Positronium on the Light-front

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Outline

- Light-front Dynamics & Basis Light-front Quantization
- Positronium
  - Motivation
  - Nonperturbative renormalization
  - Numerical Results
- Heavy quarkonium
  - Developments based on the positronium project
  - Numerical Results
- Summary and Outlook

Why Positronium

Positronium is a test bed for

- Relativistic bound state structure beyond leading Fock-sector

$$|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle + \ldots$$

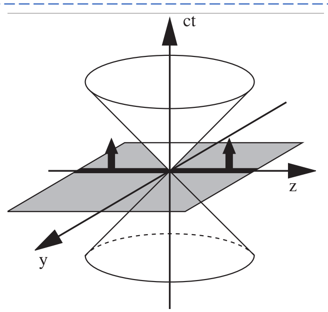
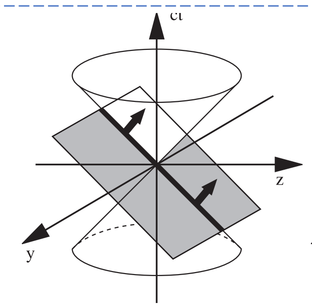
- Basis Light-front Quantization on first-principle of QED, esp., nonperturbative renormalization procedure

- Connection with one-photon-exchange effective theory

[Wiecki, et al, 2015]

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

- not just a coordinate transformation.
- a new theory in a different quantization.

Why go to light front?

- Frame independent wavefunction
- Simple vacuum structure
- Boost invariant
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^{-}|\beta\rangle = P^{-}_{\beta}|\beta\rangle$$

  - P^{-}: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^{-}_{β}: eigenvalue for $|\beta\rangle$

See James Vary's talk on Thu

See Chandan Mondal's talk on Wed

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^{b}_{n,m}(\vec{p}_{\perp})$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

Light-front QED Hamiltonian

- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$

- Light-front QED Hamiltonian from standard Legendre transformation

$$P^- = \int \mathrm{d}^2x^\perp \mathrm{d}x^- \;\; F^{\mu+}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

Light-cone gauge: $\left(A^+ = 0\right)$

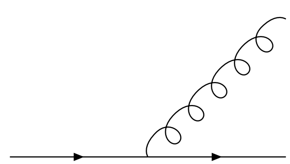
$$= \int \mathrm{d}^2x^\perp \mathrm{d}x^- \;\; \frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^\perp)^2 A^j$$

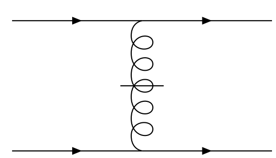
kinetic energy terms

$$+ e j^\mu A_\mu + \frac{e^2}{2} j^+ \frac{1}{(i\partial^+)^2} j^+$$

vertex interaction

instantaneous photon interaction

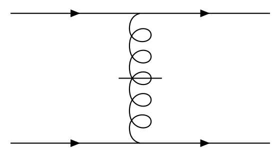
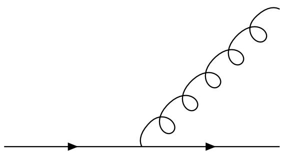
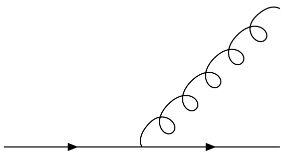




Interaction Part Of Hamiltonian

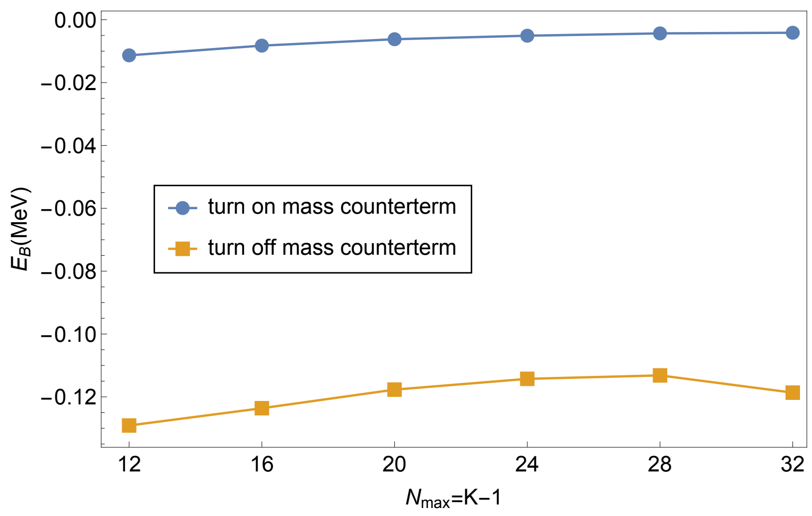
$|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle + \ldots$

