Confinement in Nuclei and the Expanding Proton

G. A. Miller, UW Seattle arXiv:1907.00110

- Nucleons are composite particles quarks, gluons
- Bound nucleons must be different that free ones
- How much? Not much, but how much?
- Today- present new approach related to experiment, lattice QCD calculations and precision nuclear structure calculations
- Key result- proton gets bigger when in nucleus

Reviews:
Sargsian et al J. Phys. G (2003) R1
Hen et al RMP 89, 045002 ,(2017)
Cloet et al arXiv:1902.10572

General approach

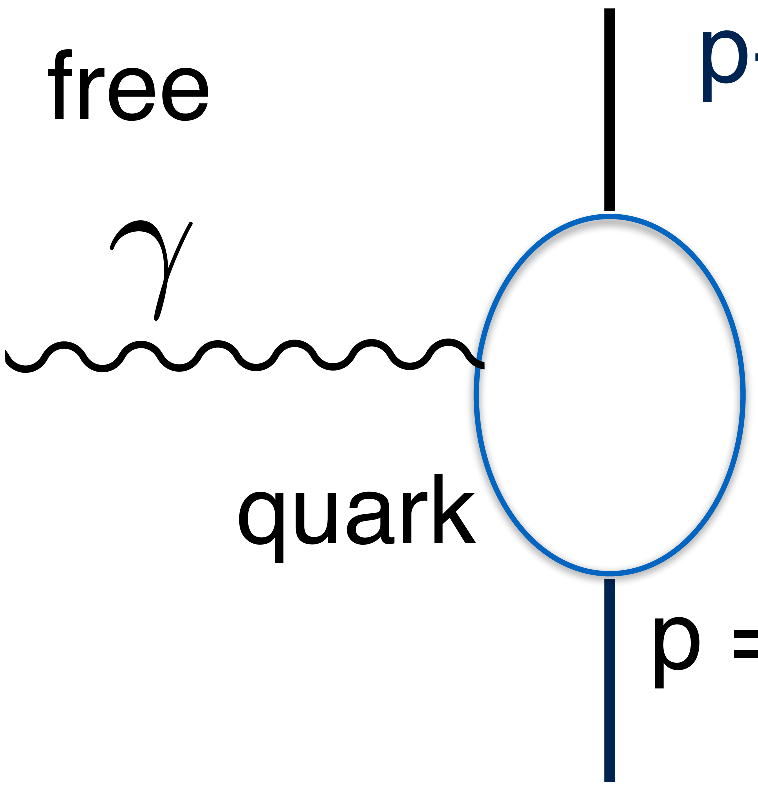
- Model free proton
- Place proton in nucleus
- Examine relevant Feynman graphs- p^2 is square of proton 4-momentum
- Expand in virtuality to first-order $V=p^2-M^2$
- Free nucleon has V=0, not so for bound nucleons

Matrix element: $\langle \mathcal{O}(p^2) \rangle \approx \langle \mathcal{O}(M^2) \rangle + (p^2 - M^2) \frac{\partial}{\partial M^2} \langle \mathcal{O}(M^2) \rangle$

Bound nucleon is a virtual nucleon

Strikman& Frankfurt, Melnitchouk & Thomas, Gross & Luiti stressed virtuality.
Present formulation is more compact, amenable to realistic nuclear structure calculations.

