Model expression for the Potential Angular Momentum in the LC-gauge

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Outline

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   - *Potential Momentum*
   - *Potential Angular Momentum*

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Proton Spin Crisis

EMC experiment $\Rightarrow h = +1/2$

[J. J. Aubert et al. (EMC), Nuc. Phys. B 259, 189 (1985)]

$$\frac{1}{2} = \langle\langle S_z^q \rangle\rangle + \langle\langle S_z^G \rangle\rangle + \langle\langle L_z \rangle\rangle$$
$$\langle\langle S_z^q \rangle\rangle = \frac{1}{2}\int_0^1 dx \Delta\Sigma(x)$$
$$\langle\langle S_z^G \rangle\rangle = \int_0^1 dx \Delta G(x)$$

COMPASS, HERMES $\Rightarrow \int_0^1 dx \Delta\Sigma(x) \approx 0.3$

[V. Y. Alexakhin et al. (COMPASS Collaboration), Phys.Lett. B 647, 8 (2007)]
[A. Airapetian et al. (HERMES Collaboration), Phys.Rev. D 75, 012007 (2007)]

PHENIX, STAR, COMPASS $\Rightarrow \int_{0.05}^{0.2} dx \Delta G(x) \approx 0.2$

[D. de Florian et al (DSSV Collaboration). Phys Rev. Lett. 113, 012001 (2014)]
[E. R. Nocera et al. (NNPDF Collaboration), Nuc. Phys. B 887, 276 (2014)]

MAIN GOAL:

- To understand $\langle\langle L_z \rangle\rangle$ and how it can be described (decomposed).

Jaffe-Manohar and Ji Decompositions

- The most common decompositions of angular momentum are the Jaffe-Manohar (JM) and Ji decompositions
- Ji:

$$\frac{1}{2}\Delta\Sigma + L_q + J_g = \frac{1}{2}$$

- Jaffe-Manohar:

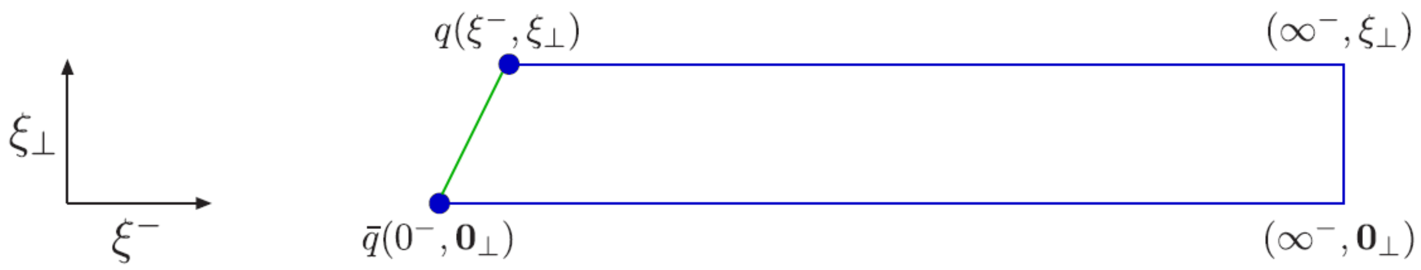
$$\frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q + \mathcal{L}_g = \frac{1}{2}$$

- Differ in their definition of Orbital Angular Momentum

$$\vec{L}_{Ji} \sim \vec{r} \times i\vec{D} \qquad \vec{\mathcal{L}}_{JM} \sim \vec{r} \times i\vec{\partial}$$

- In terms of the covariant derivative on the LC-gauge:

$$\vec{D}_\perp = \vec{\partial}_\perp + ie\vec{A}_\perp$$



[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997)]

[R. L. Jaffe, A. Manohar, Nucl. Phys. B 337, 509 (1990)]