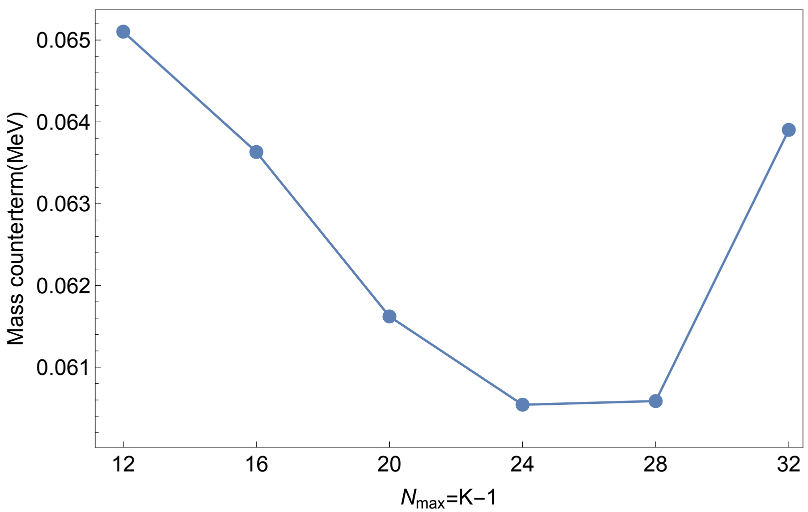
m_e=1.0MeV

H_{int}	$\|e\bar{e}\rangle$	$\|e\bar{e}\gamma\rangle$
$\langle e\bar{e}\|$		
$\langle e\bar{e}\gamma\|$		0

Mass Renormalization

α=0.3





- Mass renormalization is performed on the level single physical electron
- Mass counterterm is determined by fitting single electron mass
- Plug the physical electron and positron into the positronium.

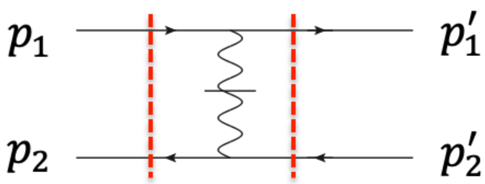
Mass counterterm is much larger than E_B

[Kaiyu Fu et al, in preparation]

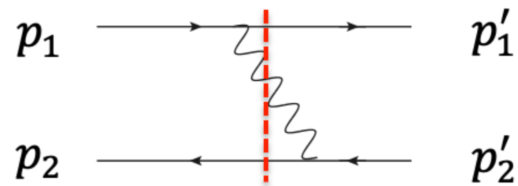
Ultraviolet Cutoff for Instantaneous Photon b_{inst}

- Mismatch between explicit and instantaneous photon interactions:

for instantaneous photon:
$p_{\rm rel} = p_1 - p_2$ not limited



for explicit photon:
$p_{\rm rel} = p_1 - p_2$ subject to N_{max} truncation



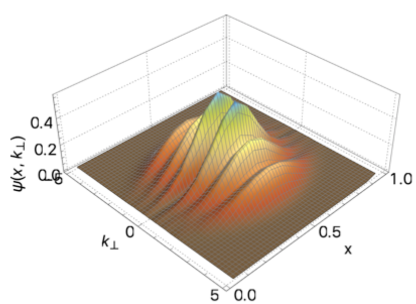
- Introduce cutoff parameter b_{inst} for instantaneous photon interaction:

$$V_{inst} \equiv \int \mathrm{d}^2 x^{\perp} \mathrm{d}x^{-} \, j^{+} \frac{1}{(i\partial^{+})^2} j^{+} \longrightarrow V_{inst} \times \exp\left(-\frac{p_{\perp}^2}{b_{\rm inst}^2}\right)$$

- $b_{\rm inst}$ is chosen by maximizing the prob. of n=m=0 HO basis state in the ground state.