Proton form factors: free and bound

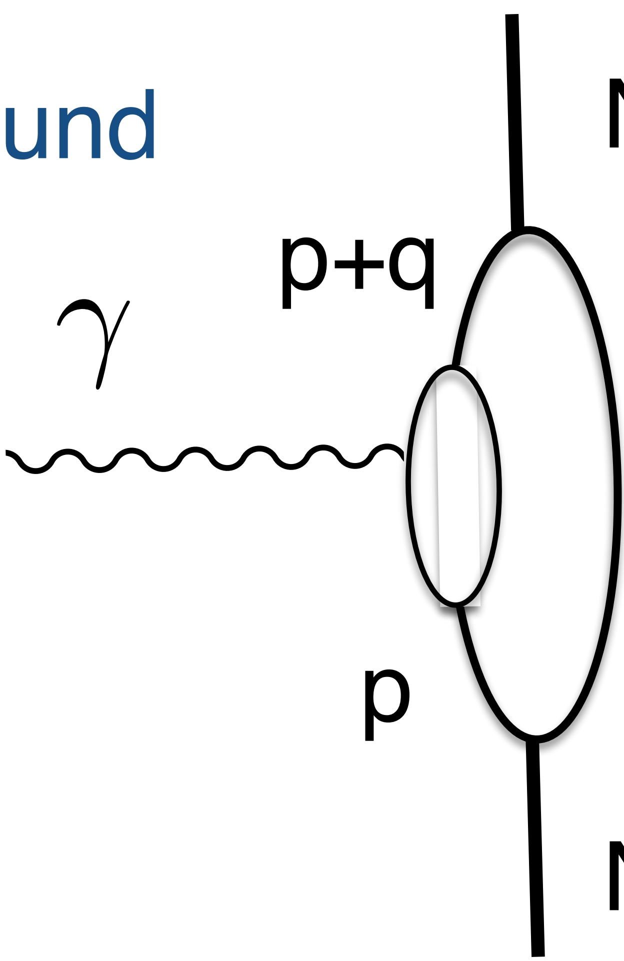


Free- space

$p^2=M^2$

$(p+q)^2=M^2$

Form factors $F_{1,2}(Q^2)$



In the medium p^2 can be anything between $\mp\infty$ $\int \frac{d^4p}{(2\pi)^4}$

$$F_{1,2}(Q^2, p^2, (p+q)^2, \not{p}, \not{p}+\not{q})$$

Other than 1,2?

Focus: Elastic Electron-Nucleus Scattering

- Pick one topic at a time, maybe easy one
- Detailed knowledge of electromagnetic form factors of nuclei -high current activity
- nuclear structure calculations of charge radii, Ruiz et al 2016
- use in atomic physics by CHROMA precision mu atoms
- Jefferson Laboratory experiment~12-14-009 electromagnetic form factors of ^{3}He and ^{3}H

Focus: Elastic Electron-Nucleus Scattering

Spinless nucleus: Standard formula

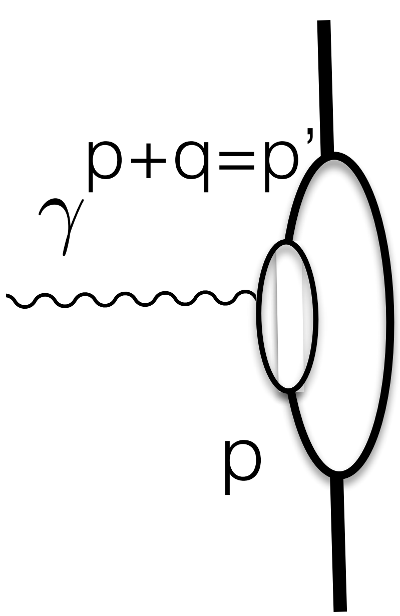
Factorization Approximation FA

$$F_A(Q^2) = F^A_{\text{point nucleon}}(Q^2)\left(F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\right)$$

Proton form factor G_E

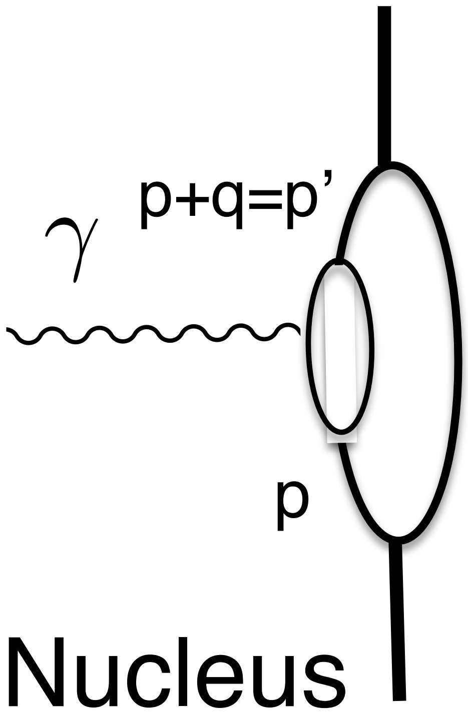
M = proton mass

- FA Assumes no dependence on p, p+q =p'
- FA Can't be completely accurate- how inaccurate?
- FA must be very accurate- widely used with no disasters
- Procedure: use models of free proton -place inside nucleus



