

Energy-Momentum Tensor and Light Cone

LC2019 - QCD on the light cone: from hadrons
to heavy ions

Ecole Polytechnique, Palaiseau, France

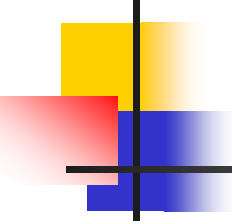
September 12, 2019

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JINR, Dubna



Main Topics

- EMT as a mechanical probe (shear)
- EMT as a gravity coupling
- EMT in hadrons/heavy ions interface



Most important classical density and quantum operator

Which single book to take to the uninhabited island?

Holy?

Shakespeare/Balzac/Goethe/Tolstoy/...?

Boat construction manual?

Single operator? EMT: probably even more
general and universal than EM current

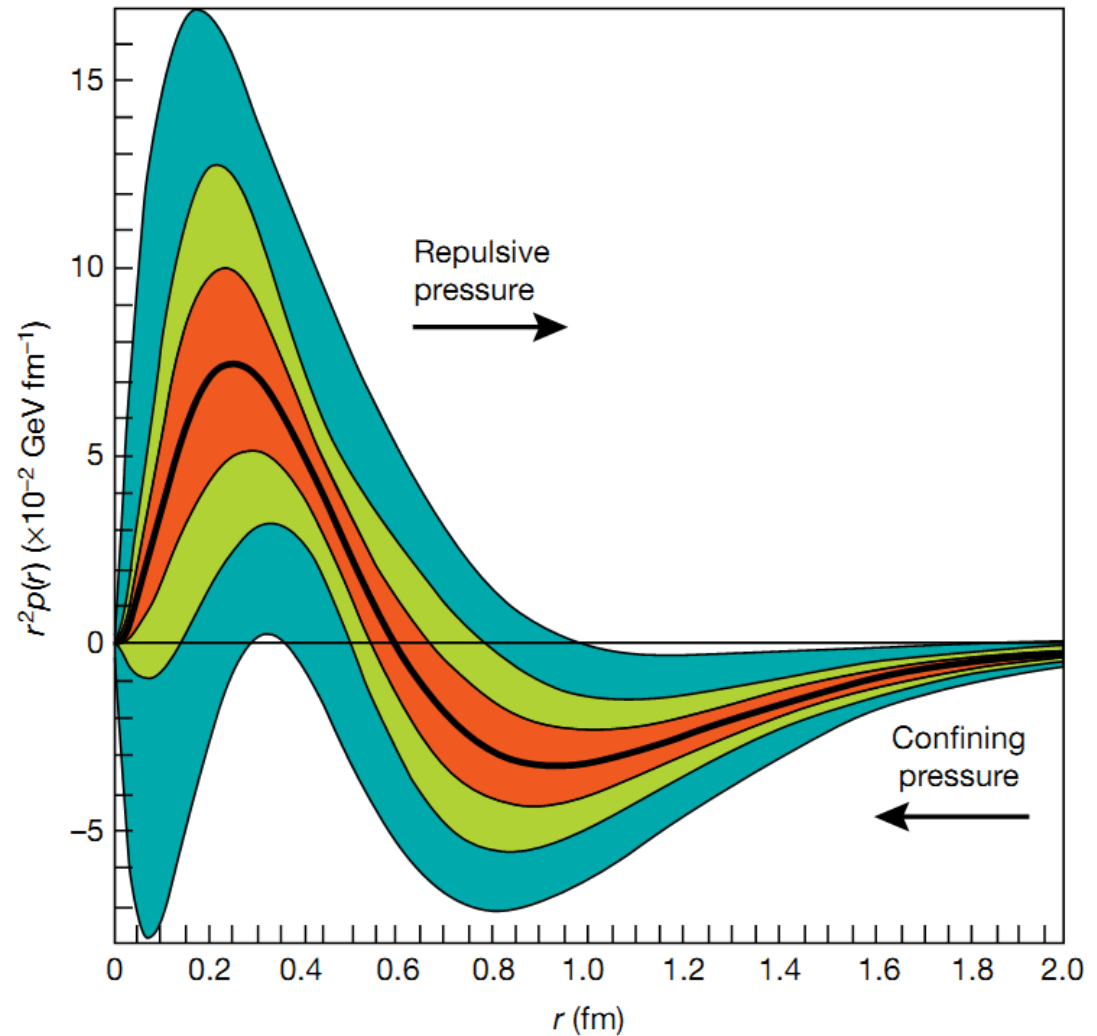
Continuous media

Hydro

Gravity

The pressure distribution inside the proton

V. D. Burkert^{1*}, L. Elouadrhiri¹ & F. X. Girod¹



Gravitational Formfactors (spin 1/2)

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M \right] u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Gravity and hadron structure: (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$
$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle



EP and hadron structure

“Microscopic” EP (coupling of gravity to EMT)

+

Conservation law

(Momentum SR to get local from LC: $\int dx \ x (\Sigma q(x) + G(x))=1$)

=

“Macroscopic” EP (universal falling) :

Tested VERY precisely



Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

$$\vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i}$$

spin dragging twice
smaller than EM

- Lorentz force – similar to EM case: factor 1/2 cancelled with 2 from same as EM

$$h_{00} = 2\phi(x) \quad \text{armor frequency}$$

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \quad \vec{H}_L = \text{rot} \vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle



Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on **quantum SPIN** – known since 1962 (Kobzarev and Okun’; rederived from conservation laws - Kobzarev and V.I. Zakharov)
- Anomalous gravitomagnetic (and electric-CP-odd) moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- Dirac eq; also for STRONG fields (Obukhov, Silenko, OT) : More general constraint (cf Lorce, Lowdon)?
- **Gravitational analog of Ji’s SR $\int dx \times (\Sigma E_q + E_G) = 0!$**



Experimental test of PNEP

Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW
LETTERS

VOLUME 68

13 JANUARY 1992

NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

If (CP-odd!) $G_{EDM}=0 \rightarrow$ constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \mathbf{B} \cdot \mathbf{S} - \zeta \hbar \boldsymbol{\omega} \cdot \mathbf{S}, \quad \zeta = 1 + \chi$$

$$|\chi(^{201}\text{Hg}) + 0.369\chi(^{199}\text{Hg})| < 0.042 \quad (95\% \text{C.L.})$$



Quantum measurement and EP

If spin is just a (pseudo) vector : EP due to Earth rotation is trivial

Crucial if measured by a device in rotating frame

Quantum measurement problem becomes practical

Cf Unruh effect in HIC (Prokhorov, OT, Zakharov'19)

Equivalence principle for moving particles

Compare gravity and acceleration: gravity provides EXTRA space components of metrics

$$h_{zz} = h_{xx} = h_{yy} = h_{00}$$

Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \quad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)

Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

Gravity vs accelerated frame for spin and helicity

Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim \mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency

Helicity flip – the same!

No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and

Flip by “gravitoelectric” field: relic neutrino?
Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

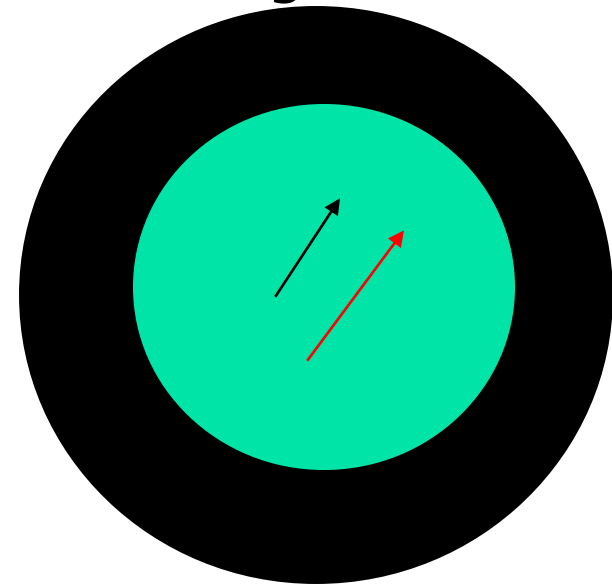
Cosmological implications of PNEP

Necessary condition for Mach's Principle (in the spirit of Weinberg's and MTW textbooks) Lense-Thirring inside massive rotating empty shell (=model of Universe)

For "flat Universe" - precession frequency equal to that of shell rotation

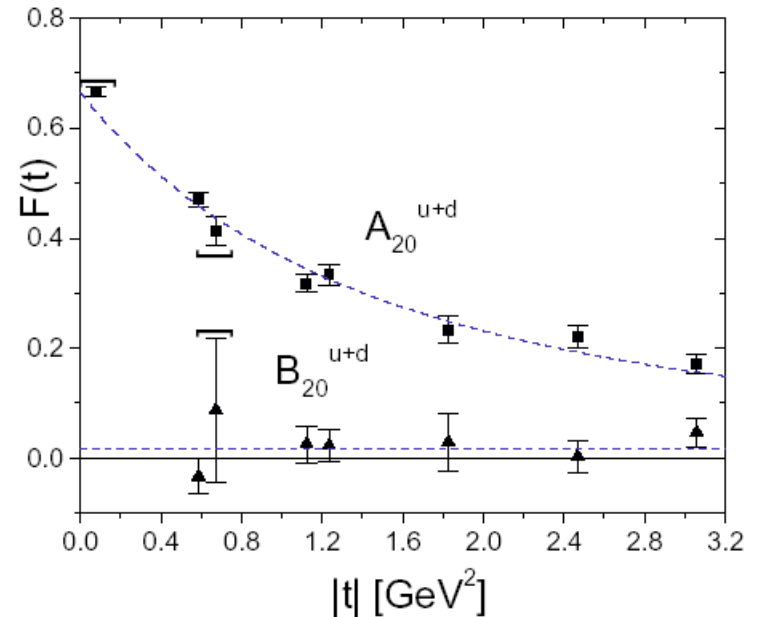
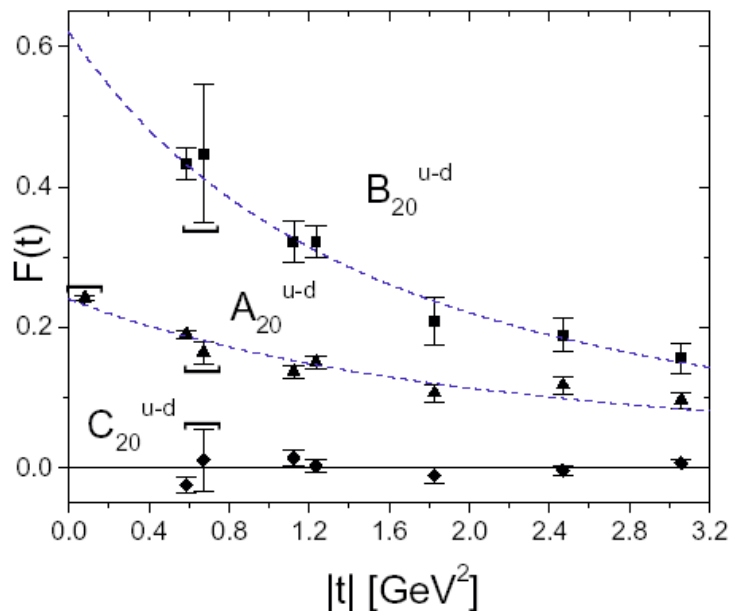
Simple observation-Must be the same for classical and quantum rotators – PNEP!