$|n_1 = 0, n_2 = 0, m_1 = 0, m_2 = 0\rangle$

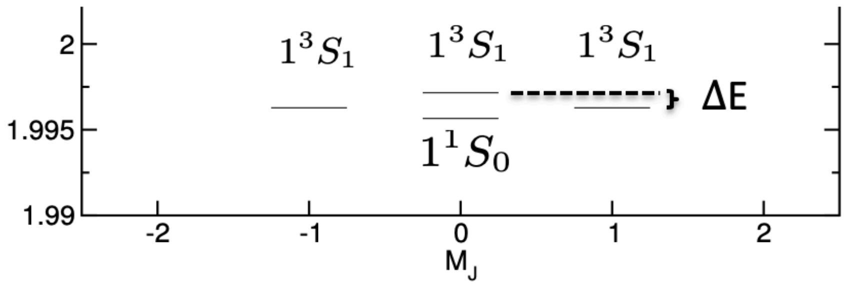
Since without $b_{\rm inst}$

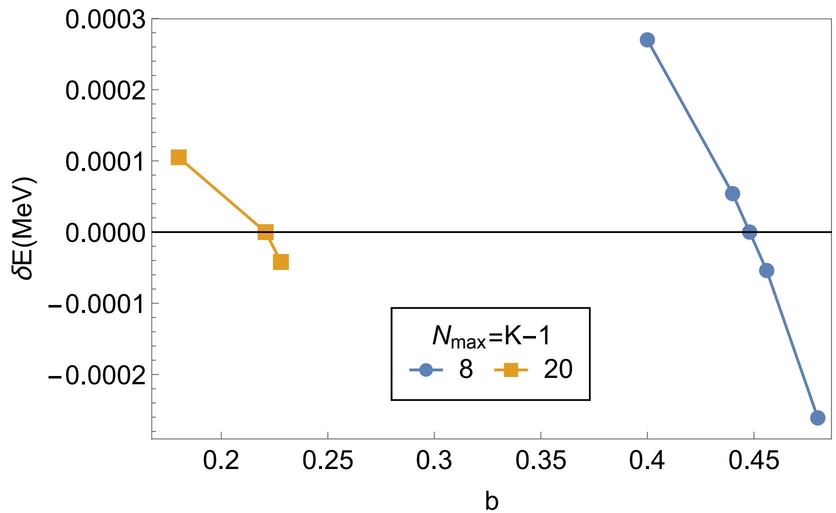


Basis b

- Suitable basis scale would make results easier to converge

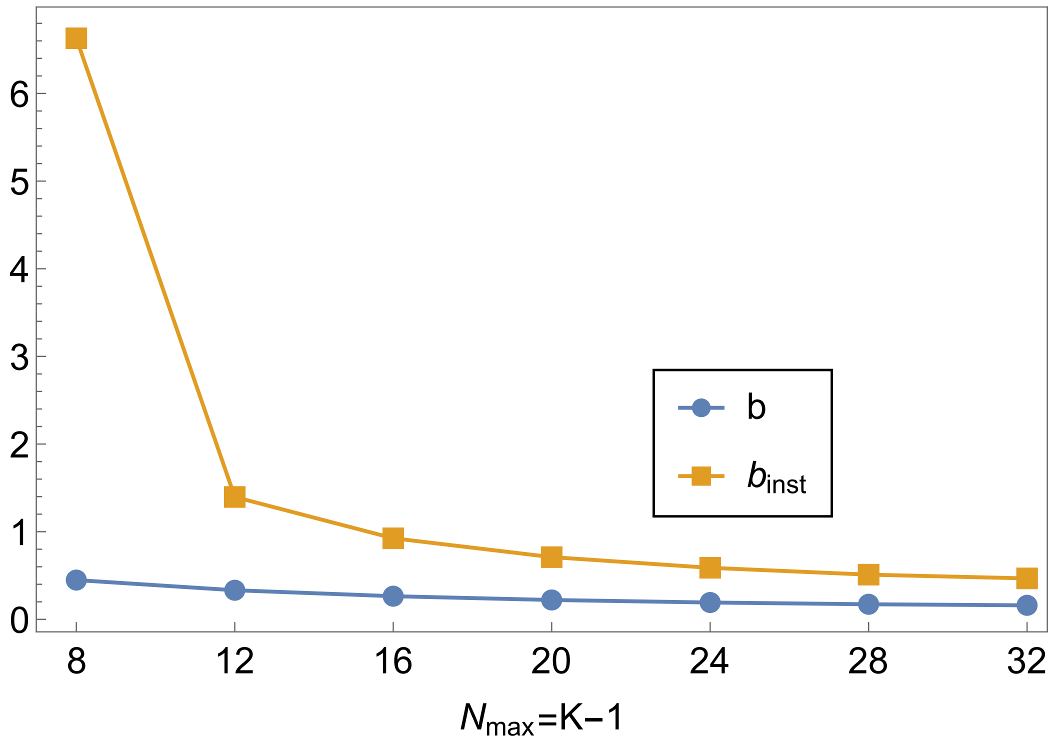
- Rotational symmetry as an indicator





A good choice will minimize the energy difference within the triplet 1^3S_1

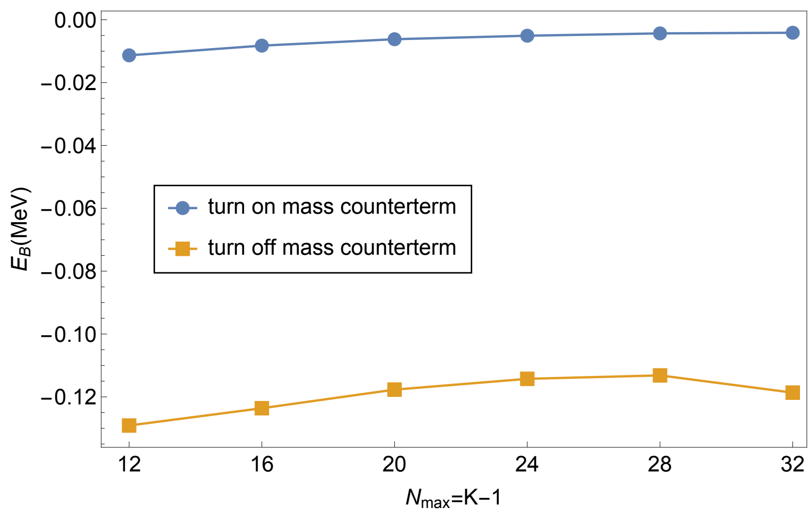
b_{inst} and Basis b dependence