Compute dependence on p^2 enters in denoms of Bethe-Saltpeter equation

Procedure: Use models of free proton-place inside nucleus



Nuclear wave functions do not depend on p^2 of nucleon, first-order expand in p^2-M^2, p'^2-M^2

Estimate: nuclear shell model, binding potential=-50 MeV

$$V/M^2 \equiv \frac{p^2-M^2}{M^2} = \frac{p'^2-M^2}{M^2} \approx -0.1$$

V is for virtuality, V/M^2 good expansion parameter

Requirements of calculations:

1. Satisfy Ward-Takahashi identity (current conservation)
2. Confinement- form factors in nuclei are real valued

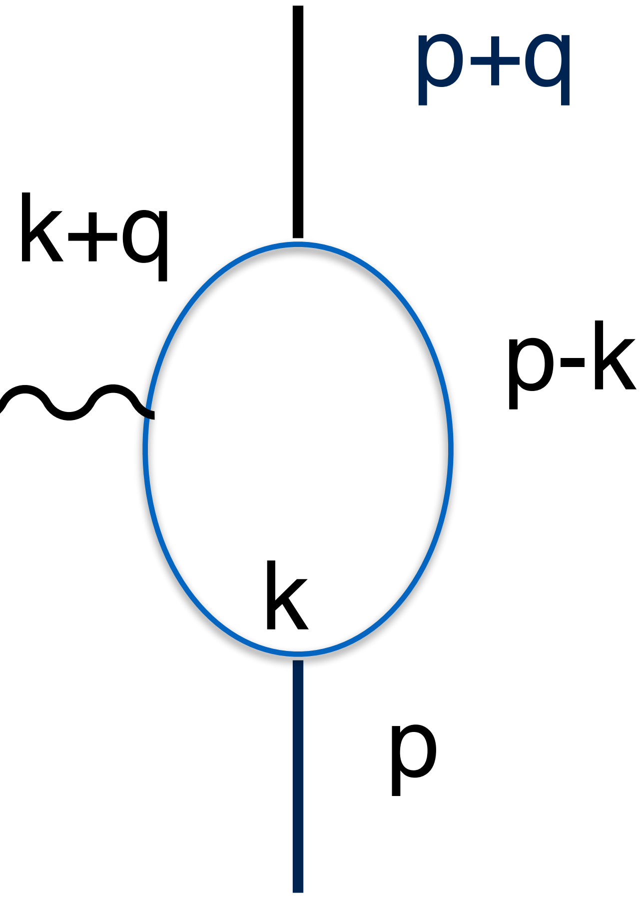
These requirements are satisfied to get the result

Result- 5 models

$$\Delta F_{1,2}(Q^2) = V \frac{\partial}{\partial M^2} F_{1,2}(Q^2, M^2)$$

Virtual nucleon is different from free nucleon

- How can proton property depend on its mass *M* ?
- In models the Mass is a parameter
- M^2 is 4-momentum^2 in Bethe-Salpeter eqns
- Results of QCD lattice calculations depend on quark mass, results presented often as function of pion mass, proton mass depends on pion mass
- First-order only: forms that survive correspond to $F_{1,2}$.



Outline of argument I

$$\Delta F_{1,2}(Q^2) = V \frac{\partial}{\partial M^2} F_{1,2}(Q^2, M^2)$$

Models:
Quark- di-quark, spin 0 quark spin 0 di-quark, scalar vertex
Quark- di-quark, spin 1/2 quark spin 0 di-quark, scalar vertex
Quark di-quark, spin 1/2 quark, spin 1 di-quark, vector vertex QED
Proton fluctuates into π^+, neutron, pseudo-vector vertex
Proton fluctuates into Δ-isobar $+\pi$, neutron, pseudo-vector vertex

