

# A new method to compute GPDs

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In collaboration with:  
N. Chouika, H. Moutarde and J. Rodriguez-Quintero

# *Part 1: Definitions and Properties*

- Generalized Parton Distributions (GPDs):

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  - ▶ are defined according to a non-local matrix element,

$$\begin{aligned} & \frac{1}{2} \int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \psi^q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle dz^- |_{z^+=0, z=0} \\ &= \frac{1}{2P^+} \left[ H^q(x, \xi, t) \bar{u} \gamma^+ u + E^q(x, \xi, t) \bar{u} \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u \right]. \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} \int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \gamma_5 \psi^q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle dz^- |_{z^+=0, z=0} \\ &= \frac{1}{2P^+} \left[ \tilde{H}^q(x, \xi, t) \bar{u} \gamma^+ \gamma_5 u + \tilde{E}^q(x, \xi, t) \bar{u} \frac{\gamma_5 \Delta^+}{2M} u \right]. \end{aligned}$$

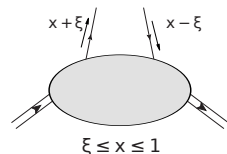
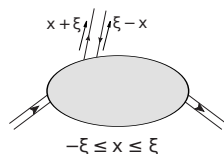
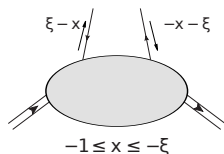
D. Müller *et al.*, Fortsch. Phys. 42 101 (1994)

X. Ji, Phys. Rev. Lett. 78, 610 (1997)

A. Radyushkin, Phys. Lett. B380, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

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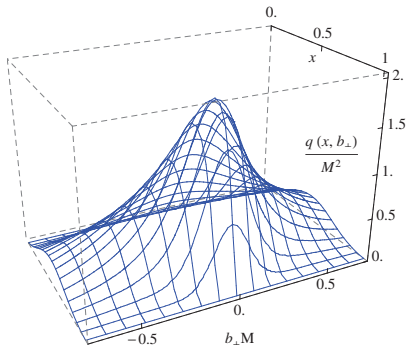


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M. Burkardt, Phys. Rev. D62, 071503 (2000)



Pion GPD in Impact  
parameter space from:  
CM *et al.*, Phys. Lett. **B741**,  
190-196 (2015)

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X. Ji, PRL 78, 610–613 (1997)

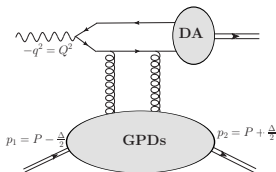
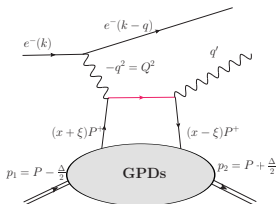
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## Bottom line

GPDs offer a priori an excellent framework  
for studying emerging phenomena in hadron physics

- Polynomiality Property:

$$\int_{-1}^1 dx x^m H^q(x, \xi, t) = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} \xi^{2j} C_{2j}^q(t) + \text{mod}(m, 2) \xi^{m+1} C_{m+1}^q(t)$$

Lorentz Covariance

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x, \xi, t) - \frac{\xi^2}{1 - \xi^2} E^q(x, \xi, t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1 - \xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)

B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)

M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

P.V. Pobilitza, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

- Polynomiality Property:

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Relativistic quantum mechanics

- Soft pion theorem (pion GPDs only)

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)  
CM *et al.*, Phys. Lett. **B741**, 190 (2015)

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Axial-Vector WTI

## Problem

There is no model (until now) fulfilling a priori all these constraints.

- GPDs are related to Double Distributions (DDs) through:

$$H(x, \xi, t) = \int_{\Omega} d\beta d\alpha (F(\beta, \alpha, t) + \xi G(\beta, \alpha, t)) \delta(x - \beta - \xi\alpha)$$

The Dirac  $\delta$  insures that the polynomiality is fulfilled, independently of our choice of  $F$  and  $G$



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Positivity property is not guaranteed, and may be violated.

