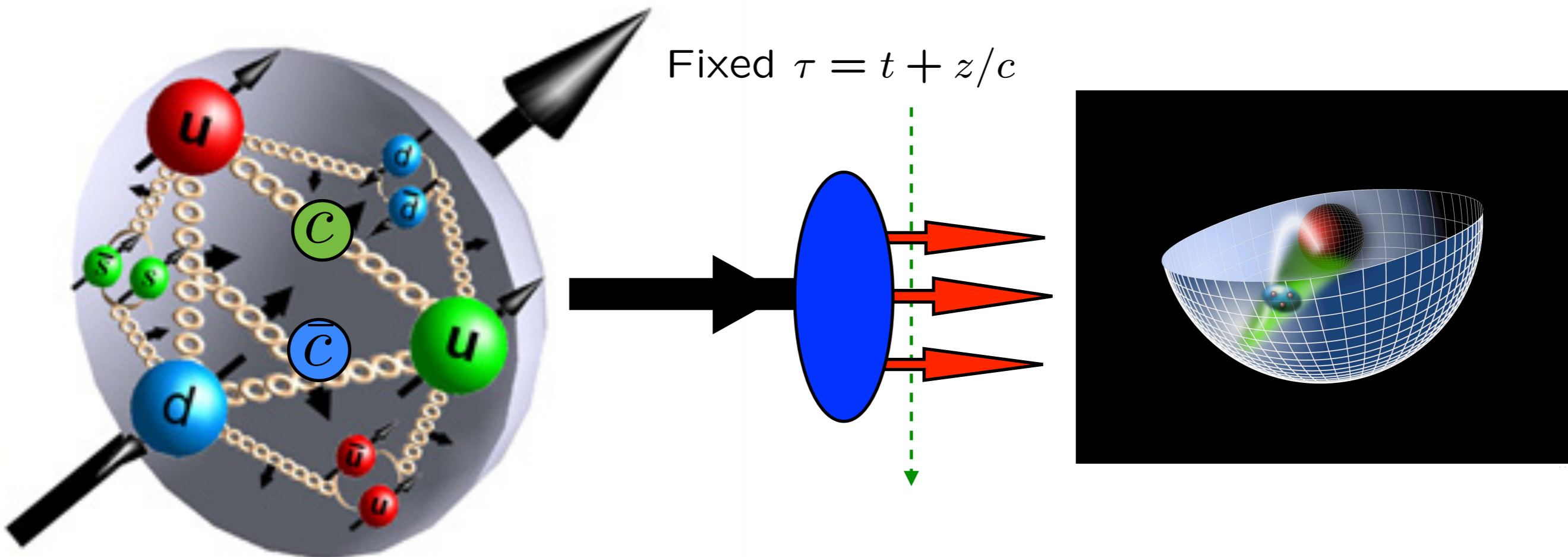


Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography and Superconformal Algebra



with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Tianbo Llu, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur



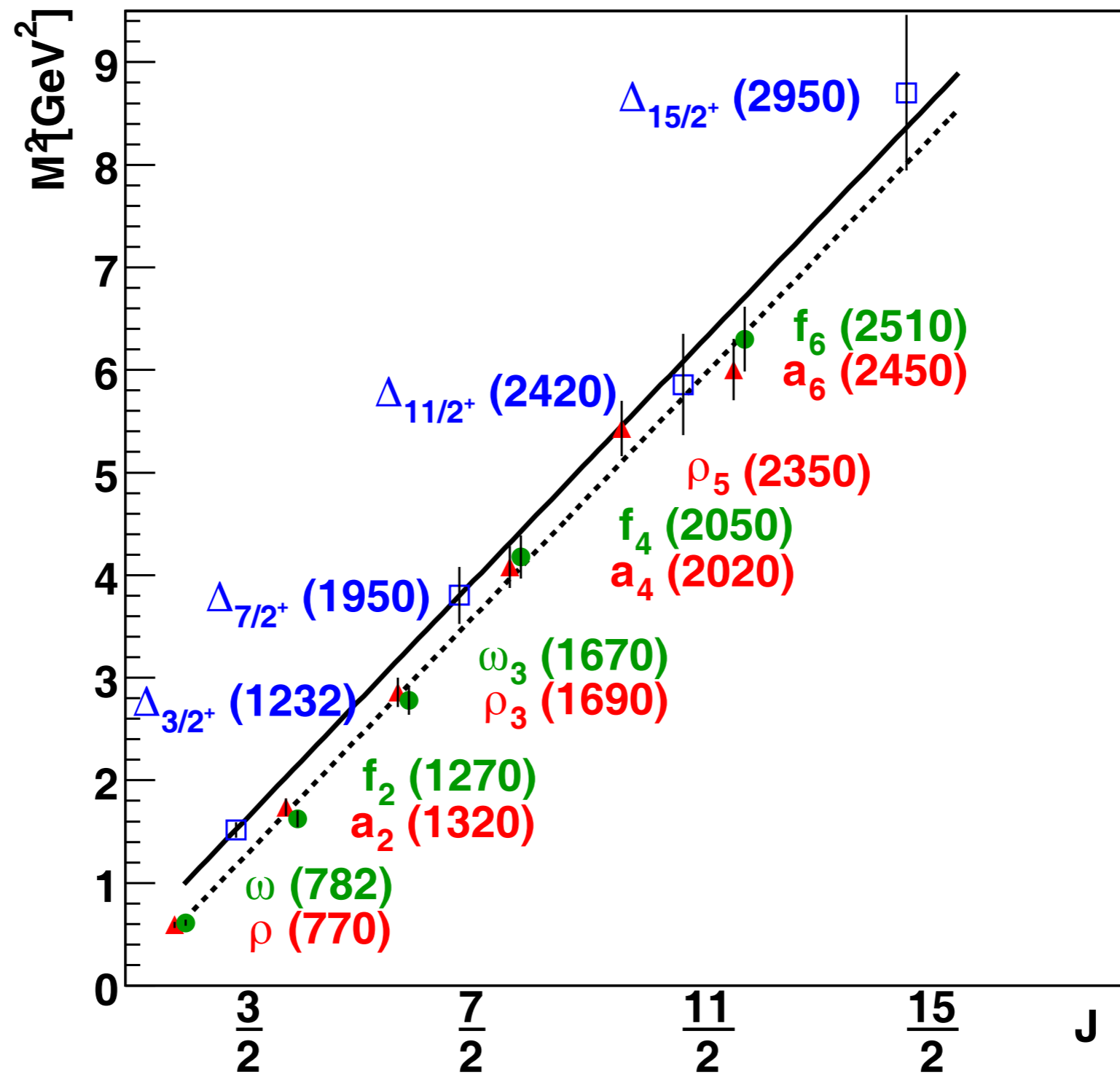
Ecole Polytechnique, Palaiseau, France



September 19, 2019

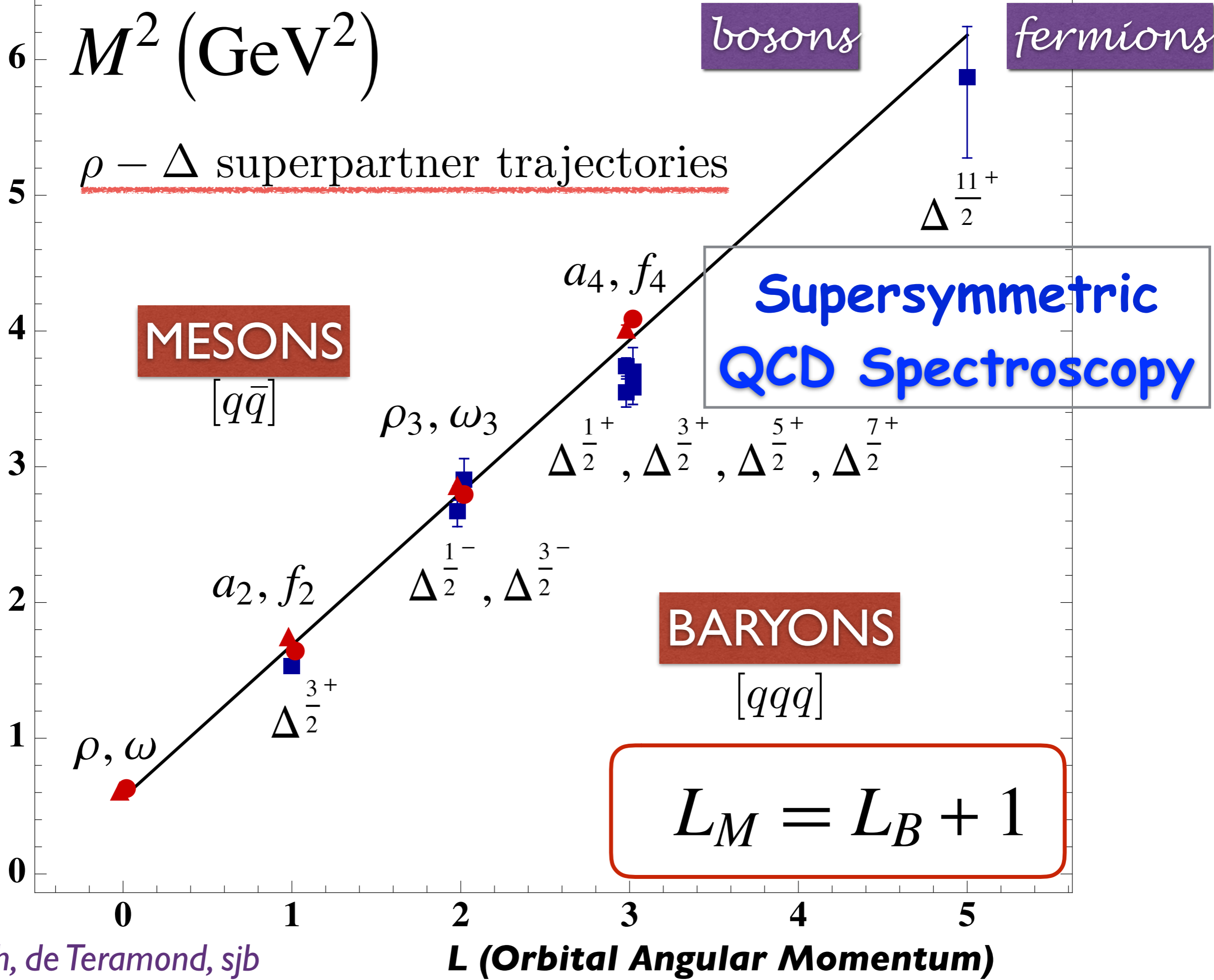
Mesons and Baryons: Same Regge Slope $M^2 \propto J$!

$M^2[\text{GeV}^2]$



The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with $J = L+S$.

E. Klempt and B. Ch. Metsch



Profound Questions for Hadron Physics

- ***Color Confinement***
- ***Origin of the QCD Mass Scale***
- ***Meson and Baryon Spectroscopy***
- ***Exotic States: Tetraquarks, Pentaquarks, Gluonium,***
- ***Universal Regge Slopes: n , L , Mesons and Baryons***
- ***Massless Pion: Quark Anti-Quark Bound State***
- ***Hadron Structure and Dynamics: QCD Coupling at all Scales***

Need a First Approximation to QCD

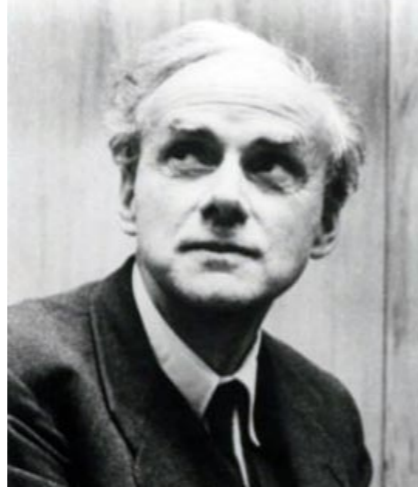
*Comparable in simplicity to
Schrödinger Theory in Atomic Physics*

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

*AdS/QCD
Light-Front Holography
Superconformal Algebra*

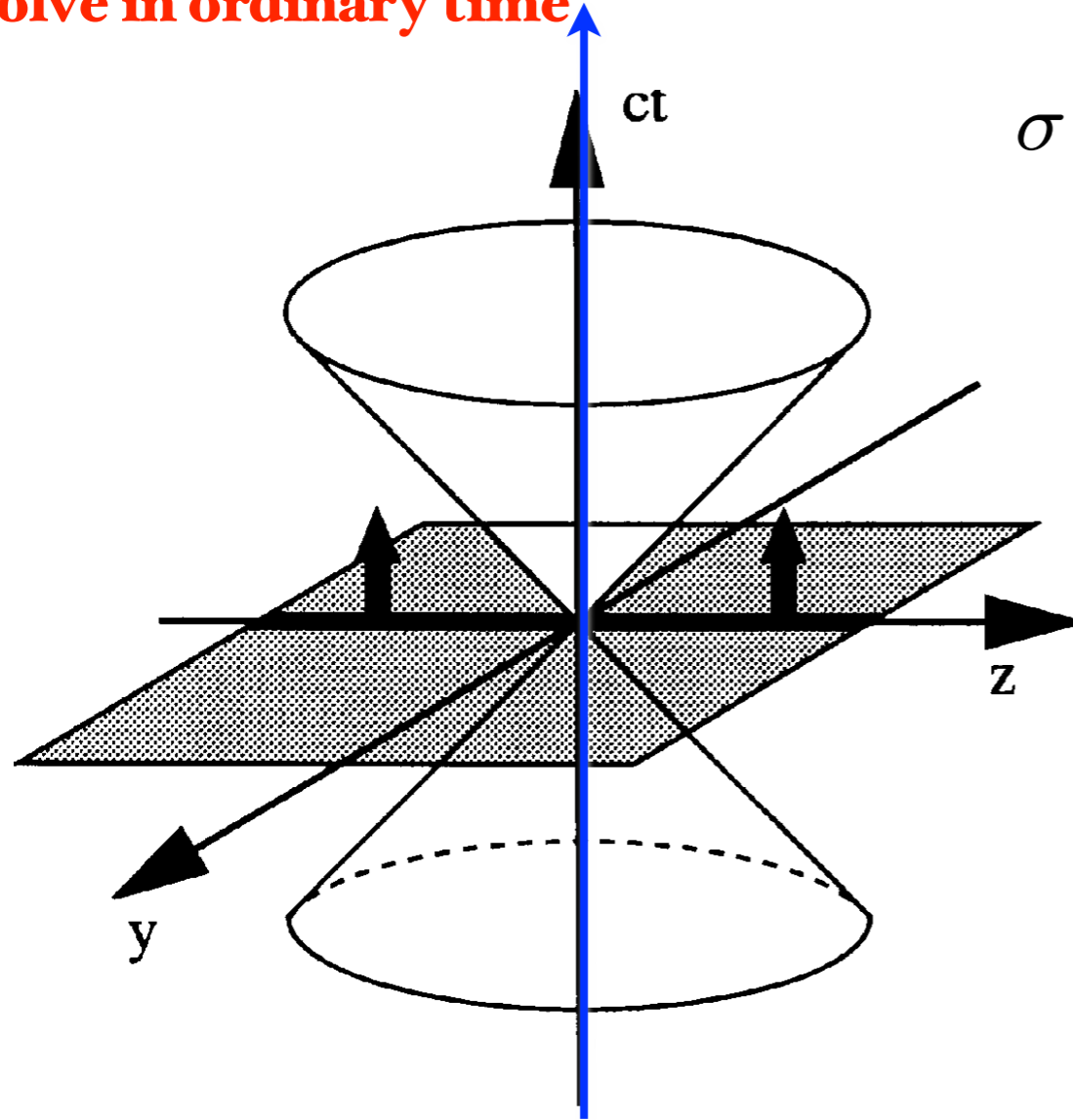
No parameters except for quark masses!



P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

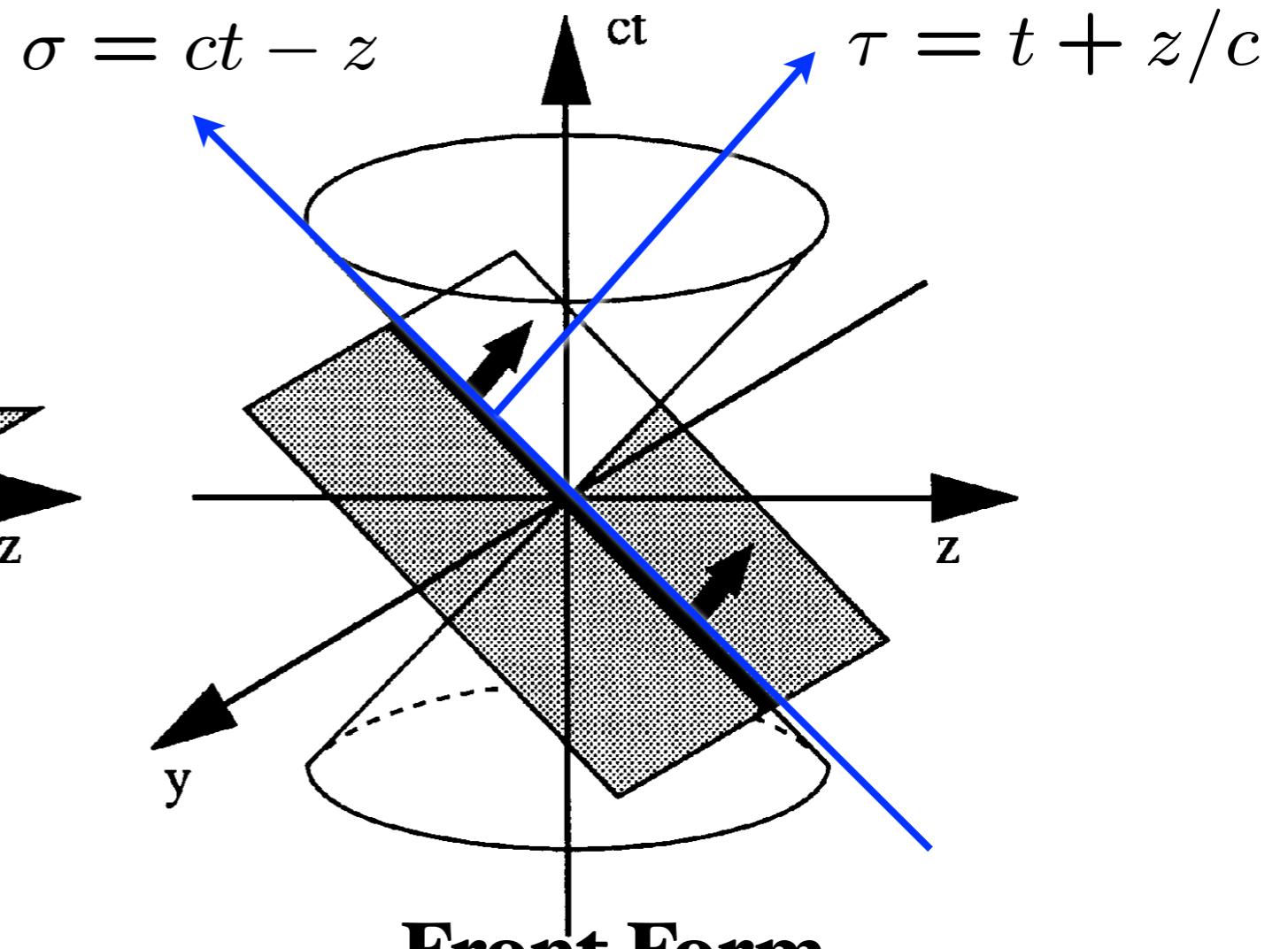
*Dirac's Amazing Idea:
The "Front Form"*

Evolve in ordinary time



Instant Form

Evolve in light-front time!



Front Form

Casual, Boost Invariant!

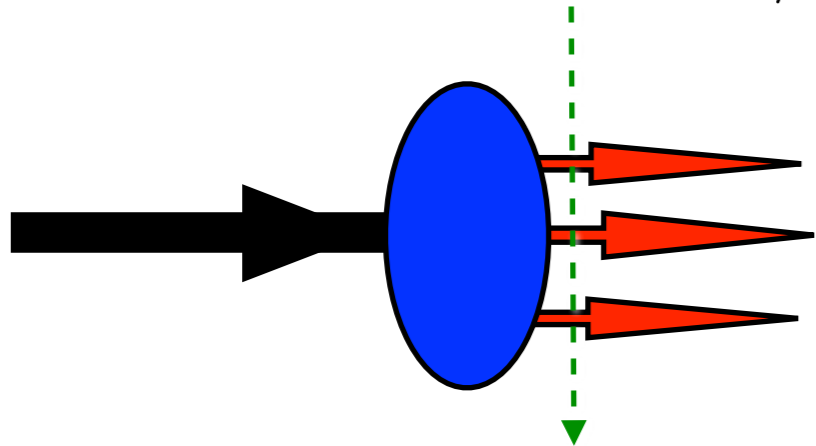
● **Trivial LF Vacuum (up to zero modes)**

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

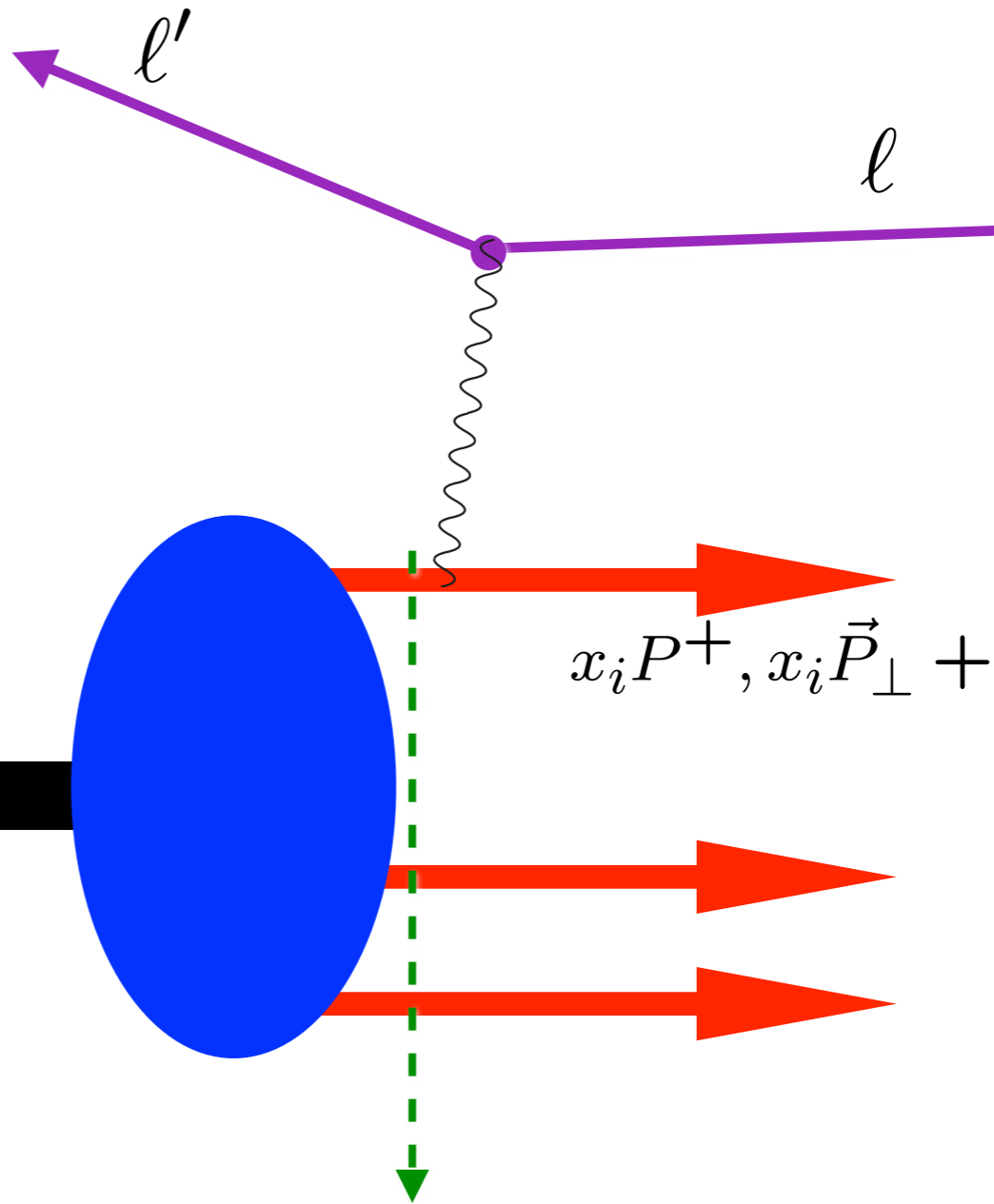
$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

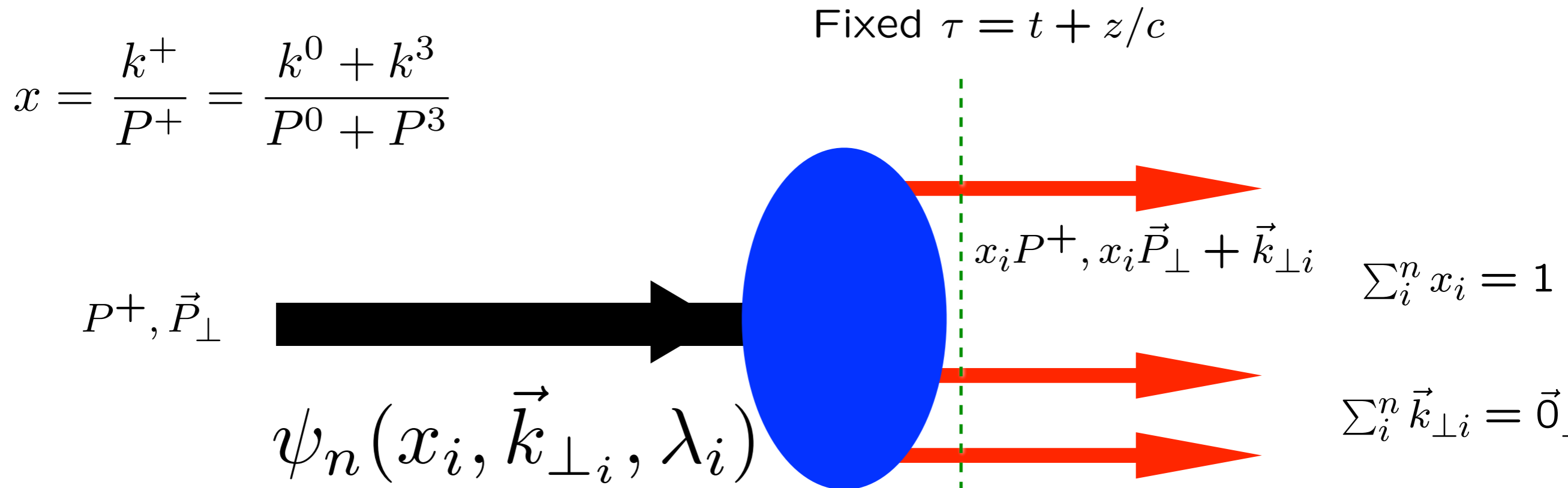
Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory



