Front-form approach to QED in an intense plane-wave field

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Outline of the talk

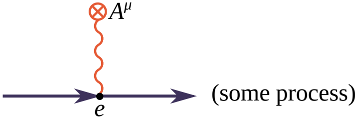
- Introduction
- Front-form approach
  - Light-cone quantization
  - Light-cone gamma-matrix basis
- Examples
  - Nonlinear Compton scattering
  - Polarization operator
  - Second-order processes

Introduction. High-intensity QED

Interaction of charge e with external EM field A^μ ($\hbar = c = 1$, a is a typical amplitude of the field):

- Low intensity (perturbative regime):

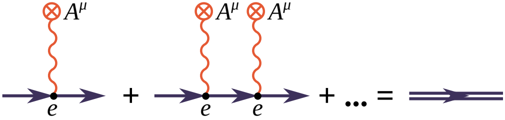
$$\xi = \frac{|e|a}{m} \ll 1.$$



Probability to interact with n photons $\propto \xi^{2n} \ll 1$.

- High intensity (nonperturbative regime):

$$\xi \gtrsim 1.$$



Nontrivial dependence on ξ.

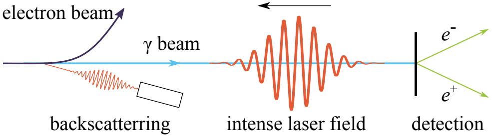
In the laboratory, the strongest EM fields of macroscopic extent are produced by optical lasers ($\omega_0 \sim 1$ eV):

- 1-PW class lasers are operational ($I \sim 10^{21}$ W/cm^2, $\xi \sim 10$):
  Apollon, Vulkan, Texas Petawatt Laser, CoReLS, ...
- 10-PW class are starting their operation ($I > 10^{22}$ W/cm^2, $\xi > 100$):
  Extreme Light Infrastructure (ELI), Apollon-10P, ...

Applications:

- Laser wakefield acceleration (compact electron/ion accelerators).
- Laboratory astrophysics.
- Tests of QED in the nonlinear regime, e.g.,

Vacuum birefringence and dichroism



SB et al., Phys. Rev. Lett. (2017)

A review: A. Di Piazza et al., Rev. Mod. Phys. (2012)

- Approximation for the laser field: classical plane wave

$$A^\mu(x) = A^\mu(\phi) = \sum_{i=1,2} a_i^\mu \psi_i(\phi), \quad \phi = k_0 x,\ k_0^2 = 0, \quad |\psi_i(\phi)| \lesssim 1.$$

The field tensor $F^{\mu\nu}(x) = F^{\mu\nu}(\phi) = \sum_{i=1,2} f_i^{\mu\nu} \psi_i'(\phi), \quad f_i^{\mu\nu} = k_0^\mu a_i^\nu - k_0^\nu a_i^\mu$.

- Special case: constant-crossed field ($\boldsymbol{E} \perp \boldsymbol{B}, |\boldsymbol{E}| = |\boldsymbol{B}|$)

$$\psi(\phi) = \phi.$$

An intense field of an arbitrary shape → locally constant-crossed field (in the rest frame of a particle).

- Application to more general field configurations:
  - laser-matter interaction (Particle-In-Cell codes).
- Potential applications:
  - astrophysical processes;
  - quark-gluon plasma.

- Light-cone coordinates are defined in a covariant way ($i, j, k, \ldots = 1, 2$):

$$\{\eta^\mu, \bar{\eta}^\mu, e_1^\mu, e_2^\mu\}, \quad \eta^2 = \bar{\eta}^2 = 0,\ \eta\bar{\eta} = 1,\ \eta e_i = \bar{\eta} e_i = 0,\ e_i e_j = -\delta_{ij}.$$

- Light-cone components of a vector v^μ:

$$v^+ = v\eta, \quad v^- = v\bar{\eta}, \quad v^1 = -ve_1, \quad v^2 = -ve_2.$$

- We choose $\eta^\mu = k_0^\mu/m$, then $\phi = mx^+$.
- Example 1: the laser wave propagates in the negative z-direction $[k_0^\mu = \omega_0(1, 0, 0, -1)]$:

$$v^+ = \frac{\omega_0}{m}(v^0 + v^3), \quad v^- = \frac{m}{2\omega_0}(v^0 - v^3), \quad v^1 = v_x, \quad v^2 = v_y.$$