- On the light front, hadronic states can be expanded on a Fock basis:

$$|P, \pi\rangle \propto \sum_{\beta} \Phi_{\beta}^{q\bar{q}} |q\bar{q}\rangle + \sum_{\beta} \Phi_{\beta}^{q\bar{q}, q\bar{q}} |q\bar{q}, q\bar{q}\rangle + \dots$$

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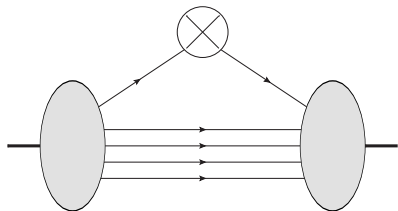
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- Non-perturbative physics is contained within the  $N$ -particle LFWFs  $\Phi^N$
- This formalism allows to recover the probabilistic picture of non-relativistic quantum mechanics

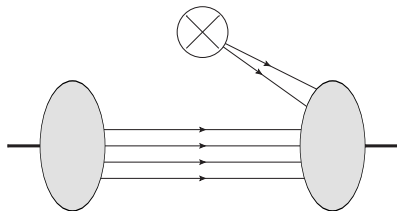
- On the light front, hadronic states can be expanded on a Fock basis

DGLAP:  $|x| > |\xi|$



- Same  $N$  LFWFs
- No ambiguity

ERBL:  $|x| < |\xi|$

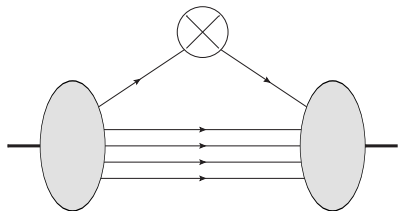


- $N$  and  $N + 2$  partons LFWFs
- Ambiguity

M. Diehl *et al.*, Nucl.Phys. B596 (2001) 33-65

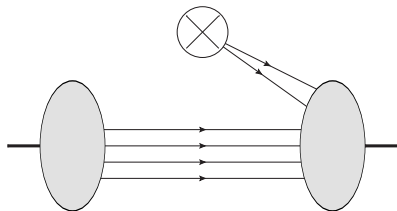
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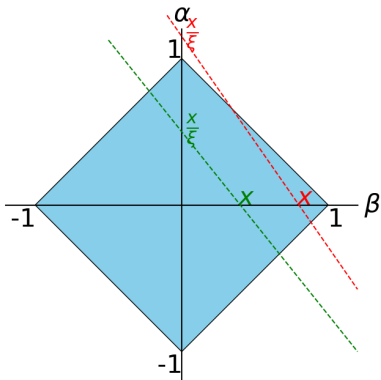
LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

*Part 2:*  
*The Inverse Radon Transform*

N.Chouika, CM, H. Moutarde, J. Rodriguez-Quintero,  
EPJC 77 (2017) no.12, 906

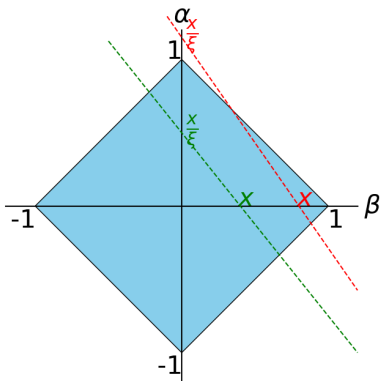


$$H(x, \xi) = \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) [F(\beta, \alpha) + \xi G(\beta, \alpha)]$$



- DGLAP (red) and ERBL (green) lines cut  $\beta = 0$  outside or inside the square
- Every point  $(\beta \neq 0, \alpha)$  contributes **both** to DGLAP and ERBL regions
- For every point  $(\beta \neq 0, \alpha)$  we can draw an infinite number of DGLAP lines.