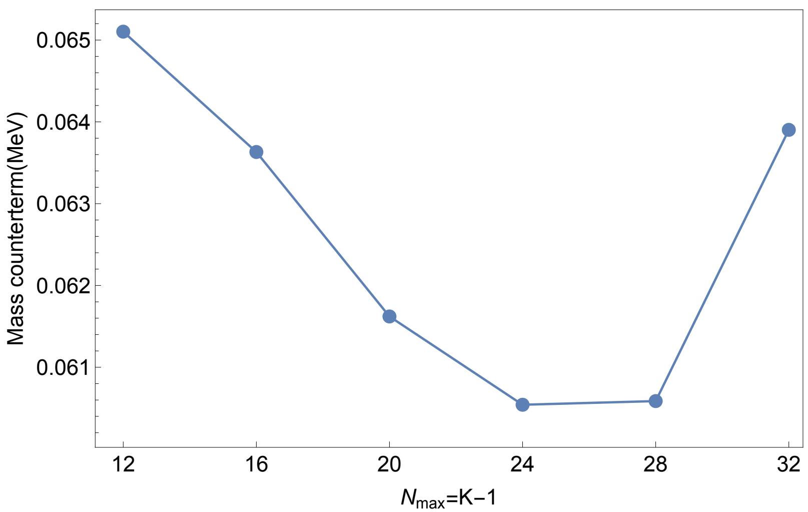


α=0.3

- For each basis truncation, we chose a suitable b_{inst} and b
- As N_{max} increases, the b_{inst} seems converge with b ?
- We may only need to deal with the photon that binds fermions when suppressing the mismatch ?

Mass Renormalization





α=0.3

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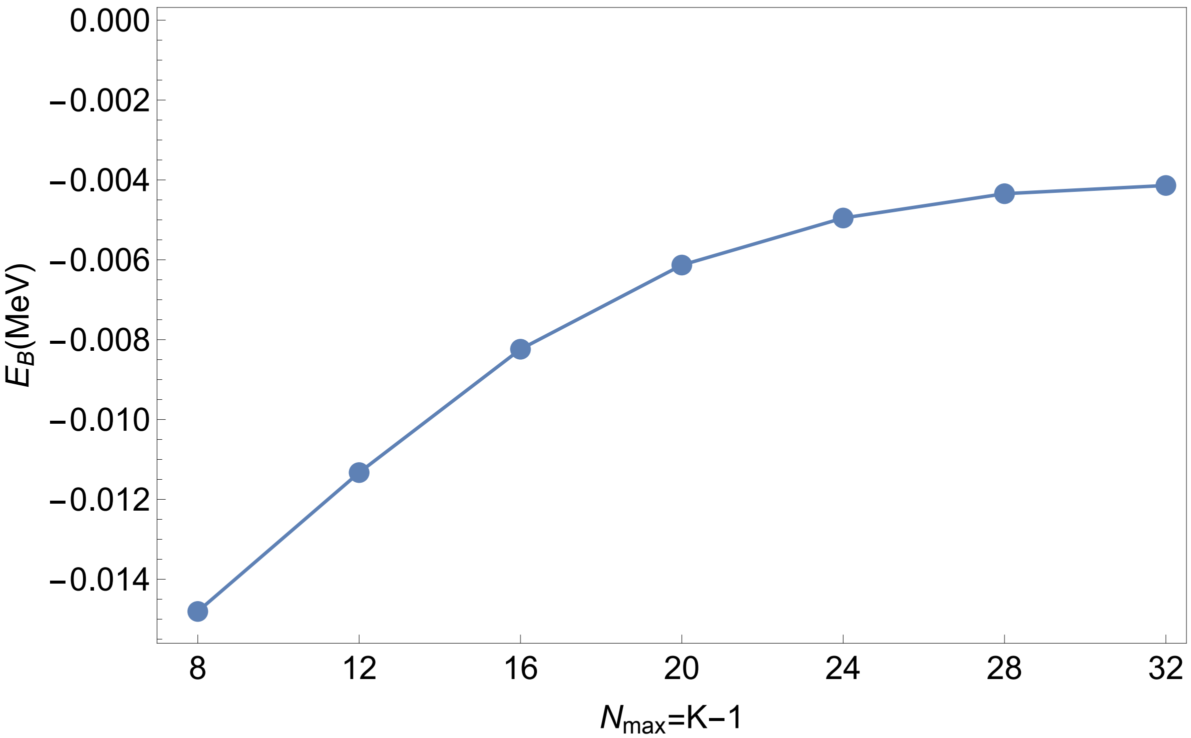
Mass counterterm is much larger than E_B

[Kaiyu Fu et al, in preparation]