- Example 2: a four-vector q^μ is such that $q^+ \neq 0$:

$$\eta^\mu = \frac{k_0^\mu}{m}, \quad \bar{\eta}^\mu = \frac{q^\mu}{q^+} - \frac{q^2\eta^\mu}{2q^{+2}}, \quad e_i^\mu = \Lambda_i^\mu = \frac{q_\lambda f_i^{\lambda\mu}}{mq^+\sqrt{-a_i^2}}.$$

S. Meuren et al., Phys. Rev. D (2013)

Front-form approach. Volkov solution

- Lightfront Hamiltonian: $H = H_0 + V_1 + V_2 + V_3$,
  - H_0 includes the interaction with the classical field $A^\mu(\phi)$ (Furry picture);
  - $V_1, V_2, V_3 \to$ the interactions with the quantized field $\mathcal{A}^\mu$.
- EOM for fermions is modified:

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi = 0.$$

- Solution (Volkov):

$$\psi_{p\sigma}(x) = K_p(\phi)u_{p\sigma}\mathrm{e}^{-ipx - i\mathcal{S}_p(\phi)},$$

$$K_p(\phi) = [\gamma\pi_p(\phi) + m]\frac{\gamma^+}{2p^+}, \quad \mathcal{S}_p(\phi) = \int_{-\infty}^{\phi} \mathrm{d}\beta \, [\pi_p^-(\beta) - p^- + eA^-(\beta)].$$

Dressed momentum $\pi_p^\mu(\phi)$ is the solution of the Lorentz equation in the field $A^\mu(\phi)$:

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \frac{e}{m}F^{\mu\nu}(\phi)p_\nu, \qquad \pi_p^\mu(\phi) = p^\mu - eA^\mu(\phi) + \eta^\mu\left(\frac{epA(\phi)}{p^+} - \frac{e^2A^2(\phi)}{2p^+}\right).$$

D. Mustaki et al., Phys. Rev. D (1991); S. J. Brodsky et al., Phys. Rep. (1998); W. H. Furry, Phys. Rev. (1951); D. M. Volkov, Z. Phys. (1935); A. Hartin, Phys. Rev. D (2016)

- The fermion field is expanded in Volkov states:

$$\psi(x) = \sum_{\sigma} \int \frac{d^2 p^{\perp}}{(2\pi)^2} \int_0^{\infty} \frac{dp^+}{2\pi} \left[a_{p\sigma} \psi_{p\sigma}(x) + b^{\dagger}_{p\sigma} \psi^{(-)}_{p\sigma}(x) \right].$$

(Note: Volkov states are complete only in the subspace of the '+' projections)

- Time-dependent perturbation theory (S-matrix approach).
- Dressed states vs Dressed vertices:

  - Three momentum components are conserved at each vertex ($+$, $\perp$);
  - Γ^{μ} contains nontrivial prefactor (up to 5 gamma matrices) and phase (all integrated).

- Usual gamma-matrix basis:

$$\{1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}, \quad \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3,\ \sigma^{\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2.$$

- Light-cone gamma-matrix basis:

$$\{1, \gamma^1, \gamma^2, \gamma^1\gamma^2\} \times \{\gamma^+, \gamma^-, \Lambda_+, \Lambda_-\},$$

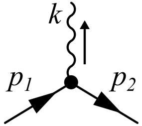
where

$$\gamma^+ = \gamma\eta, \quad \gamma^- = \gamma\bar{\eta}, \quad \gamma^i = -\gamma e_i, \quad \Lambda_\pm = \gamma^\mp\gamma^\pm/2.$$

- Some properties:
  - $\{\gamma^\pm, \gamma^i\} = 0$;
  - $\gamma^+\gamma^+ = \gamma^-\gamma^- = 0, \quad \gamma^+\gamma^-\gamma^+ = 2\gamma^+, \quad \gamma^-\gamma^+\gamma^- = 2\gamma^-$;
  - $\Lambda_+ + \Lambda_- = 1$;
  - $\mathrm{Tr}\{\gamma^{i_1}\gamma^{i_2}\ldots\gamma^{i_n}\gamma^+\gamma^-\} = \mathrm{Tr}\{\gamma^{i_1}\gamma^{i_2}\ldots\gamma^{i_n}\gamma^-\gamma^+\} = \mathrm{Tr}\{\gamma^{i_1}\gamma^{i_2}\ldots\gamma^{i_n}\}$.