Jaffe-Manohar and Ji Decompositions

- The *potential momentum* ($\vec{k}_{pot}$) is the difference between Ji and JM momentum:
  $$\langle \vec{k}_\perp \rangle_{Ji} - \langle \vec{k}_\perp \rangle_{JM} = -e_q \langle \int d^3r \, \bar{\Psi}(\vec{r}) \gamma^+ \vec{A}_\perp(\vec{r}) \Psi(\vec{r}) \rangle$$
- The *potential angular momentum* (L_{pot}) is the difference between Ji and JM OAM:
  $$\langle L \rangle_{Ji} - \langle \mathcal{L} \rangle_{JM} = -e_q \langle \int d^2r_\perp \, \bar{\Psi}(r) \gamma^+ (\vec{r}_\perp \times \vec{A}_\perp(\vec{r}))_z \Psi(r) \rangle$$
- Explicit calculations found that the difference $\langle L \rangle_{Ji} - \langle \mathcal{L} \rangle_{JM}$ vanishes at one-loop perturbation theory
- A lattice calculation of both decompositions of quark OAM depicts a JM OAM "significantly enhanced" compared to Ji OAM.
- The scale dependence of L_{pot} was recently studied for the first time, where its magnitude seems to be suppressed by the evolution in the large energy-scale region

[X. Ji et al., Phys. Rev. D 93, 54013 (2016)]
[Engelhardt et al., arXiv:1901.00843 (2019)]
[Y. Hatta, X. Yao, arXiv:1906.07744 (2019)]

Single Spin Asymmetries

- Single Spin Asymmetries (SSA) are sensitive to the orbital momentum of quarks
- The Sivers SSA is interpreted as an effect of the orbital motion of (unpolarized) quarks in a transversely polarized nucleon
- The Sivers function $f_{1T}^{\perp}$ can be related to average transverse momentum via

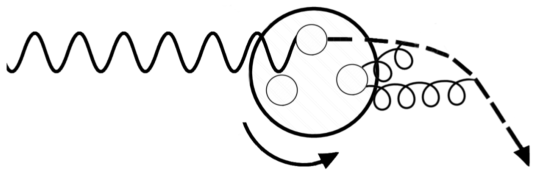
$$\langle k_{\perp}^{j} \rangle = \epsilon^{ij} S_{\perp}^{i} \int dx \, d^2 k_{\perp} \frac{k_{\perp}^{2}}{2M^2} f_{1T}^{\perp}(x, k_{\perp}^{2})$$

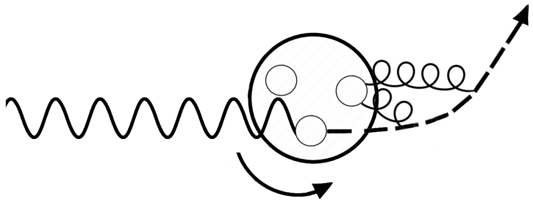
- A nonzero Sivers asymmetry can arise as a consequence of initial/final state interactions (ISI/FSI)
- In SIDIS, $f_{1T}^{\perp}$ is associated with FSI through gluon exchange
- The mechanism that causes transverse SSA is similar in nature to the mechanism that causes the change in the OAM of the struck quark.

[M. Burkardt, Phys. Rev. D 88, 014014 (2013)]

Lensing function and torque

- The attracting FSI bends the observed hadrons into a direction opposite of the impact position.
- FSI are responsible for a “chromomagnetic lensing” effect on the struck quark
- It is postulated that this mechanism is responsible for the torque generated on the struck quark with respect to the spectator system
- The possibility of factorizing $f_{1T}^{\perp}$ into a distortion effect times a *Lensing function* has not been proven in a model-independent way.