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Is it possible to recover the DDs from the DGLAP region only?

- We can define a  $D$ -term such that:

$$\int_{-1}^1 dx x^m (H(x, \xi) - D(x/\xi)) = \sum_{i \text{ even}}^m (2\xi)^i C_{m,i},$$

yielding the **Ludwig-Helgason** consistency conditions.

- From **Hertle theorem** (1983), we know that  $H - D$  is in the range of the Radon transform and that:

$$H(x, \xi) = D(x/\xi) + \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) F_D(\beta, \alpha)$$

This allows us to identify the DD  $F_D$  with the Radon transform of  $H - D$ . This has been first noticed by O. Teryaev (PLB510 2001 125).

- It should be possible to use the **limited** Radon inverse transform to obtain the DD and thus the ERBL part.

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NB: This is equivalent to fixing the DD to the Polyakov-Weiss scheme. The same argument can be done in other schemes, but the  $D$ -term remains ambiguous without additional assumptions.

- Since DD are compactly supported, we can use the **Boman and Todd-Quinto theorem** which tells us

$$H(x, \xi) = 0 \quad \text{for } (x, \xi) \in \text{DGLAP} \Rightarrow F_D(\beta, \alpha) = 0 \quad \text{for all } (\beta \neq 0, \alpha) \in \Omega$$

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

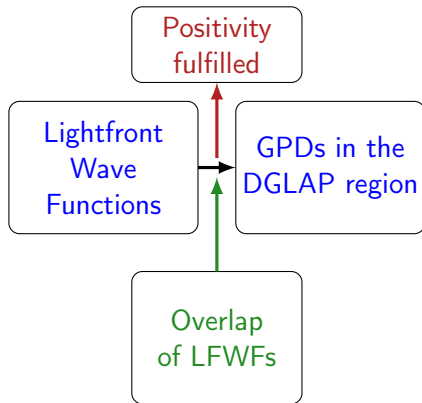
insuring the uniqueness of the extension up to  $D$ -term like terms.

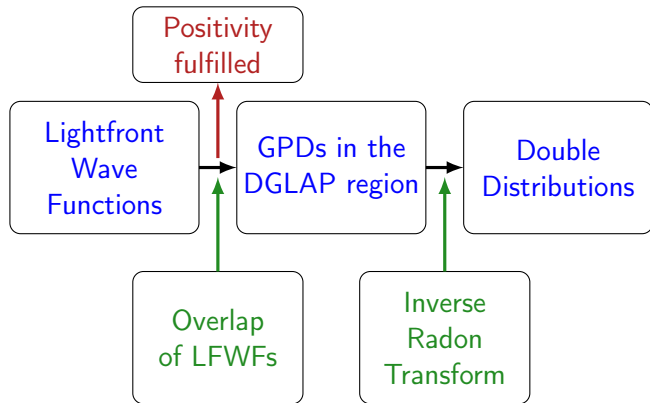
- The DGLAP region almost completely characterises the entire GPD.

## New modeling strategy

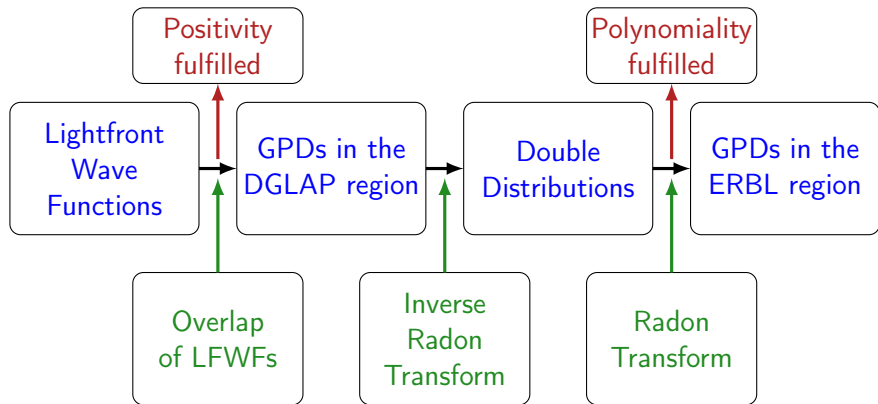
- Compute the DGLAP region through overlap of LFWFs  
 $\Rightarrow$  **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform  
 $\Rightarrow$  **fulfilment of the polynomiality property**