Feynman graph denominator $= D$

$D = (k^2 - m_q^2)((k+q)^2 - m_q^2)((p-k)^2 - m_d^2)$ parameters x, y, z

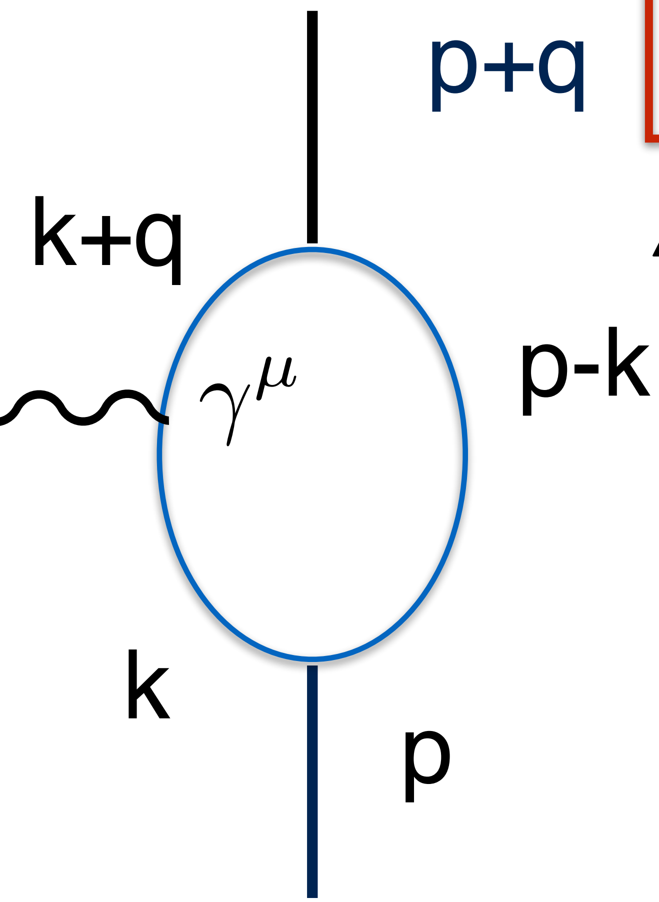
Combine and add and subtract M^2 in appropriate places

$D \to (k^2 - \Delta)^3$

$$\Delta = xyQ^2 + m_q^2(x+y) + zm_d^2 - M^2 z(1-z) + Vz(1-z) = (1 - V\frac{\partial}{\partial M^2})\Delta_{\rm on}$$

Model-specific scattering amps $\sim \frac{1}{\Delta^n}$ (No log Δ appear)

$$1/\Delta \approx 1/\Delta_{\rm on}(1 + \frac{V}{\Delta_{\rm on}}\frac{\partial}{\partial M^2}\Delta_{\rm on})$$



Outline of argument II

$$\Delta F_{1,2}(Q^2) = V \frac{\partial}{\partial M^2} F_{1,2}(Q^2, M^2)$$

Models:
Quark- di-quark, spin 0 quark spin 0 di-quark, scalar vertex
Quark- di-quark, spin 1/2 quark spin 0 di-quark, scalar vertex
Quark di-quark, spin 1/2 quark, spin 1 di-quark, vector vertex QED
Proton fluctuates into π^+, neutron, pseudo-vector vertex
Proton fluctuates into Δ-isobar $+\pi$, neutron, pseudo-vector vertex

Numerator forms: $\not p,\ \not p', p^\mu, p'^\mu = (p+q)^\mu$ ....

$$\not p = M + (\not p - M) = M + \frac{p^2 - M^2}{\not p + M} \approx \text{(first} - \text{order)} : M + \frac{V}{2M} = (1 + V\frac{\partial}{\partial M^2})M$$