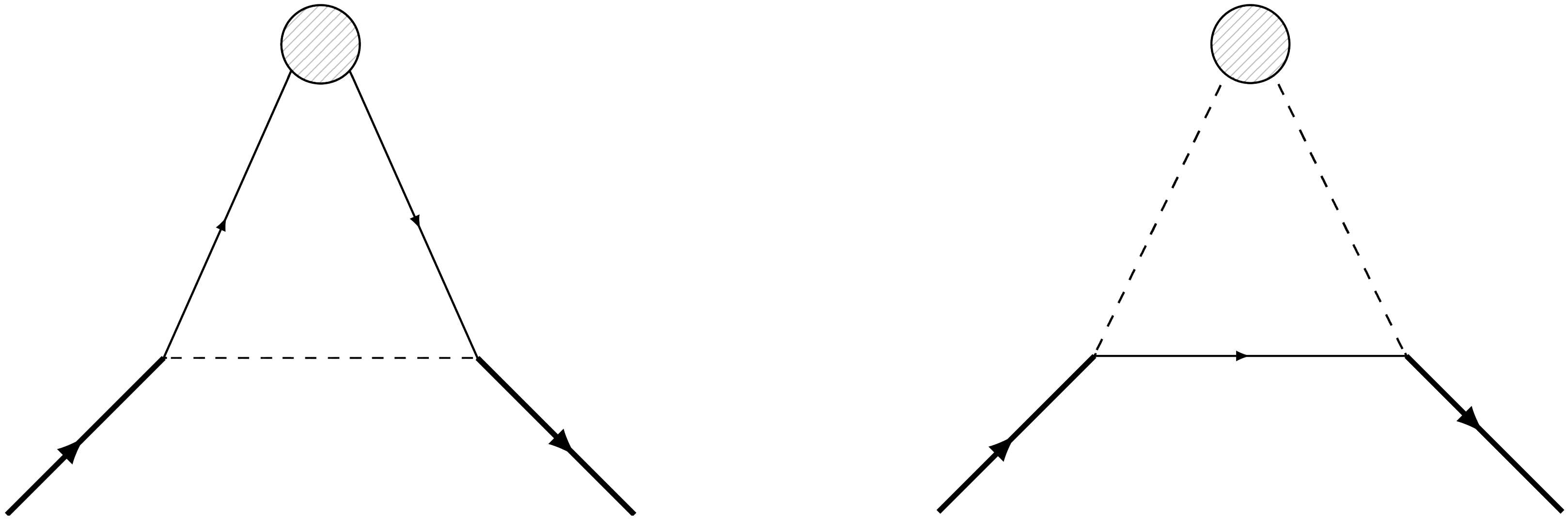




[M. Burkardt, Nucl. Phys. A 735, (2004) 185.]
[Pasquini, Rodini & Bacchetta, arXiv:1907.06960v2 (2019)]

Scalar Diquark Model

- The proton splits into a quark and a diquark structure. While the active quark interacts with the photon, the diquark acts as the 'spectator' and vice versa.
- Only QED is evaluated for Initial/Final State Interactions.
- A simple model that provides analytic results and estimations of several observables.
- Explicit Lorentz covariance is maintained.
- We compute and compare the magnitude of the effect of ISI/FSI on the transverse momentum and OAM of the struck quark.

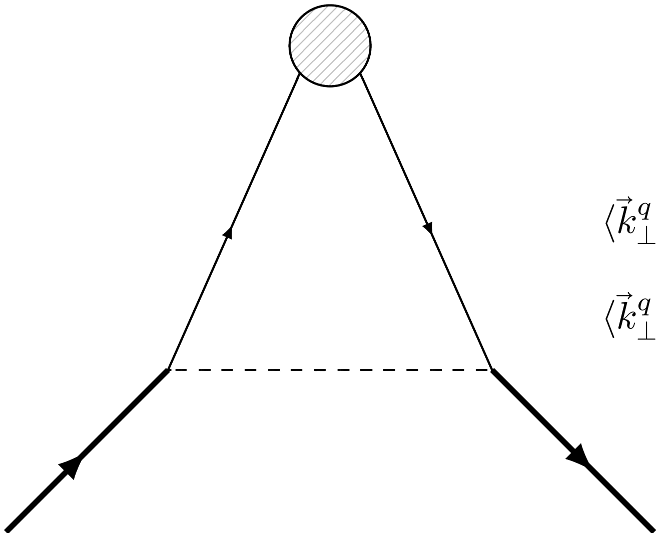


Potential Momentum

- The potential momentum corresponds to the difference between Ji and JM decompositions:

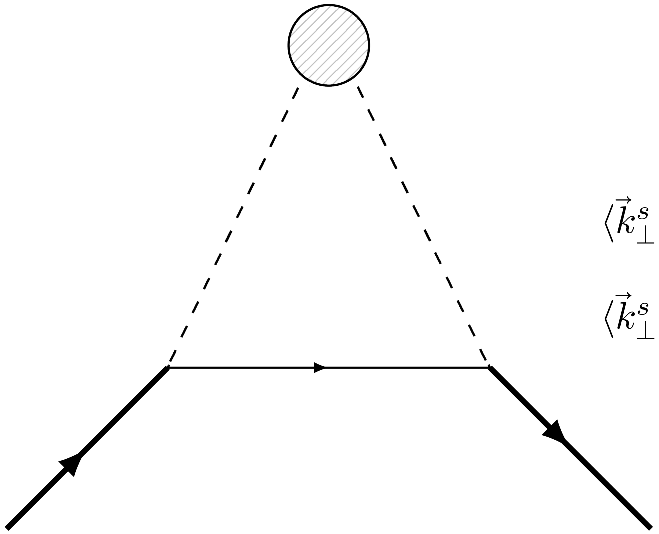
$$\langle k_\perp^i \rangle_{Ji} - \langle k_\perp^i \rangle_{JM} = \frac{1}{2}\, e \,\langle \int d^3 r\, \bar{\Psi}(r) \gamma^+ \vec{A}^i(r) \Psi(r) \rangle$$