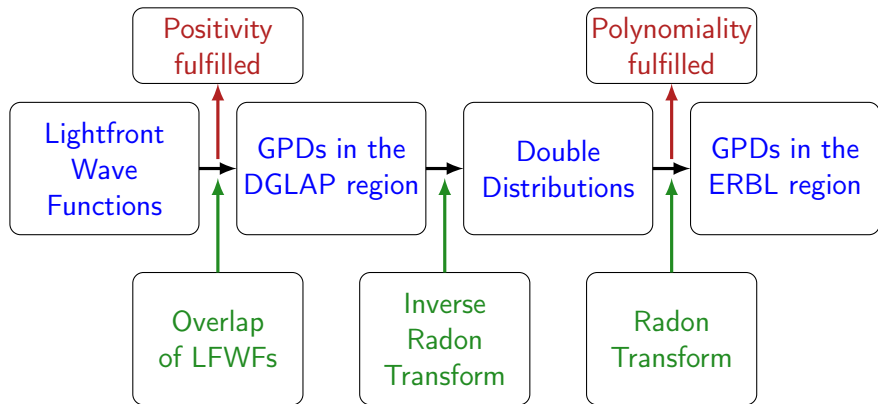
Lightfront  
Wave  
Functions









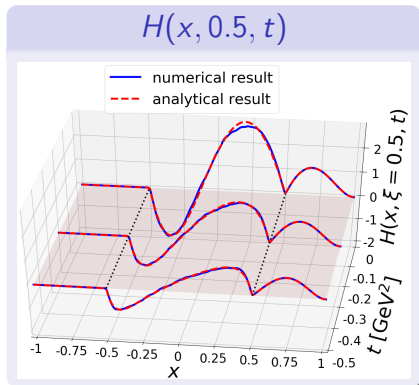
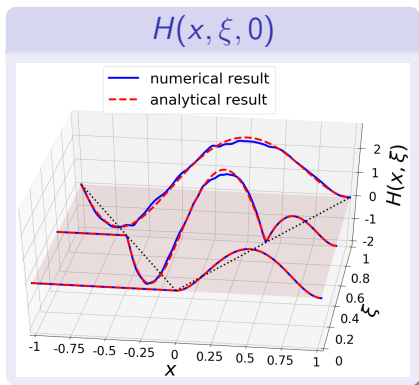


Reshuffling of the series in the ERBL region  
→ Polynomiality is fulfilled at every order in  $N$ .

*Part 3:*  
*An example on the pion*

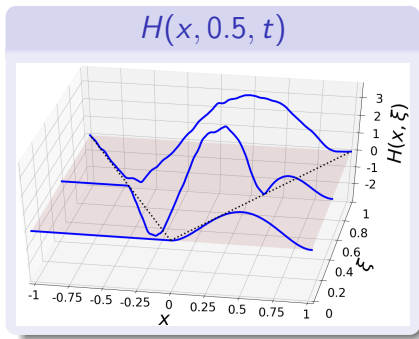
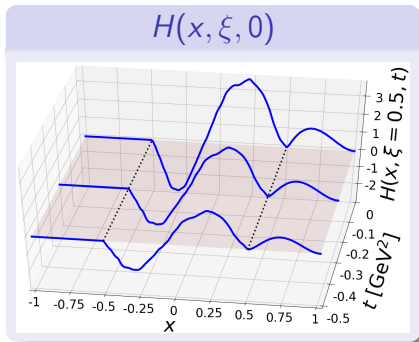
N.Chouika, CM, H. Moutarde, J. Rodriguez-Quintero,  
PLB 780 (2018) 287-293