Eigenstate of LF Hamiltonian

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

Invariant under boosts! Independent of P^μ

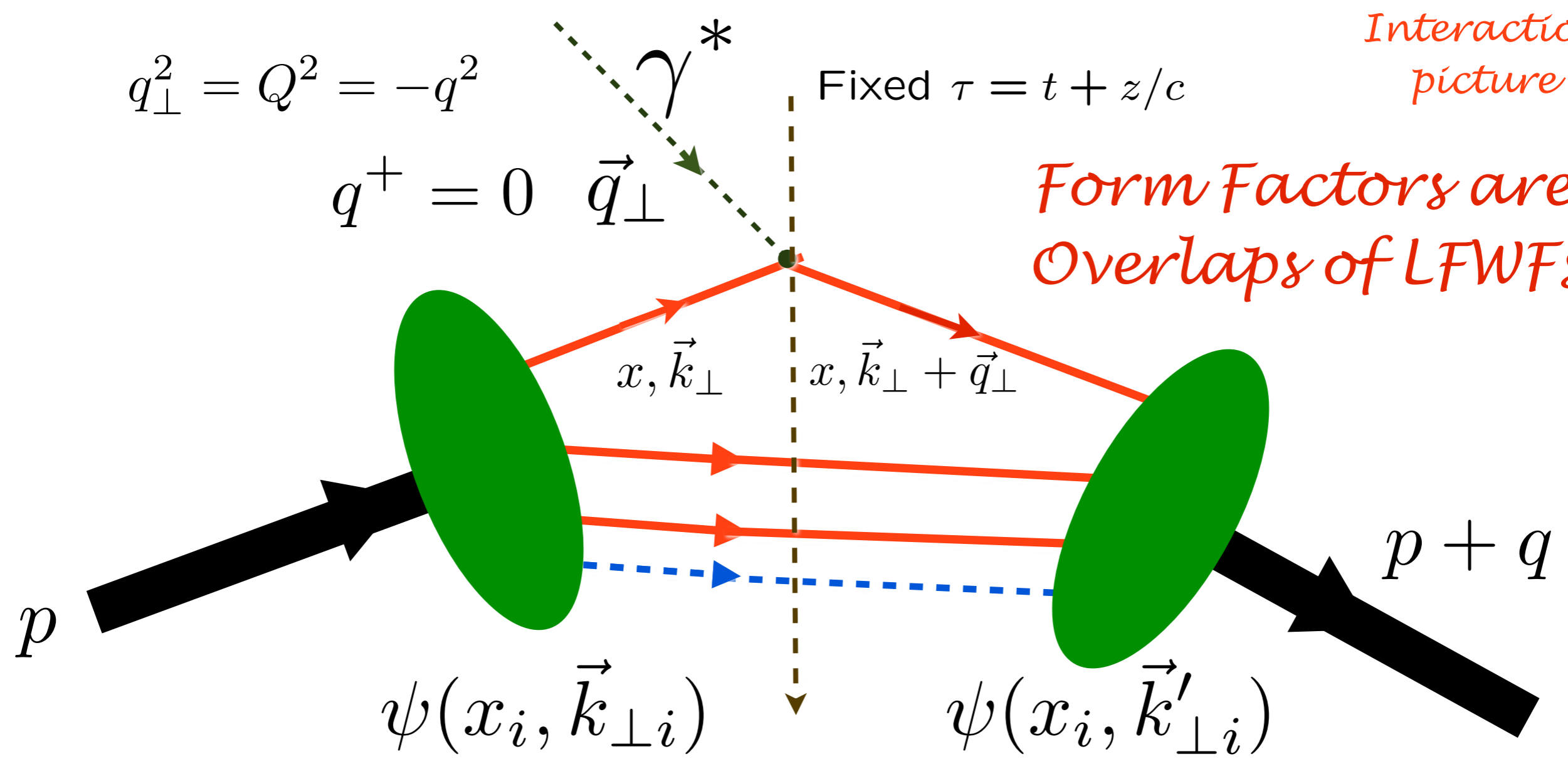
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

**Drell & Yan, West
Exact LF formula!**

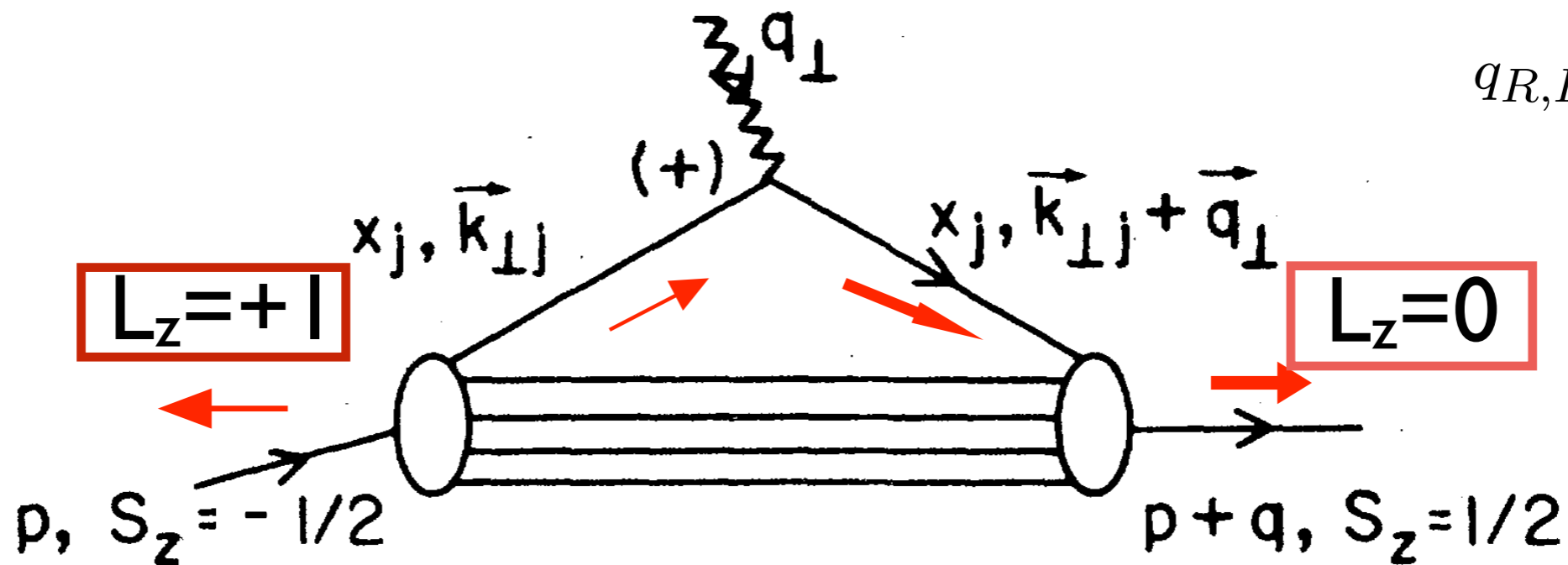
Drell, sjb

Exact LF Formula for Pauli Form Factor

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times \quad \text{Drell, sjb}$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \quad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$



Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

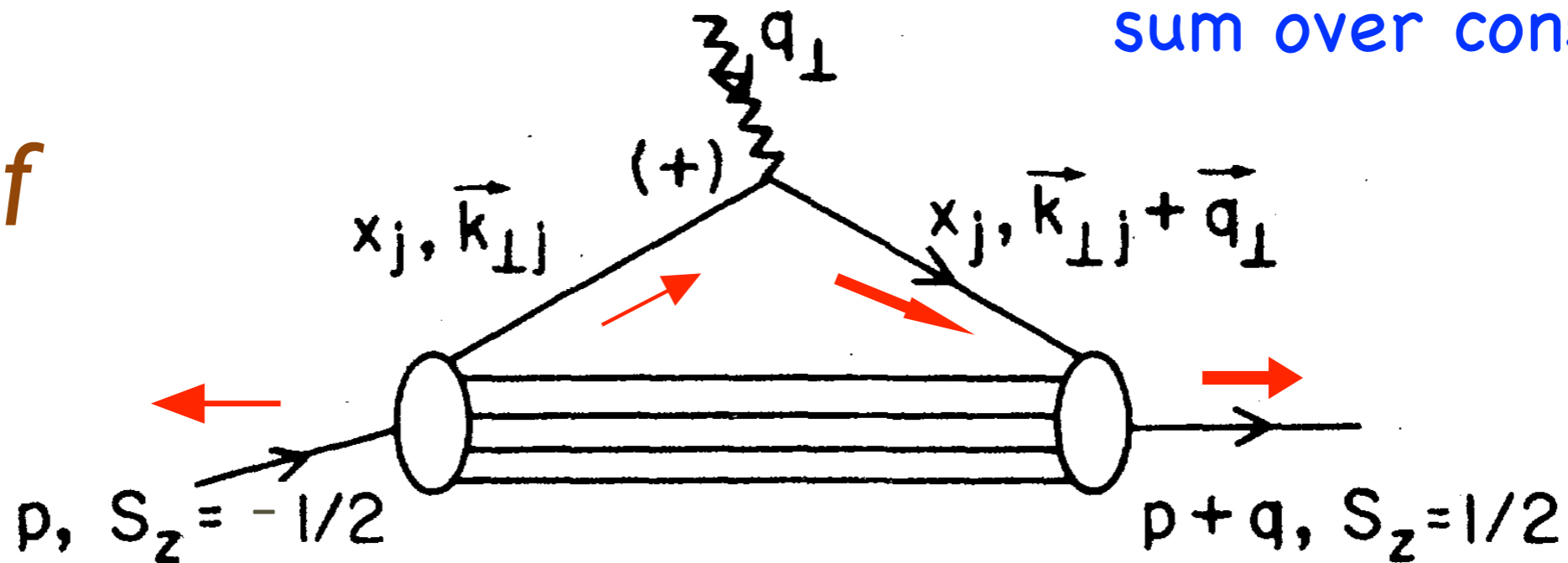
Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Terayev, Okun: $B(0)$ Must vanish because of Equivalence Theorem

graviton

sum over constituents

LF Proof



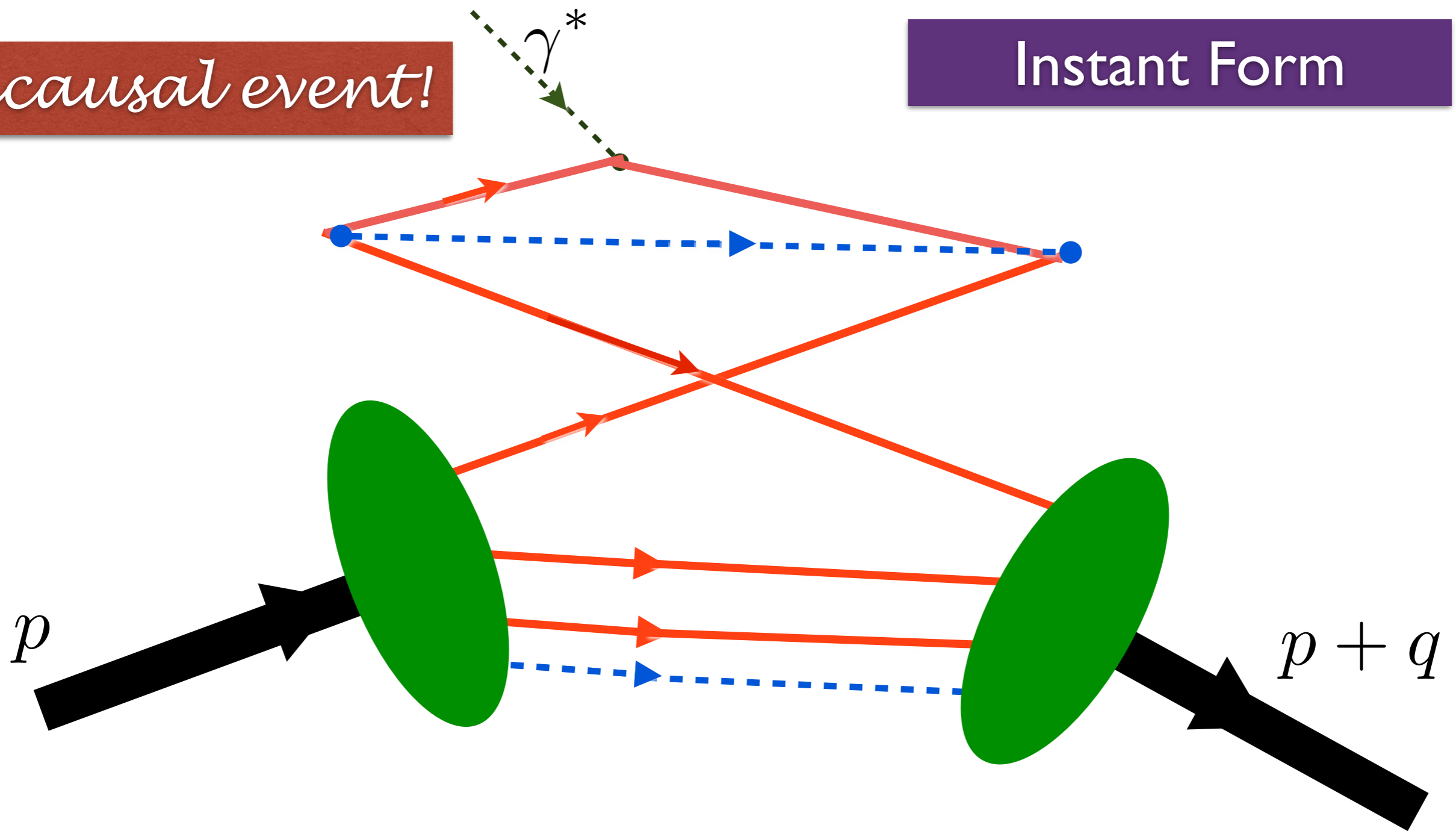
$$B(0) = 0$$

Each Fock State

Vanishing Anomalous gravitomagnetic moment $B(0)$

acausal event!

Instant Form



Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form

Boost are dynamical in instant form

Advantages of the Dirac's Front Form for Hadron Physics

Poincare' Invariant

Physics Independent of Observer's Motion

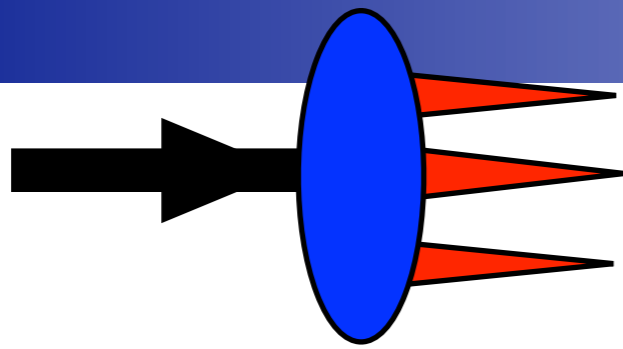


- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, ~~no pancakes!~~

Penrose, Terrell, Weisskopf

- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial up to zero modes
- Implications for Cosmological Constant

Roberts, Shrock, Tandy, sjb



Light-Front Wavefunctions underly hadronic observables

Lorce, Pasquini

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

GTMDs

$$x, \vec{k}_{\perp}, \vec{b}_{\perp}$$

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

Transverse density in position space

Weak transition form factors

TMDs

$$x, \vec{k}_{\perp}$$

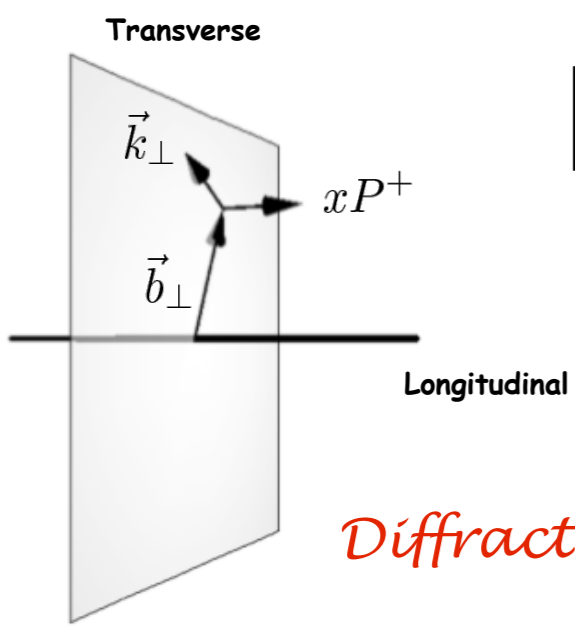
TMFFs

$$\vec{k}_{\perp}, \vec{b}_{\perp}$$

GPDs

$$x, \vec{b}_{\perp}$$

DGLAP, ERBL Evolution Factorization Theorems



TMSDs

$$\vec{k}_{\perp}$$

PDFs

$$x,$$

FFs

$$\vec{b}_{\perp}$$

- $\int d^2 b_{\perp}$
- $\int dx$
- $\int d^2 k_{\perp}$

Diffractive DIS from FSI

Charges

Sivers, T-odd from lensing

Exact frame-independent formulation of nonperturbative QCD!

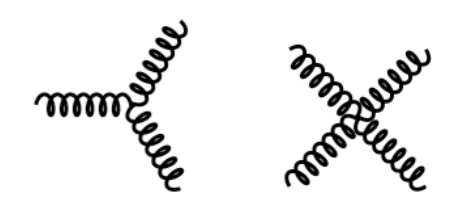
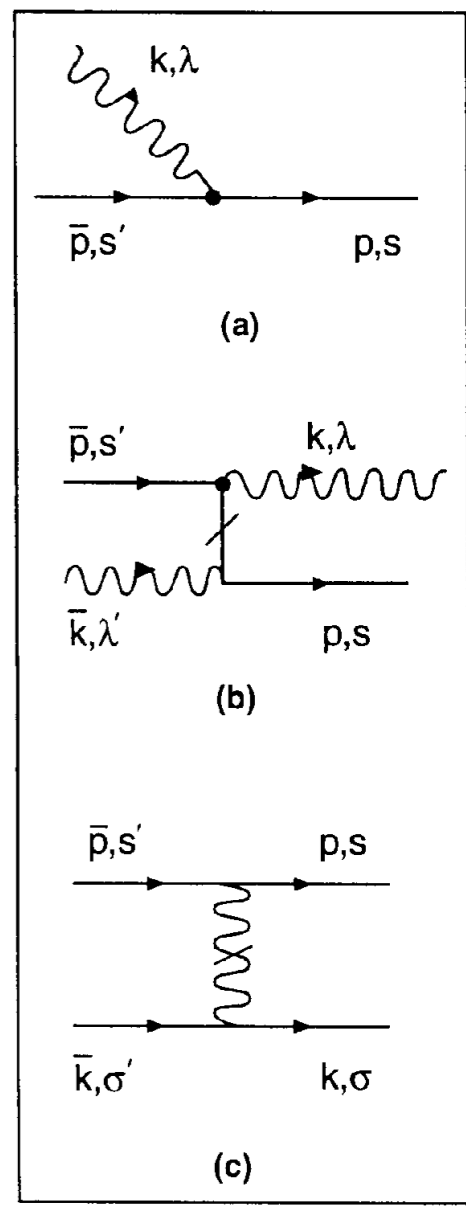
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



H_{LF}^{int}

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

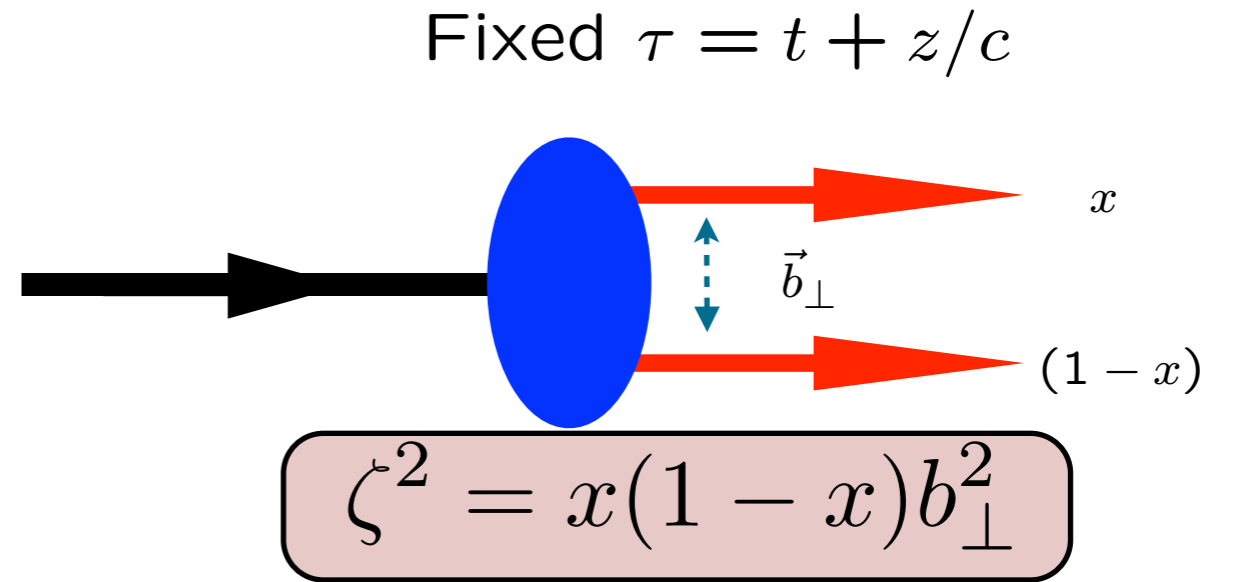
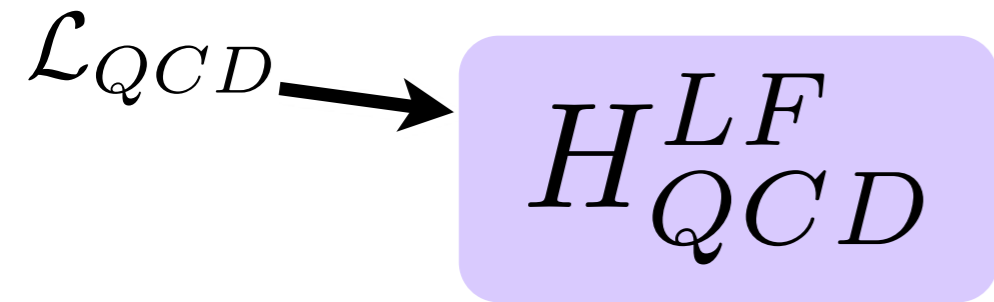
LFWFs: Off-shell in P- and invariant mass

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}_{initial}^2 - \mathcal{M}_{intermediate}^2 + i\epsilon} H_I + \dots$$

- “History”: Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!
- Wick Theorem applies, but few amplitudes since all $k^+ > 0$.
- J_z Conservation at every vertex $\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n$ at order g^n
- Unitarity is explicit K. Chiu, Lorcé, sjb
- Loop Integrals are 3-dimensional $\int_0^1 dx \int d^2 k_\perp$
- hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_{\perp}) = M^2 \psi_{LF}(x, \vec{k}_{\perp})$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$$m_q = 0$$

AdS/QCD:

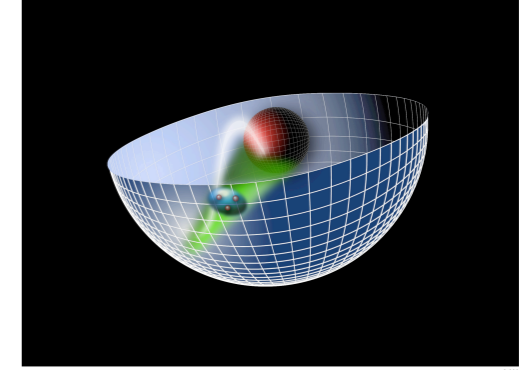
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Sums an infinite # diagrams

AdS₅



- Isomorphism of $SO(4, 2)$ of **conformal QCD** with the group of **isometries** of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure ←

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

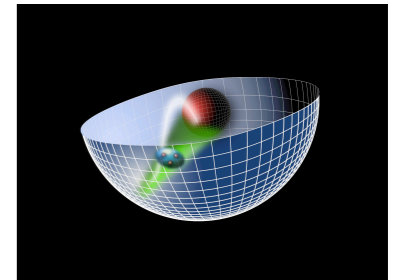
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS₅ as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

• de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

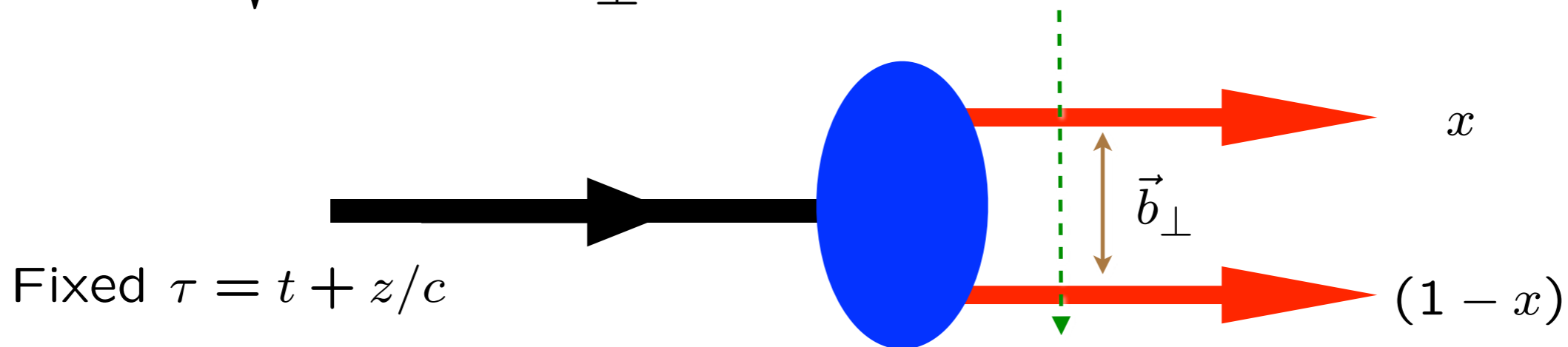
Derived from variation of Action for Dilaton-Modified AdS₅

Identical to Single-Variable Light-Front Bound State Equation in ζ !