- A non-zero potential momentum requires a transversely polarized target
- Can be regarded as the change in $k_\perp$ experienced by the struck quark due to ISI/FSI



$$\langle \vec{k}_\perp^q \rangle_{Ji} = \vec{0}_\perp$$

$$\langle \vec{k}_\perp^q \rangle_{JM} = \vec{0}_\perp$$

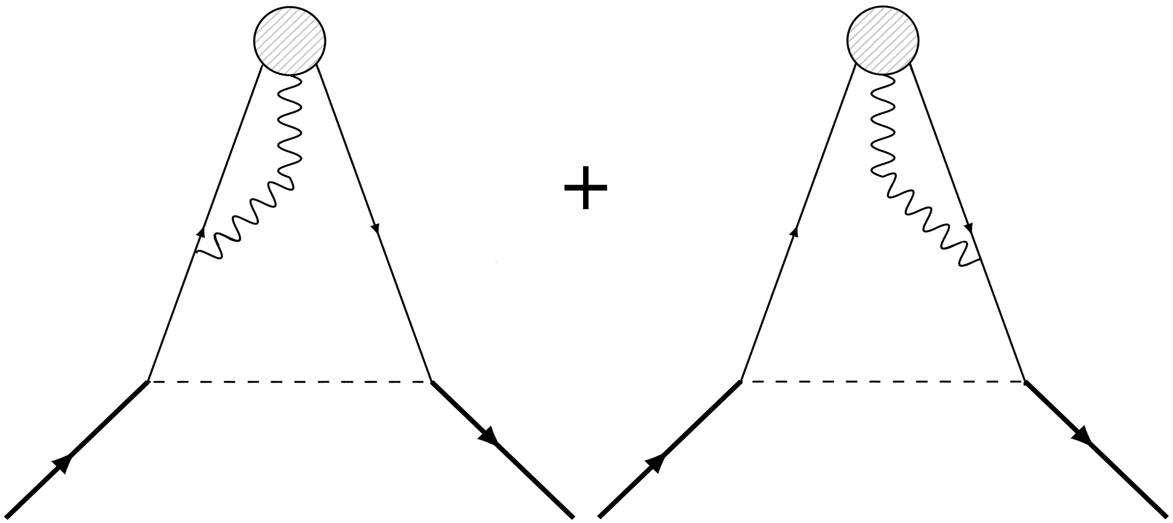


$$\langle \vec{k}_\perp^s \rangle_{Ji} = \vec{0}_\perp$$

$$\langle \vec{k}_\perp^s \rangle_{JM} = \vec{0}_\perp$$

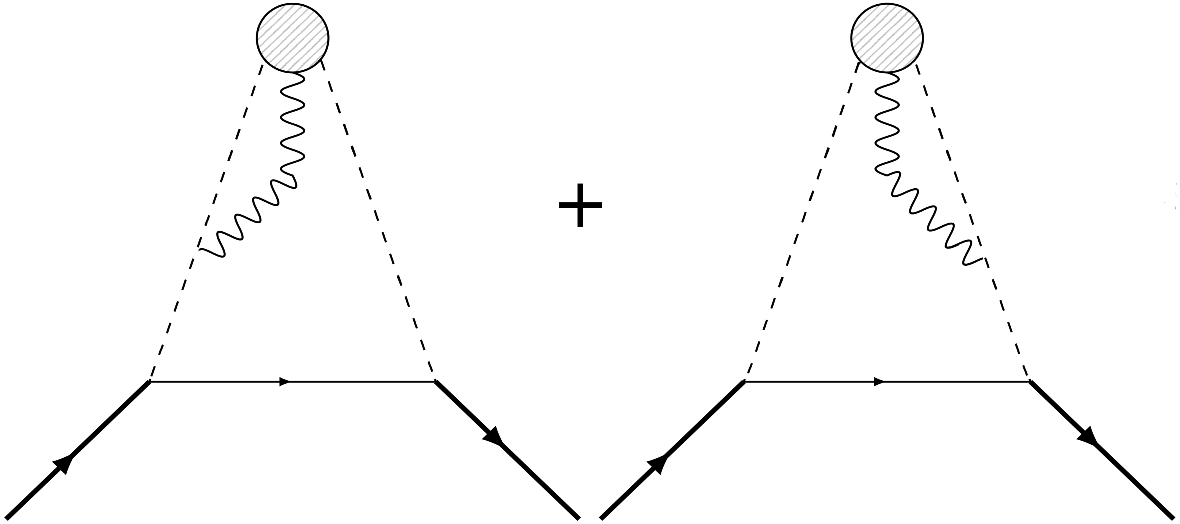
[M. Burkardt, Phys. Rev. D 66, 114005 (2002)]