SB, PhD Thesis (2019)

Front-form approach. Three-point dressed vertex



The vertex function:

$$\Gamma^{\mu}_{21}(k) = \int \mathrm{d}x^+ \, K^{\mu}_{21}(\phi) \, \exp\left[i(p_2^- + k^- - p_1^-)x^+ + i\mathcal{S}_2(\phi) - i\mathcal{S}_1(\phi) \right],$$

where

$$K^{\mu}_{21}(\phi) = \overline{K}_2(\phi)\gamma^{\mu}K_1(\phi) = \left[S^{\mu}_{21}(\phi) + V^{i\mu}_{21}\gamma_i + T^{\mu}_{21}(\phi)\gamma^1\gamma^2 \right]\gamma^+.$$

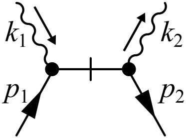
The coefficients:

$$S^{\mu}_{21}(\phi) = \frac{1}{2p_2^+p_1^+}\left\{ p_2^+\pi_1^{\mu}(\phi) + p_1^+\pi_2^{\mu}(\phi) - \left[\pi_2(\phi)\pi_1(\phi) - m^2\right]g^{\mu+} \right\},$$

$$V^{i\mu}_{21} = \frac{m}{2p_2^+p_1^+}\left[(p_2^+ - p_1^+)g^{\mu i} - (p_2^i - p_1^i)g^{\mu+} \right],$$

$$T^{\mu}_{21}(\phi) = -\frac{1}{2p_2^+p_1^+}\epsilon_{\nu\rho\varkappa\lambda}g^{\mu\nu}\eta^{\rho}\pi_2^{\varkappa}(\phi)\pi_1^{\lambda}(\phi).$$

"Seagull": $\bar{u}_2 \dfrac{-ie^2}{2(p_1^+ + k_1^+)} \Gamma_{21}^{\mu\nu}(k_2, k_1)\, u_1 \epsilon_2^{*\mu} \epsilon_1^{\nu}$,

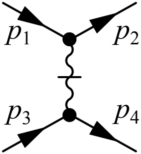


$$\Gamma_{21}^{\mu\nu}(k_2, k_1) = \int \mathrm{d}x^+ K_{21}^{\mu\nu}(\phi) \exp\left[i(p_2^- + k_2^- - p_1^- - k_1^-)x^+ + i\mathcal{S}_2(\phi) - i\mathcal{S}_1(\phi)\right],$$

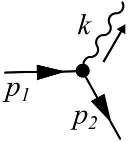
$$K_{21}^{\mu\nu}(\phi) = \overline{K}_2(\phi)\gamma^\mu\gamma^+\gamma^\nu K_1(\phi) = \left[S_2^\mu + V_2^{i\mu}(\phi)\gamma_i\right]\gamma^+\left[S_1^\nu + V_1^{j\nu}(\phi)\gamma_j\right],$$

$$S_p^\mu = \frac{m}{p^+} g^{\mu+}, \qquad V_p^{i\mu}(\phi) = \frac{1}{p^+}\left[p^+ g^{\mu i} - \pi_p^i(\phi) g^{\mu+}\right].$$

"Self-interaction": $\int \mathrm{d}x^+\, \bar{u}_4 \Gamma_{43}(\phi) u_3 \dfrac{-ie^2}{(p_2^+ - p_1^+)^2} \bar{u}_2 \Gamma_{21}(\phi) u_1$,



$$\Gamma_{21}(\phi) = \gamma^+ \exp\left[i(p_2^- - p_1^-)x^+ + i\mathcal{S}_2(\phi) - i\mathcal{S}_1(\phi)\right].$$