Ground State Binding Energy



$$E_B = M_P - 2M_e$$

- E_B: binding energy of positronium
- M_P: Invariant mass of positronium
- M_e: Invariant mass of free electron

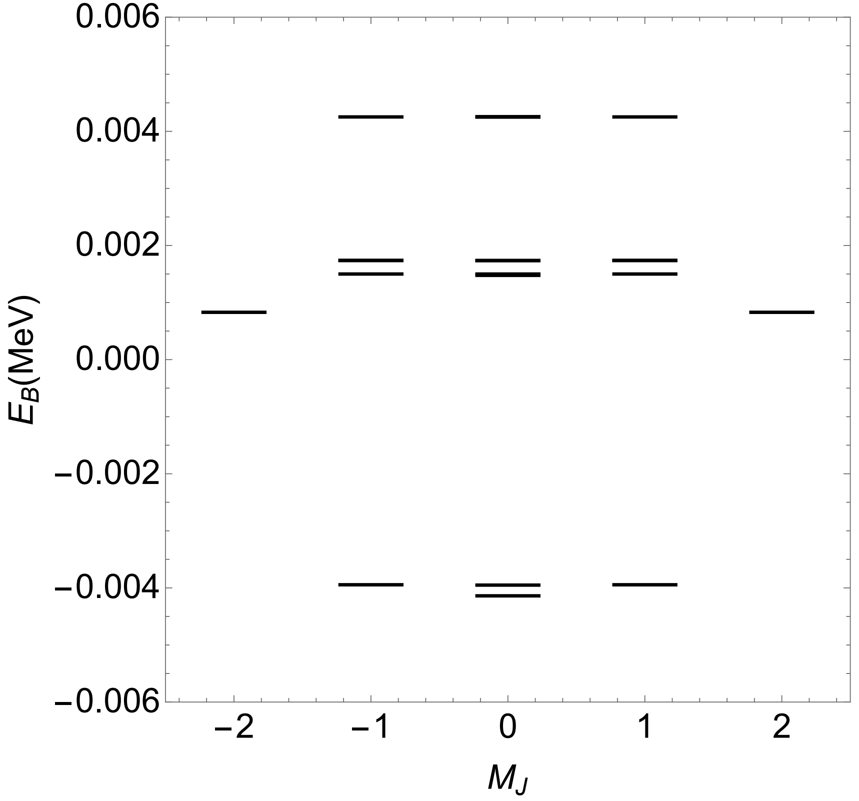
$\alpha = 0.3$

Binding energy looks convergent. nontrivial

[Kaiyu Fu et al, in preparation]

Energy spectrum

$N_{max} = 32, K = 33$

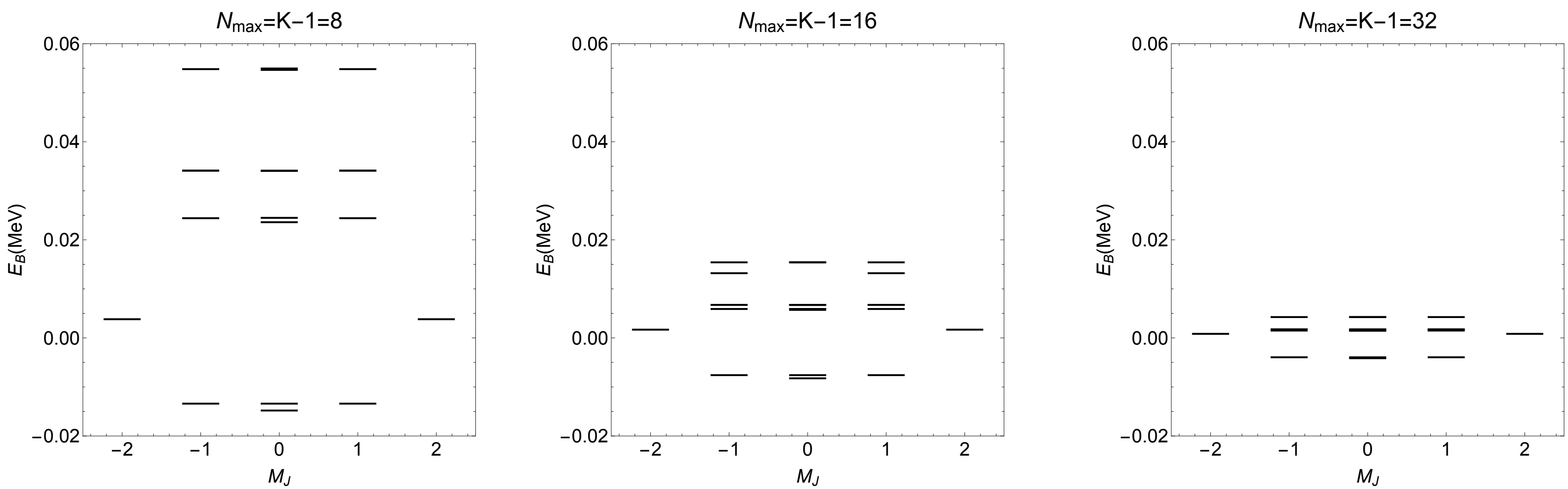


α=0.3

lowest 8 states of Mj=0 : parity and charge conjugation parity agree with hydrogen atom.