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- Numerical implementation can be challenging due to noise



LFWF model from N. Chouika et al., PLB 780 (2018) 287-293

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Gaussian LFWF model (typical of AdS/QCD models)

# *Toward a Nucleon Wave Function*

Part 4:

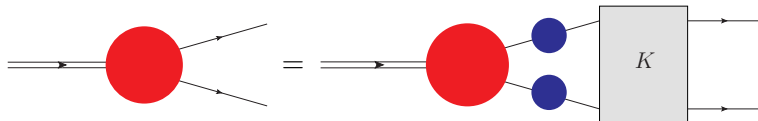
CM, J. Segovia, L. Chang, C.D. Roberts,  
Phys.Lett. B783 (2018) 263-267

- Dyson-Schwinger equations (DSEs) are an infinite set of equations relating the  $N$ -point functions among each other.
- Truncating this set yields a non-perturbative approximation of QCD Green functions.



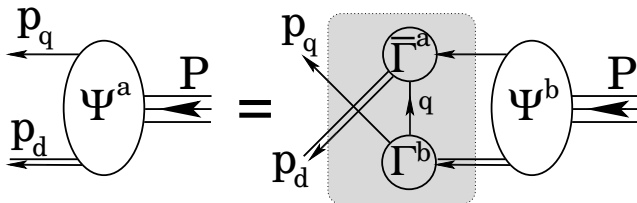


- Coupling between DSEs and bound-state equations (Bethe-Salpeter)  
→ Bound state description with effective dressed quarks
- Kernel truncation needs to be consistent with the underlying symmetries of the theory
- For baryons, a full relativistic 3-body equation needs to be solved  
→ few people are able to solve this problem today



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G. Eichmann *et al.*, *Prog.Part.Nucl.Phys.* 91 (2016) 1-100

- Algebraic parametrisation inspired by the results obtained from DSEs and Faddeev equations.
- It is based on Nakanishi representation, which is completely general.
- This is an exploratory work: we want to know what we can or cannot do.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD one.

- Operator point of view for every DA (and at every twist):

$$\langle 0 | \epsilon^{ijk} \left( u_{\uparrow}^i(z_1) C \not{n} u_{\downarrow}^j(z_2) \right) \not{n} d_{\uparrow}^k(z_3) | P, \lambda \rangle \rightarrow \varphi(x_1, x_2, x_3),$$

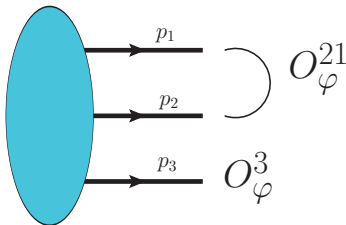
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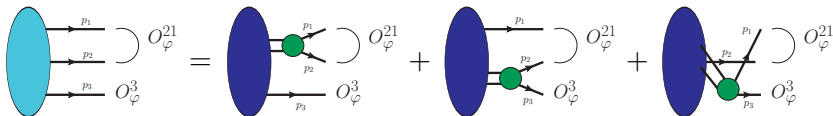


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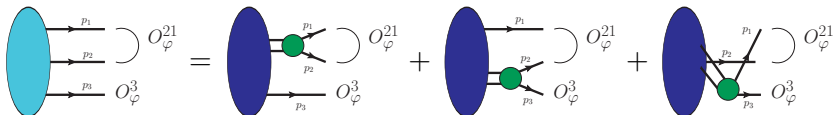


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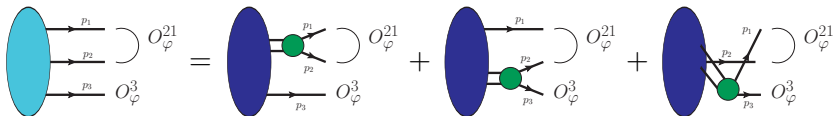