Potential Momentum



$$\langle \vec{k}_\perp^q \rangle_{Ji} = \vec{0}_\perp$$

$$\langle \vec{k}_\perp^q \rangle_{JM} = \vec{0}_\perp$$



$$\langle \vec{k}_\perp^s \rangle_{Ji} = \vec{0}_\perp$$

$$\langle \vec{k}_\perp^s \rangle_{JM} = \vec{0}_\perp$$

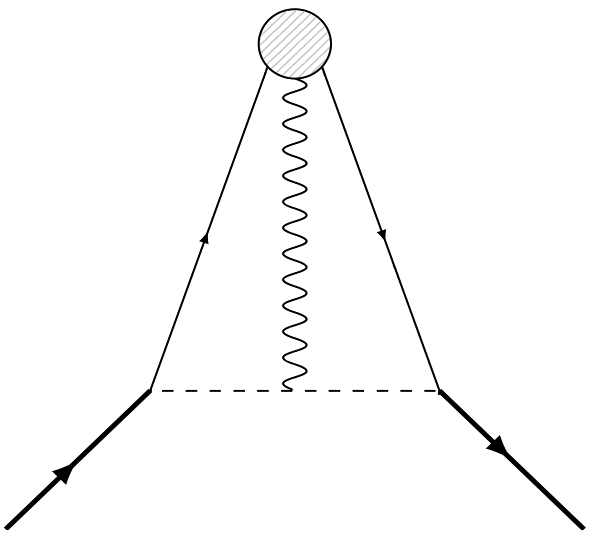
Potential Momentum

Momentum has to be conserved!

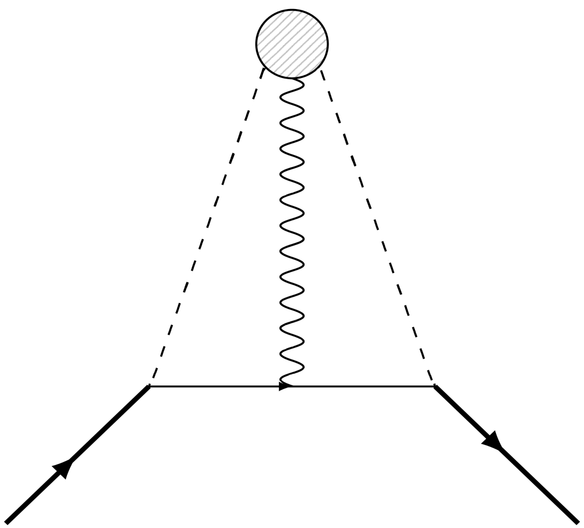
Baron Münchhausen by Oskar Herrfurth

Potential Momentum

- Requires at least a one-photon exchange



$$\langle \vec{k}_\perp^q \rangle_{Ji} = \vec{0}_\perp$$
$$\langle \vec{k}_\perp^q \rangle_{JM} \neq \vec{0}_\perp$$



$$\langle \vec{k}_\perp^s \rangle_{Ji} = \vec{0}_\perp$$
$$\langle \vec{k}_\perp^s \rangle_{JM} \neq \vec{0}_\perp$$

- Burkardt sum rule is fulfilled order by order $\sum_{a=q,s} \langle k_\perp^a \rangle = 0_\perp$
- The potential momentum corresponds to JM transverse momentum:

$$\langle k_\perp^i \rangle_{JM} = -\tfrac{1}{2} e \langle \int d^3 r \, \bar{\Psi}(r) \gamma^+ \vec{A}^i_{phys}(r) \Psi(r) \rangle$$
$$= \frac{1}{6} \left(\frac{g}{4\pi\epsilon} \right)^2 \frac{\epsilon_T^{ij} s_\perp^j}{(4\pi)^2} (3m_q + M) \pi e_s e_q + \mathcal{O}(\epsilon^{-1})$$