[Kaiyu Fu et al, in preparation]

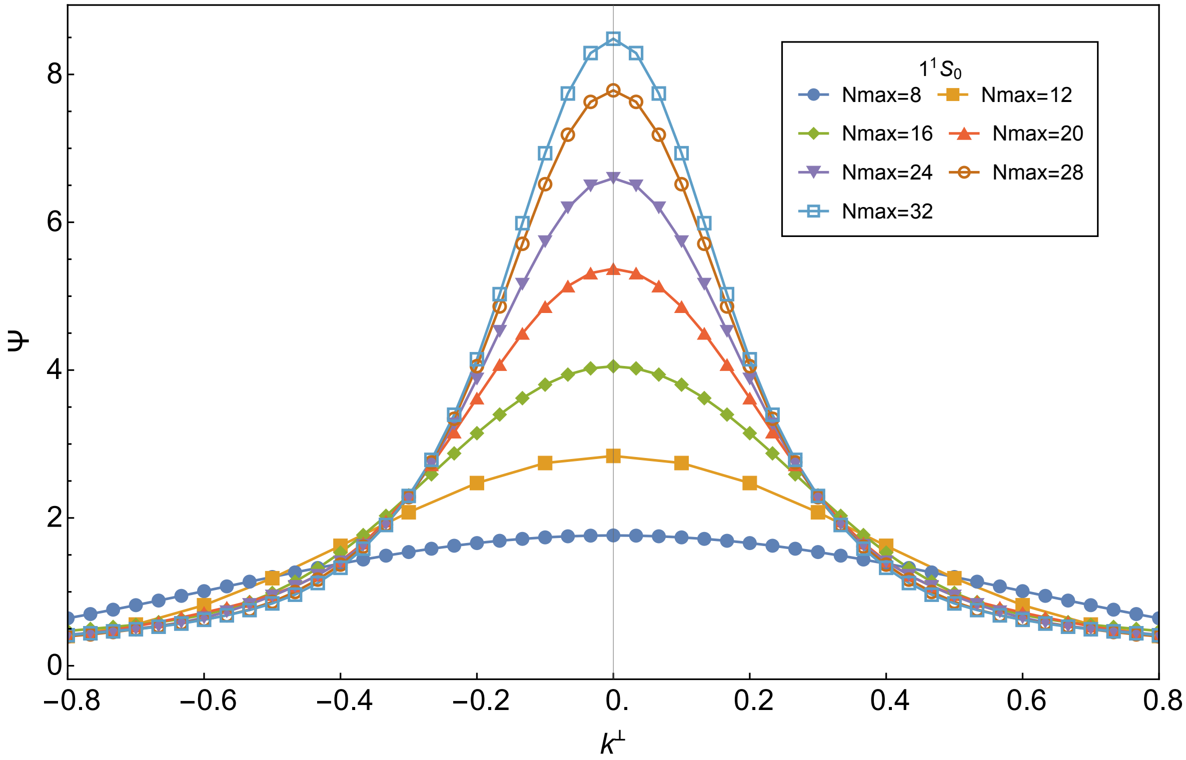
$\alpha = 0.3$



- The highest state in each column is a component of 2^3P_2
- They have similar binding energy in large basis size
- Rotational symmetry is restoring

[Kaiyu Fu et al, in preparation]