More elaborate models - Tests for cosmology ?!



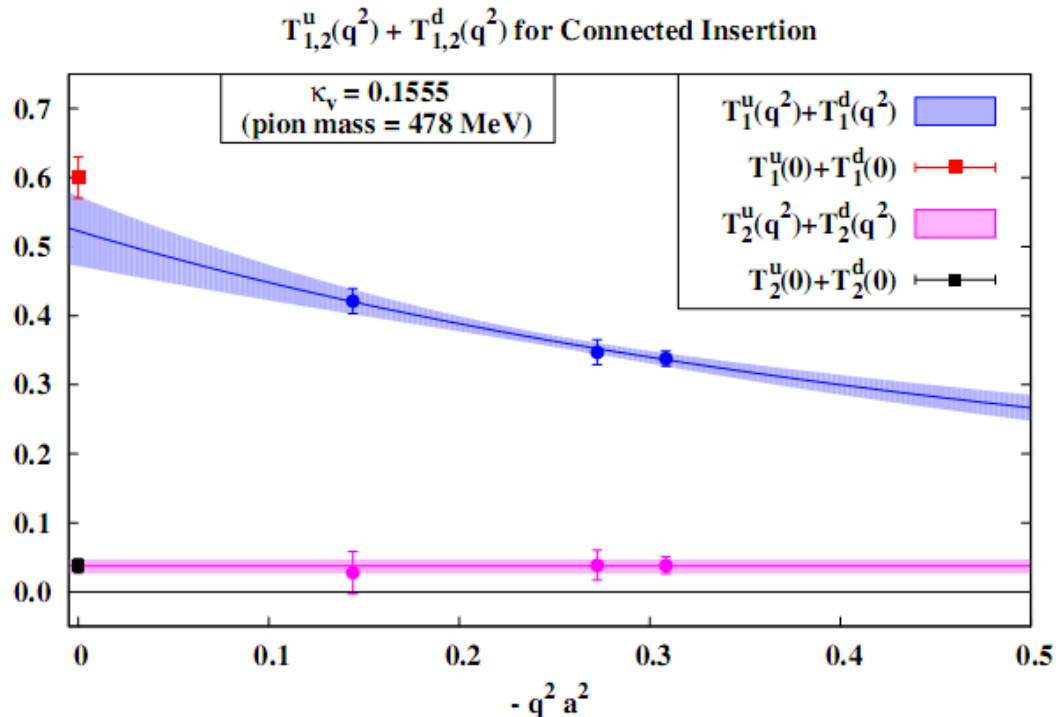
Generalization of Equivalence principle

- Various arguments: $AGM \approx 0$ separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence

Principle=Exact EquiPartition

- In QED, pQCD – violated (Brodsky et al)
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smallness of (nucleon “cosmological constant”) C_{bar}



One more gravitational formfactor (related to D-term)

- *Quadrupole*

$$\langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + \dots$$

- *Cf vacuum matrix element – cosmological constant*

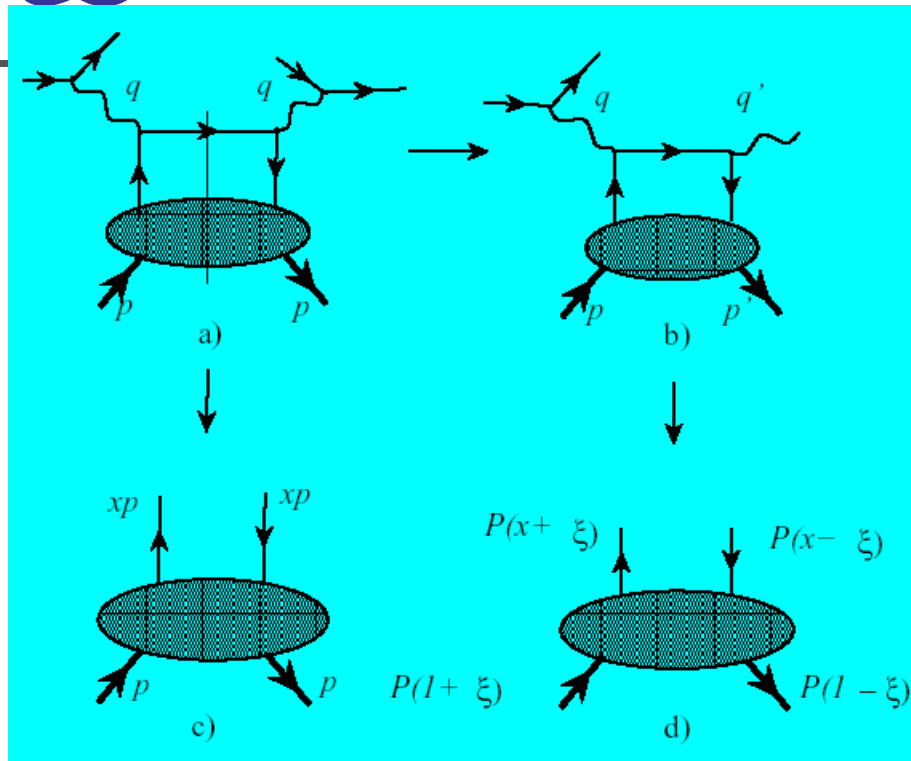
$$\langle 0 | T^{\mu\nu} | 0 \rangle = \Lambda g^{\mu\nu}$$