Potential Momentum

- Is non-zero and indirectly determines JM transverse momentum. At $\mathcal{O}(g^2 e_q e_s)$ it is given by:

$$\langle k_\perp^i \rangle_{JM} = \frac{1}{6}\left(\frac{g}{4\pi\epsilon}\right)^2 \frac{\epsilon_T^{ij} s_\perp^j}{(4\pi)^2}(3m_q + M)\pi e_s e_q + \mathcal{O}(\epsilon^{-1})$$

- $\langle \vec{k}_\perp \rangle(x) \sim (\vec{P} \times \vec{S}) f^{\mathcal{W}}$ is naive T-odd
- $f^{\mathcal{W}}$ has to be naive T-odd, i.e., $T:\ \mathcal{W} \longmapsto \mathcal{W}'$
- $\langle \bar{\Psi}\gamma^+ \vec{D}_\perp \Psi \rangle \ - \ \langle \bar{\Psi}\gamma^+ \vec{\partial}_\perp \Psi \rangle \ = \ \langle \bar{\Psi}\gamma^+ ieA_\perp \Psi \rangle$
- Non-vanishing $k_\perp$ requires a non-vanishing Sivers function

$$\begin{aligned} f_{1T}^{\perp q}(x, k_\perp^2) &= \frac{e_s e_q g^2}{4(2\pi)^4} \frac{(1-x)(m_q + xM)M}{k_\perp^2(k_\perp^2 + \tilde{m}^2)} \ln \frac{k_\perp^2 + \tilde{m}^2}{k_\perp^2} \\ \tilde{m}^2 &= x(1-x)\left(-M^2 + m_q^2/x + m_s^2/(1-x)\right) \end{aligned}$$

[X. Ji & F. Yuan, Phys.Lett. B 543, 66 (2002)]

Physical interpretation

- Possible chromodynamic lensing mechanism that can provide a torque

$$\int d^2k_\perp \frac{k_\perp^2}{2M^2} f_{1T}^\perp(x, k_\perp^2) \;\propto\; \int d^2b_\perp \mathcal{J}(x, b_\perp)(S_T \times \partial_{b_\perp})_z \mathcal{E}^q(x, b_\perp^2)$$

$$\mathcal{J}^{q,i}_{SDM}(x, b_\perp) \;=\; \frac{e_q e_s}{4\pi} \frac{(1-x)b_\perp^i}{b_\perp^2}$$

- The lensing function $\mathcal{J}(x, b_\perp)$ accounts for the effect of ISI/FSI
- The difference between Ji and JM decompositions appears at two-loop level
- This supports the interpretation of such a difference as originating from the torque exerted by the spectator system on the struck quark

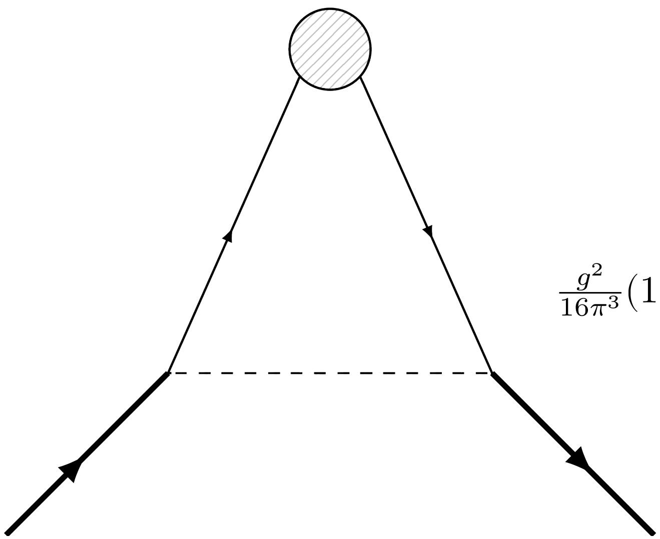
[M. Burkardt, Nucl. Phys. A 735, 185 (2004)]

[S. Meissner, A. Metz, & K. Goeke, Phys. Rev. D 76, 034002 (2007)]