Wavefunction Projection



Wavefunction at x=0
x: longitudinal momentum fraction

Dominant parity: ↑↓−↓↑

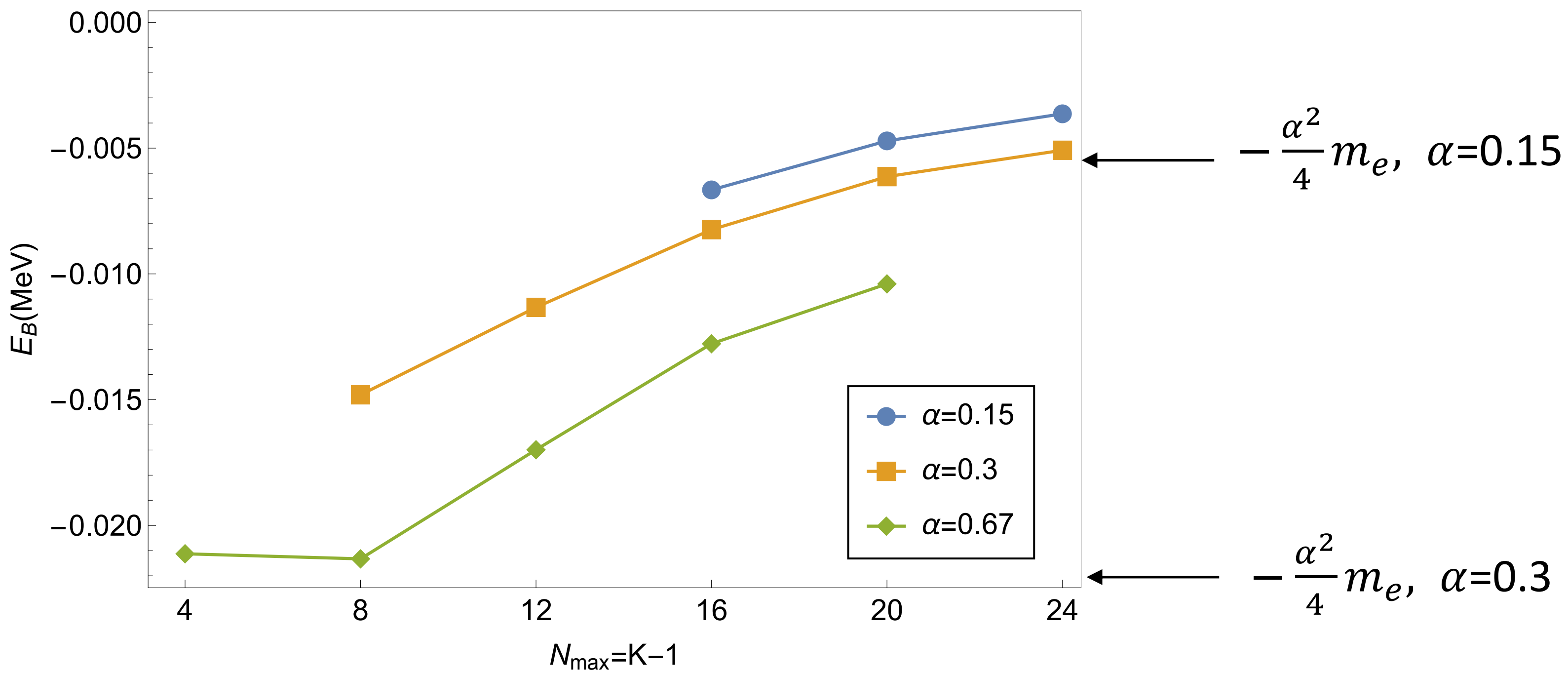
α=0.3

- The convergence of wavefunction looks promising.

[Kaiyu Fu et al, in preparation]