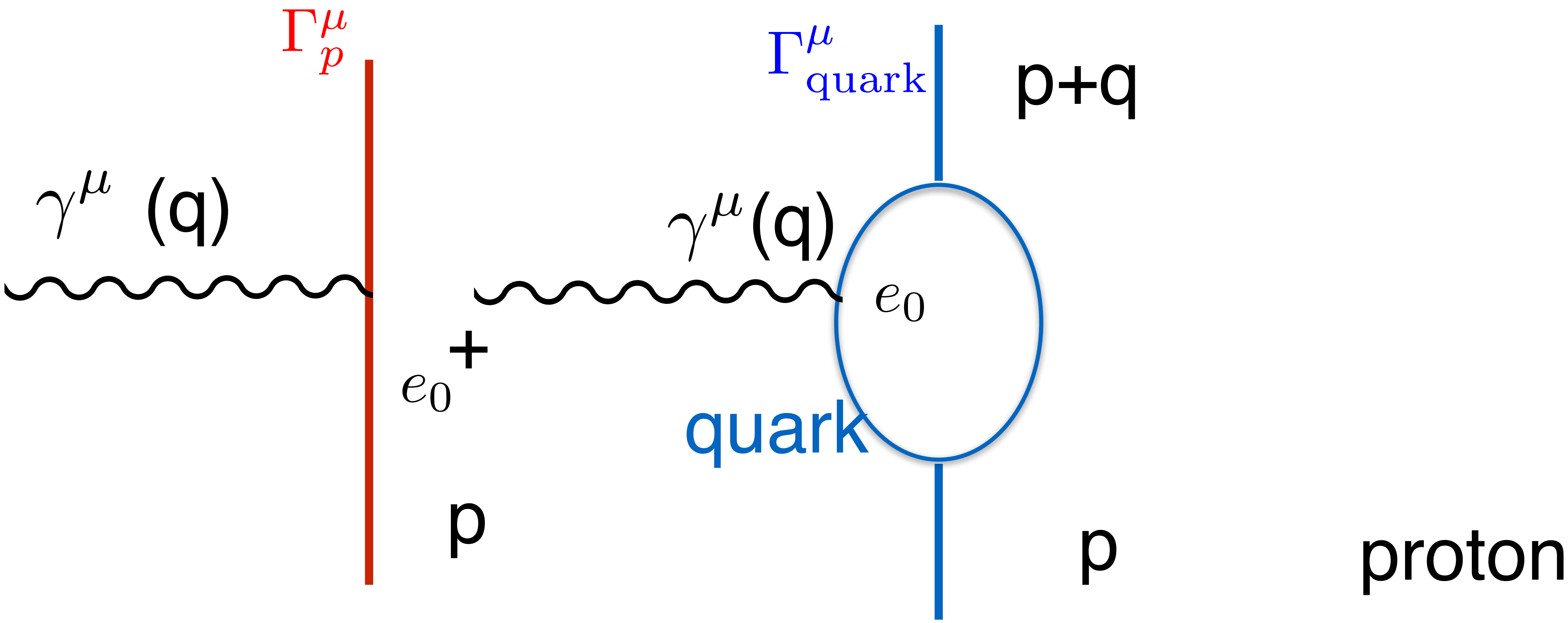
do same for $\not p'$

p^μ : Take $\mu = +$, $q^+ = 0$ Drell-Yan frame, or $\mu = 0$, $q^0 = 0$ Breit frame

$2p^\mu = \gamma^\mu \not p + \not p' \gamma^\mu + I\sigma^{\mu\nu} q_\nu$
All terms involving virtuality add up to result:

$$\Delta F_{1,2} = V \frac{\partial}{\partial M^2} F_{1,2}(Q^2)$$

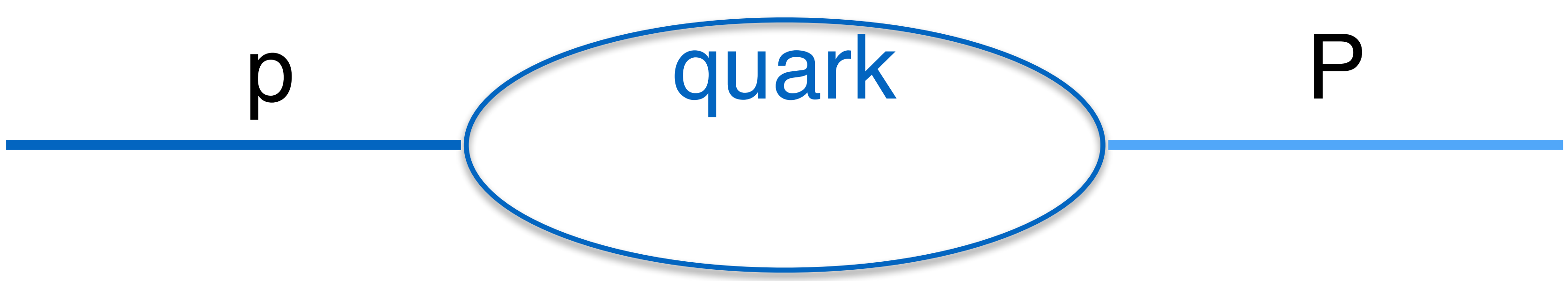
Ward-Takashi identity satisfied



$$\Gamma^\mu = \Gamma_p^\mu + \Gamma^\mu_{\rm quark}$$

$$q \cdot \Gamma = \not{p} + \not{q} - (\Sigma((p+q)^2) - \Sigma(p^2))) = S^{-1}(p+q) - S^{-1}(p)$$