$$\Lambda = C(q^2)q^2$$

- *Inflation \sim annihilation ($q^2 > 0$) OT'15*

- *How to measure experimentally – DVCS (and DVMP?) : talk of P. Sznajder*

QCD Factorization for DIS and DVCS



- *Manifestly spectral*

$$\mathcal{H}(x_B) = \int_{-1}^1 dx \frac{H(x)}{x - x_B + i\epsilon}$$

- *Extra dependence on ξ*

$$\mathcal{H}(\xi) = \int_{-1}^1 dx \frac{H(x, \xi)}{x - \xi + i\epsilon}$$



Holographic property (OT'05)

■ *Factorization
Formula*

->

- *Analyticity ->
Imaginary part ->
Dispersion relation:*

$$\mathcal{H}(\xi) = \int_{-1}^1 dx \frac{H(x, \xi)}{x - \xi + i\epsilon}$$

$$\mathcal{H}(\xi) = \int_{-1}^1 dx \frac{H(x, x)}{x - \xi + i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^1 dx \frac{H(x, x) - H(x, \xi)}{x - \xi + i\epsilon}$$

- *"Holographic"
equation*

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^1 H(x, \xi) dx (x - \xi)^{n-1} = \text{const}$$



Holographic property - II

- *Directly follows from double distributions*

$$H(z, \xi) = \int_{-1}^1 dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y)$$

- *Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term $G(x, y)$*

$$\begin{aligned} \Delta \mathcal{H}(\xi) &= \int_{-1}^1 dx \int_{|x|-1}^{1-|x|} dy \frac{G(x, y)}{1-y} \\ &= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^1 dz \frac{D(z)}{z - 1} = \text{const} \end{aligned}$$



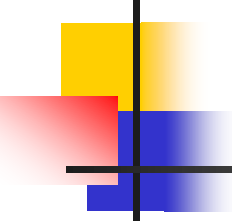
From D-term to pressure

- *Inverse -> 1st moment (model)*
- *Kinematical factor – moment of pressure $C \sim \langle p r^4 \rangle$
($\langle p r^2 \rangle = 0$) *M.Polyakov'03**

$$T_{\mu\nu}^Q(\vec{r}, \vec{s}) = \frac{1}{2E} \int \frac{d^3\Delta}{(2\pi)^3} e^{i\vec{r}\cdot\vec{\Delta}} \langle p', S' | \hat{T}_{\mu\nu}^Q(0) | p, S \rangle$$

$$T_{ij}(\vec{r}) = s(r) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}$$

- *Possible justification: Born gravitational scattering*
- *Stable equilibrium $C > 0$: Holds for quarks (or leptons) in photon*
- *Rest frame? T-channel states also?*



Spin 1 EMT (full analysis: Cosyn, Cotogno, Freese, Lorce) and **inclusive** processes

- Forward matrix element \rightarrow density matrix
- Contains **P-even** term: tensor polarization
- Symmetric and **traceless**: correspond to (average) **shear** forces
- For spin $1/2$: P-odd vector polarization requires another vector (q) to form vector product



SUM RULEs

- Efremov, OT'81 : zero sum rules:
- 1st moment: also in parton model by Close and Kumano (90)
- 2nd moment (forward analog of Ji's SR)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT'09

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

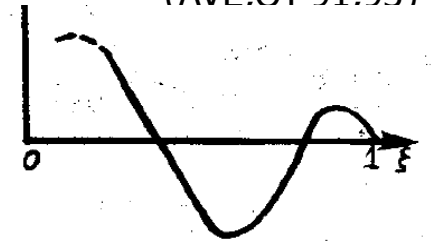
$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\int_0^1 C_i^T(x) dx = 0$$

$$\langle P, S | \bar{\psi}(0) \gamma^\nu D^{\nu_1} \dots D^{\nu_n} \psi(0) | P, S \rangle_{\mu^2} = i^{-n} M^2 S^{\nu\nu_1} P^{\nu_2} \dots P^{\nu_n} \int_0^1 C_q^T(x) x^n dx \quad (\text{AVE.OT'91.93})$$

$$\sum_q \langle P, S | T_i^{\mu\nu} | P, S \rangle_{\mu^2} = 2P^\mu P^\nu (1 - \delta(\mu^2)) + 2M^2 S^{\mu\nu} \delta_1(\mu^2)$$

$$\langle P, S | T_g^{\mu\nu} | P, S \rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu} \delta_1(\mu^2)$$