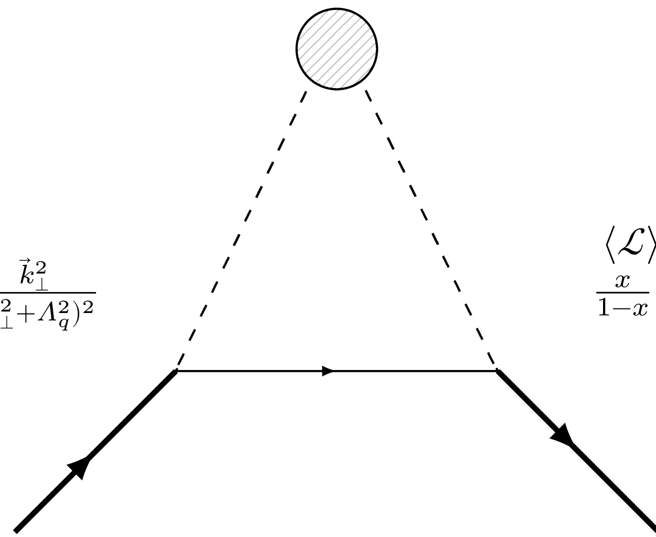
Orbital Angular Momentum

- In the LC-gauge we can compute (Ji / JM) OAM for a longitudinally polarized target as:

$$\langle L_{Ji} \rangle = \langle \mathcal{L}_{JM} \rangle = \langle \int d^2 r_\perp \, \bar{\Psi}(\vec{r}) \gamma^+ \vec{r}_\perp \times i\partial_\perp \Psi(\vec{r}) \rangle$$



$$\langle \mathcal{L} \rangle_q(x) = \frac{g^2}{16\pi^3}(1-x)^2 \int d^2 k_\perp \frac{\vec{k}_\perp^2}{(\vec{k}_\perp^2 + \Lambda_q^2)^2}$$



$$\langle \mathcal{L} \rangle_s(x) = \frac{x}{1-x} \langle \mathcal{L} \rangle_q(x)$$

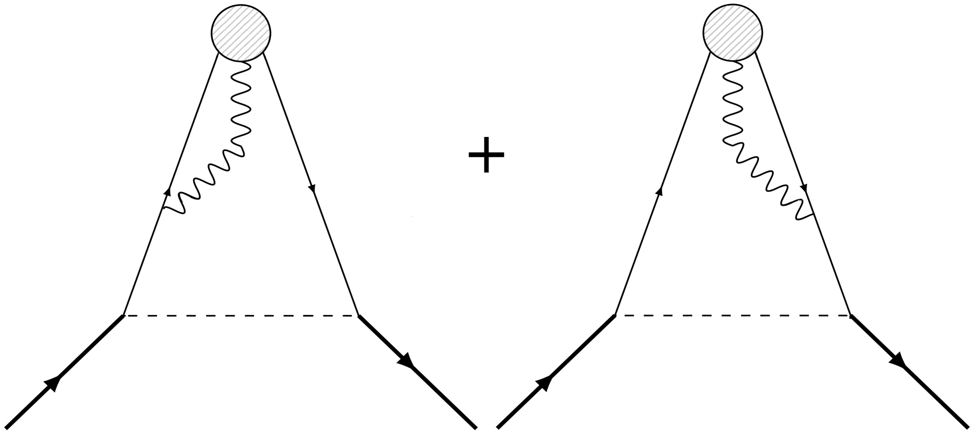
$$J(x) = \langle \mathcal{L} \rangle_q(x) + \langle \mathcal{L} \rangle_s(x) + \tfrac{1}{2} g_1^q(x) \implies J = \int dx J(x) = \tfrac{1}{2}$$

[C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B 776, 38 (2018)]

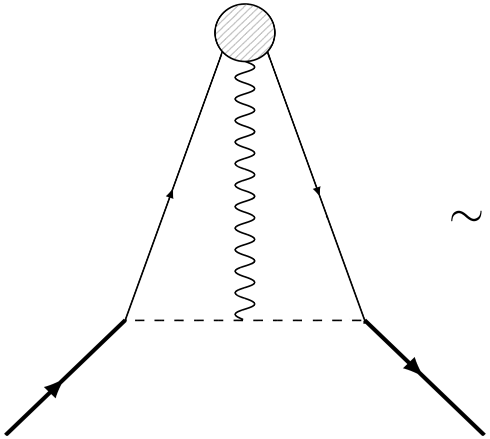
Potential Angular Momentum

- A two-loop calculation of potential OAM is in progress:

$$L_{pot} = -e_q \langle \int d^2 r_\perp \, \bar{\Psi}(\vec{r}) \gamma^+ (\vec{r}_\perp \times \vec{A}_\perp(\vec{r}))_z \Psi(\vec{r}) \rangle$$



$$= \; -\frac{1}{15}\left(\frac{g}{4\pi\epsilon}\right)^2 \frac{e_s e_q}{(4\pi)^2} + \mathcal{O}(\epsilon^{-1})$$