$\Sigma(p) =$

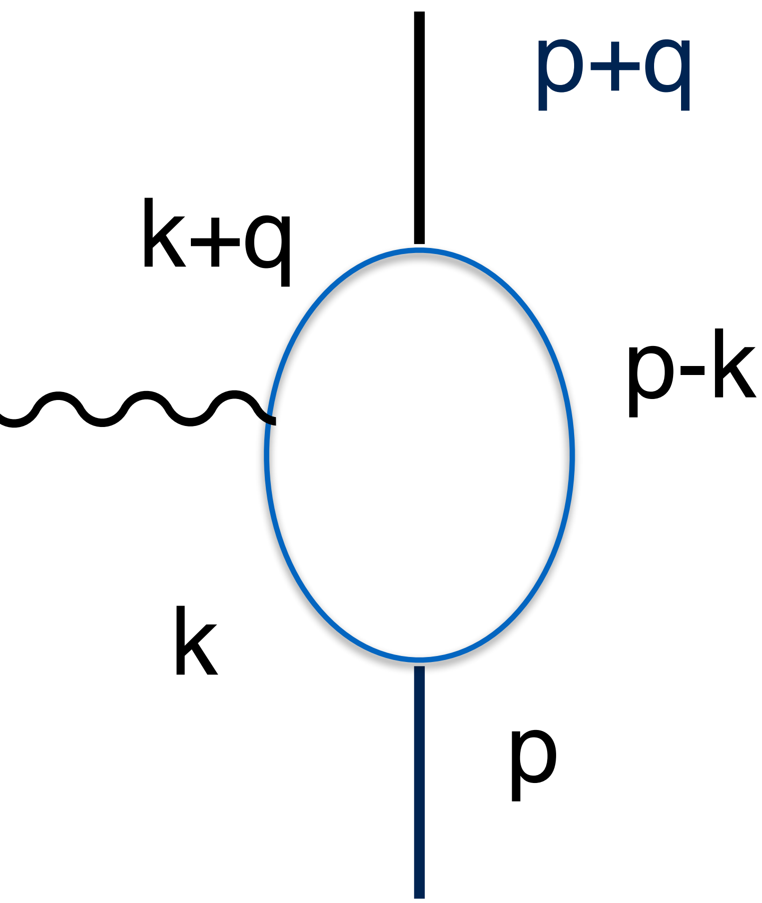


WT : current is conserved in nuclei

for all (p+q)², p², $(\not{p} + \not{q})$, $\not{p}$

Confinement I

Feynman graph denominator $= D$



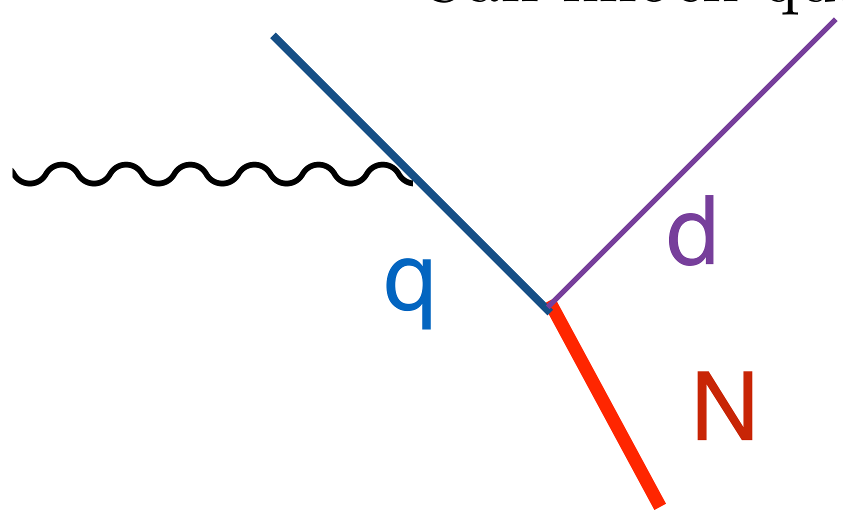
$D \to (k^2 - \Delta)^3$

$\Delta = xyQ^2 + m_q^2(x+y) + zm_d^2 - M^2 z(1-z) + Vz(1-z) = (1 - V\frac{\partial}{\partial M^2})\Delta_{\text{on}}$

If $p^2 = M^2$, $(p+q)^2 = M^2$, $V = 0$ and $\Delta > 0$ if $m_q + m_d > M$ stability condition