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

$LF(3+1) \longleftrightarrow AdS_5$

Light-Front Holographic Dictionary

 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$
 $\zeta = \sqrt{x(1-x)b_\perp^2} \longleftrightarrow z$


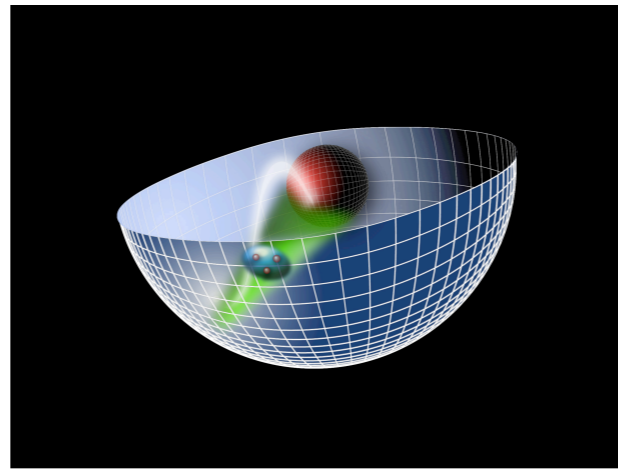
$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Single variable ζ

***Unique
Confinement Potential!***

*Conformal Symmetry
of the action*

Confinement scale:

$$\kappa \simeq 0.5 \text{ GeV}$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

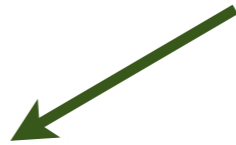
GeV units external to QCD: Only Ratios of Masses Determined

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential



- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

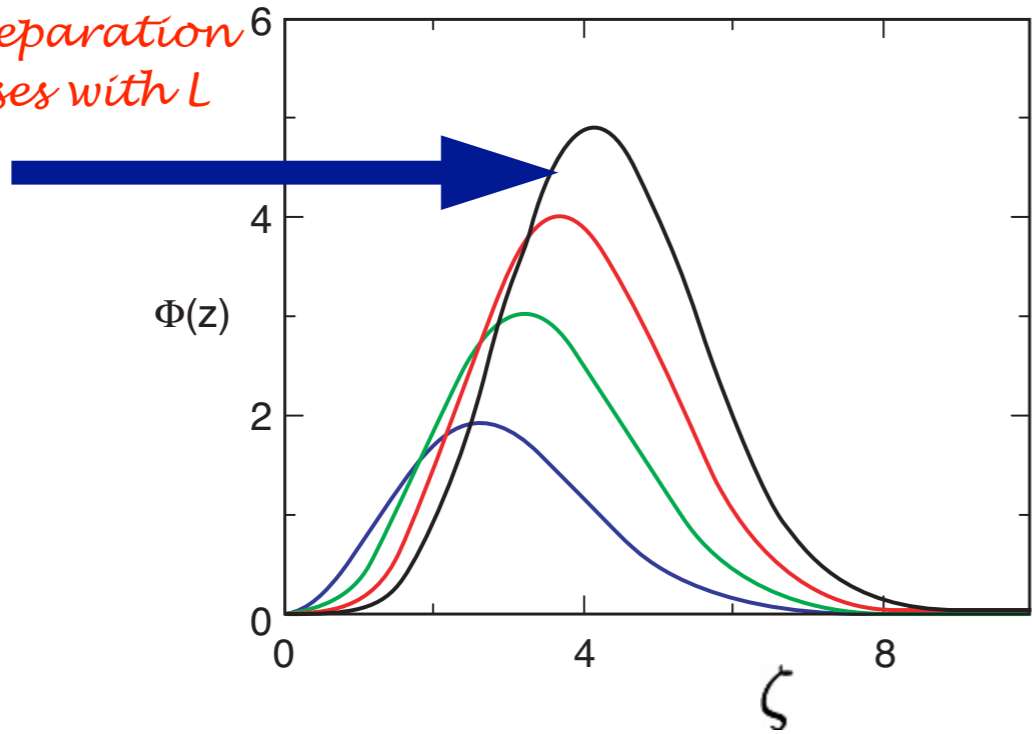
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$M_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

Quark separation increases with L



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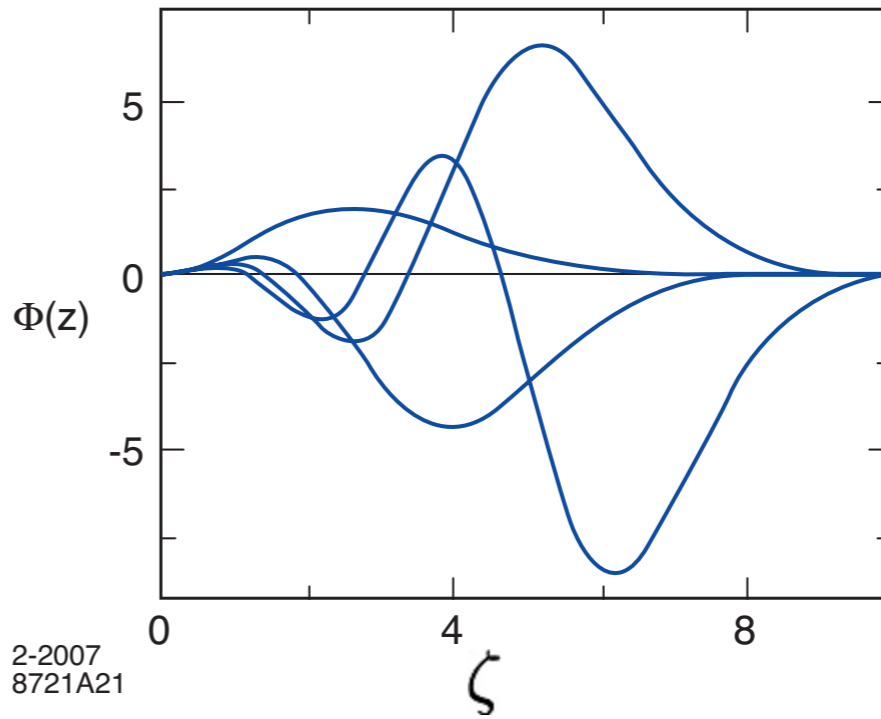
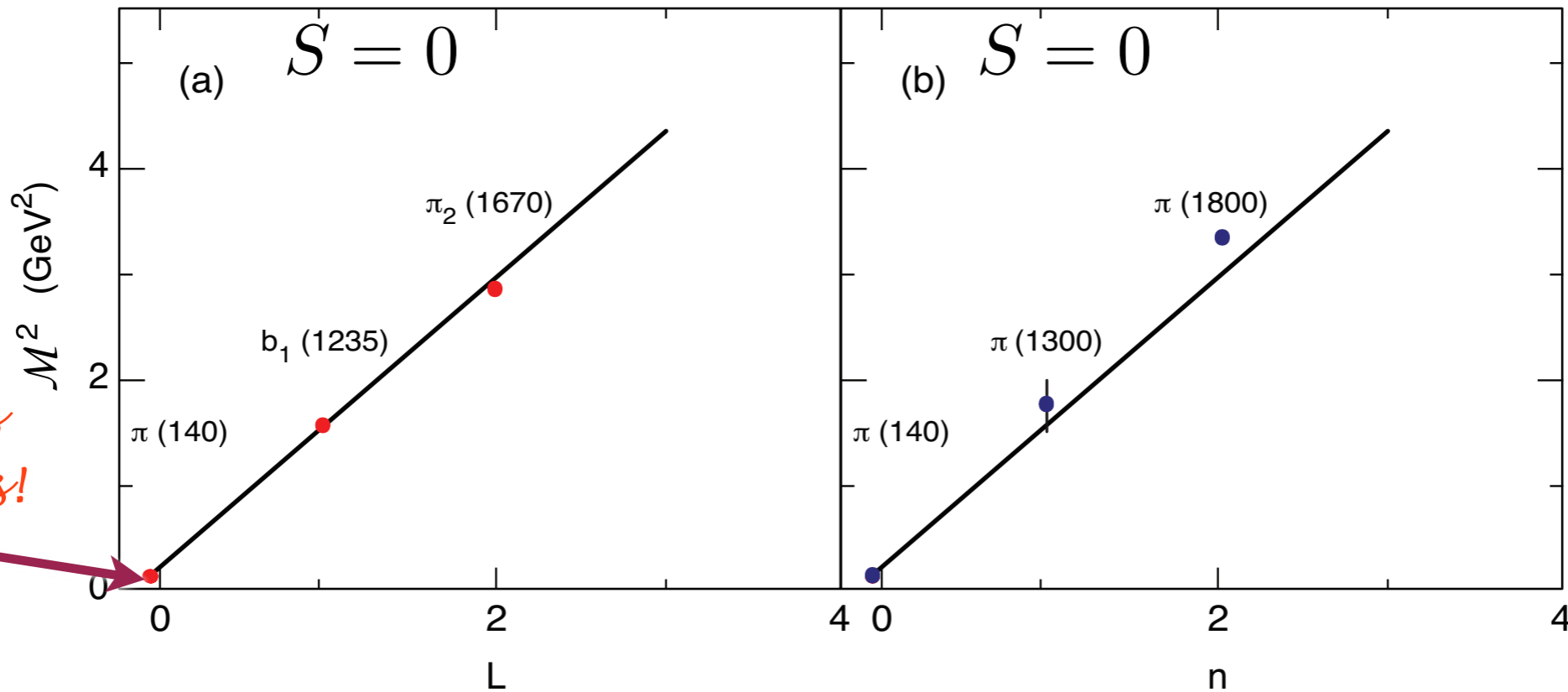


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Same slope in n and L !

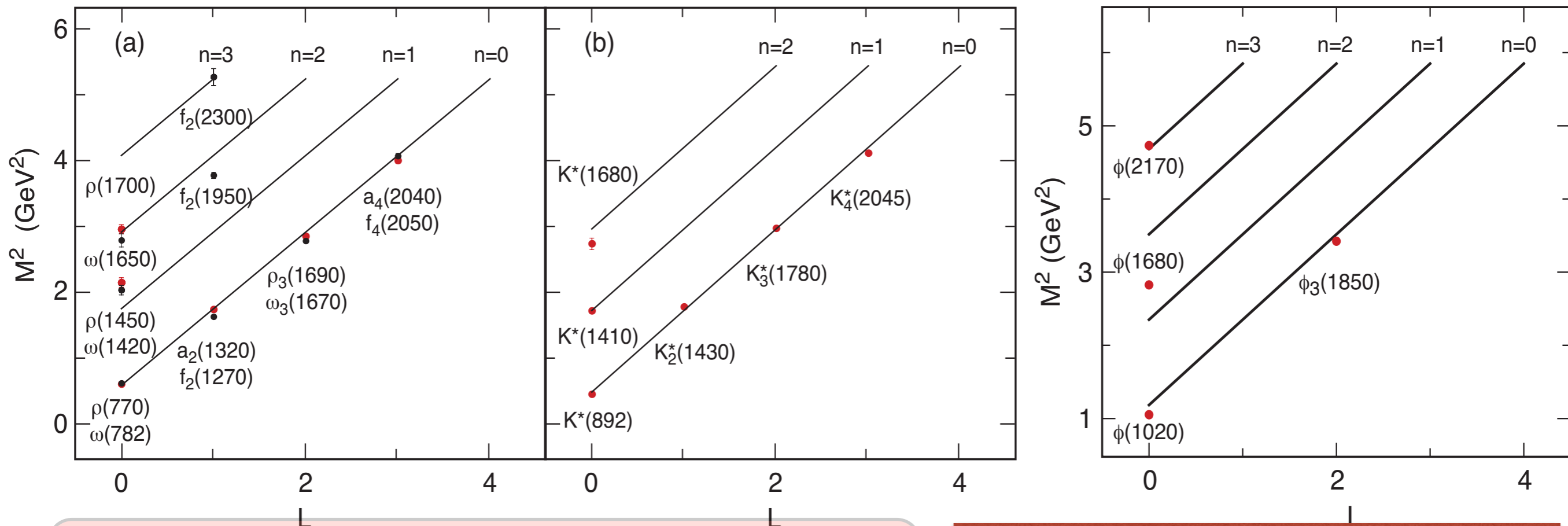
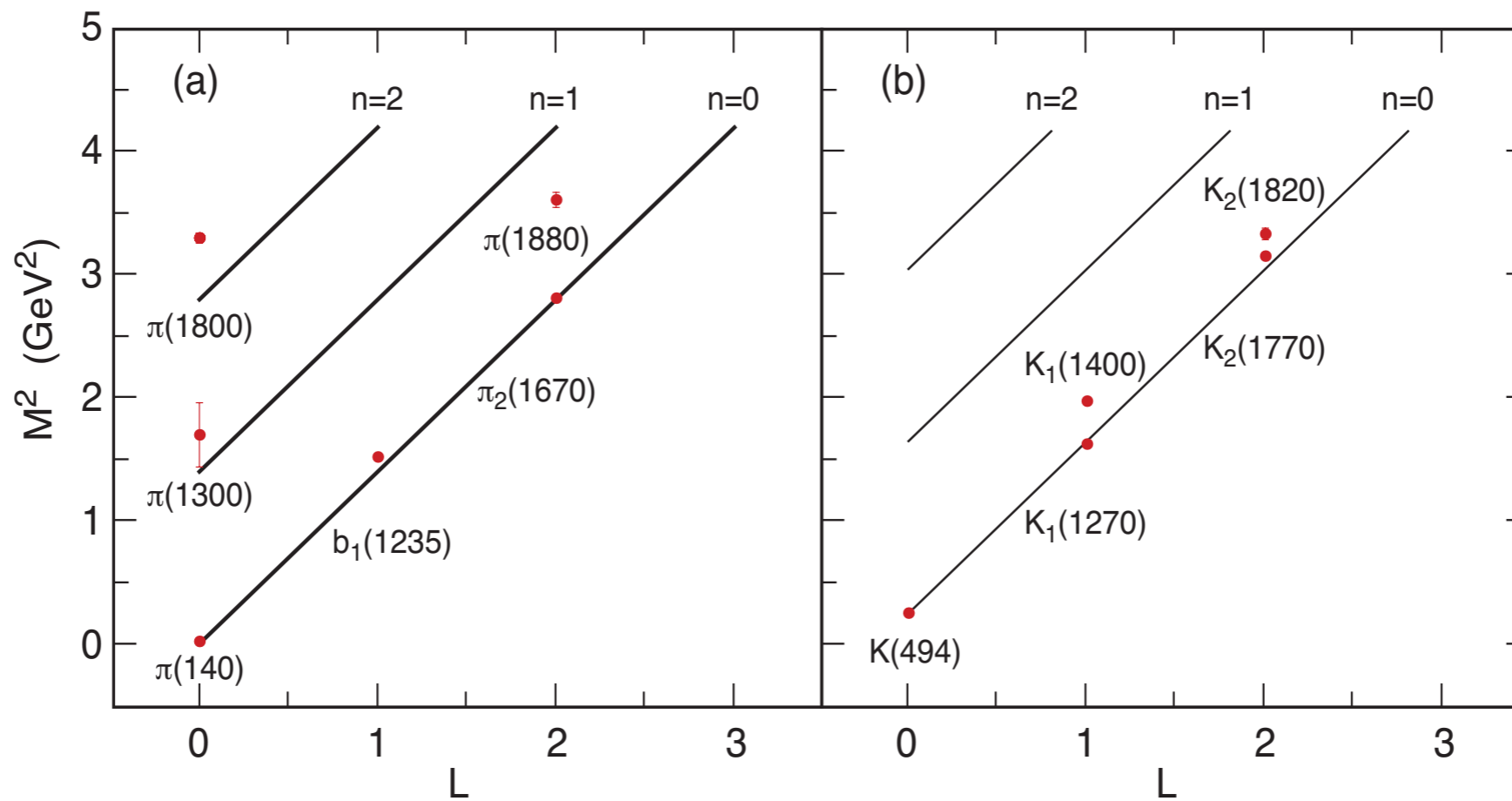
Soft Wall Model



Pion has zero mass!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

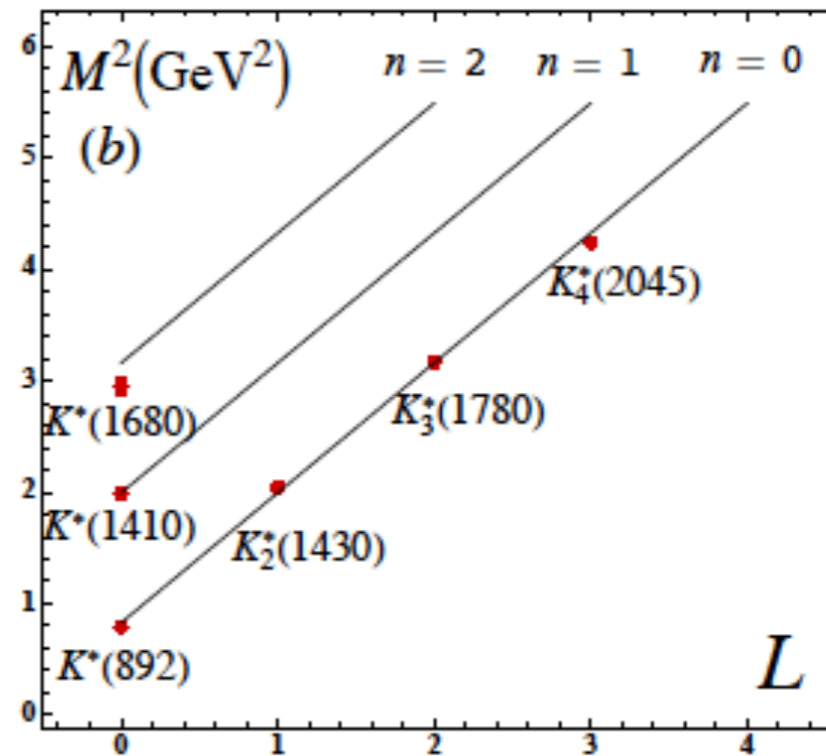
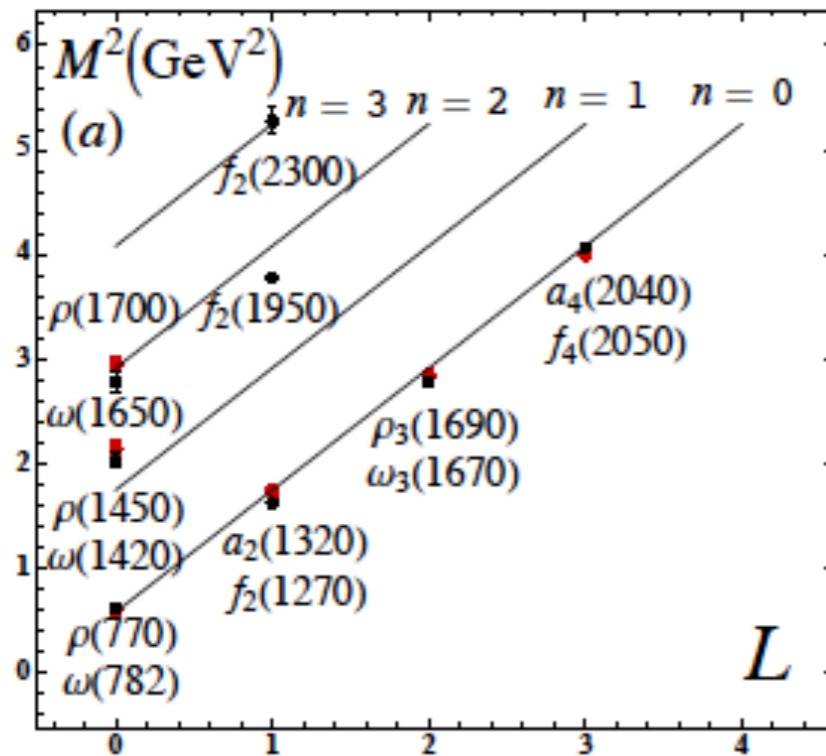
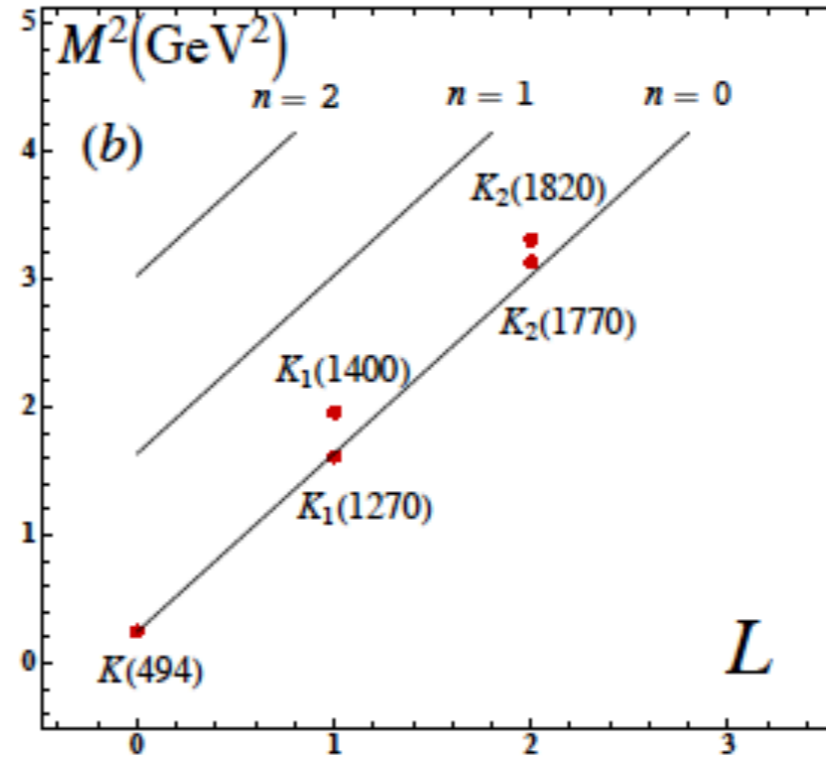
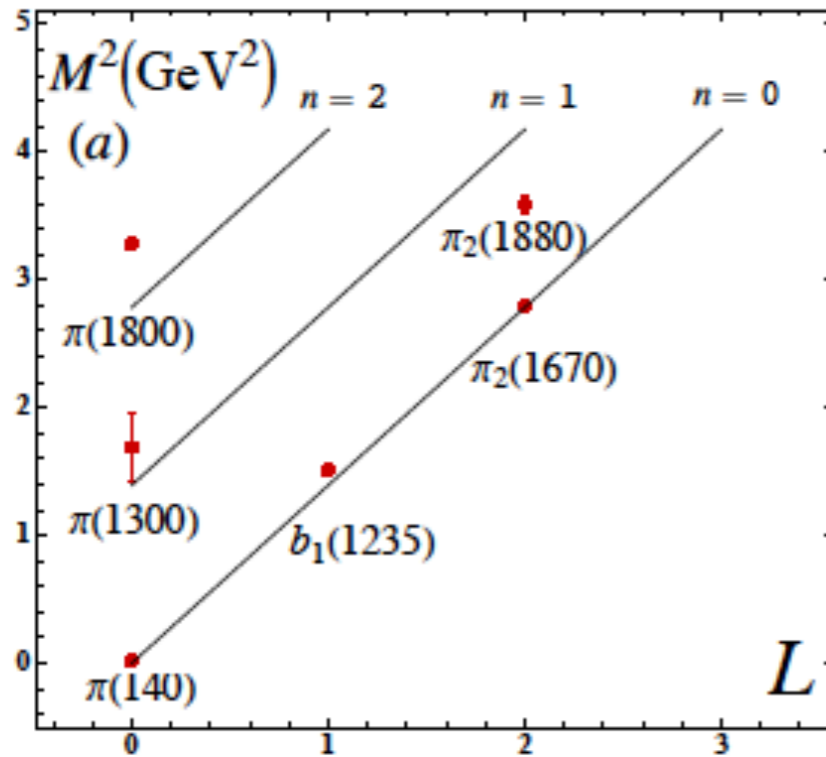


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

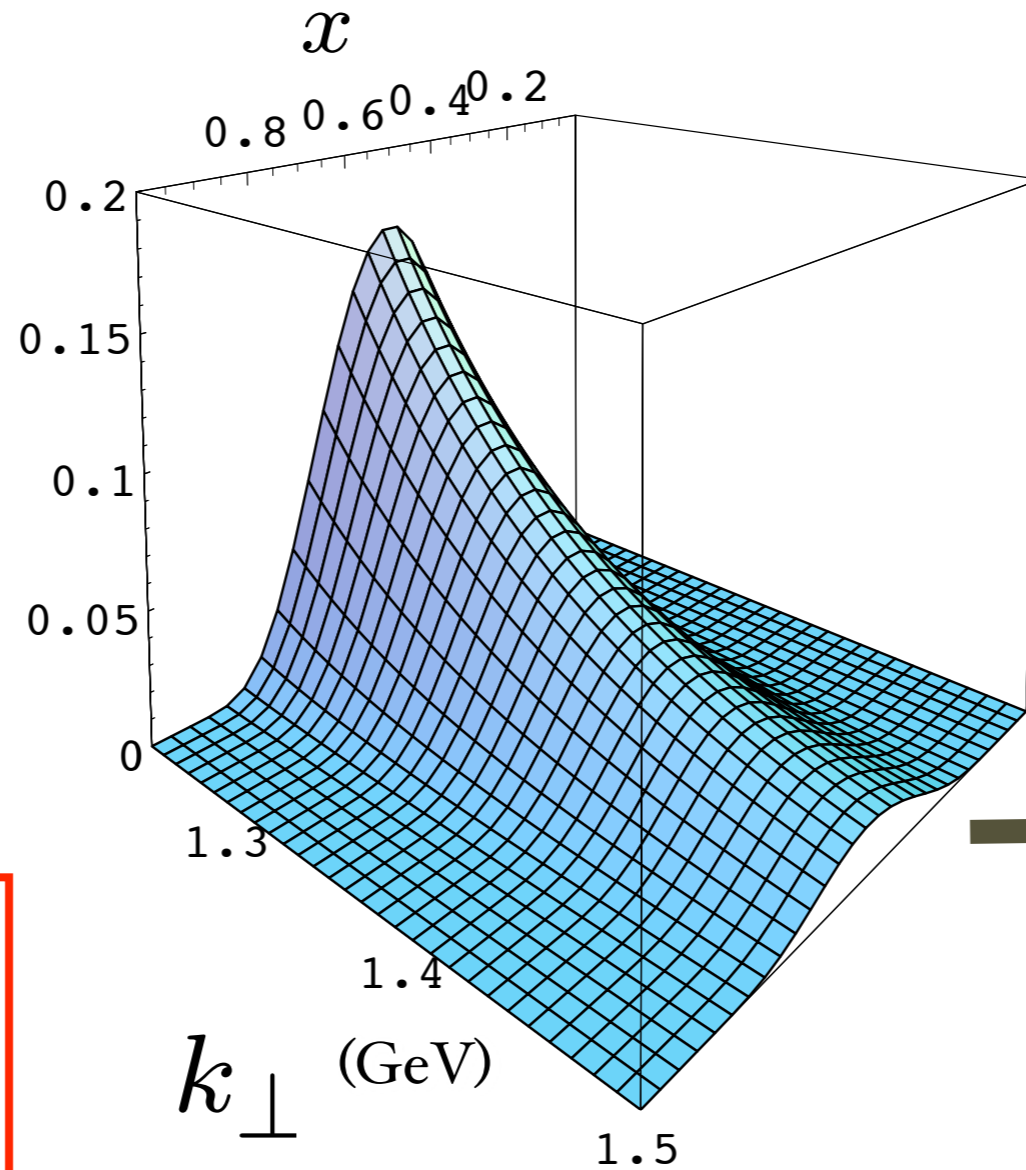
from LF Higgs mechanism



Prediction from AdS/QCD: Meson LFWF

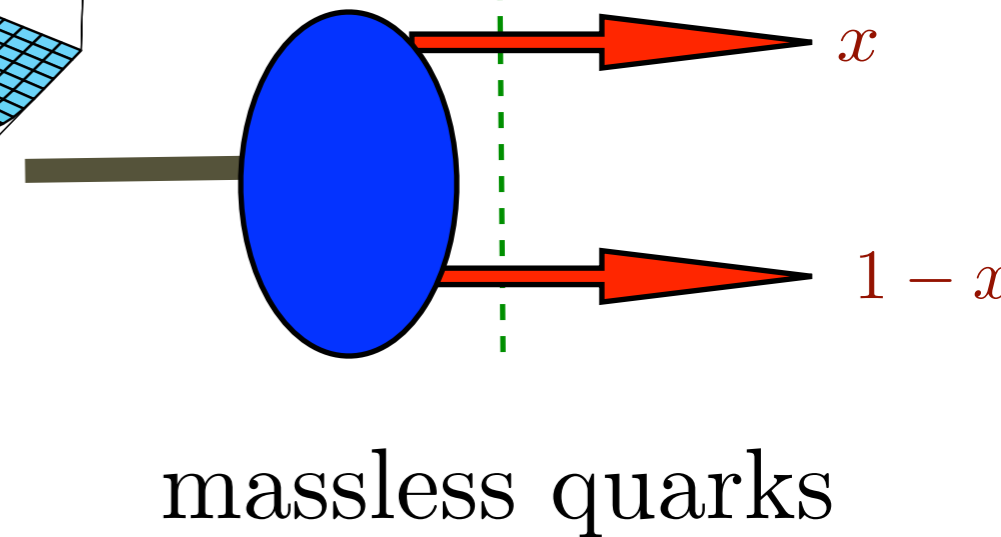
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_{\perp}^2)$$



de Teramond,
Cao, sjb

“Soft Wall”
model



Note coupling

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE! C. D. Roberts et al.