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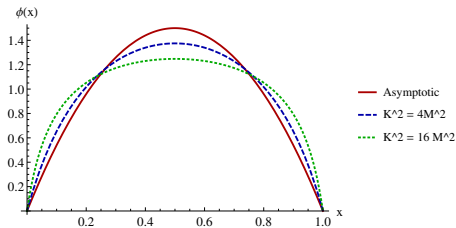
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- The operator then selects the relevant component of the wave function.
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)

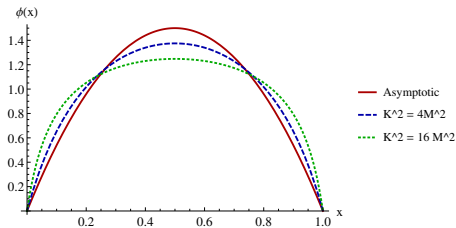
$$\phi(x) \propto 1 - \frac{M^2 \ln \left[ 1 + \frac{K^2}{M^2} x(1-x) \right]}{K^2 x(1-x)}$$

Scalar diquark

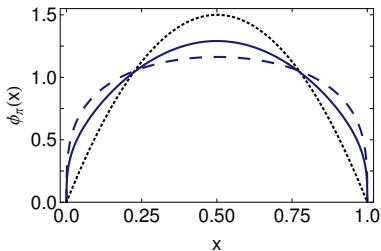


$$\phi(x) \propto 1 - \frac{M^2 \ln \left[ 1 + \frac{K^2}{M^2} x(1-x) \right]}{K^2 x(1-x)}$$

## Scalar diquark



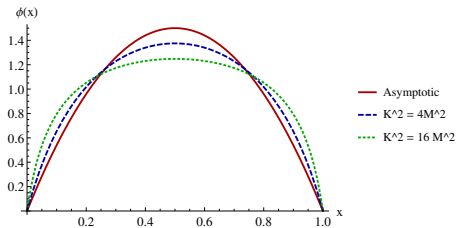
## Pion



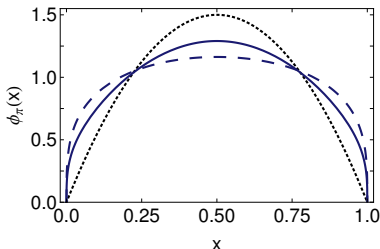
Pion figure from L. Chang et al., PRL 110 (2013)

$$\phi(x) \propto 1 - \frac{M^2 \ln \left[ 1 + \frac{K^2}{M^2} x(1-x) \right]}{K^2 x(1-x)}$$

### Scalar diquark

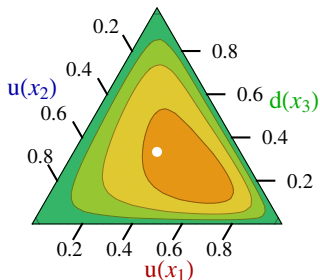


### Pion

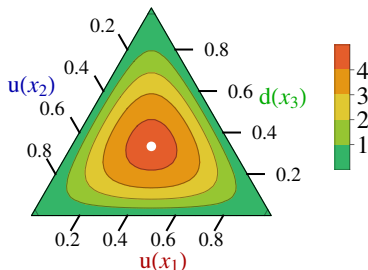


Pion figure from L. Chang et al., PRL 110 (2013)

- This result provides a broad and concave meson DA parametrisation
- The endpoint behaviour remains linear