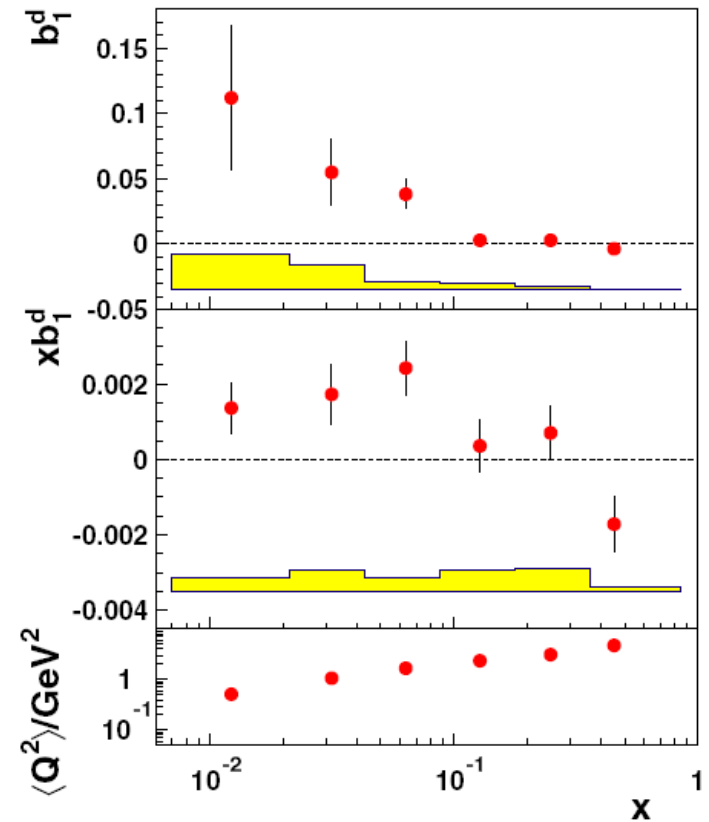


$$\sum_q \int_0^1 C_i^T(x) x dx = \delta_1(\mu^2) = 0 \quad \text{for ExEP}$$

HERMES – data on tensor spin structure function

PRL 95, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective glue \ll sea)