Provides Connection of Confinement to Hadron Structure

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

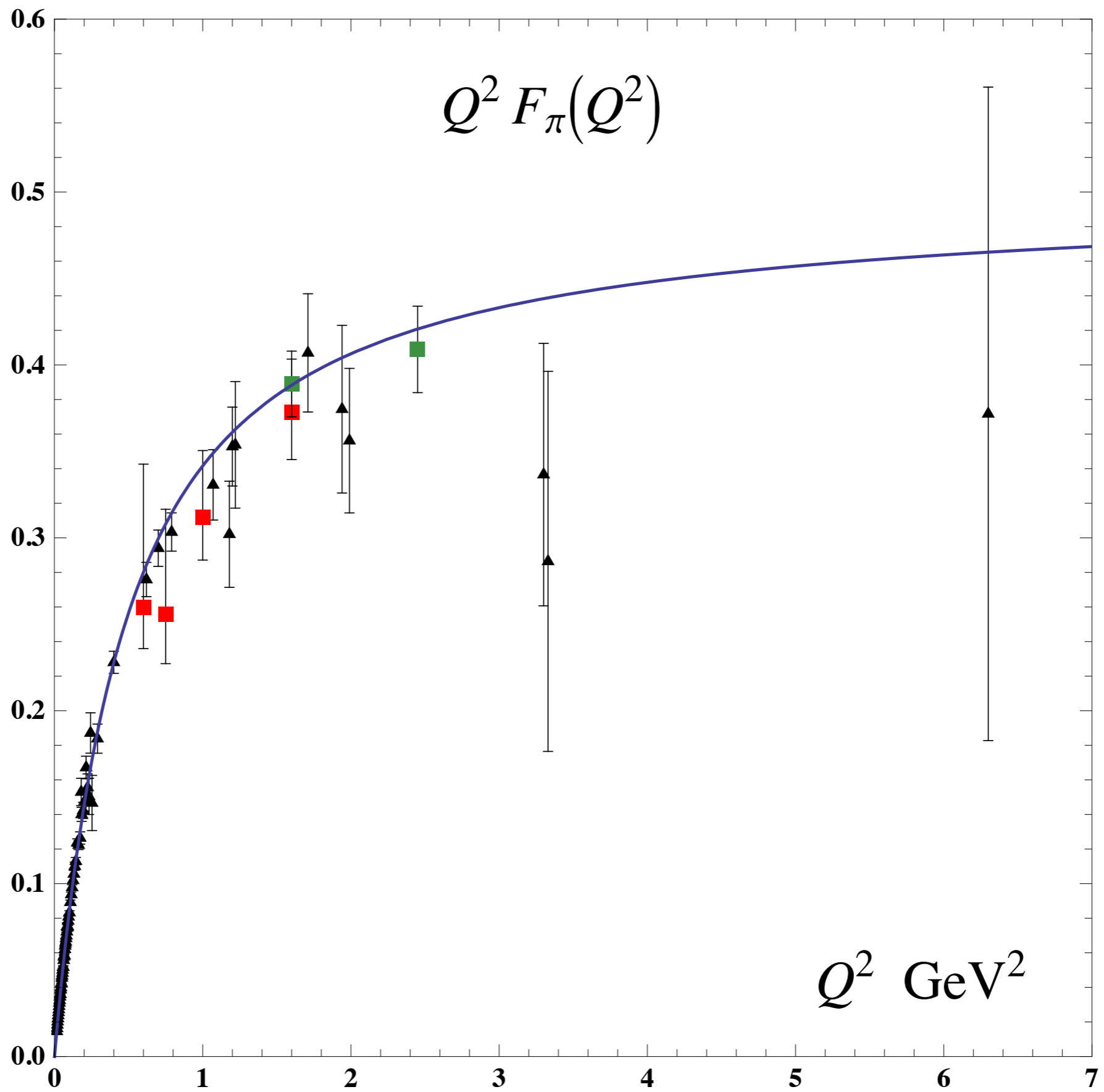
- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

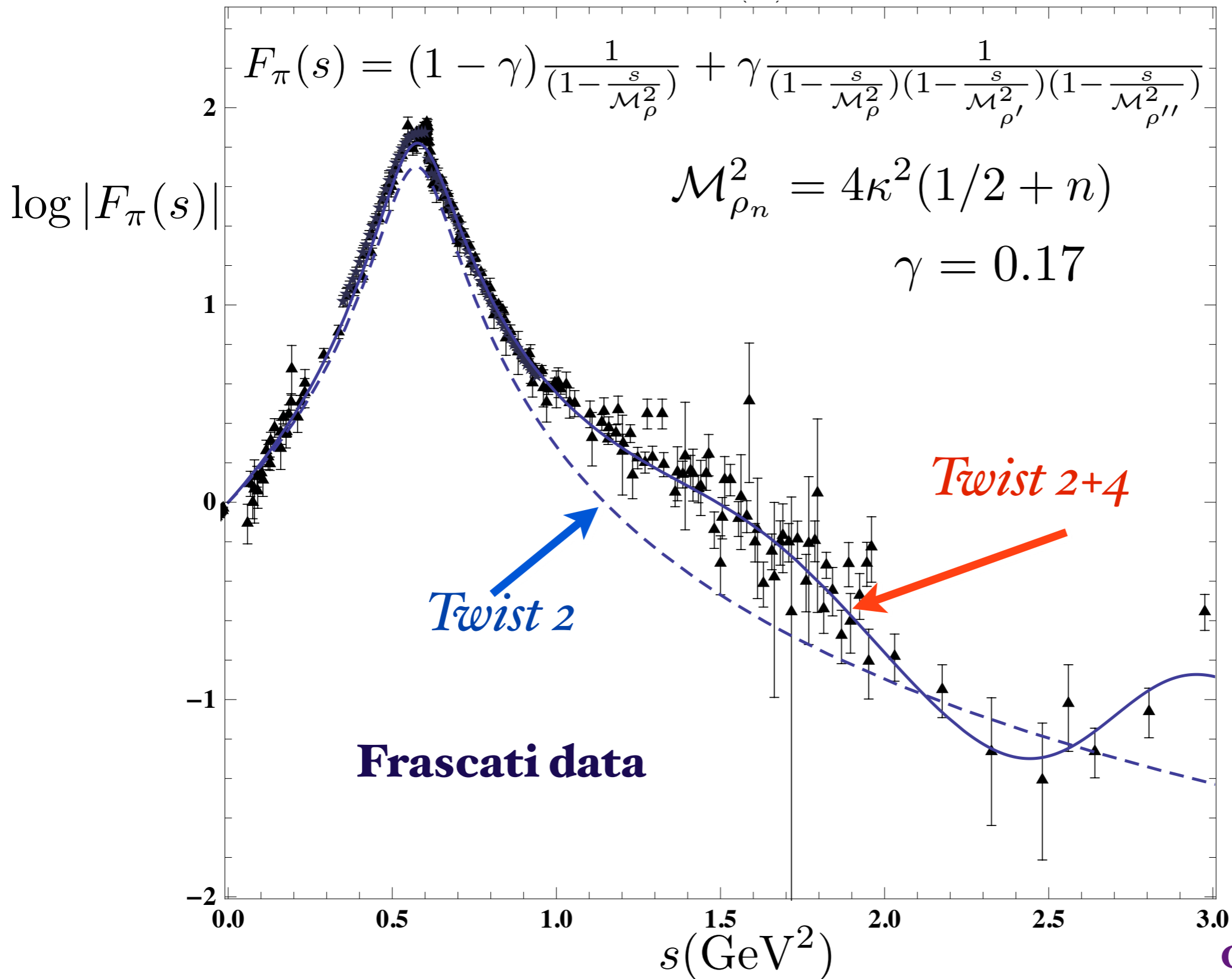
the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

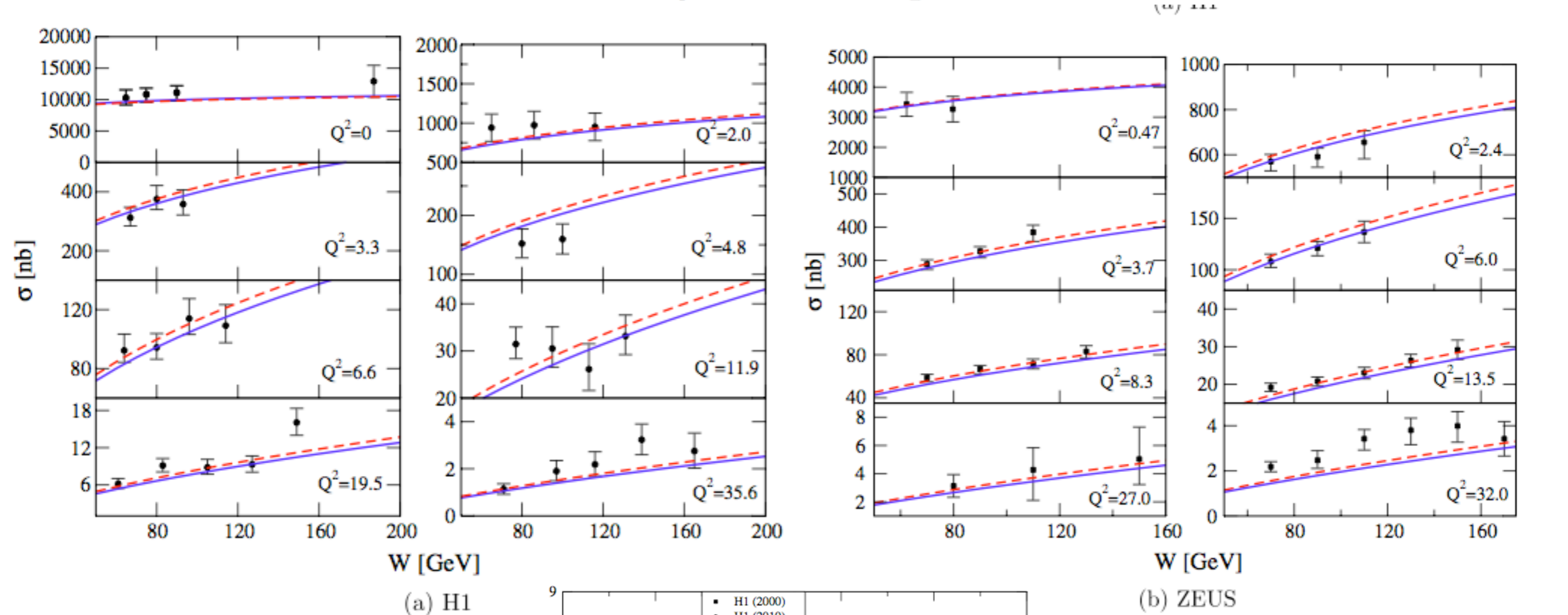


Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

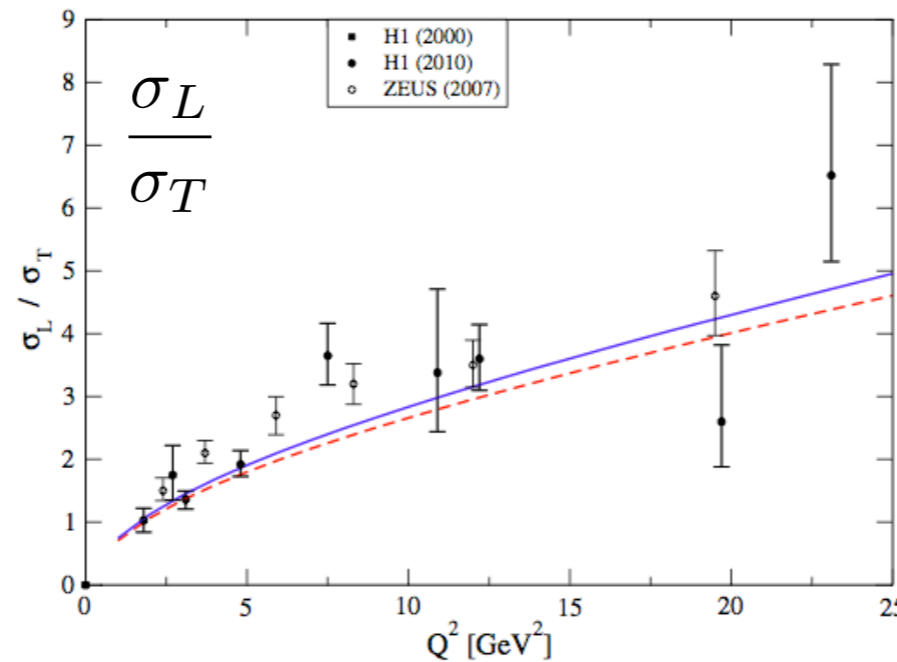
14% four-quark probability

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



**J. R. Forshaw,
R. Sandapen**

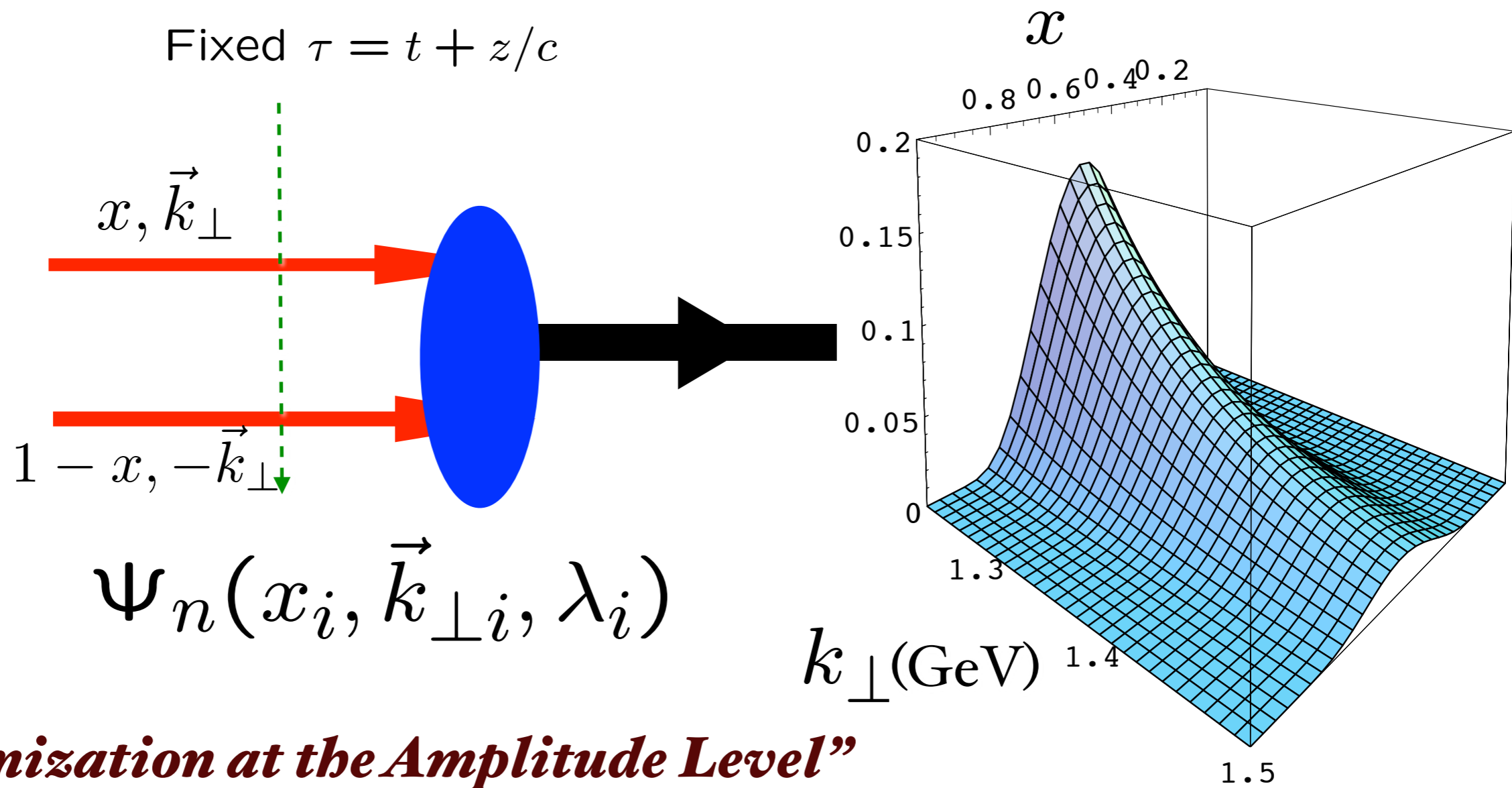
$$\gamma^* p \rightarrow \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

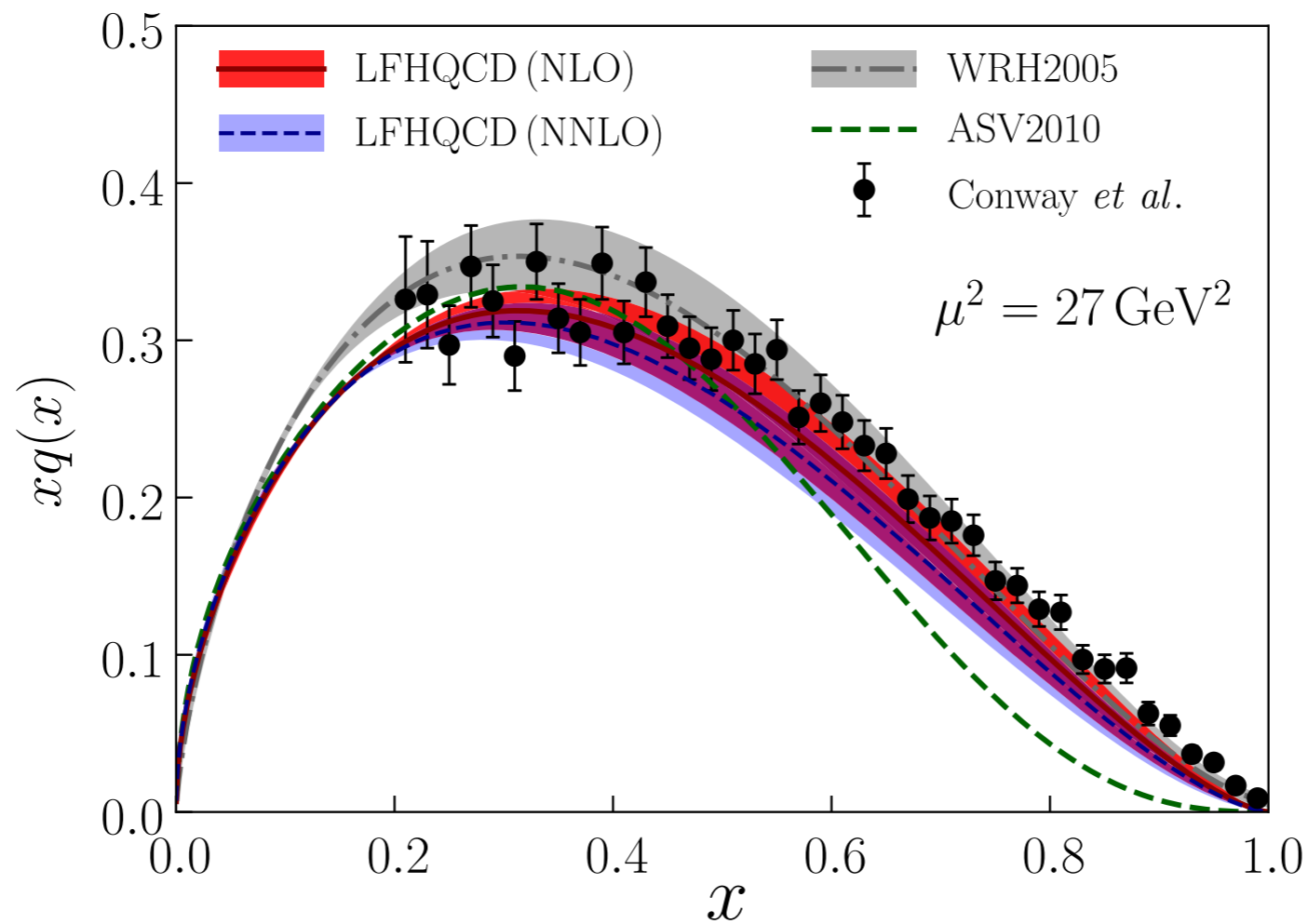
• *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}_{q\bar{q}}^2$



“Hadronization at the Amplitude Level”

Boost-invariant LFWF connects confined quarks and gluons to hadrons



Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2 \text{ GeV}$ at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15 \text{ GeV}$ at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks



Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

**QCD does not know what MeV units mean!
Only Ratios of Masses Determined**

- **de Alfaro, Fubini, Furlan:** *Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!*

Unique confinement potential!

● **de Alfaro, Fubini, Furlan** (*dAFF*)

$$G|\psi(\tau)\rangle = i\frac{\partial}{\partial\tau}|\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left(\frac{2tw + v}{\sqrt{4uw - v^2}} \right),$$

- **Identify with difference of LF time $\Delta x^+ / P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Stan Brodsky

LC2019

**Supersymmetric Features of Hadron Physics
from Superconformal Algebra and Light-Front Holography**

SLAC
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19 Sept 2019

Light-Front Holography: First Approximation to QCD

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

Stan Brodsky

LC2019

*Supersymmetric Features of Hadron Physics
from Superconformal Algebra and Light-Front Holography*

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Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \quad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \quad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}$, $S \simeq \sqrt{K}$

Consider $R_w = Q + wS$; w : dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \quad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B$, $w = \kappa^2$ $\lambda = \kappa^2$

Eigenvalue of G : $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+ \quad \left. \vphantom{\psi_J^+} \right\}$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^- \quad \left. \vphantom{\psi_J^-} \right\}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1) \quad \mathbf{S=1/2, P=+}$$

Meson Equation

$$\lambda = \kappa^2$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$\mathbf{S=0, P=+}$
Same κ !

$S=0, I=I$ Meson is superpartner of $S=1/2, I=I$ Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

- Eigenvalues

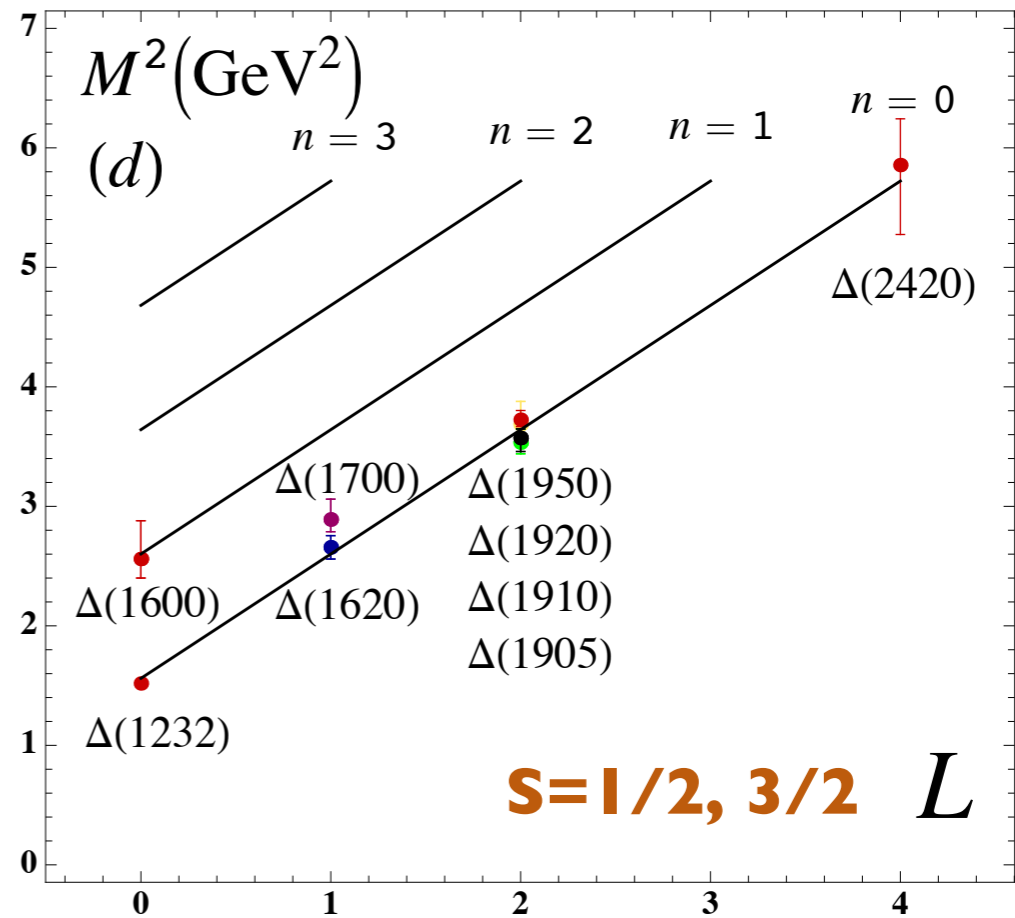
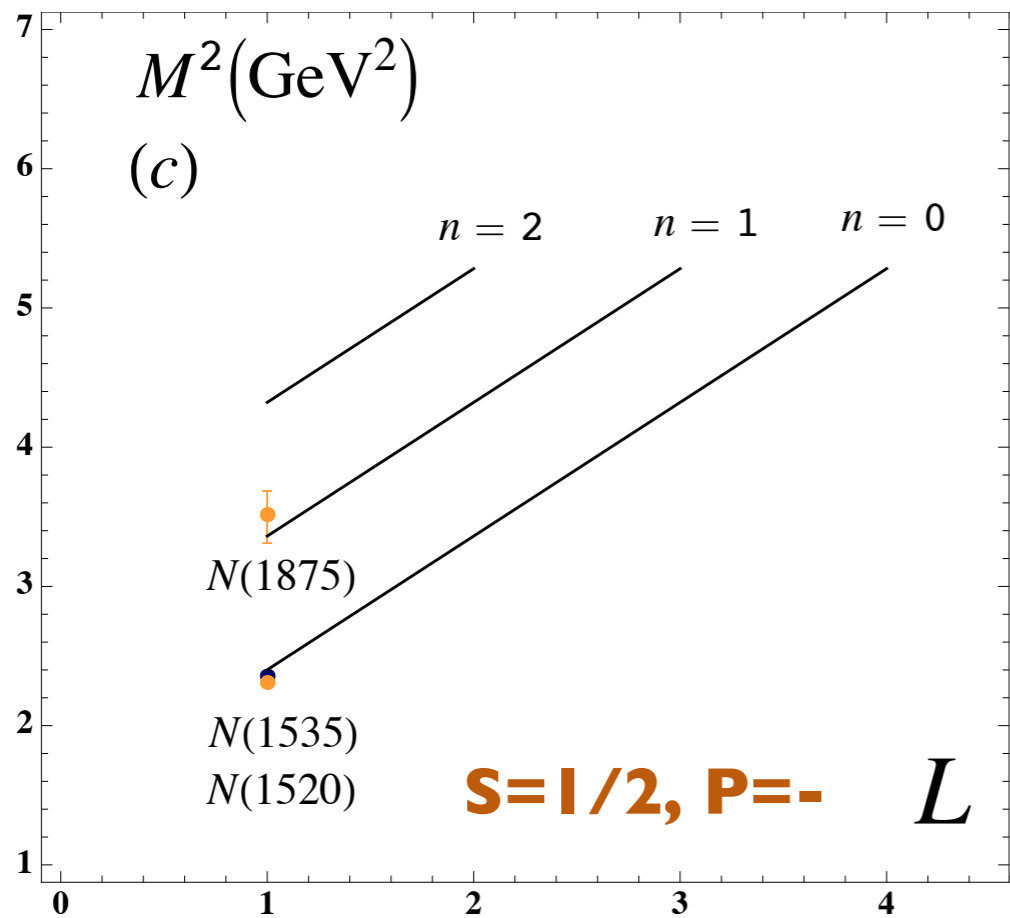
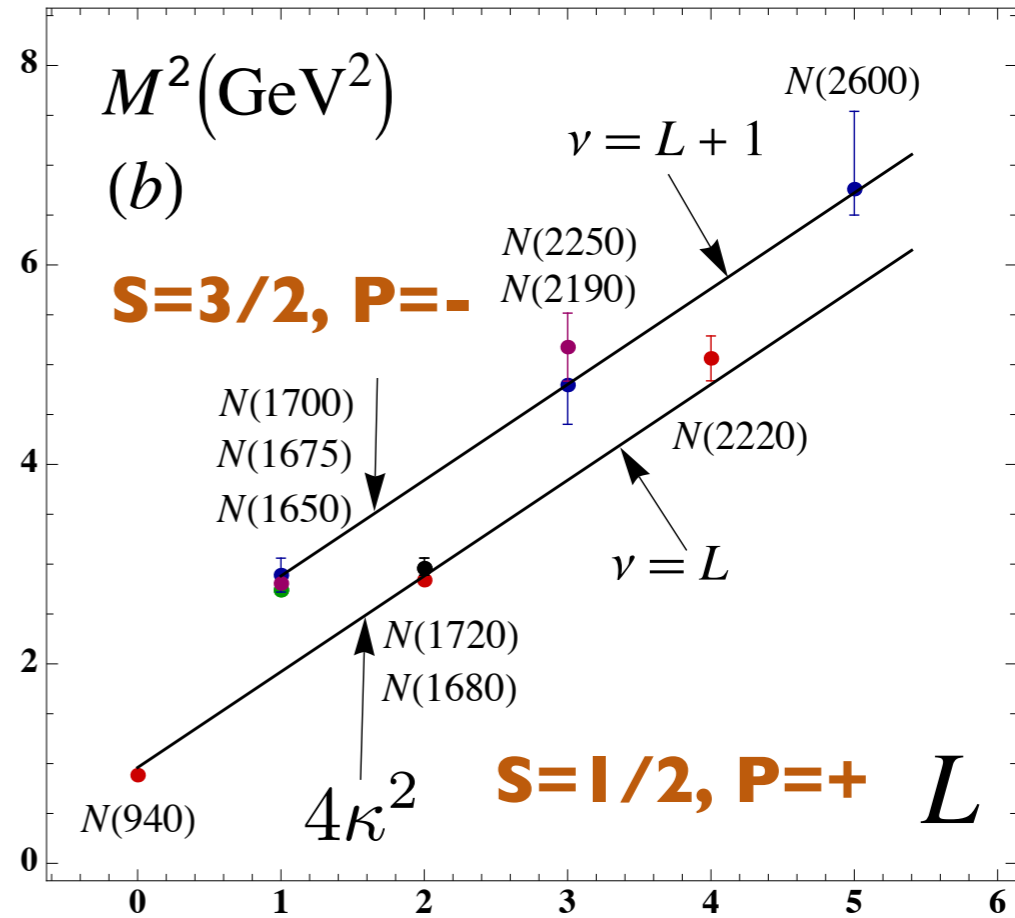
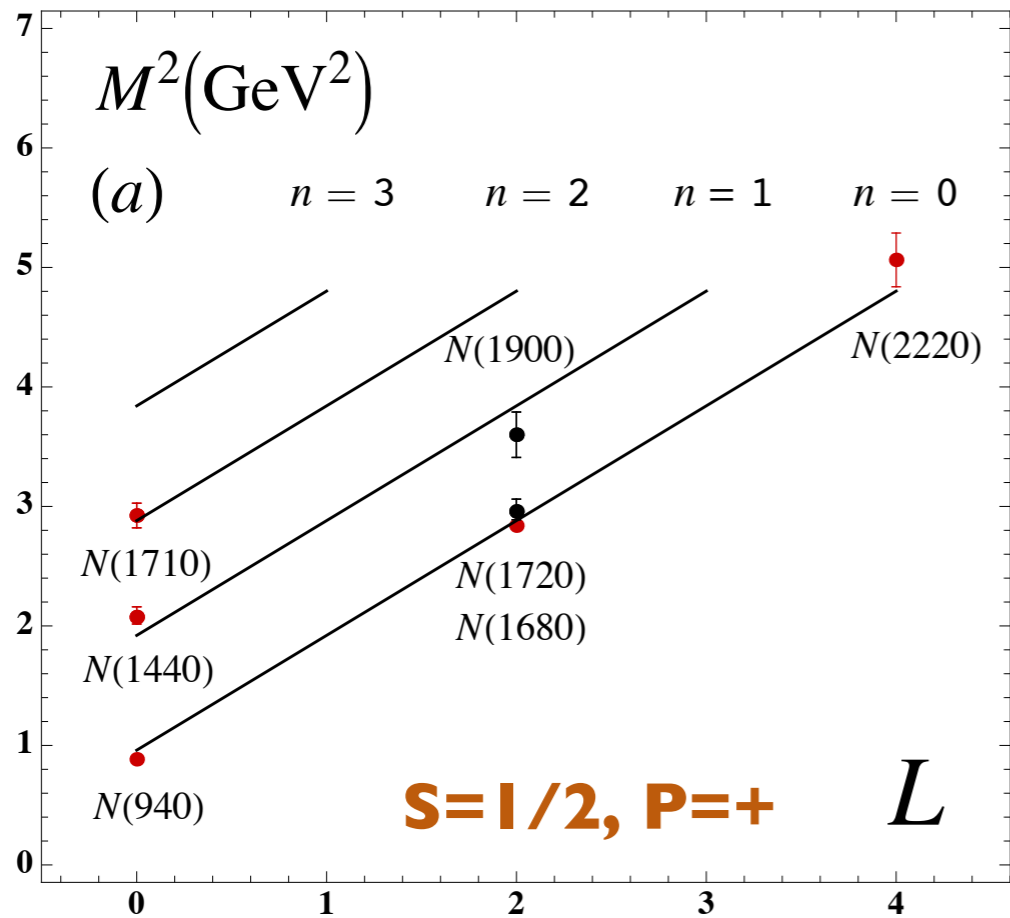
$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

