Light Cone 2019

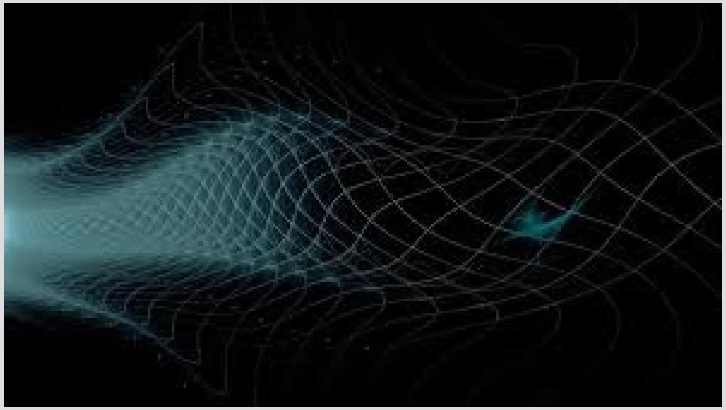
Poincaré constraints on the gravitational form factors

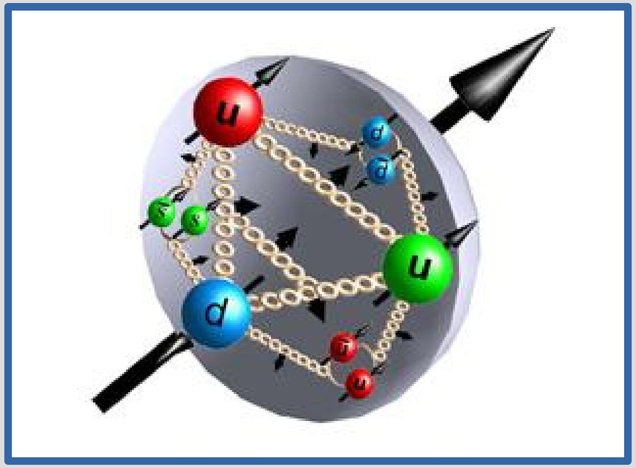
Peter Lowdon

(Ecole Polytechnique)

Outline

1. The Gravitational form factors
2. Constraints for massive states
3. Arbitrary state generalisation
4. Summary & outlook





[Iowa State University (2013)]

1. The Gravitational form factors

- The energy-momentum tensor (EMT) matrix elements encode many different dynamical properties, including:
  - *Quantum corrections to the gravitational motion of particles*
  - *Distribution of mass and angular momentum within hadrons*
- The EMT matrix elements can be decomposed into a series of form factors → these fully parametrise the **non-perturbative** information

Define on-shell states:

$$|p, m; M\rangle = \delta_M^{(+)}(p)|p, m\rangle \equiv 2\pi\,\theta(p^0)\,\delta(p^2 - M^2)|p, m\rangle$$

$$\langle p', m'; M|T^{\mu\nu}(0)|p, m; M\rangle = \overline{\eta}_{m'}(p')O^{\mu\nu}(p', p)\eta_m(p)\,\delta_M^{(+)}(p')\,\delta_M^{(+)}(p),$$

"Generalised" polarisation tensors (GPTs)

$$O^{\mu\nu}(p', p) = \bar{p}^{\{\mu}\bar{p}^{\nu\}}A(q^2) + i\bar{p}^{\{\mu}S^{\nu\}\rho}q_\rho\,G(q^2) + \cdots$$

Lorentz generator

1. The Gravitational form factors

→ **What are the constraints on these form factors?**

- Most previous studies chose to focus on massive (canonical spin) states with lower spin, in particular spin 0, ½ or 1
- Analyses often suffered from technical issues, such as the incorrect treatment of non-normalisable states or boundary terms, as detailed in: [Bakker, Leader, Trueman, hep-ph/0406139].
- A novel approach was developed in [PL, Chiu, Brodsky, 1707.06313] for the **spin-½** case in which the EMT matrix elements were treated rigorously, using their properties as **distributions**

  → Established that the $q \to 0$ limit of $A(q^2)$ and $G(q^2)$ are fixed by the Poincaré transformation properties of the states alone

Central question: *What about states with arbitrary spin?*

2. Constraints for massive states

- Use "distributional matching procedure" [Cotogno, Lorcé, PL, 1905.11969]:

Step 1: Construct rigorous expressions for the Lorentz charge operators in terms of the EMT components

$$J^i = \frac{1}{2}\epsilon^{ijk} \lim_{\substack{d\to 0\\ R\to\infty}} \int \mathrm{d}^4x\, f_{d,R}(x)\left[x^j T^{0k}(x) - x^k T^{0j}(x)\right],$$

$$K^i = \lim_{\substack{d\to 0\\ R\to\infty}} \int \mathrm{d}^4x\, f_{d,R}(x)\left[x^0 T^{0i}(x) - x^i T^{00}(x)\right],$$

Need smearing with appropriate test functions

Step 2: Use these definitions, together with the EMT form factor decomposition, to write the rotation and boost generator matrix elements in terms of these form factors

$$\langle p', m'; M|J^i|p, m; M\rangle = -i\epsilon^{ijk}\bar{p}^k(2\pi)^4\delta^{(+)}_M(\bar{p})\left[\delta_{m'm}\,\partial^j\delta^4(q) - \partial^j[\overline{\eta}_{m'}(p')\eta_m(p)]\big|_{q=0}\,\delta^4(q)\right]A(q^2) + \frac{1}{2}\epsilon^{ijk}(2\pi)^4\delta^{(+)}_M(\bar{p})\left[\overline{\eta}_{m'}(\bar{p})S^{jk}\eta_m(\bar{p})\right]\delta^4(q)\,G(q^2)$$

$$\langle p', m'; M|K^i|p, m; M\rangle = i(2\pi)^4\delta^{(+)}_M(\bar{p})\left[\delta_{m'm}\,(\bar{p}^0\partial^i - \bar{p}^i\partial^0)\delta^4(q) - \bar{p}^0\partial^i[\overline{\eta}_{m'}(p')\eta_m(p)]\big|_{q=0}\,\delta^4(q)\right]A(q^2) + (2\pi)^4\delta^{(+)}_M(\bar{p})\left[\overline{\eta}_{m'}(\bar{p})S^{0i}\eta_m(\bar{p})\right]\delta^4(q)\,G(q^2)$$

2. Constraints for massive states

<u>Step 3</u>: Use the transformation properties of on-shell states under rotations and boosts...

$$U(\alpha)|p,k;M\rangle = \sum_l \mathcal{D}^{(s)}_{lk}(\alpha)|\Lambda(\alpha)p,l;M\rangle,$$

...to write an arbitrary spin representation for the rotation and boost matrix elements in terms of the rest frame spin $\Sigma^i_{m'm}(k) = \overline{\eta}_{m'}(k)\, J^i\, \eta_m(k)$

$$\langle p',m';M|J^i|p,m;M\rangle = (2\pi)^4 \delta^{(+)}_M(\bar{p}) \left[\Sigma^i_{m'm}(k) - \delta_{m'm}\, i\epsilon^{ijk}\bar{p}^k \frac{\partial}{\partial q_j}\right] \delta^4(q)$$

$$\langle p',m';M|K^i|p,m;M\rangle = (2\pi)^4 \delta^{(+)}_M(\bar{p}) \left[-\frac{\epsilon^{ijk}\bar{p}^j}{\bar{p}^0 + M}\, \Sigma^k_{m'm}(k) + \delta_{m'm}\, i\left(\bar{p}^0 \frac{\partial}{\partial q_i} - \bar{p}^i \frac{\partial}{\partial q_0}\right)\right] \delta^4(q)$$

<u>Step 4</u>: Compare the two different representations!

$\rightarrow$ Implies the constraint $A(q^2)\,\delta^4(q) = G(q^2)\,\delta^4(q) = \delta^4(q)$

...which is simply: $\boldsymbol{A(0)=1}$ and $\boldsymbol{G(0)=1}$

2. Constraints for massive states

- Identical form factor constraints obtained from boost and rotation generators $\rightarrow$ ***not generator specific!***
- In fact, one can instead use the covariant operator basis
  - Pauli-Lubanski W^μ $\rightarrow$ implies: $\boldsymbol{G(0)=1}$
  - Covariant boost $B^\mu = ½(S^{\nu\mu}P_\nu + P_\nu S^{\nu\mu})$ $\rightarrow$ implies: $\boldsymbol{A(0)=1}$

The constraints are non-perturbative and **independent of both the spin and internal structure of the states** in the matrix elements $\rightarrow$ *fixed purely from Poincaré covariance of states*

Implications:

- Spin universality of GPD sum rules:

$$P^z = \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x\, x H_1^a(x,0,0) = A(0) = 1,$$

$$J^z = \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x\, x H_2^a(x,0,0) = G(0) = 1.$$