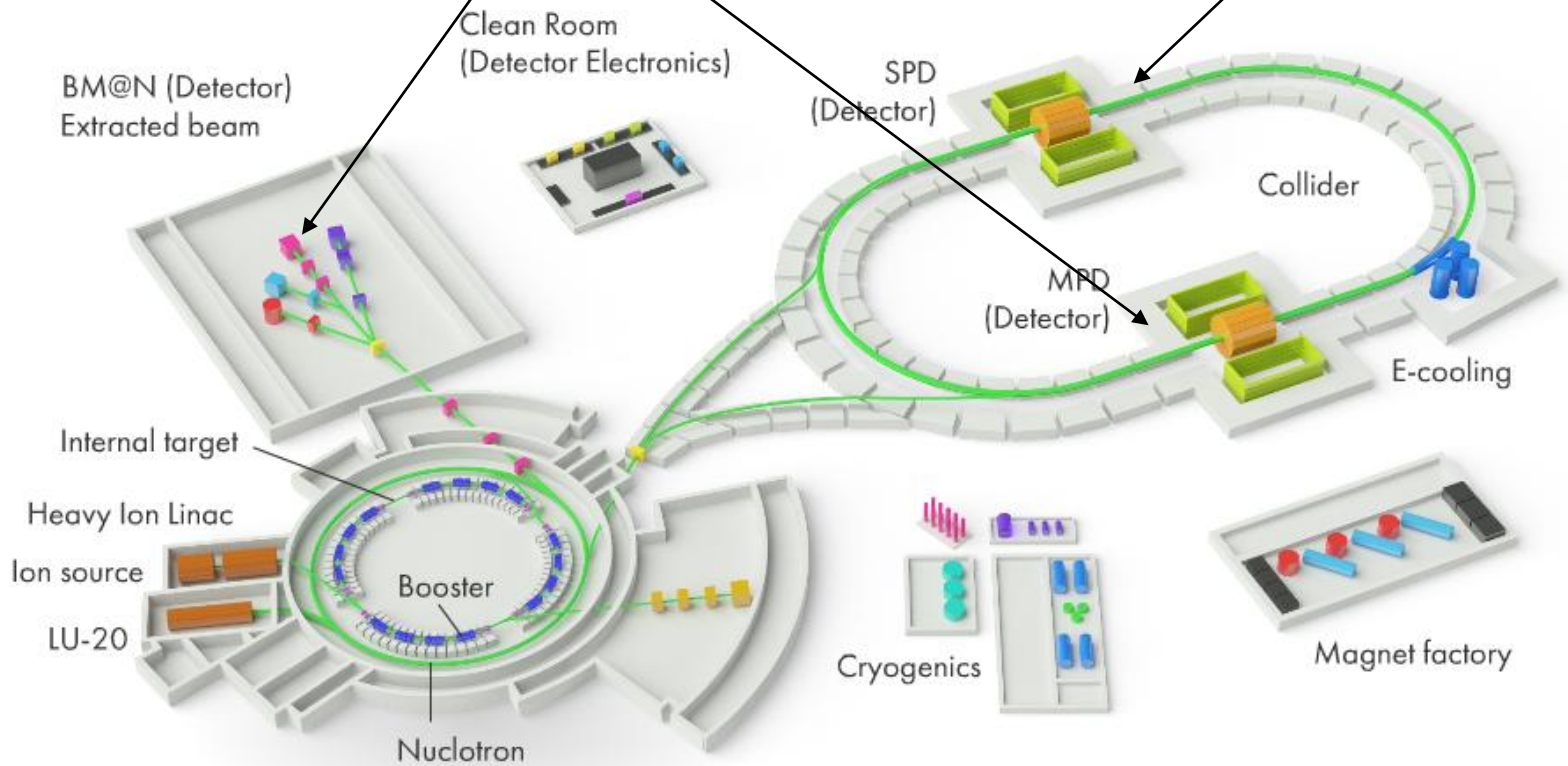




Where else to test?

- COMPASS
- EIC
- DY@J-PARC:
(Song,Kumano:1902.04712)
- However: ET'81-**any** hard process
- Suggestion: **hadronic** tensor SSA

NICA: heavy ions and hadrons





Tensor polarized beams

- Opportunity: NICA@JINR with polarized **hadronic** beams
- Polarized deuterons is easier to accelerate: no depolarizing resonances
- DY, J/ψ (+**hadronic** SSA)



Vector vs Tensor SSA

- Vector: $A = (\sigma(+)-\sigma(-))/(\sigma(+)+\sigma(-))$

Tensor:

$A =$

$$(\sigma(+)+\sigma(-)-2\sigma(0))/(\sigma(+)+\sigma(-)+\sigma(0))$$

- Inclusive pion production: (T-odd) vector SSA may be also excluded by summing $\sigma(L)+\sigma(R)$



Shear: viscosity?!

From spherically symmetric object to fluid
(EoS!)

$$T^{\mu\lambda} = (e+p) v^\mu v^\lambda - p g^{\mu\lambda}$$

$V^\mu = P^\mu/M$: correct normalization but no
coordinate dependence

Another suggestion:

$$V^\mu = (P^\mu + a(t) k_T^\mu) / (M^2 + a^2(t) k_T^2)^{1/2}$$

Viscosity: $\sim \eta p^{[\mu} \Delta^{\lambda]}$

Naïve T-oddness: phases



Viscosity in GDA channel

Possibility to study gravitational FFs in time-like region (Kumano, Song, OT'18)

Viscosity (new!): will correspond to **Exotic $J^{PC}=1^-$** + meson (already studied: Anikin, Pire, Szymanowski, OT, Wallon'06)

$\pi\eta$ pairs observation instead of $\pi\pi$ required

Smallness of viscosity: related to smallness of T-odd GPDs and exotic GDAs ?!



Another appearance of T-oddness in EMT: Burkardt SR

T-invariance : antisymmetry of twist 3 gluonic pole matrix element nullifies its contribution to EMT

BUT Pole prescription (dynamics!) provides ("T-odd") symmetric part (OT'14)!

SR:
$$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$

$$\sum \int dx T(x, x) = 0$$

Also EP!

ExEP: approximate validity separately for quarks and gluons: smallness of deuteron Sivers function



Conclusions

- EMT: appears in various (zero) SRs for various LC distributions
- EMT: link between QCD, gravity and HIC
- Gravitational Ffs; probes of coupling of quarks and gluons to gravity separate: EP/ExEP: gravity-proof confinement?!
- Tensor polarization: way to test of gravitational coupling of quarks and gluons in inclusive processes: shear
- Tests in DY and **hadronic** TSSA at NICA are possible
- Shear viscosity: relation to exotic hybrid mesons?!



BACKUP



D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + \dots$$

- Moment of D-term – positive

- Vacuum – Cosmological Constant

$$\langle 0 | T^{\mu\nu} | 0 \rangle = \Lambda g^{\mu\nu}$$

- 2D effective CC – negative in scattering, positive in annihilation

$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobinsky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe



C vs Cbar

- Cancellations of Cbars – negative pressure (cf Chaplygin gas)
- Cancellation in vacuum; Pauli (divergent), Zel'dovich (finite)
- Flavour structure of pressure: DVMP!



Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \leq |X_B|$

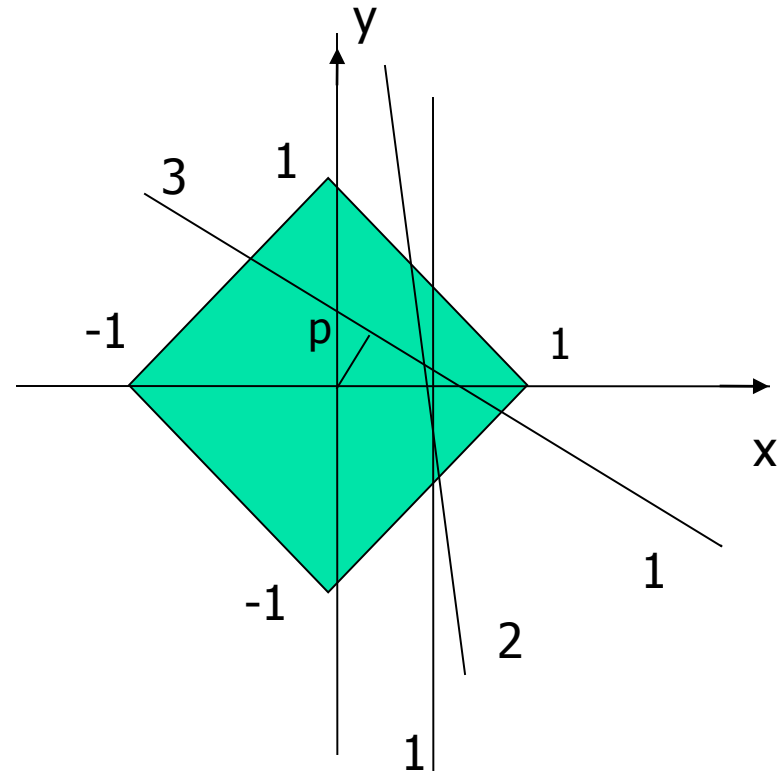
$$H(x_B) = -\int_{-1}^1 dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x, \xi)$
- Solved by using of Double Distributions Radon transform

$$H(z, \xi) = \int_{-1}^1 dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y)$$

Double distributions and their integration

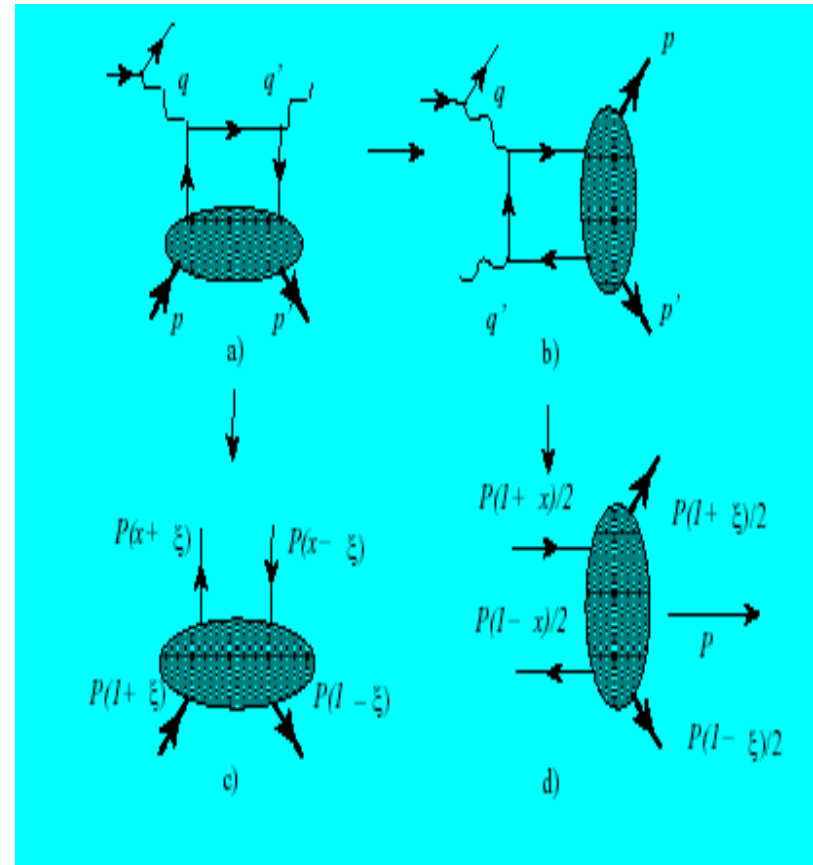
- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$
("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
- line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography



$$\begin{aligned}
 f(x, y) &= -\frac{1}{2\pi^2} \int_0^\infty \frac{dp}{p^2} \int_0^{2\pi} d\phi |\cos\phi| (H(p/\cos\phi + x + ytg\phi, t g\phi) - H(x + ytg\phi, t g\phi)) = \\
 &= -\frac{1}{2\pi^2} \int_{-\infty}^\infty \frac{dz}{z^2} \int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))
 \end{aligned}$$

Crossing for DVCS and GPD

- DVCS \rightarrow hadron pair production in the collisions of real and virtual photons
- GPD \rightarrow Generalized Distribution Amplitudes
- Duality between s and t channels
(Polyakov, Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = - \int_{-1}^1 dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = - \int_{-1}^1 dx \sum_{n=0}^{\infty} H(x, \xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power $n+1$
- As a result, analyticity is preserved: only non-positive powers of ξ appear