*Quark Chiral
Symmetry of
Eigenstate!*

Nucleon: Equal Probability for L=0, 1

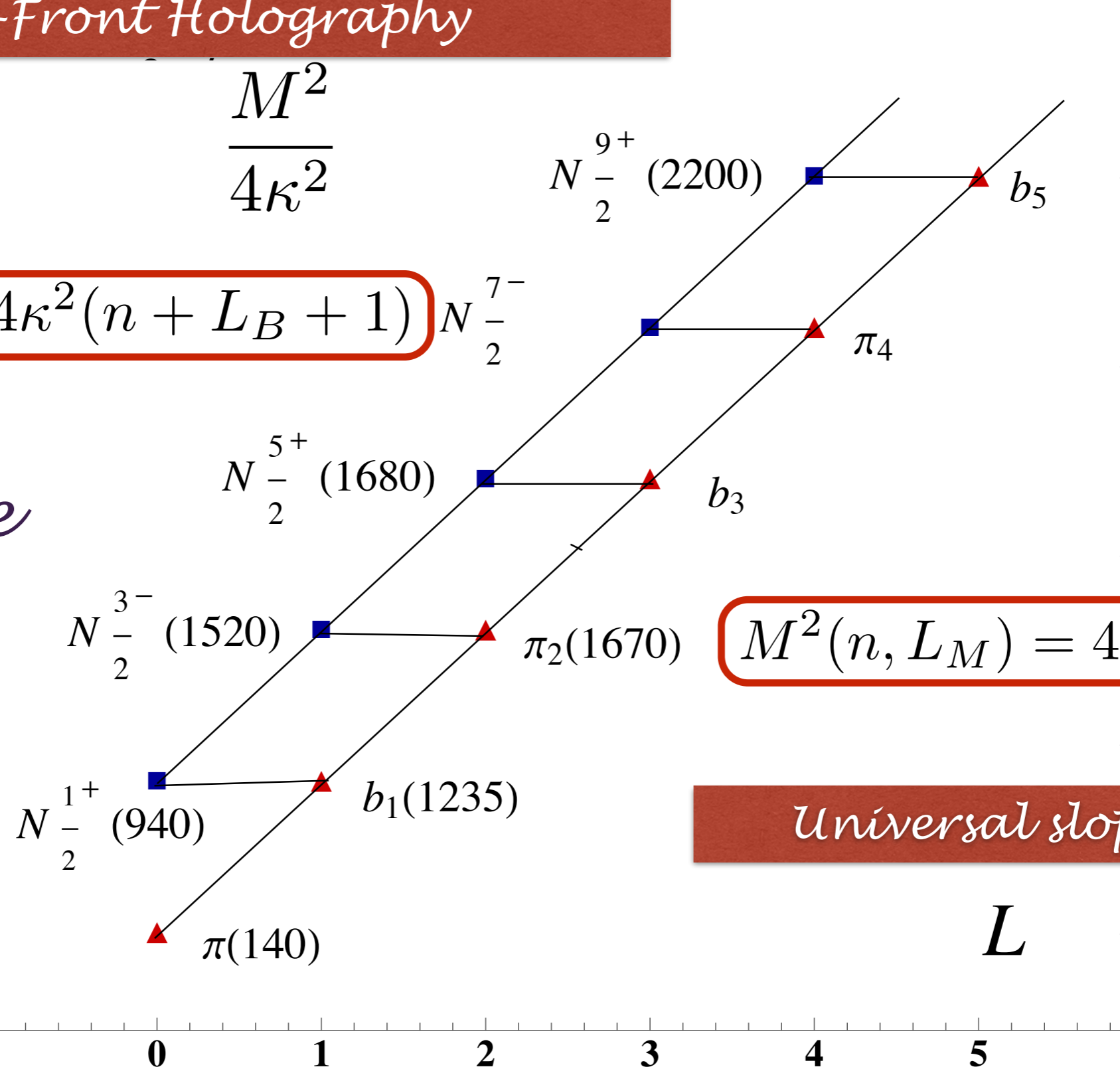
$$J^z = +1/2 : \frac{1}{\sqrt{2}} [|S_q^z = +1/2, L^z = 0\rangle + |S_q^z = -1/2, L^z = +1\rangle]$$

Nucleon spin carried by quark orbital angular momentum



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

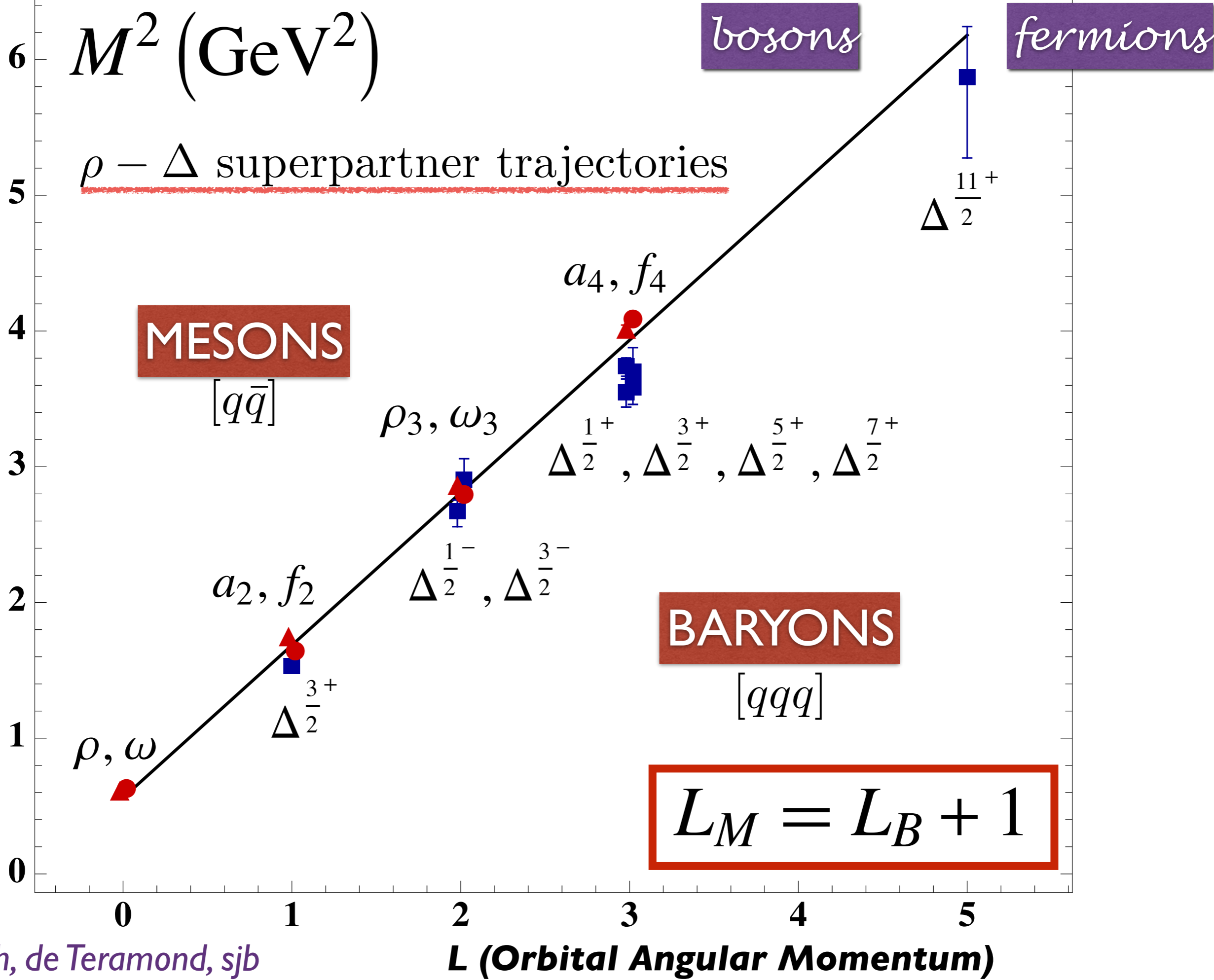


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

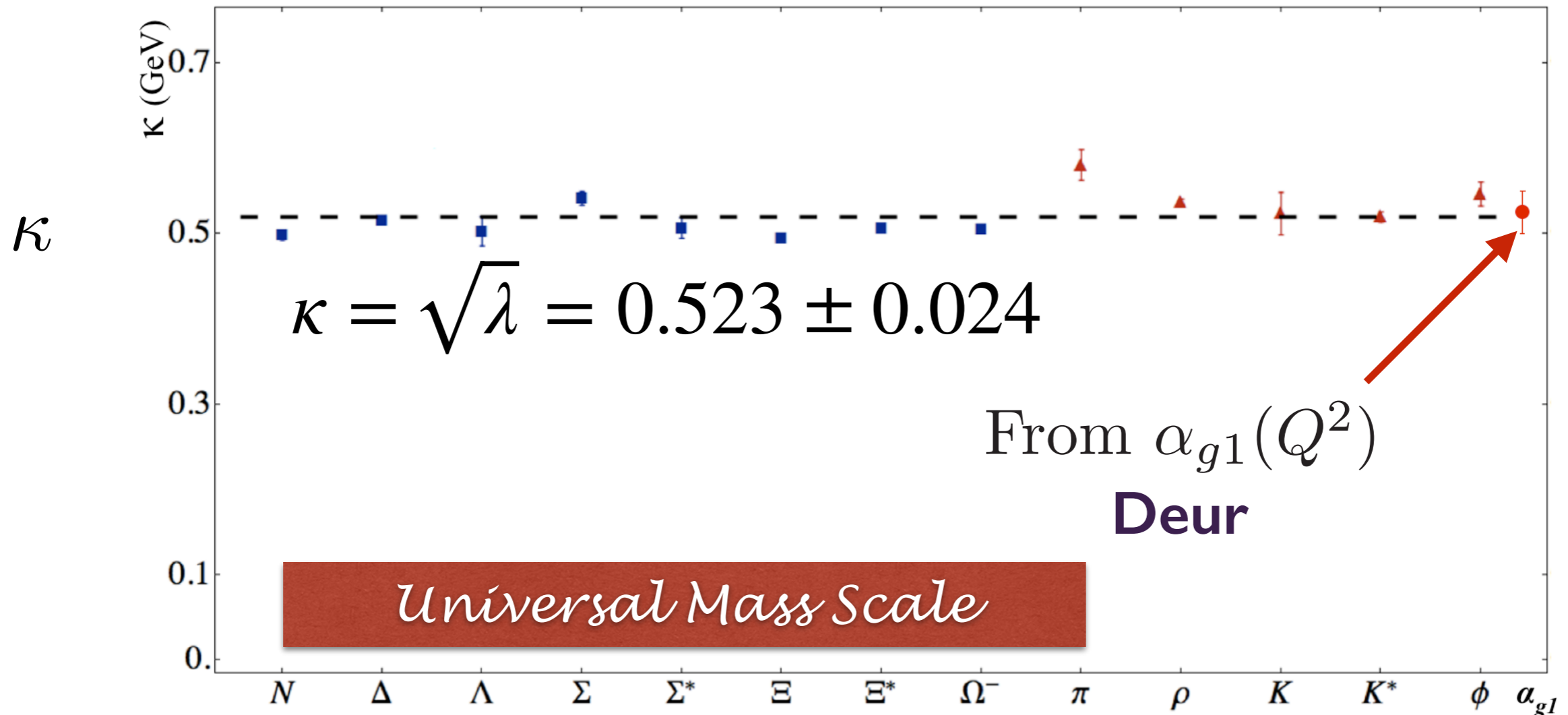
$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon
Mass Degeneracy
for $L_M=L_B+1$**



$$\lambda = \kappa^2$$

$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



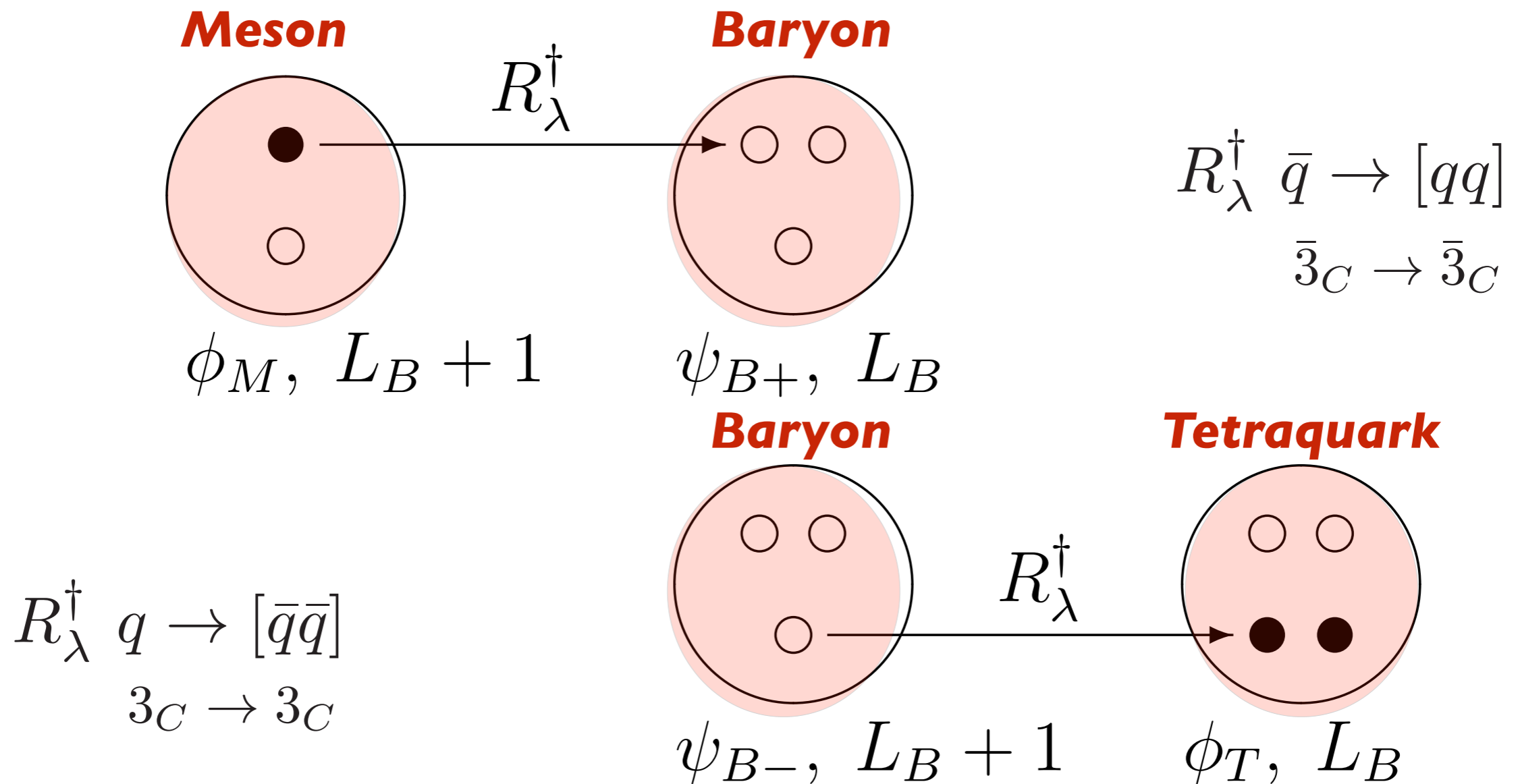
**Fit to the slope of Regge trajectories,
including radial excitations**

**Same Regge Slope for Meson, Baryons:
Supersymmetric feature of hadron physics**

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



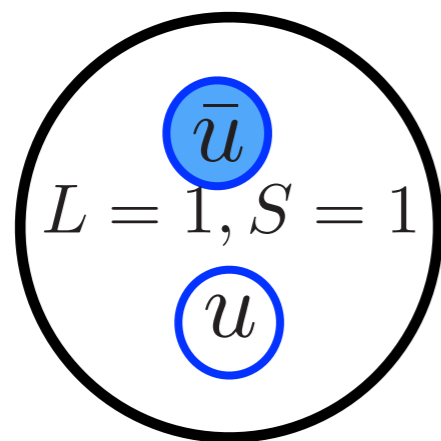
Proton: $|u[ud]\rangle$ Quark + Scalar Diquark
 Equal Weight: $L=0, L=1$

Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \begin{array}{l} \bar{q} \rightarrow (qq) \\ \bar{3}_C \rightarrow \bar{3}_C \end{array} \quad S = 1$$

Vector () + Scalar [] Diquarks

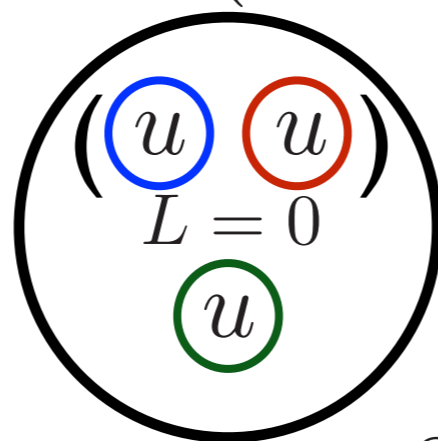
$f_2(1270)$



$$J^{PC} = 2^{++}$$

Meson

$\Delta^+(1232)$



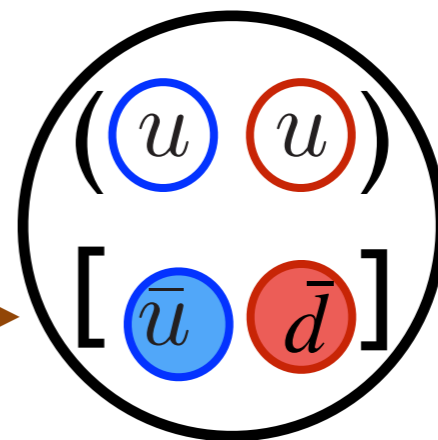
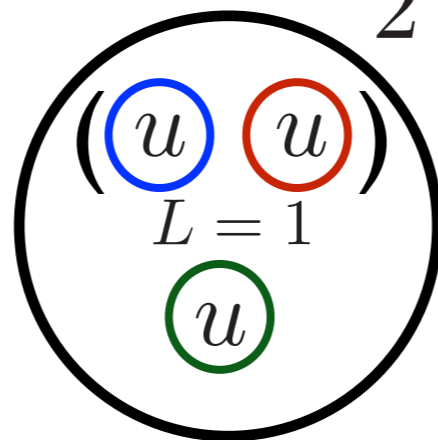
$$J^P = \frac{3}{2}^+$$

Baryon

Tetraquark

$$J^{PC} = 1^{++}$$

$a_1(1260)$



$$\begin{array}{l} S = 0 \\ L = 0 \end{array}$$

$$R_\lambda^\dagger \begin{array}{l} q \rightarrow [\bar{q}\bar{q}] \\ 3_C \rightarrow 3_C \end{array}$$

Meson			Baryon			Tetraquark		
$q\text{-cont}$	$J^{P(C)}$	Name	$q\text{-cont}$	J^P	Name	$q\text{-cont}$	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}}^-(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}}^-(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770), \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320), f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690), \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}}^-(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}}^-(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040), f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}}^+(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)?$
$\bar{q}s$	0^{-+}	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	1^{+-}	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	0^{++}	$K_0^*(1430)$
$\bar{q}s$	2^{-+}	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	1^{-+}	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	0^{-+}	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	1^{+-}	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$ $f_0(980)$
$\bar{s}q$	1^{-+}	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	2^{++}	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	1^{++}	$K_1(1400)$
$\bar{s}q$	3^{-+}	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	2^{-+}	$K_2(\sim 1700)?$
$\bar{s}q$	4^{++}	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	3^{++}	$K_3(\sim 2070)?$
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$ $a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)?$
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)?$
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	1^{++}	$K_1(\sim 1700)?$

Meson

Baryon

Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen,
sjb

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- **Universal quark light-front kinetic energy**

$$\Delta\mathcal{M}_{LFKE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal quark light-front potential energy**

$$\Delta\mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$$

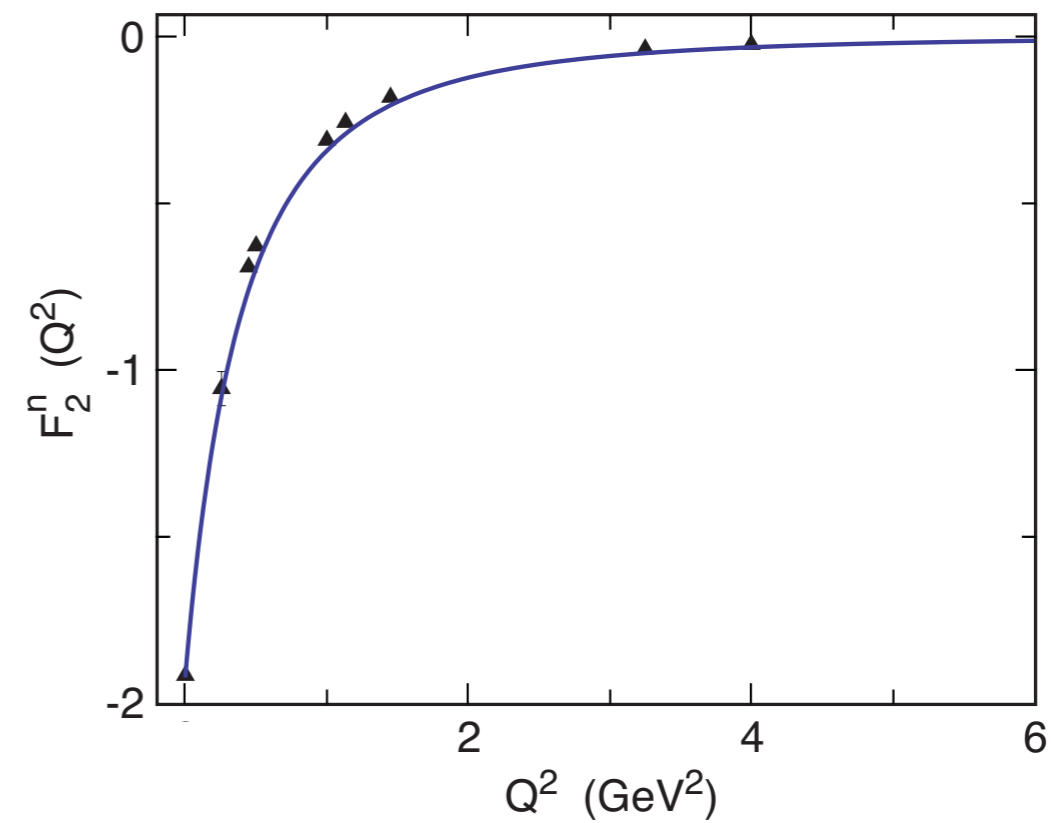
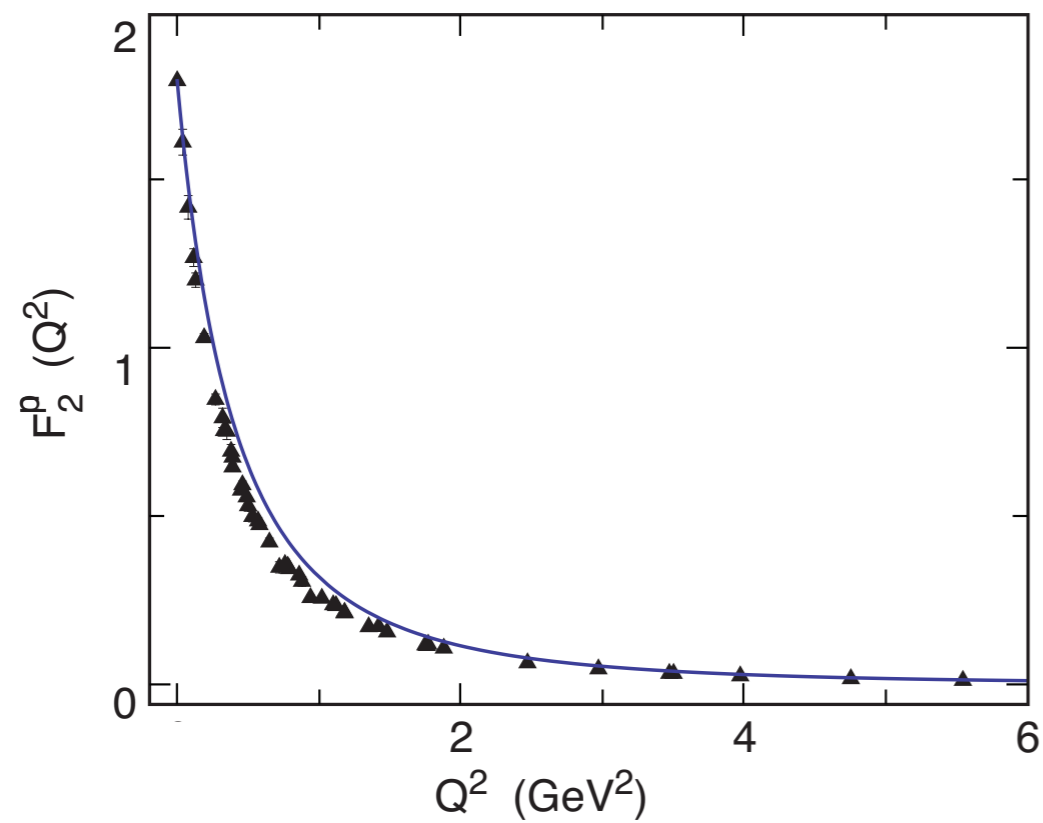
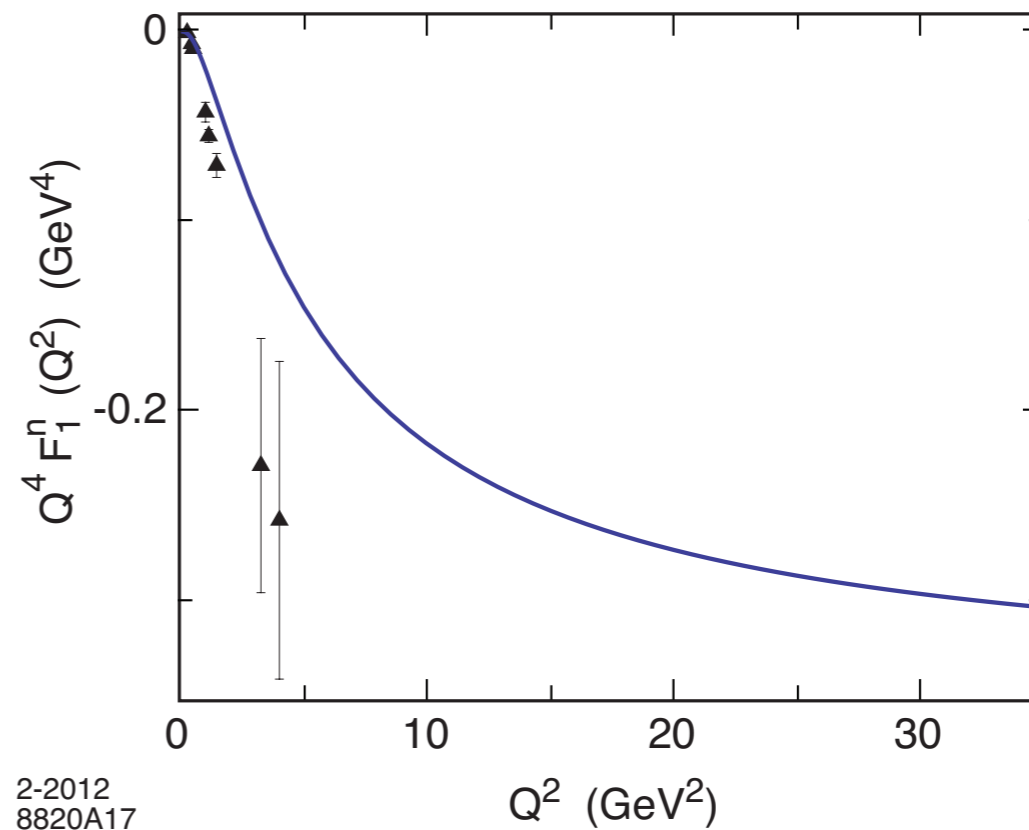
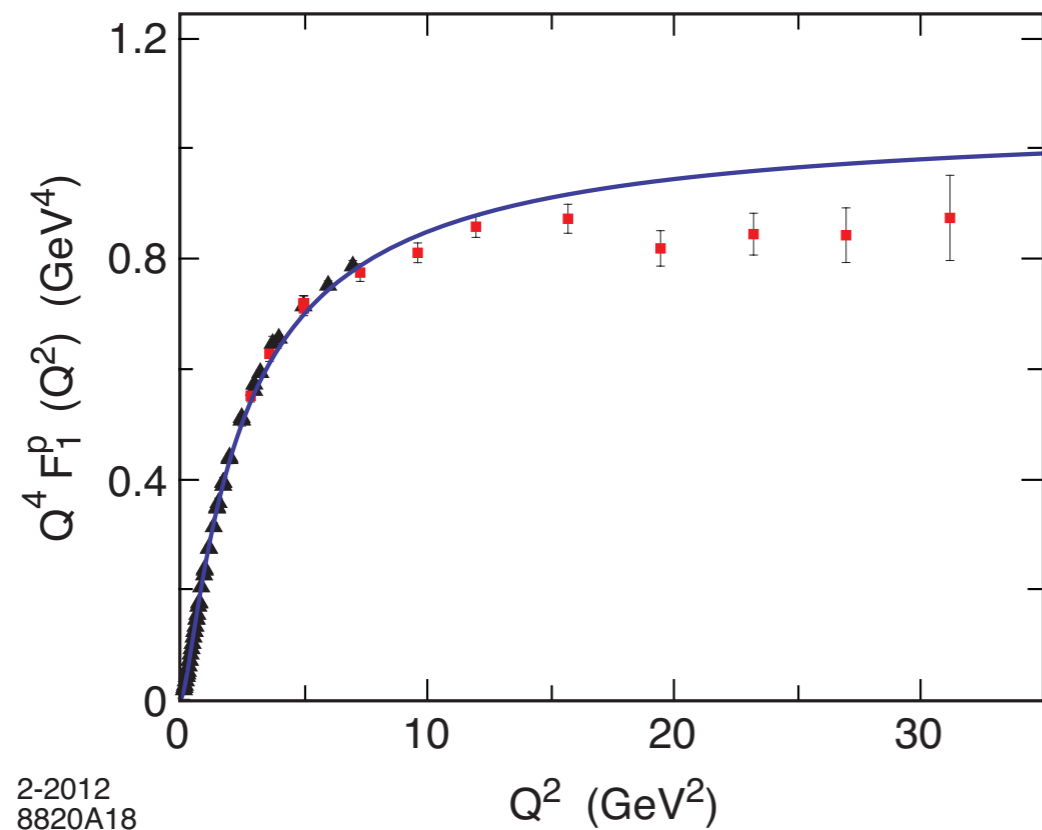
- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**