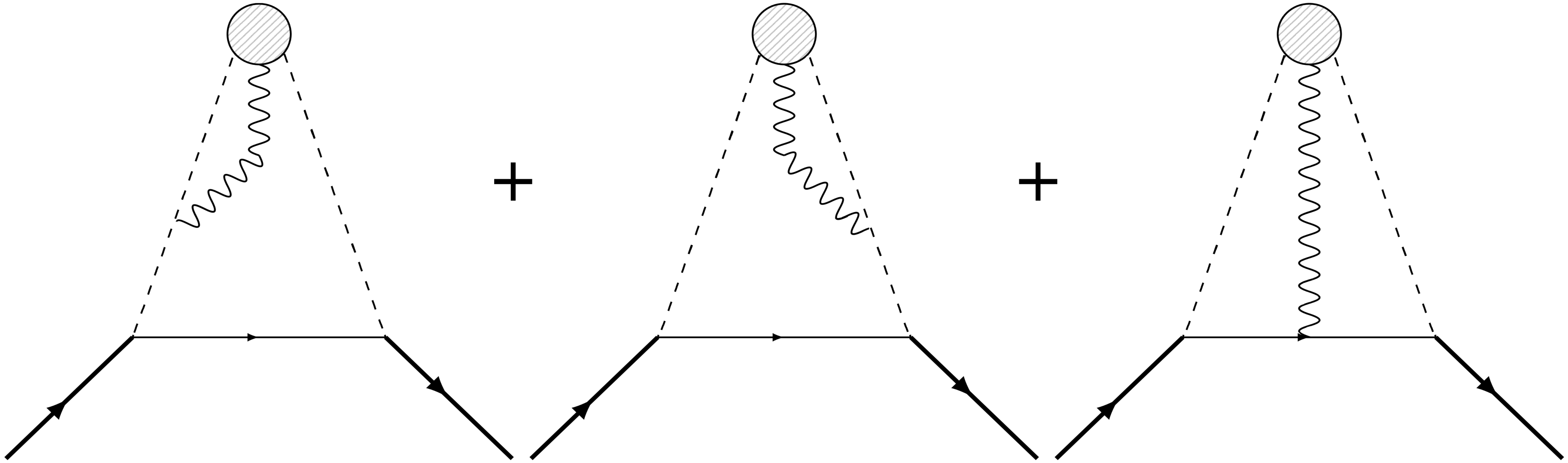


$$\sim -\left(\frac{g}{4\pi\epsilon}\right)^2 \frac{e_s e_q}{(4\pi)^2} + \mathcal{O}(\epsilon^{-1})$$

$$\frac{|\vec{k}^q_{pot}|}{L^q_{pot}} \sim (3m_q + M)$$

Potential Angular Momentum

- Compute potential OAM at two-loop order for the scalar diquark sector:



- Crosscheck for Ji Sum Rule at this order in perturbation theory

$$J(x) = \langle \mathcal{L} \rangle_q(x) + \langle \mathcal{L} \rangle_s(x) + \tfrac{1}{2} g_1^q(x) \implies J = \int dx J(x) = \tfrac{1}{2}$$

Conclusions and Outlook

Conclusions:

- The potential momentum was computed for the diquark model.
- The difference between Ji and JM decompositions appears at two-loop level.
- Ji and JM decompositions for OAM were obtained up to one-loop for both q and s-sectors.
- Ji and JM decompositions for OAM were obtained at two-loop level for the q-sector.
- We provided an estimate of $\langle L^q_{pot} \rangle$ within the SDM.

Outlook: $\langle L^s_{pot} \rangle$ has to be evaluated!

- Obtain analytical expressions for s-sector.
- Crosscheck for Ji sum rule.
- Address more complex/realistic models.

QUESTIONS?