- For off shell protons $M^2 - V(p^2) > (m_q + m_d)^2$ Δ can go through zero
- corresponds to knocking a quark out of proton.
- proton self-energy $\Sigma(p^2)$ becomes complex
- zeros of Δ cause havoc in numerical integration

Can knock quark out if $s = (p+q)^2 > (m_q + m_d)^2$



Confinement II

- Lengthy literature based on non-perturbative Schwinger-Dyson equations (e.g. Roberts & Williams hep-ph/9403224)
- Solutions for quark self-energy obtain complex masses, appearing in complex-conjugate pairs
- Use scalar di-quark masses as complex conjugate poles. Result is same using complex mass and take real part at the end of the calculation
- Nucleon self-energy is real-valued in all models

Application: radius of proton I

$$r_E^2 \equiv -6G'_E(0)$$

G. A. Miller, PRC99, 035202

$$\delta r_E^2 = V \frac{\partial r_E^2}{\partial M^2} > 0$$

$$\frac{\partial r_E^2}{\partial M^2} < 0 \text{ as computed in ALL models}$$

$$\delta r_E^2 = (-)(-) > 0$$

Proton radius grows when proton is bound in a nucleus!

Negative derivative in all computed models
Next examine other models

Application: radius of proton II

$$r_E^2 \equiv -6G'_E(0)$$

G. A. Miller, PRC99, 035202

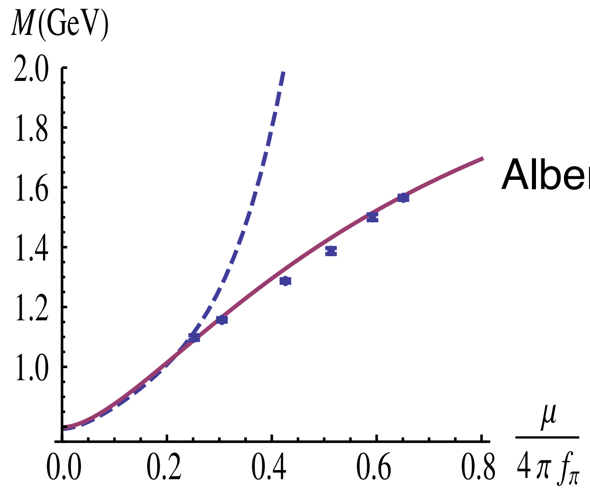
- MIT bag model (Chodos, Jaffe, Johnson, Thorn PRD 10, 2599)
  $R \propto \frac{1}{M} \rightarrow \frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial M^2} = -1$
- Non-rel harmonic oscillator quark model (Isgur, Karl) PRD 20,1191
  $b^2 \propto \frac{1}{m_q} \rightarrow, \frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial M^2} = -\frac{1}{2}$
- pion cloud model Beg $\frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial m_\pi^2} \propto -\frac{M^2}{m_\pi^2}$

  Beg, Zepeda PRD6,2912
- M increases with increasing m_π
- models show that $\frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial M^2}$ is negative, so is the virtuality, V

$$\delta r_E^2 = V\,\frac{\partial r_E^2}{\partial M^2} > 0$$

Application radius of proton II-Lattice

$$r_E^2(m_\pi^2),\ M^2(m_\pi^2)$$



Alberg, Miller PRL108,172001

$$\delta r_E^2 = V\frac{\partial r_E^2}{\partial M^2} = V\frac{\partial r_E^2}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial M^2}$$

Product = (negative)*(negative)*(positive)

Proton radius increases in the medium

Y.~C.~Jang, R.~Gupta, H.~W.~Lin, B.~Yoon and T.~Bhattacharya,
``Nucleon Electromagnetic Form Factors in the Continuum Limit from 2+1+1-flavor Lattice QCD,''
arXiv:1906.07217 [hep-lat].

$$\delta r_E = -\frac{V}{M^2}(1.1 \pm 0.1)\,\mathrm{fm}$$

V/M^2 = -.1 at center of nucleus

13% increase in size of proton

Big number-

Shifting the quark mass acts in the same way as an attractive scalar potential that causes non-zero virtuality.