$$\Delta\mathcal{M}_{spin}^2 = 2\kappa^2(L + 2S + B - 1)$$

hyperfine spin-spin

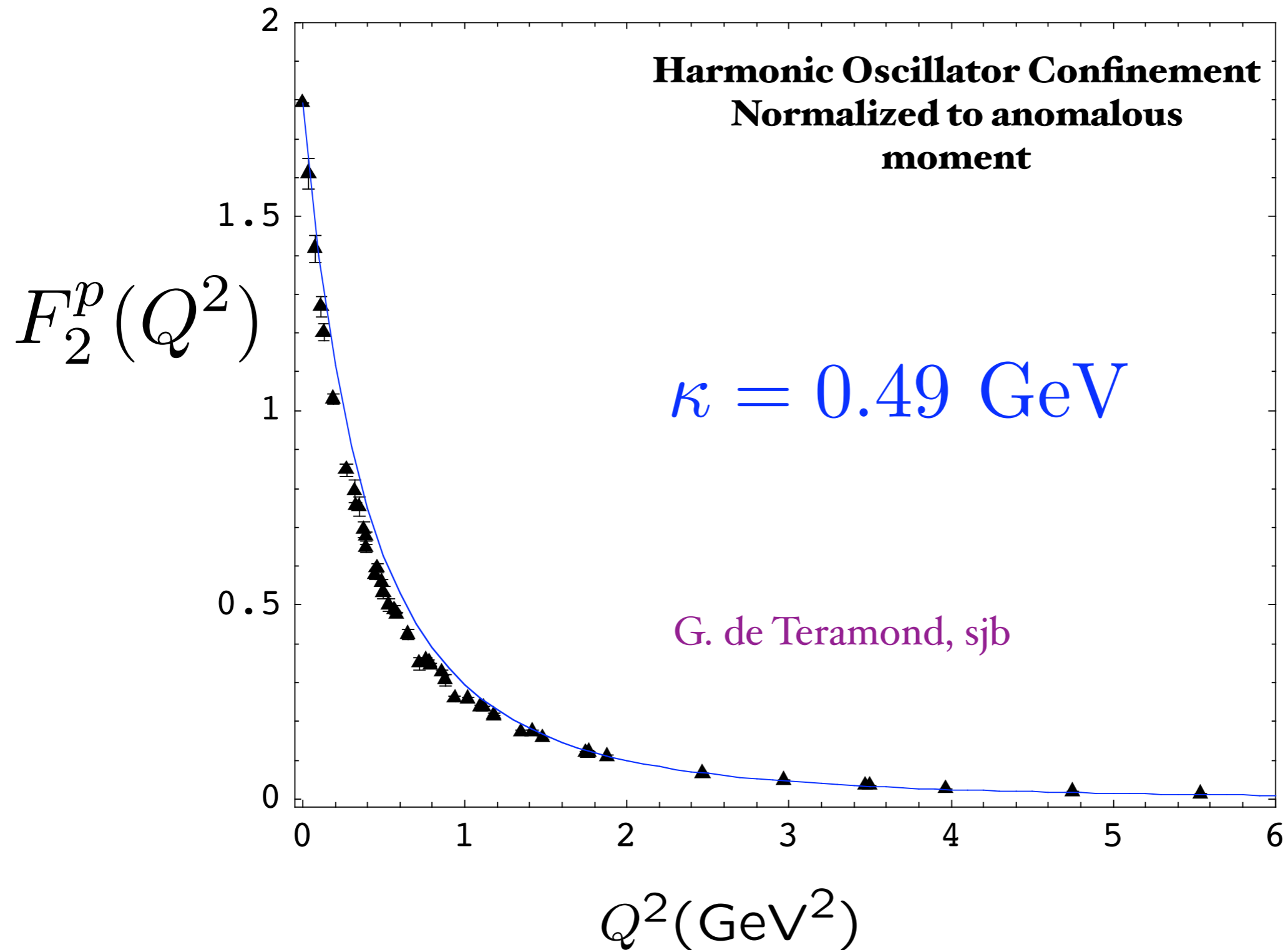
**Equal:
Virial
Theorem**

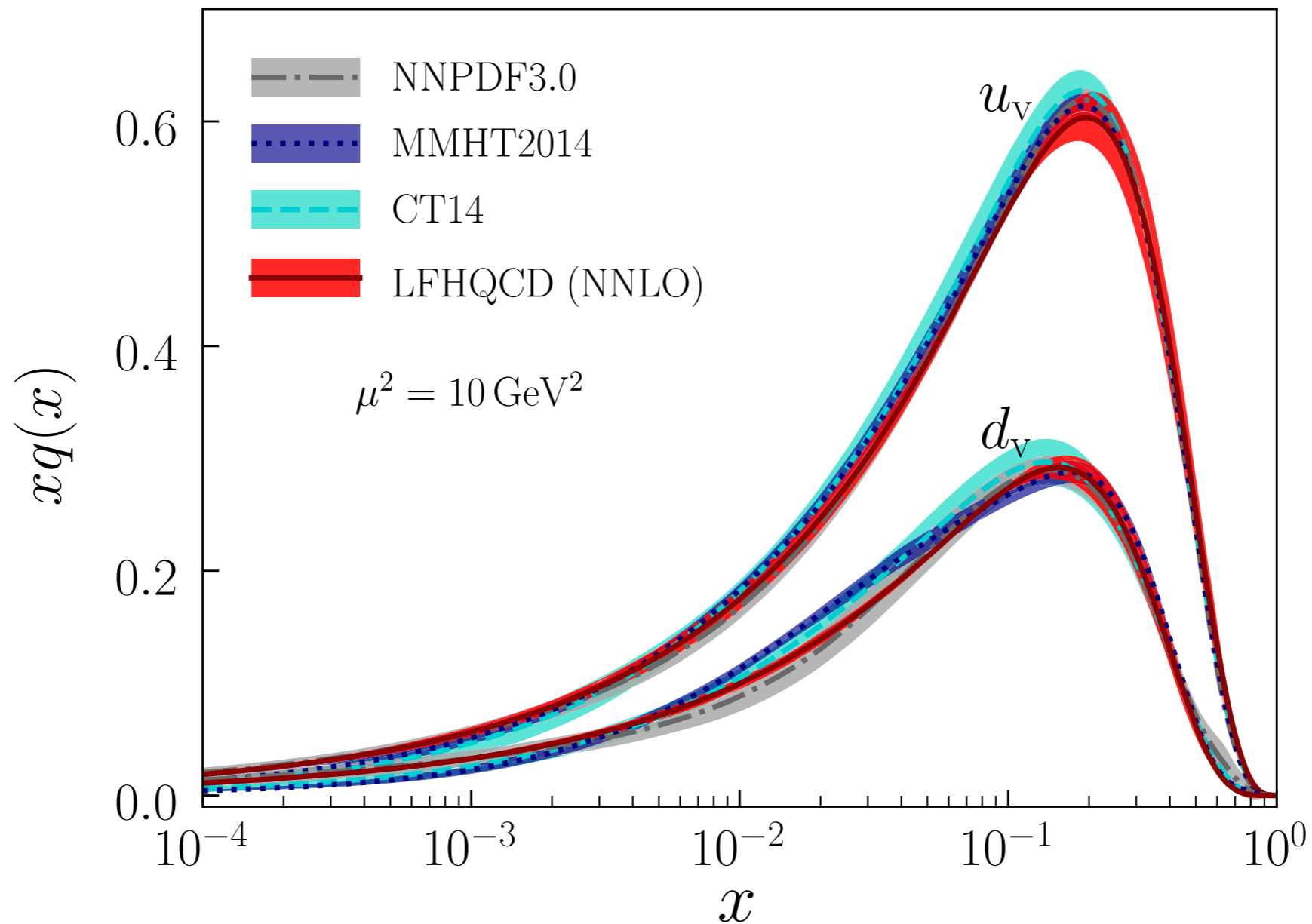
Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs





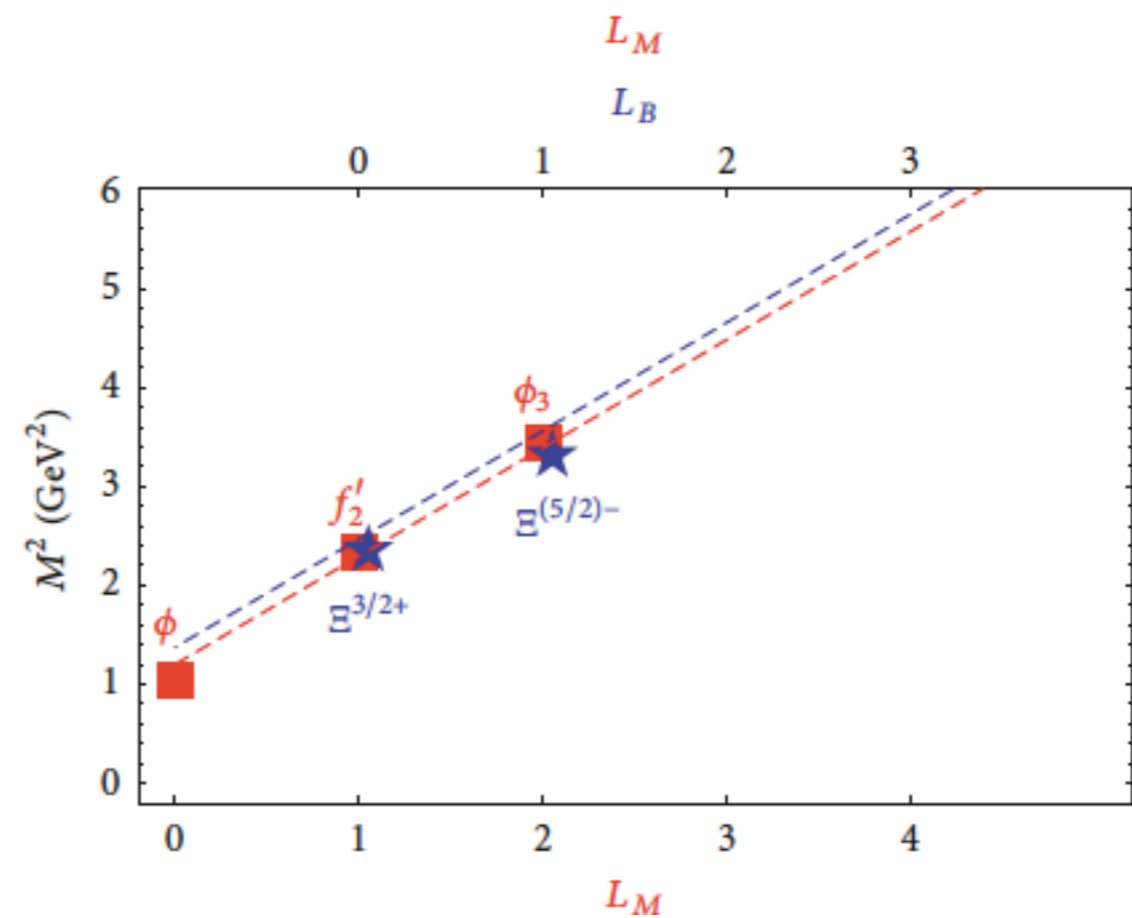
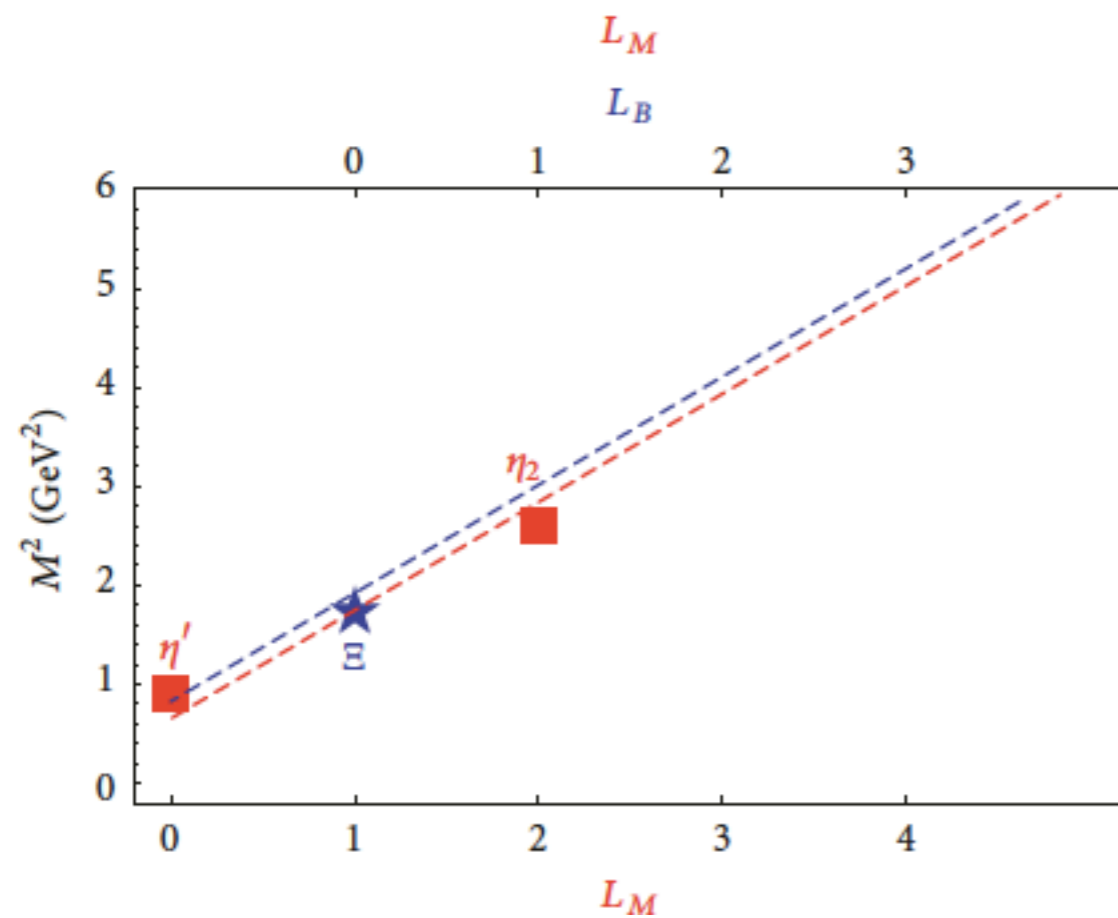
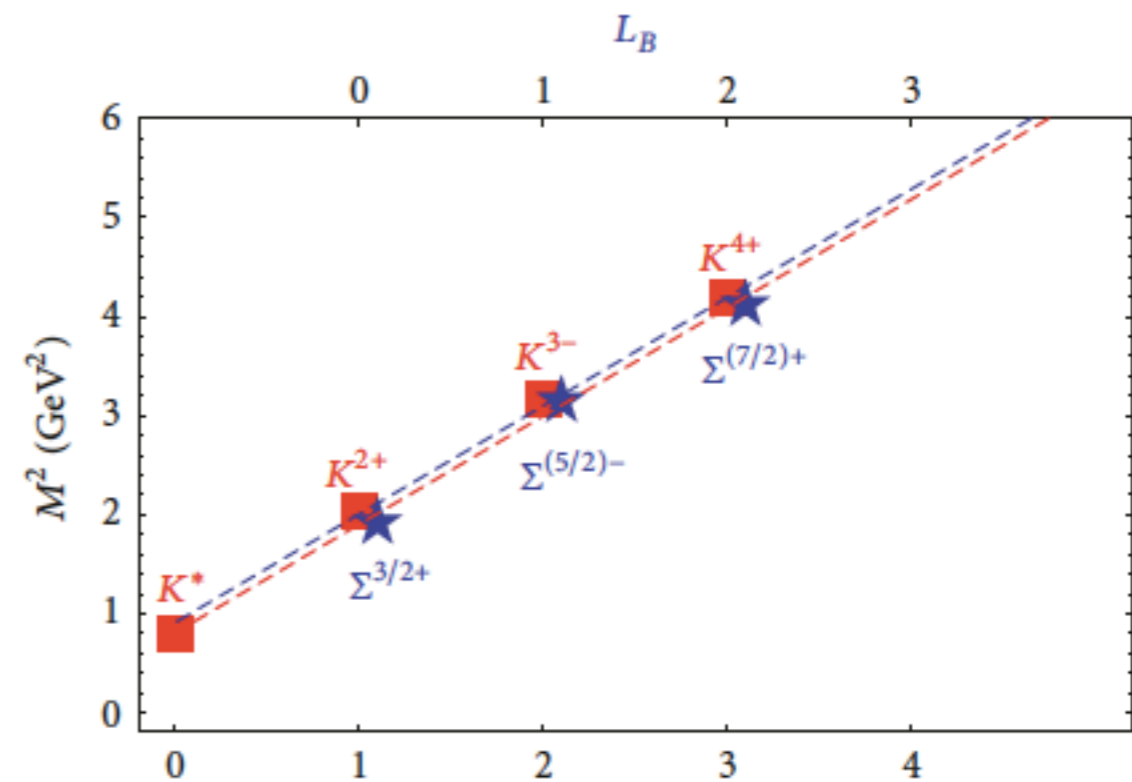
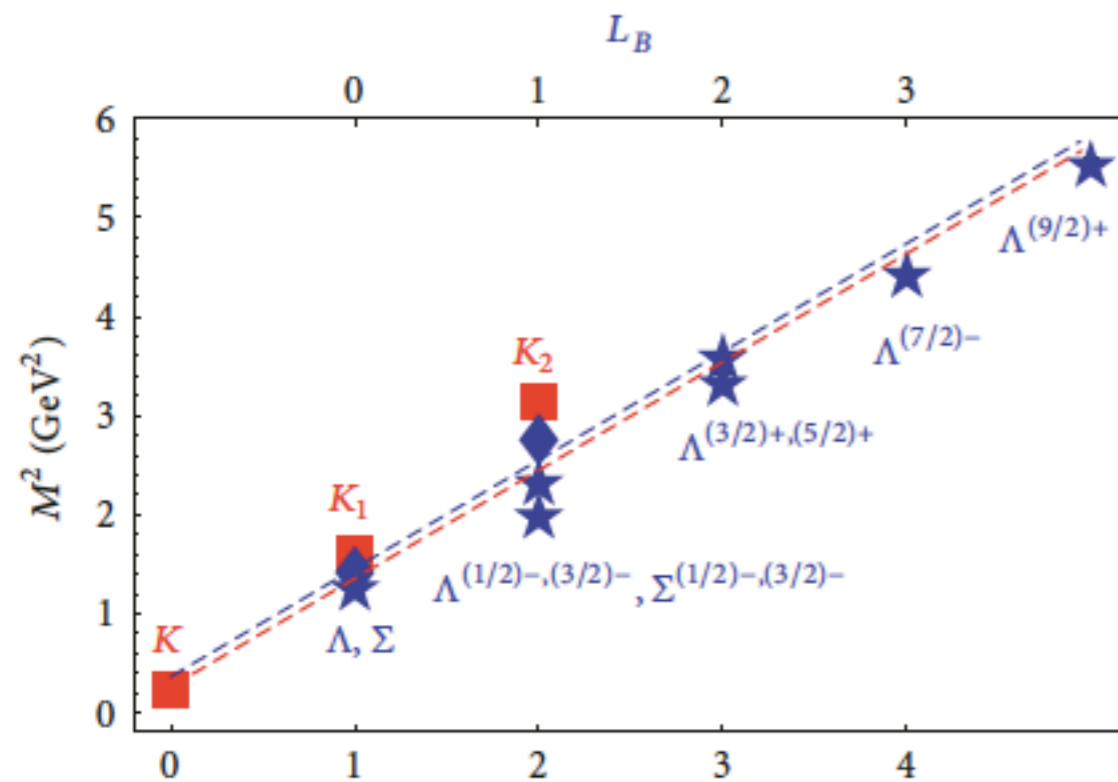
Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

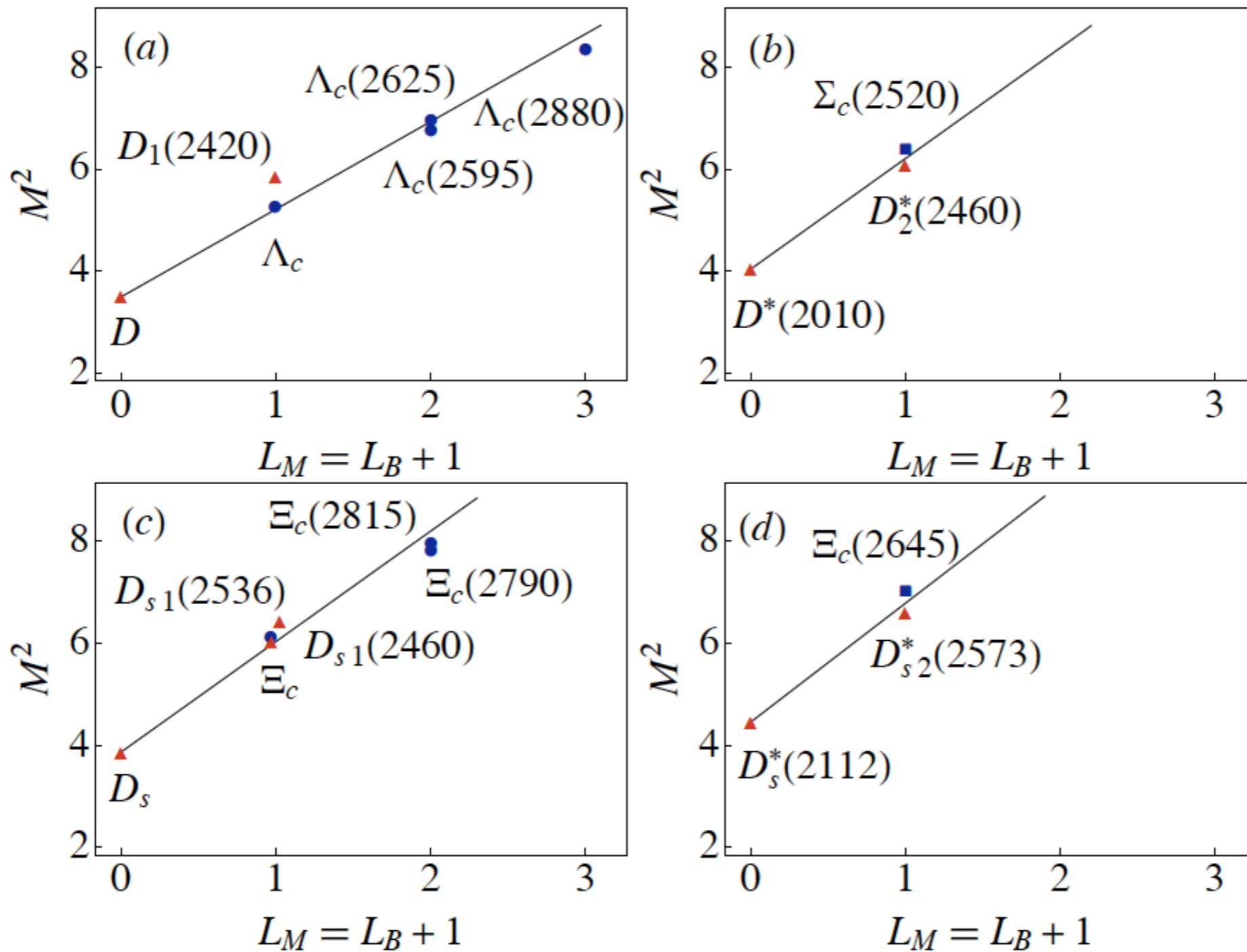
Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

PHYSICAL REVIEW LETTERS 120, 182001 (2018)