- Average/sum over the polarization states (the initial electron is unpolarized):

$$|S_{fi}|^2 \to \frac{1}{4}\sum M_{fi}(\phi_2)M_{fi}(\phi_1)^* = -e^2\mathcal{T}_{21}(\phi_2,\phi_1),$$

$$\mathcal{T}_{21}(\phi_2,\phi_1) = \frac{1}{4}\mathrm{Tr}\{(\gamma p_2 + m)K^{\mu}_{21}(\phi_2)(\gamma p_1 + m)K^{\nu}_{12}(\phi_1)\}g_{\mu\nu}.$$

- Insert the expressions for the dressed vertex:

$$\mathcal{T}_{21}(\phi_2,\phi_1) = \frac{1}{4}\mathrm{Tr}\Big\{(\gamma p_2 + m)\left[S^{\mu}_{21}(\phi_2) + V^{i\mu}_{21}\gamma_i + T^{\mu}_{21}(\phi_2)\gamma^1\gamma^2\right]\gamma^+ \\ \times(\gamma p_1 + m)\left[S^{\nu}_{12}(\phi_1) + V^{j\nu}_{12}\gamma_j + T^{\nu}_{12}(\phi_1)\gamma^1\gamma^2\right]\gamma^+\Big\}g_{\mu\nu}.$$

- Continue:

$$\mathcal{T}_{21}(\phi_2,\phi_1) = \frac{1}{2} p_2^+ p_1^+ \mathrm{Tr}\Big\{ \Big[S_{21}^{\mu}(\phi_2) + V_{21}^{i\mu}\gamma_i + T_{21}^{\mu}(\phi_2)\gamma^1\gamma^2 \Big] \times \Big[S_{12}^{\nu}(\phi_1) + V_{12}^{j\nu}\gamma_j + T_{12}^{\nu}(\phi_1)\gamma^1\gamma^2 \Big] \Big\} g_{\mu\nu}.$$

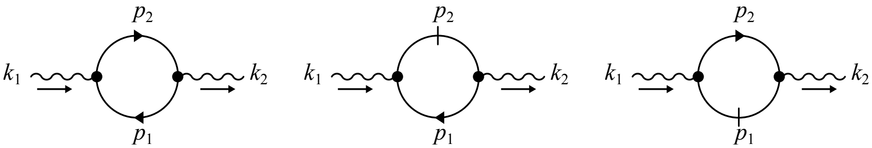
- The trace is trivial:

$$\mathcal{T}_{21}(\phi_2,\phi_1) = 2p_2^+ p_1^+ \Big[S_{21}^{\mu}(\phi_2) S_{12}^{\nu}(\phi_1) + V_{21}^{i\mu} V_{12}^{j\nu} g_{ij} - T_{21}^{\mu}(\phi_2) T_{12}^{\nu}(\phi_1) \Big] g_{\mu\nu}.$$

- Make the contractions with $g_{\mu\nu}$:

$$\mathcal{T}_{21}(\phi_2,\phi_1) = 2m^2 - \frac{1}{2}\left(\frac{p_2^+}{p_1^+} + \frac{p_1^+}{p_2^+} \right) \Delta_1^2(\phi_2,\phi_1),$$

$$\Delta_1^{\mu}(\phi_2,\phi_1) = \pi_1^{\mu}(\phi_2) - \pi_1^{\mu}(\phi_1).$$



- The noninstantaneous-noninstantaneous term:

$$\mathcal{P}^{\mu\nu}_{\text{nn}} \to \mathcal{T}^{\mu\nu}_{\text{nn}} \propto \left[S^{\mu}_{21}(\phi_2)S^{\nu}_{12}(\phi_1) + V^{i\mu}_{21}V^{j\nu}_{12}g_{ij} - T^{\mu}_{21}(\phi_2)T^{\nu}_{12}(\phi_1)\right].$$

- Complete and orthogonal basis sets for the left- and right-hand sides [remember that $k_{2\mu}\mathcal{P}^{\mu\nu}(k_2, k_1) = \mathcal{P}^{\mu\nu}(k_2, k_1)k_{1\nu} = 0$]:

$$\left\{k^{\mu}_2, \Lambda^{\mu}_1, \Lambda^{\mu}_2, Q^{\mu}_2\right\}, \quad \left\{k^{\nu}_1, \Lambda^{\nu}_1, \Lambda^{\nu}_2, Q^{\nu}_1\right\},$$

$$\Lambda^{\mu}_i = \frac{q_\lambda f^{\lambda\mu}_i}{mq^+\sqrt{-a^2_i}}, \quad Q^{\mu}_i = \frac{k^2_i\eta^{\mu} - q^+k^{\mu}_i}{q^+}, \quad q^{\mu} = k^{\mu}_1.$$