So lattice calculations with ~10 MeV steps in quark masses would be very interesting

Nuclei with $A = 3$

Standard procedure (Pieper:2001mp) $R_A^2 = R_{\rm pt}^2 + r_E^2$

(CiofidegliAtti:2007ork) : $V/M^2 = -0.073$, $\delta r_E = -\frac{V}{M^2}(1.1 \pm 0.1)$ fm $\rightarrow$ $\delta r_E = 0.08$ fm.

$R_{\rm pt} = 1.54$ fm (Piarulli:2012bn) r_E 0.84 fm $\rightarrow$ 0.92 fm gives 2% increase in computed ^{3}H charge radius. The 2% $\approx$ present experimental uncertainties

Future experiments aim for better than 1%. $\delta r_E = 0.08$ fm $\gg$ changes caused by meson exchange currents or variations in cutoffs of chiral perturbation theory (Piarulli:2012bn).

This expansion is testable.

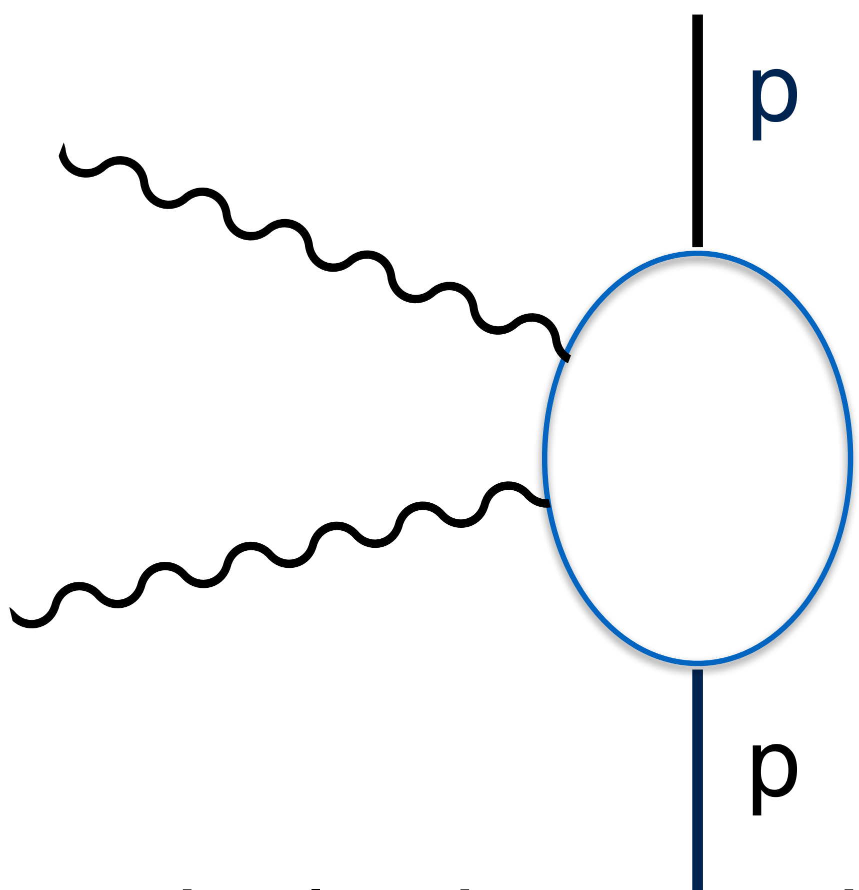
Conjecture- maybe

- Calculations here for electromagnetic form factors

- Perhaps all one-body matrix elements *O* obey the same theorem:

$$\langle \mathcal{O}(p^2) \rangle \approx \langle \mathcal{O}(M^2) \rangle + (p^2 - M^2) \frac{\partial}{\partial M^2} \langle \mathcal{O}(M^2) \rangle$$

Lattice



Deep inelastic scattering as a function of p^2 ?

Summary

- Nucleons are modified when bound in nuclei- they are Virtual Nucleons
- Elastic electron scattering result 4 models $\Delta F_{1,2}(Q^2) = V\frac{\partial}{\partial M^2}F_{1,2}(Q^2)$
- models respect current conservation, nucleon self-energy real-valued
- Maybe other matrix elements: $\langle\mathcal{O}(p^2)\rangle \approx \langle\mathcal{O}(M^2)\rangle + (p^2 - M^2)\frac{\partial}{\partial M^2}\langle\mathcal{O}(M^2)\rangle$
- Opens door to lattice QCD calculations
- precision nuclear structure calculations can compute virtuality

New approach to medium modifications of proton structure