Supersymmetry across the light and heavy-light spectrum



Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

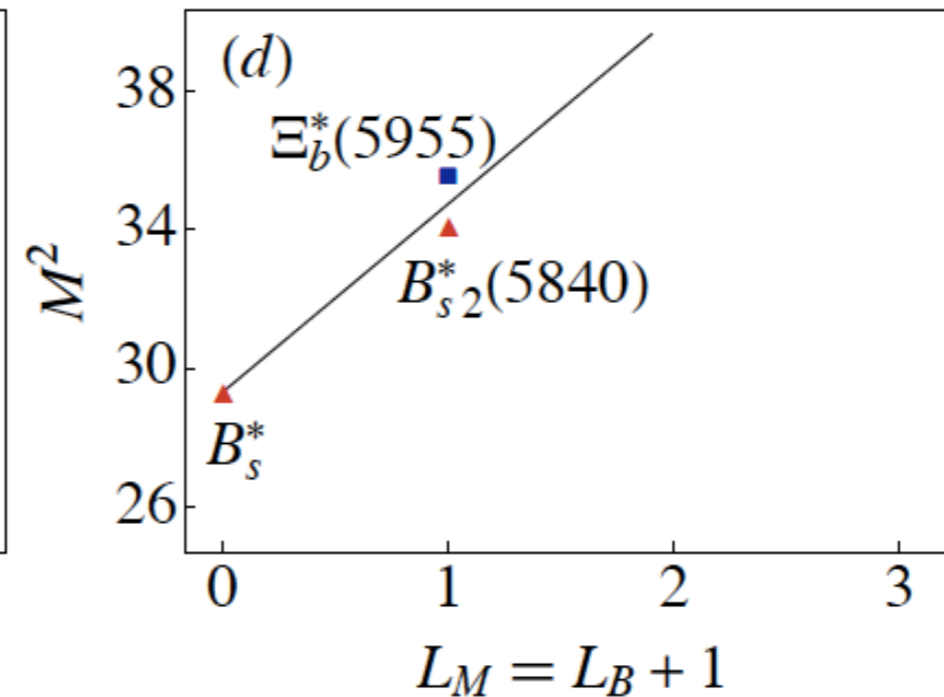
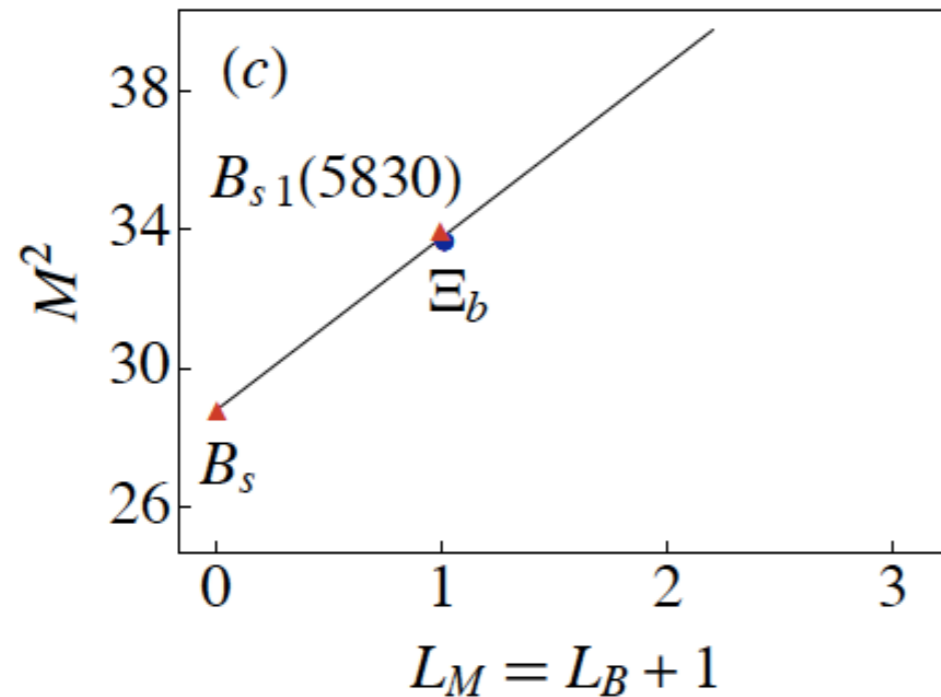
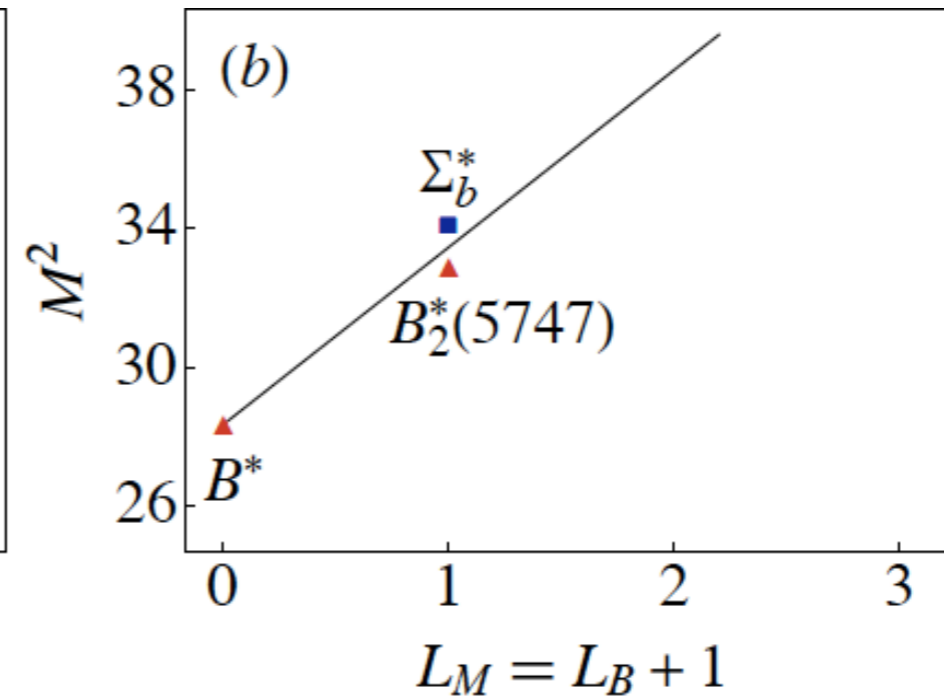
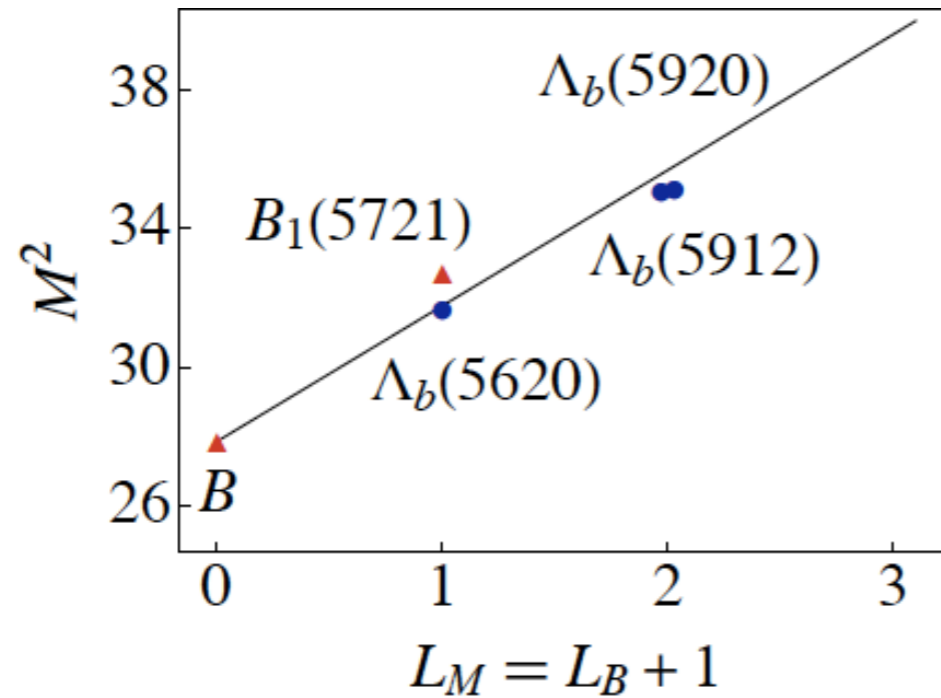
Meson			Baryon			Tetraquark		
q -cont	$J^{P(C)}$	Name	q -cont	J^P	Name	q -cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$\bar{D}_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

M. Nielsen, sjb

predictions

beautiful agreement!

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

[H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

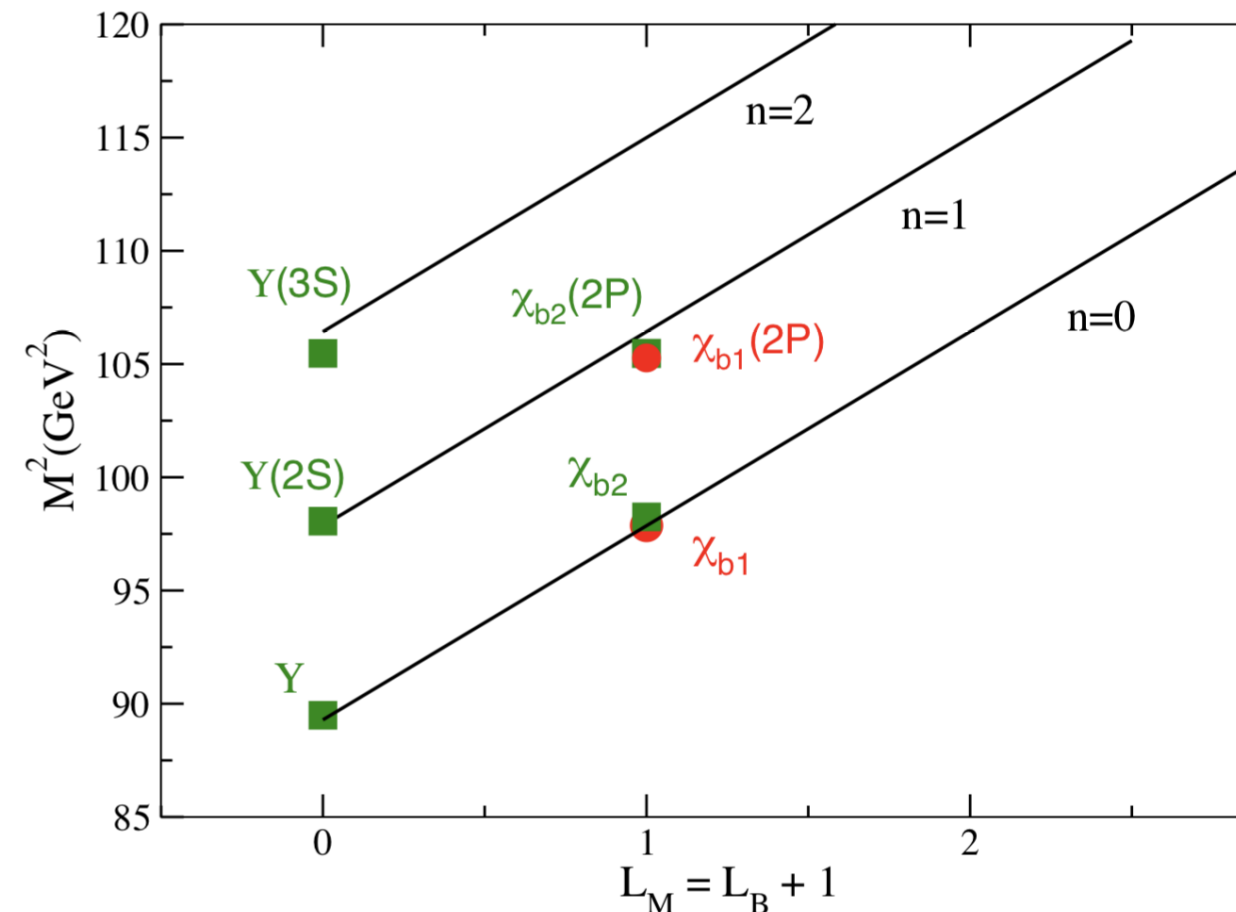
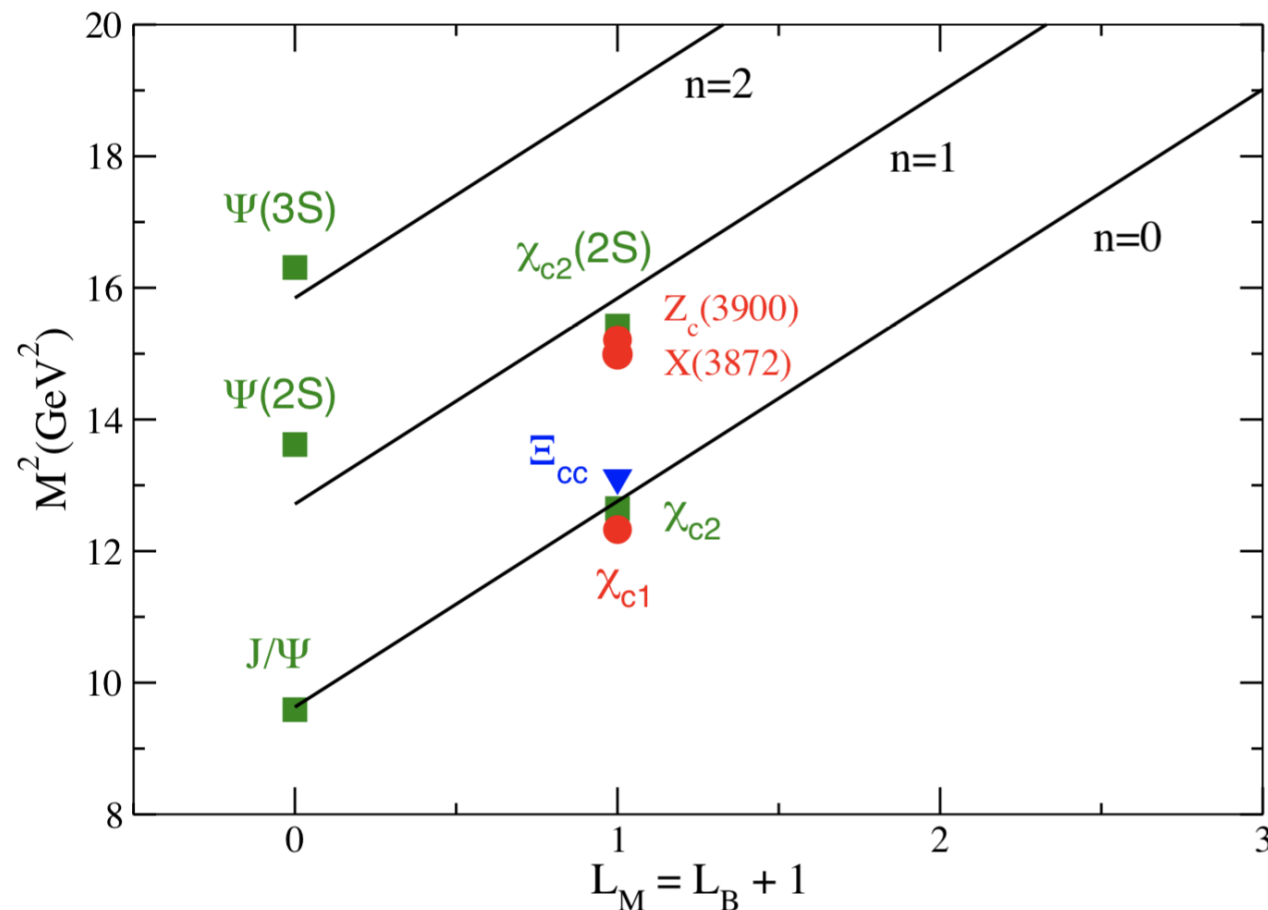
- Extension to the double-heavy hadronic sector

[M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

[M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

[L. Zou, H. G. Dosch, GdT, S. J. Brodsky, arXiv:1901.11205 [hep-ph]]



Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_c$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

de Téramond, Dosch, Lorcé, sjb

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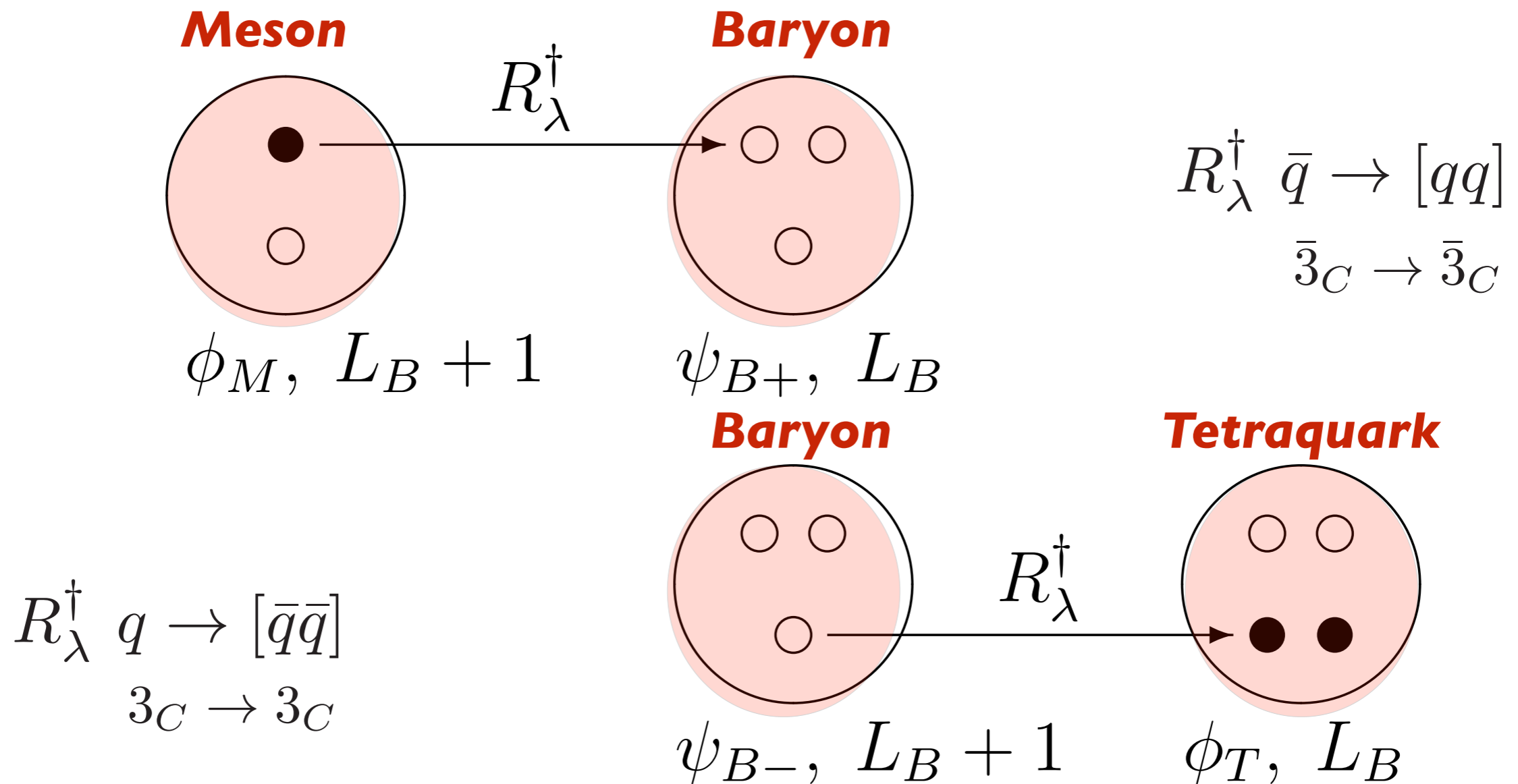


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Superconformal Algebra

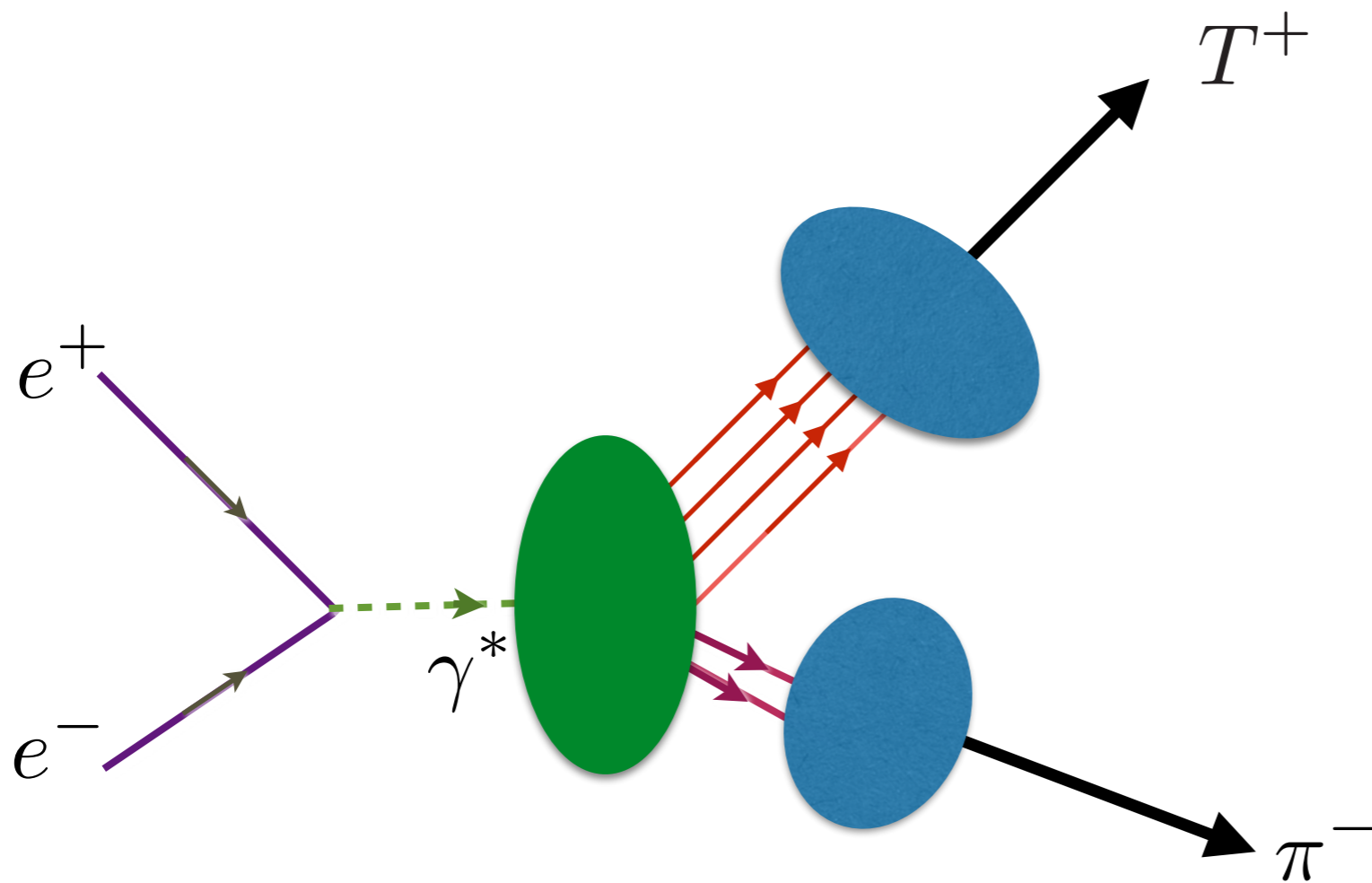
2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: $|u[ud]\rangle$ Quark + Scalar Diquark
 Equal Weight: $L=0, L=1$

$$\sigma(e^+e^- \rightarrow MT) \propto \frac{1}{s^{N-1}} \quad N = 6$$



Use counting rules to identify composite structure

Lebed, sjb

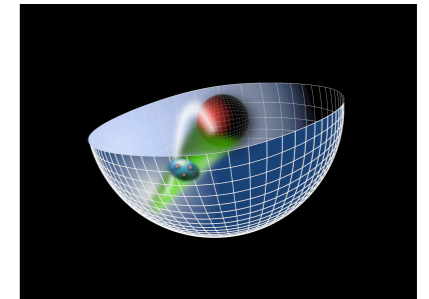
Underlying Principles

- **Polncarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time τ**

- **Causality: Information within causal horizon: Light-Front**

- **Light-Front Holography: $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce mass scale κ while retaining the Conformal Invariance of the Action (dAFF)**

“Emergent Mass”

- **Unique Dilaton in AdS_5 : $e^{+\kappa^2 z^2}$**

- **Unique color-confining LF Potential $U(\zeta^2) = \kappa^4 \zeta^2$**

- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson $q\bar{q} \leftrightarrow$ Baryon $q[qq] \leftrightarrow$ Tetraquark $[qq][\bar{q}\bar{q}]$

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Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \quad S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

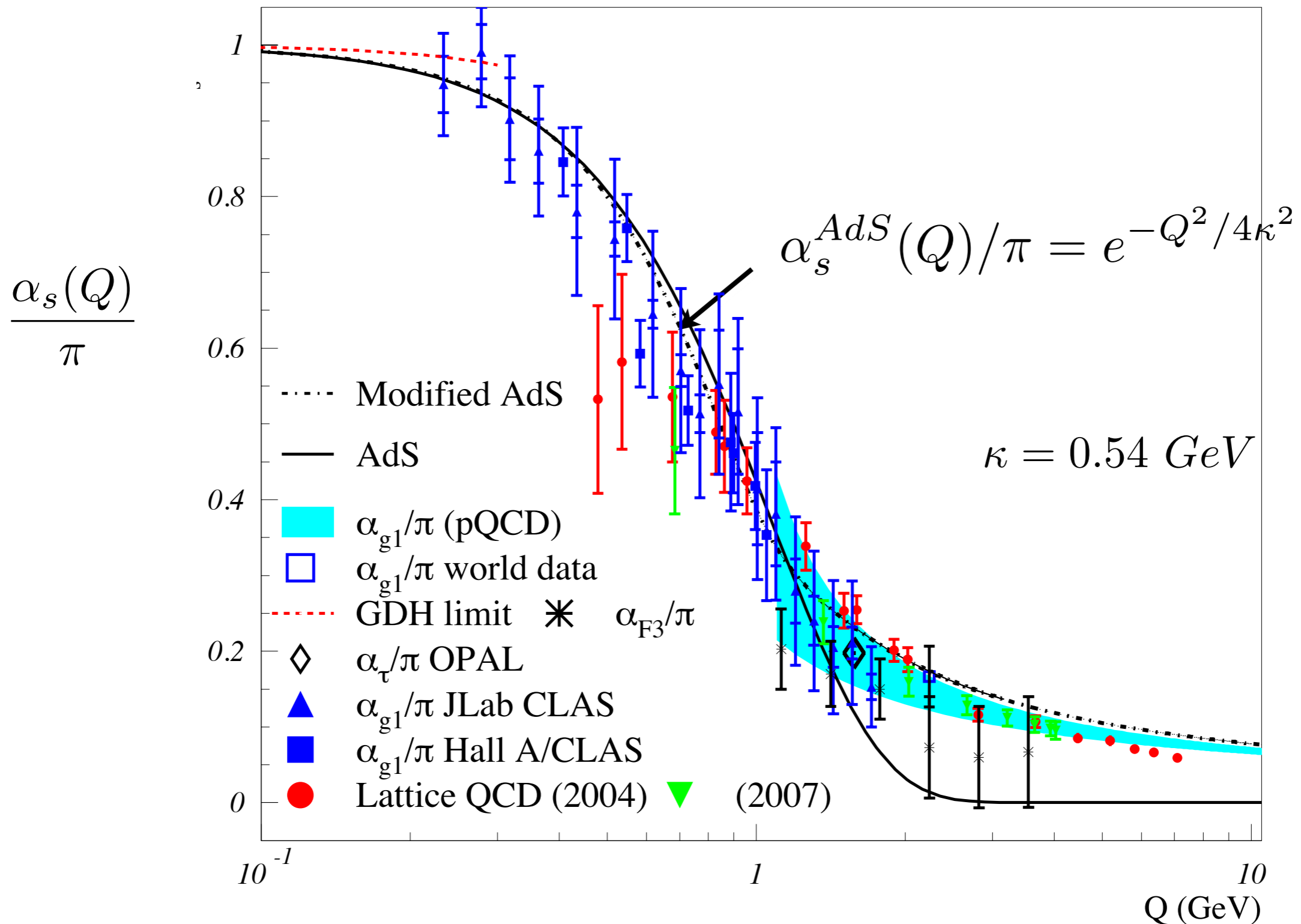
Bjorken sum rule defines effective charge

$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large Q^2**
- **Computable at large Q^2 in any pQCD scheme**
- **Universal β_0, β_1**

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1 \text{ GeV}$

$$e^\varphi = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

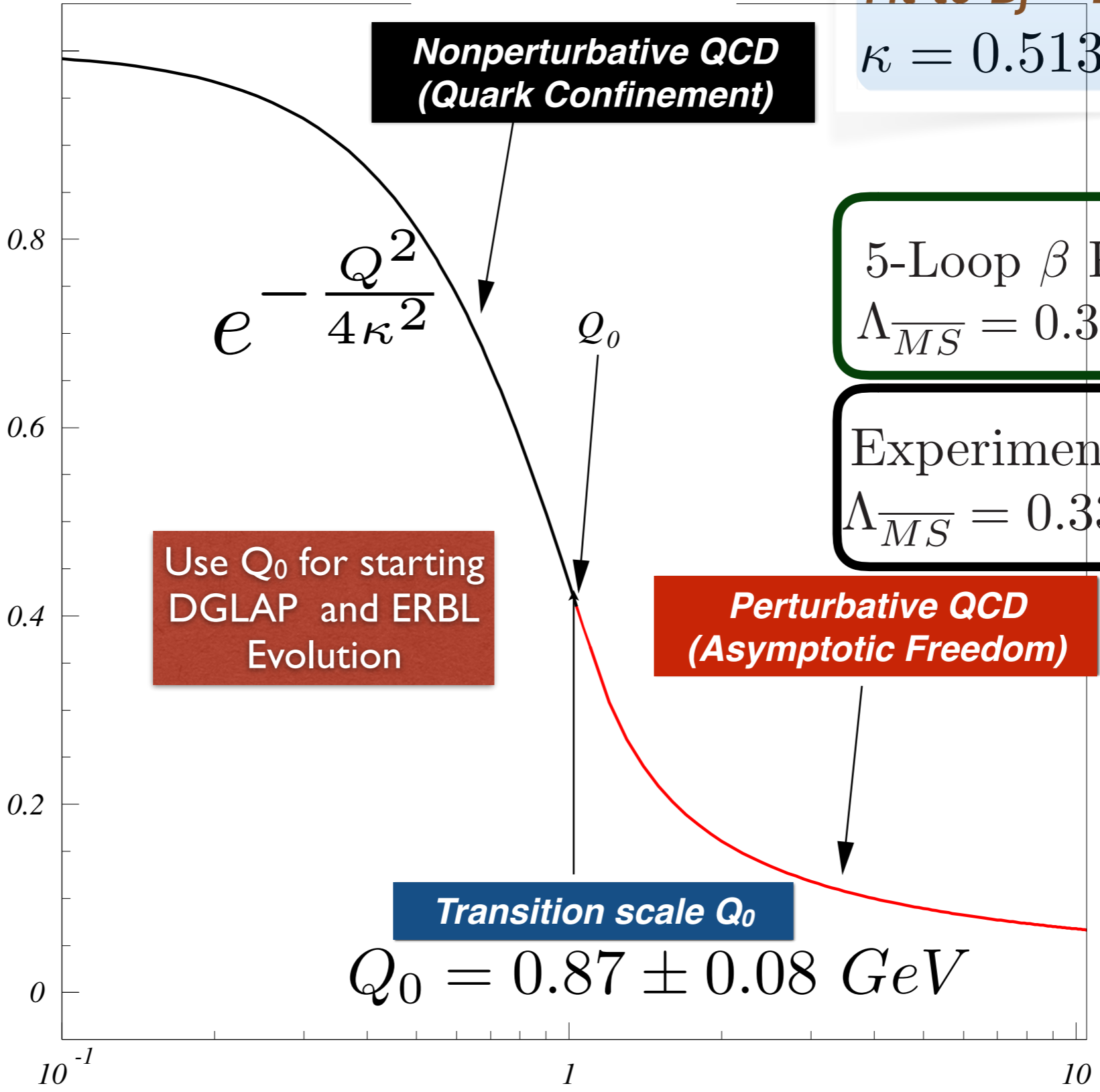
$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

All-Scale QCD Coupling

Fit to Bj + DHG Sum Rules:
 $\kappa = 0.513 \pm 0.007 \text{ GeV}$

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$



5-Loop β Prediction:
 $\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$

Experiment:
 $\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV}$

Use Q_0 for starting
 DGLAP and ERBL
 Evolution

**Perturbative QCD
 (Asymptotic Freedom)**

Transition scale Q_0

$$Q_0 = 0.87 \pm 0.08 \text{ GeV}$$

$$\lambda \equiv \kappa^2$$

Reverse Dimensional Transmutation!