V. N. Baier et al., Sov. Phys. JETP (1976); S. Meuren et al., Phys. Rev. D (2013)

- The coefficients:

$$\Lambda_k S_{21}(\phi) = \frac{p_2^+ + p_1^+}{2p_2^+ p_1^+}\pi_1^k(\phi), \quad \Lambda_k V_{21}^i = \frac{m(p_2^+ - p_1^+)}{2p_2^+ p_1^+} g^{ik},$$
$$\Lambda_k T_{21}(\phi) = -\frac{p_2^+ - p_1^+}{2p_2^+ p_1^+}\epsilon^{kl}\pi_{1l}(\phi), \quad \eta S_{21}(\phi) = 1, \quad \eta V_{21}^i = \eta T_{21}(\phi) = 0.$$

- Symmetry relations:

$$S_{12}^\mu(\phi) = S_{21}^\mu(\phi), \quad V_{12}^{i\mu} = -V_{21}^{i\mu}, \quad T_{12}^\mu(\phi) = -T_{21}^\mu(\phi).$$

- The result:

$$\mathcal{T}_{\rm nn}^{\mu\nu} = a^{12}\Lambda_1^\mu\Lambda_2^\nu + a^{21}\Lambda_2^\mu\Lambda_1^\nu + b^{12}\Lambda_1^\mu\Lambda_1^\nu + b^{21}\Lambda_2^\mu\Lambda_2^\nu + c_5 Q_2^\mu Q_1^\nu$$
$$+ d^1(\phi_1)Q_2^\mu\Lambda_1^\nu + d^2(\phi_1)Q_2^\mu\Lambda_2^\nu + d^1(\phi_2)\Lambda_1^\mu Q_1^\nu + d^2(\phi_2)\Lambda_2^\mu Q_1^\nu.$$

- General structure:

$$\mathcal{T}_{4321} \propto \mathrm{Tr}\Big\{ \Big(S_{43}^{\varkappa} + V_{43}^{i\varkappa}\gamma_i + T_{43}^{\varkappa}\gamma^1\gamma^2\Big)\Big(S_{32}^{\lambda} + V_{32}^{j\lambda}\gamma_j + T_{32}^{\lambda}\gamma^1\gamma^2\Big)$$
$$\times \Big(S_{21}^{\mu} + V_{21}^{k\mu}\gamma_k + T_{21}^{\mu}\gamma^1\gamma^2\Big)\Big(S_{14}^{\nu} + V_{14}^{l\nu}\gamma_l + T_{14}^{\nu}\gamma^1\gamma^2\Big)\Big\} g_{\varkappa\nu} g_{\lambda\mu}.$$

- Evaluate in pairs, then

$$\mathcal{T}_{4321} \propto \Big(S_{3221}S_{1443} + V_{3221}^{i}V_{1443}^{j}g_{ij} - T_{3221}T_{1443}\Big).$$

- The result can be written in a relatively compact form, via parameters, analogous to Mandelstam variables in vacuum. Relevant momentum combinations:

$$Z_p^{\mu}(\phi,\phi') = \Big[\pi_p^{\mu}(\phi) + \pi_p^{\mu}(\phi')\Big]/2, \quad \Delta_p^{\mu}(\phi,\phi') = \pi_p^{\mu}(\phi) - \pi_p^{\mu}(\phi').$$

Summary

- High intensities of EM fields, which are accessible at experiments, require the field to be treated nonperturbatively.
- QED in an intense plane-wave field can be conveniently formulated in a complete front-form fashion, with the help of light-cone quantization and the light-cone gamma-matrix basis.
- Front-form treatment of the bispinor part allows to write the final result in a compact form, expressed via parameters, similar to Mandelstam variables in the vacuum case.

Thank you for your attention!