Holographic property - II

- Directly follows from double distributions

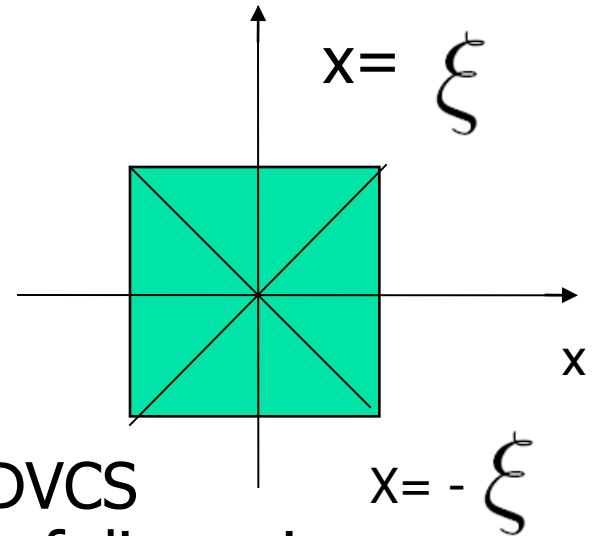
$$H(z, \xi) = \int_{-1}^1 dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y)$$

- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term $G(x, y)$

$$\begin{aligned} \Delta \mathcal{H}(\xi) &= \int_{-1}^1 dx \int_{|x|-1}^{1-|x|} dy \frac{G(x, y)}{1 - y} \\ &= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^1 dz \frac{D(z)}{z - 1} = \text{const} \end{aligned}$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!
- Strategy (now adopted) of GPD's studies: start at diagonals
(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants



Pressure in hadron pairs production

- Back to GDA region
- \rightarrow moments of $H(x,x)$ - define the coefficients of powers of cosine! $- 1/$
- Higher powers of cosine ξ in t-channel – threshold in s-channel
- Larger for pion than for nucleon pairs because of less fast decrease at $x \rightarrow 1$
- Stability defines the sign of GDA and (via soft pion theorem) DA: work in progress

$$\begin{aligned} \mathcal{H}(\xi) &= - \int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x, \xi) \frac{x^n}{\xi^{n+1}} \\ &= - \int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x, x) \frac{x^n}{\xi^{n+1}} + \Delta \mathcal{H}. \end{aligned}$$

Analyticity of Compton amplitudes in energy plane (Anikin, OT'07)

- Finite subtraction implied

$$\operatorname{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi} \mathcal{P} \int_{\nu_0}^{\infty} \frac{d\nu'^2}{\nu'^2} \frac{\operatorname{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2 - \nu^2)} + \Delta \quad \Delta = 2 \int_{-1}^1 d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta_{\text{CQM}}^p(2) = \Delta_{\text{CQM}}^n(2) \approx 4.4, \quad \Delta_{\text{latt}}^p \approx \Delta_{\text{latt}}^n \approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!
- Duality (sum of squares vs square of sum; proton: $4/9+4/9+1/9=1$)?!



Loss of stability?

- $D=0$ -> extra node required (cf tensor distribution - Efremov, OT- mechanical analogy – c.m. and c.i.)
- Smooth decrease – two extra nodes
- + + + + -----
- + + + + + + + ----- + + + + + -----
- $J=2$ (Talk of Barbara Pasquini, comment by Maxim – zeros of Bessel functs?!)



BACKUP



Is D-term independent?

- Fast enough decrease at large energy -

$$> \quad \text{Re } \mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi} \int_{\nu_0}^{\infty} d\nu'^2 \frac{\text{Im } \mathcal{A}(\nu')}{\nu'^2 - \nu^2} + \mathbf{C}_0.$$

$$\begin{aligned} \mathbf{C}_0 &= \Delta - \frac{\mathcal{P}}{\pi} \int_{\nu_0}^{\infty} d\nu'^2 \frac{\text{Im } \mathcal{A}(\nu')}{\nu'^2} \\ &= \Delta + \mathcal{P} \int_{-1}^1 dx \frac{H^{(+)}(x, x)}{x}. \end{aligned}$$

- FORWARD limit of Holographic equation

$$\begin{aligned} \Delta &= \mathcal{P} \int_{-1}^1 dx \frac{H^{(+)}(x, 0) - H^{(+)}(x, x)}{x} & \mathbf{C}_0(t) &= 2\mathcal{P} \int_{-1}^1 dx \frac{H(x, 0, t)}{x} \\ &= 2\mathcal{P} \int_{-1}^1 dx \frac{H(x, 0) - H(x, x)}{x}, \end{aligned}$$



“D – term” 30 years before...

- Cf Brodsky, Close, Gunion'72 (**seagull** \sim **pressure**) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?



Vector mesons and EEP

- $J=1/2 \rightarrow J=1$. QCD SR calculation of Rho's AMM gives g close to 2.
- Maybe because of similarity of moments
- $g-2 = \langle E(x) \rangle$; $B = \langle xE(x) \rangle$
- Directly for charged Rho (combinations like $p+n$ for nucleons unnecessary!). Not reduced to non-extended EP:



EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides $g=2$ identically!
- Experimental test at time –like region possible



EEP and Siverson function

- Siverson function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements



EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**) : $x f_T(x) : xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$



EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} J_{S\sigma}^5 + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T_B^{\mu\rho} - x^\rho T_B^{\mu\nu}$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J_{5S}^\alpha x^\nu) = 0$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P | J_{5S}^\alpha | P + q \rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P | J_{5S}^\gamma | P + q \rangle$$

$$\langle P, S | J_\mu^5(0) | P + q, S \rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2, \\ q^2 G_2|_0 = 0$$