- AGM $B(0)=G(0)\text{-}A(0)$ vanishes for particles of **any** spin

3. Arbitrary state generalisation

- Relativistic spin states are **convention dependent**

$$|p,\sigma\rangle = U(L(p))|k,\sigma\rangle \qquad \Lambda(L(p))k = p$$

- Defined by choice of Lorentz transformation and reference vector

  (i) *"Canonical spin state"* $\rightarrow$ $k=(m,0,0,0)$, $L_c(p)$ = pure boost

  (ii) *"Wick helicity state"* $\rightarrow$ $k=(\kappa,0,0,\kappa)$, $L_w(p)$ = z-boost & rotation

- Results derived in most of the literature, including [Cotogno, Lorcé, PL, 1905.11969], assumed massive canonical spin states

  $\rightarrow$ **What happens for arbitrary states?**

- It turns out [Lorcé, PL, 1908.02567] that one can apply an analogous matching procedure

$$\eta_\sigma(p) = D(L(p))\eta_\sigma(k)$$

Key: need to take derivative wrt to momentum components of D(L(p))

3. Arbitrary state generalisation

- For the matrix element $\langle p',\sigma';M|\widetilde{U}(J^i)|p,\sigma;M\rangle = (2\pi)^4\delta^{(+)}_M(\bar{p})\,\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q),$

  one can write this in the state-independent form:

$$\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q) = -\,i\epsilon^{ijk}\bar{p}^k\delta_{\sigma'\sigma}\,\partial^j\delta^4(q)A(q^2) + i\epsilon^{ijk}\bar{p}^k\,\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(\frac{\partial L^{-1}(\bar{\boldsymbol{p}})}{\partial\bar{p}_j}L(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q)A(q^2)$$
$$+\,\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(L^{-1}(\bar{\boldsymbol{p}})J^iL(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q)\,G(q^2),$$

Lie algebra representation of D

- Similarly, the transformation properties of the states under **rotations** implies the general representation:

"Wigner rotation"

$$W(\mathcal{R}_i,\bar{p}) = L^{-1}\left(\Lambda\left(\mathcal{R}_i\right)\bar{p}\right)e^{-i\beta J^i}L(\bar{p}).$$

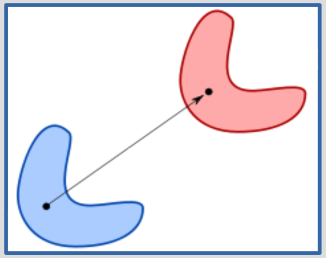
$$\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q) = -i\epsilon^{ijk}\bar{p}^k\delta_{\sigma'\sigma}\,\partial^j\delta^4(q) + i\,\delta^4(q)\left.\frac{d}{d\beta}\right|_{\beta=0}\overline{\eta}_{\sigma'}(k)D(W(\mathcal{R}_i,\bar{p}))\eta_\sigma(k).$$

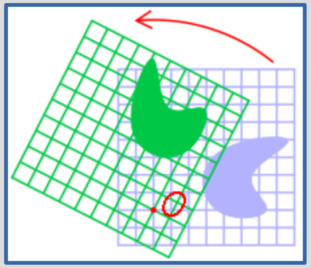
$$\mathcal{J}^i_{\sigma'\sigma}(\bar{p},q) = -\,i\epsilon^{ijk}\bar{p}^k\delta_{\sigma'\sigma}\,\partial^j\delta^4(q) + i\epsilon^{ijk}\bar{p}^k\,\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(\frac{\partial L^{-1}(\bar{\boldsymbol{p}})}{\partial\bar{p}_j}L(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q)$$
$$+\,\overline{\eta}_{\sigma'}(k)\widetilde{D}\left(L^{-1}(\bar{\boldsymbol{p}})J^iL(\bar{\boldsymbol{p}})\right)\eta_\sigma(k)\,\delta^4(q).$$

$\rightarrow$ Comparing these expressions implies: ***A*(0)=1** and ***G*(0)=1**

4. Summary & outlook

- By adopting a distributional approach one can prove on a ***non-perturbative level*** that Poincaré symmetry alone is responsible for the $q \to 0$ behaviour of $A(q^2)$ and $G(q^2)$





- These constraints hold ***independently*** of the internal properties of the states (internal structure, spin convention, mass, spin representation)

- Constraints imply: $\rightarrow$ GPD spin sum rules are ***state universal***

  $\rightarrow$ AGM vanishes for ***any*** particle

- Results could potentially have important implications in the context of gravitational scattering