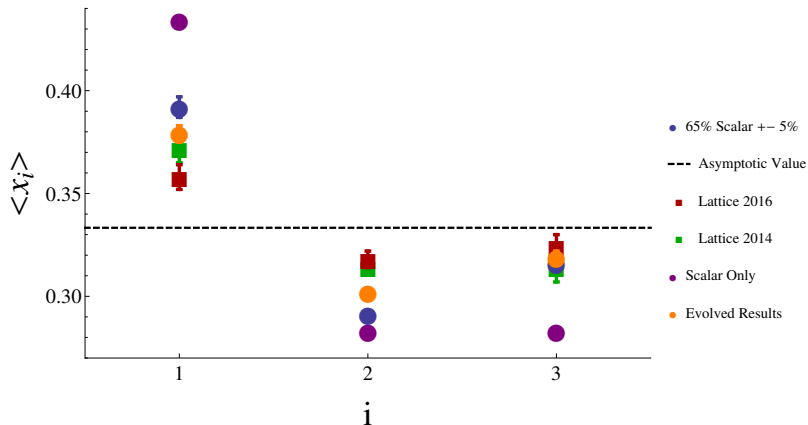
Our Result



Asymptotic DA

- Results evolved from 0.51 to 2 GeV with both scalar and AV diquark
- Nucleon DA is skewed compared to the asymptotic one
- It is also broader than the asymptotic results
- These properties are consequences of our quark-diquark picture
- Can be extended to the radial excitations (Roper)

$$\langle x_i \rangle_\varphi = \int \mathcal{D}x \ x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

# *Conclusion*

## GPDs Theory

- We can now fulfil **positivity** and **polynomiality** a priori.
- We have a **systematic** way to do it.

## Nakanishi Parametrisation

- Simple algebraic Nakanishi-like models for the pion and nucleon.
- Improvement toward more realistic Nakanishi weights is on-going.
- Algebraic models have their **successes** and their **limitations**.
- Aim : numerical solution from **Dyson-Schwinger Equations**.

## Phenomenology

- Final goal: DVCS/TCS/DVMP **cross sections**.
- Use **PARTONS** to achieve it.



Thank you for your attention

Back up slides

<http://partons.cea.fr>

## PARTONS

PARTonic Tomography Of Nucleon Software

[Main Page](#) [Reference documentation +](#)

Search

## Main Page

## What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments. A detailed description of the project can be found [here](#).



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## Get PARTONS

Here you can learn how to get your own version of PARTONS. We offer two ways.

You can use our provided virtual machine with an out-of-the-box PARTONS runtime and development environment. This is the easiest way to start your experience with PARTONS.

[Using PARTONS with our provided Virtual Machine](#)

You can also build PARTONS by your own on either GNU/Linux or Mac OS X. This is useful if you want to have PARTONS on your computer without using the virtualization technology or if you want to use PARTONS on computing farms.

[Using PARTONS on GNU/Linux](#)[Using PARTONS on Mac OS X](#)

## Configure PARTONS

If you are using our [virtual machine](#), you will find all configuration files set up and ready to be used. However, if you want to tune the configuration or if you have installed PARTONS by your own, this tutorial will be helpful for



$$H(x, \xi, t) = (1-x) \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) h_P(\beta, \alpha, t)$$
$$h_P(\beta, \alpha, t) = \frac{15}{2} \theta(\beta) \left[ 1 + \frac{-t}{4M^2} ((1-\beta)^2 - \alpha^2) \right]^{-3}$$
$$\times \left[ 1 - 3(\alpha^2 - \beta^2) - 2\beta + \frac{-t}{4M^2} (1 - (\alpha^2 - \beta^2)^2 - 4\beta(1-\beta)) \right],$$

From the algebraic DD we can deduce the GPD in ERBL region

$$H(x, \xi, 0)|_{|x| \leq \xi} = \frac{15(1-x)(\xi^2 - x^2)}{2\xi^3(1+\xi)^2} (x + 2x\xi + \xi^2),$$

- Use of a  $P_1$  (planar by pieces) basis
- We have to trade of precision and noise:  
In ill-posed inverse problem, small errors coming from our discretisations can trigger significant increases in the numerical noise.