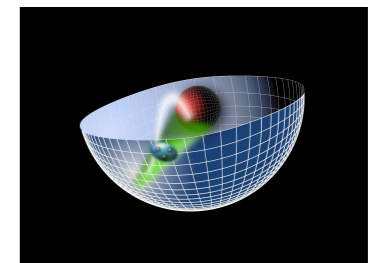
Q (GeV)

\overline{MS} scheme

Underlying Principles

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- **Unique Dilaton in AdS_5 : $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson $q\bar{q} \leftrightarrow$ Baryon $q[qq] \leftrightarrow$ Tetraquark $[qq][\bar{q}\bar{q}]$

What is PMC ?

Principle of Maximum Conformality

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$

Choose μ_R^{init} ; arbitrary initial renormalization scale

Identify $\{\beta_i^R\}$ – terms using n_f – terms
through the PMC – BLM correspondence principle

order-by-order ↓

Shift scale of α_s to μ_R^{PMC} to eliminate $\{\beta_i^R\}$ – terms

Conformal Series

Result is independent of μ_R^{init} and scheme at fixed order

*Xing-Gang Wu, Martin Mojaza
Leonardo di Giustino, Sfb*

PMC-BLM – one

Phys. Rev. Lett. **109**, 042002 (2012)

R_δ -scheme – two

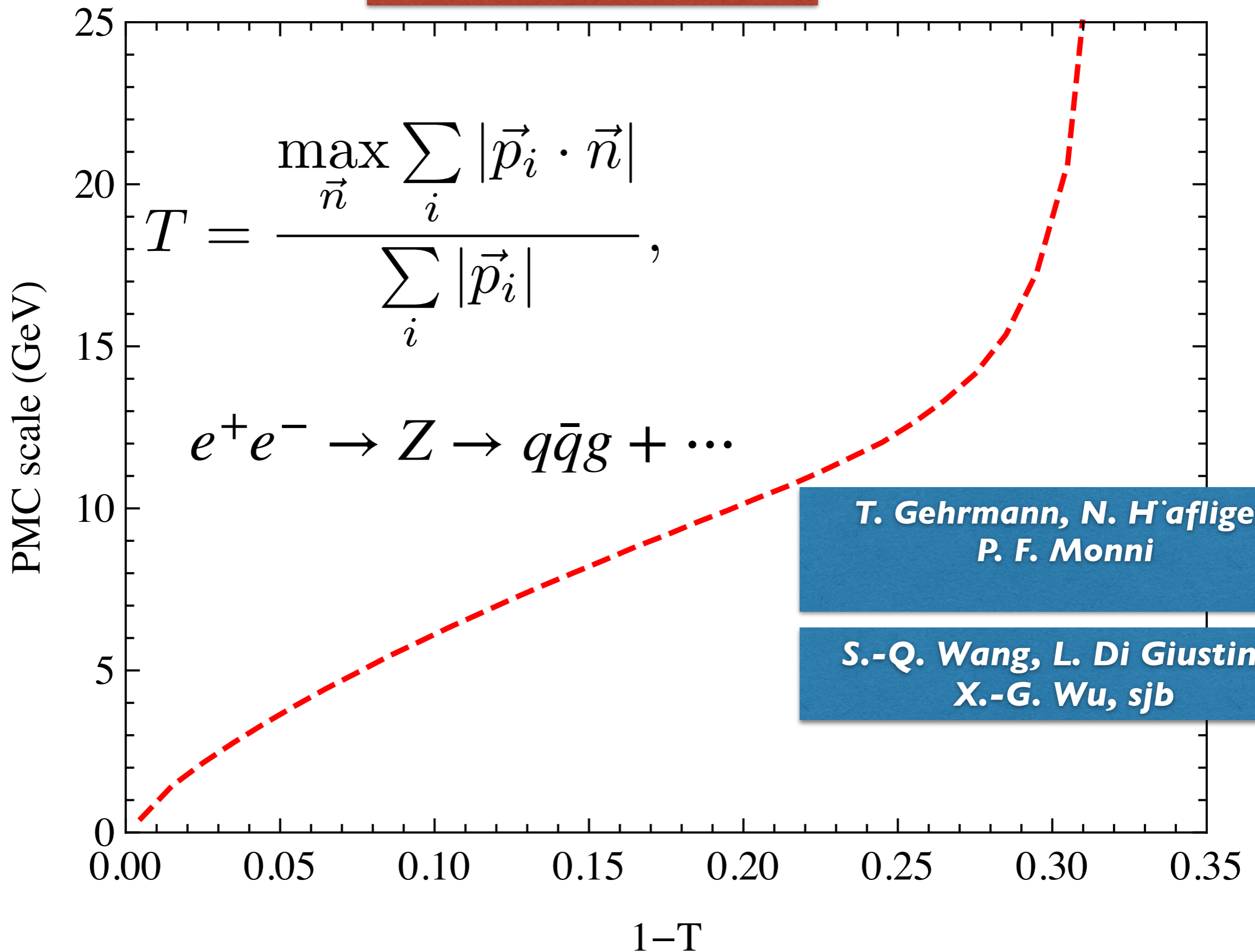
Phys. Rev. Lett. **110**, 192001 (2013)

Eliminate β -terms

**n_f dependence of pQCD series does not
uniquely identify the β terms**

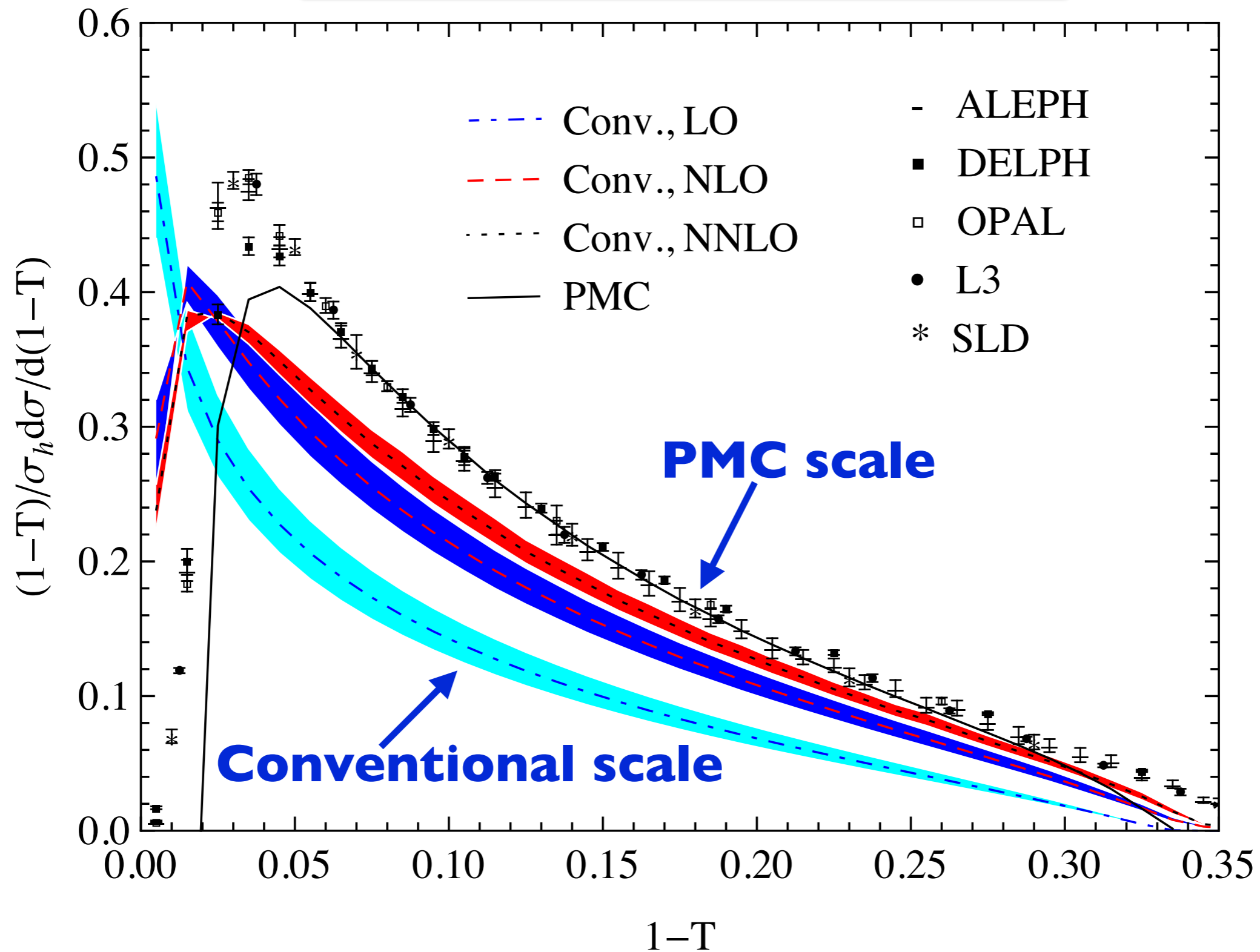
Renormalization scale depends on the thrust

Not constant !

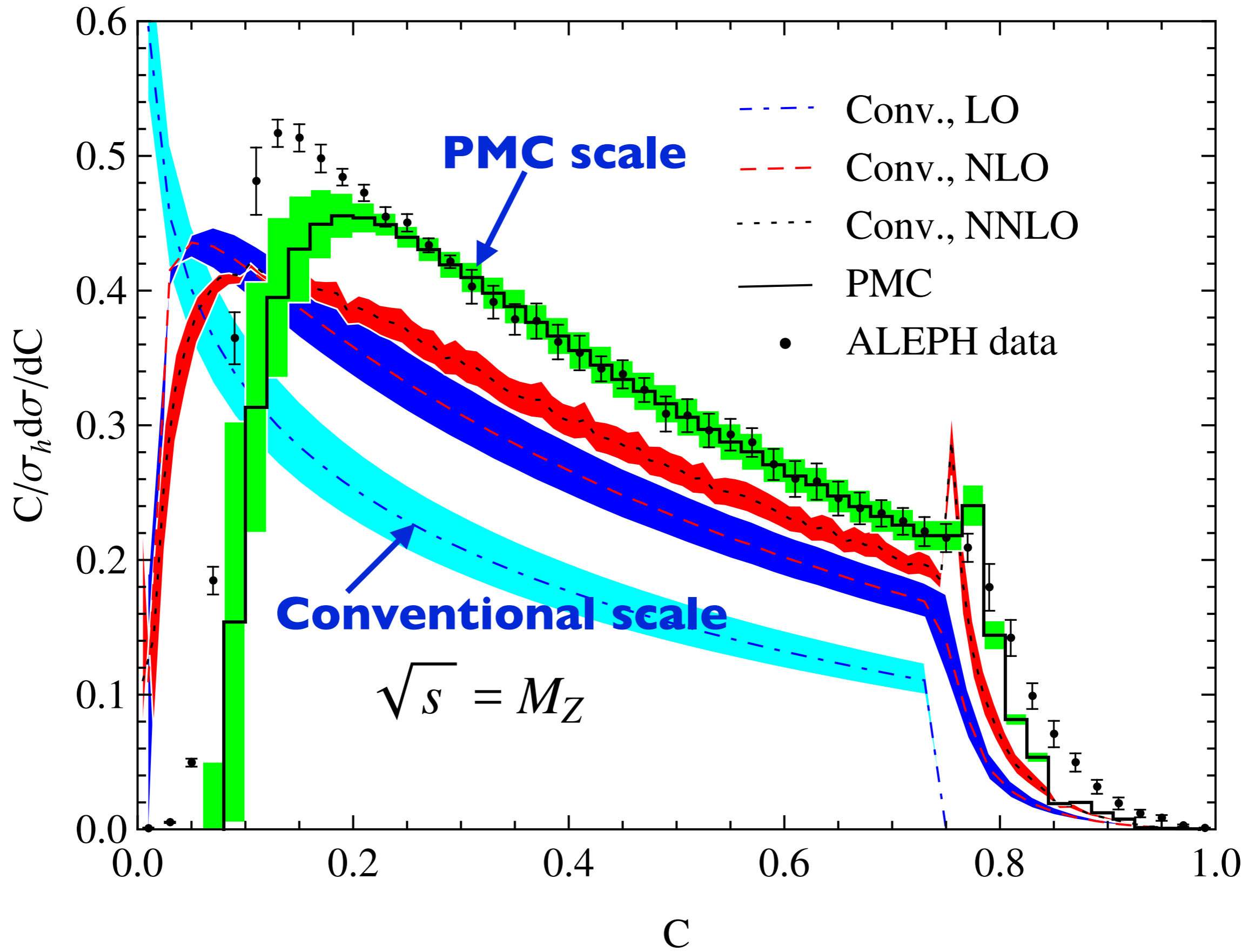


T. Gehrmann, N. H'afliker, P. F. Monni

S.-Q. Wang, L. Di Giustino, X.-G. Wu, sjb

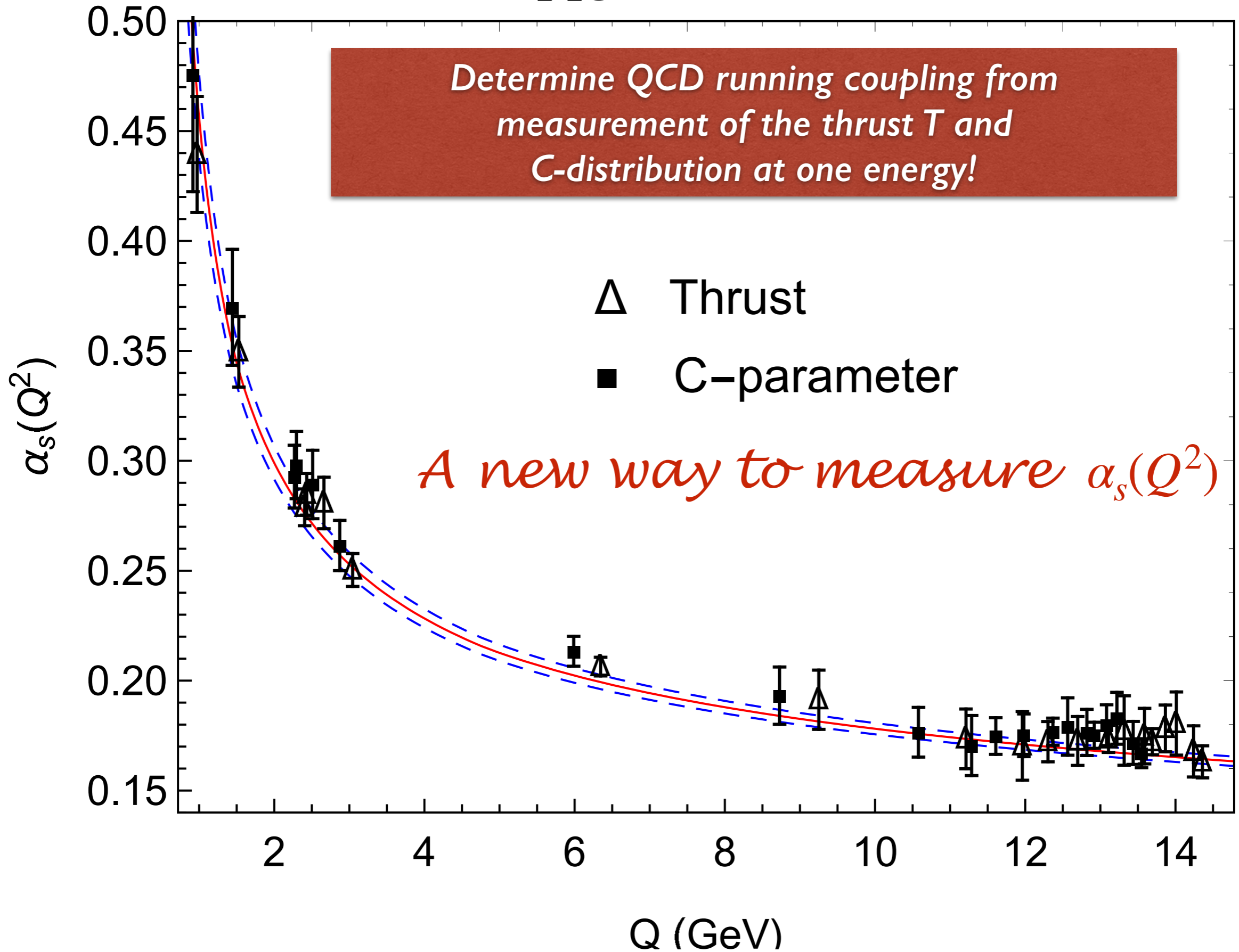


Principle of Maximum Conformality (PMC)



Principle of Maximum Conformality (PMC)

$e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}g + \dots$ $\alpha_s(Q^2)$ in \overline{MS} scheme

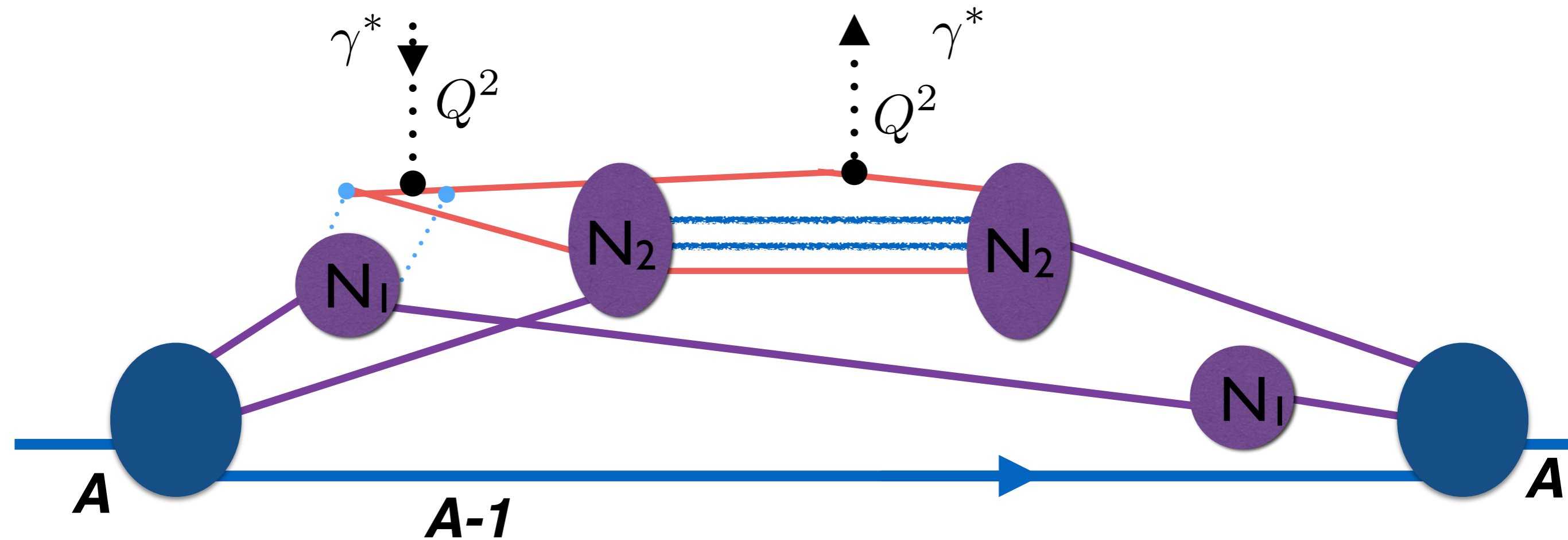


Features of BLM/PMC

- **Predictions are scheme-independent at every order**
- **Matches conformal series**
- **No $n!$ Renormalon growth of pQCD series**
- **New scale appears at each order; n_F determined at each order - matches virtuality of quark loops**
- **Multiple Physical Scales Incorporated (Hoang, Kuhn, Tuebner, sjb)**
- **Rigorous: Satisfies all Renormalization Group Principles**
- **Realistic Estimate of Higher-Order Terms**
- **Reduces to standard QED scale $N_C \rightarrow 0$**
- **GUT: Must use the same scale setting procedure for QED, QCD**
- **Eliminates unnecessary theory error**
- **Maximal sensitivity to new physics**
- **Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsmann, sjb)**
- **PMC Reduces to BLM at NLO: Example: BFKL intercept (Fadin, Kim, Lipatov, Pivovarov, sjb)**

Illustrates the LF time sequence

$$q^+ = 0 \quad q_{\perp}^2 = Q^2 = -q^2$$



Front-Face Nucleon N_1 struck

Front-Face Nucleon N_1 not struck

One-Step / Two-Step Interference

Study Double Virtual Compton Scattering $\gamma^* A \rightarrow \gamma^* A$

Cannot reduce to matrix element of local operator! No Sum Rules! Liuti, Schmidt sjb

- Unlike shadowing, anti-shadowing from Reggeon exchange is flavor specific;
- Each quark and anti-quark will have distinctly different constructive interference patterns
- The flavor dependence of antishadowing explains why anti-shadowing is different for electron (neutral electro-magnetic current) vs. neutrino (charged weak current) DIS reactions.
- Test of the explanation of antishadowing: Bjorken-scaling leading-twist charge exchange DDIS reaction $\gamma^*p \rightarrow nX^+$ with a rapidity gap due to $I=1$ Reggeon exchange
- The finite path length due to the on-shell propagation of V^0 between N_1 and N_2 contributes a finite distance $(\Delta z)^2$ between the two virtual photons in the DVCS amplitude.

The usual “handbag” diagram where the two $J^\mu(x)$ and $J^\nu(0)$ currents acting on an uninterrupted quark propagator are replaced by a local operator $T^{\mu\nu}(0)$ as $Q^2 \rightarrow \infty$, is inapplicable in deeply virtual Compton scattering from a nucleus since the currents act on different nucleons.

Δz^2 does not vanish as $\frac{1}{Q^2}$.

OPE and Sum Rules invalid for nuclear pdfs

Invariance Principles of Quantum Field Theory

- **Polncarè Invariance:** Physical predictions must be independent of the observer's Lorentz frame: *Front Form*
- **Causality:** Information within causal horizon: *Front Form*
- **Gauge Invariance:** Physical predictions of gauge theories must be independent of the choice of gauge
- **Scheme-Independence:** Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — *Principle of Maximum Conformality (PMC)*
- **Mass-Scale Invariance:** *Conformal Invariance of the Action (DAFF)*

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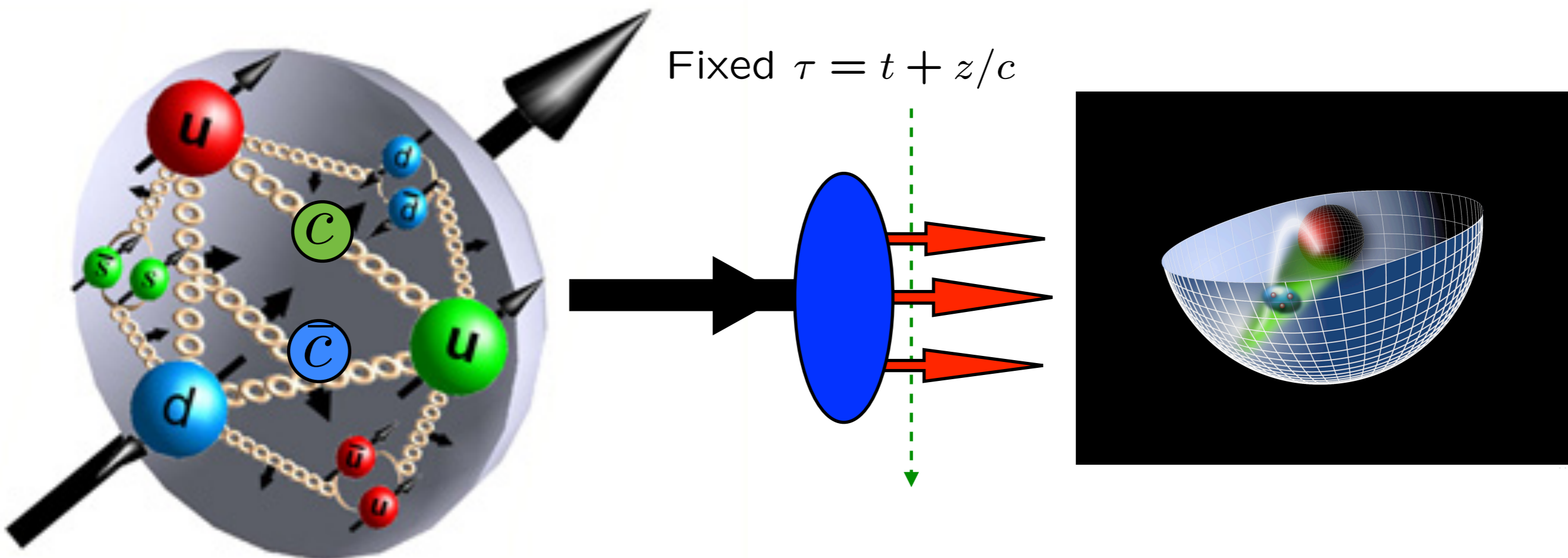
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Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography and Superconformal Algebra



with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Tianbo Llu, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur



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September 19, 2019