Results of different α

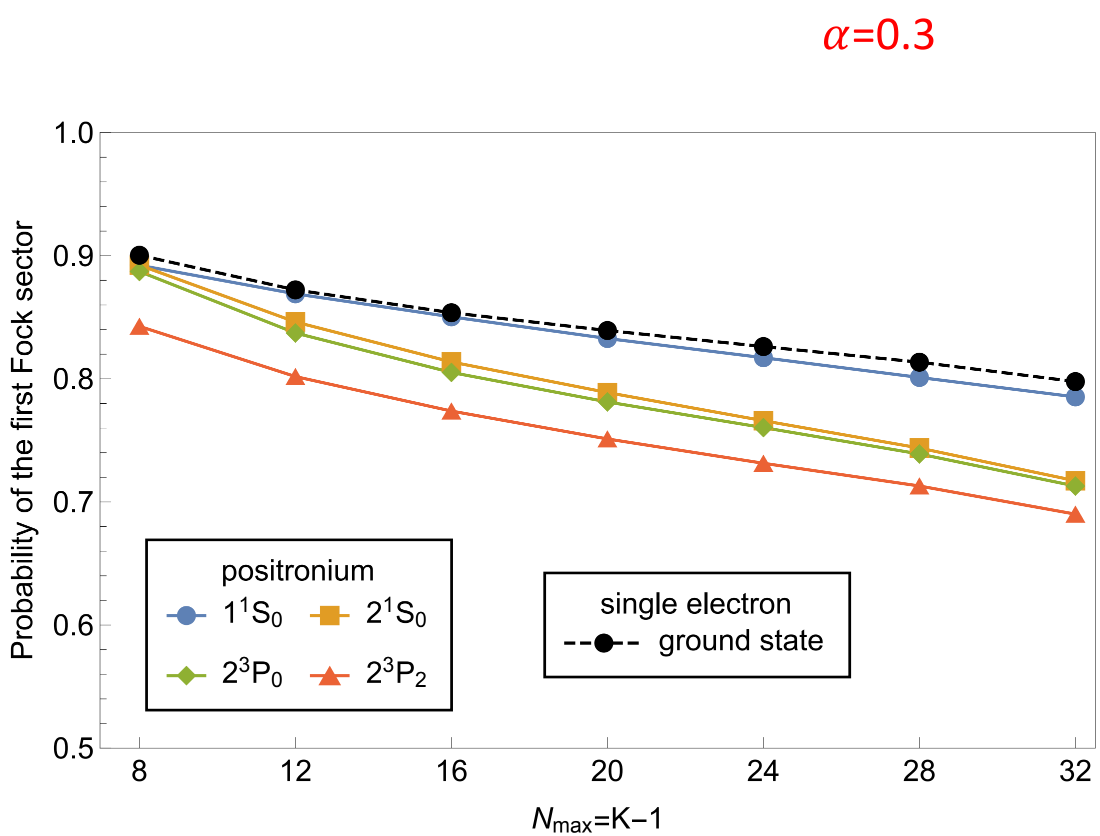
Binding energy



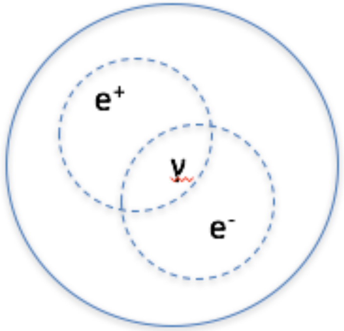
- Compare with hydrogen atom, our results are underbound
- But at small α, our results are closer to the prediction of hydrogen atom formula

[Kaiyu Fu et al, in preparation]

Probability Of $|e^+e^-\rangle$



- Interaction mediated through photon.
- Finite probability to find photon



$1-$ probability of $|e^+e^-\rangle$: the probability to find photon

Excited states have larger $|e\bar{e}\gamma\rangle$ component

[Kaiyu Fu et al, in preparation]

Photon Distribution In Positronium

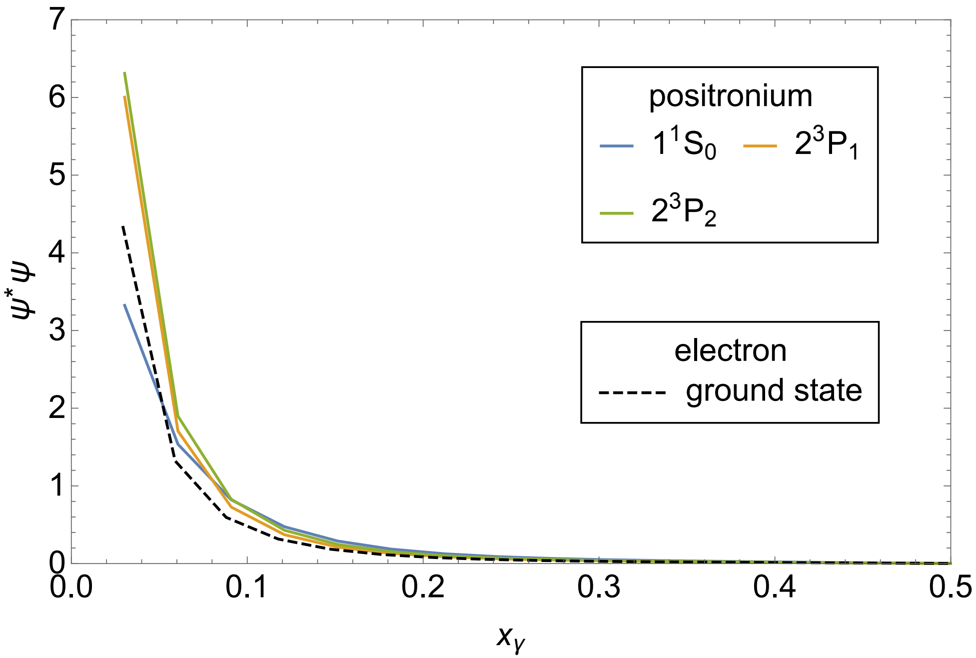
Positronium: $N_{max} = 32, K = 33$

$\alpha = 0.3, b = 0.16$

electron: $N_{max} = 32, K = 17$

$\alpha = 0.3, b = 0.16/\sqrt{2}$

x/2 and 2f(x/2) for electron



Normalization:

$$\int_0^1 f(x)dx = norm2$$

$norm2$ is the probability of the second Fock sector

in this case,

$$norm2_{Ps} = 0.215$$
$$norm2_e = 0.21$$

[Kaiyu Fu et al, in preparation]

- In excited states photons have larger probability at small-x region
- Photon is massless, so peak is at small-x region

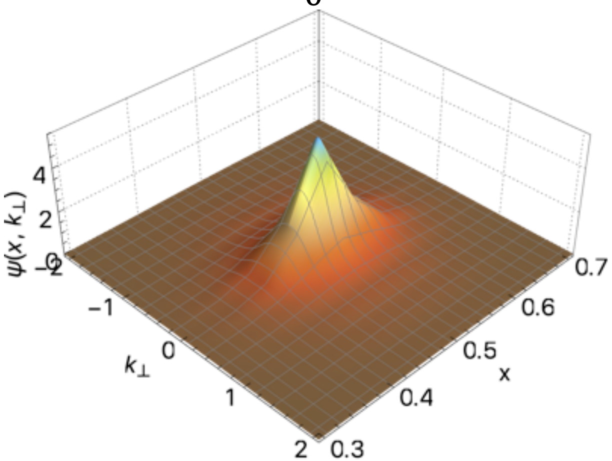
Wavefunction

This work

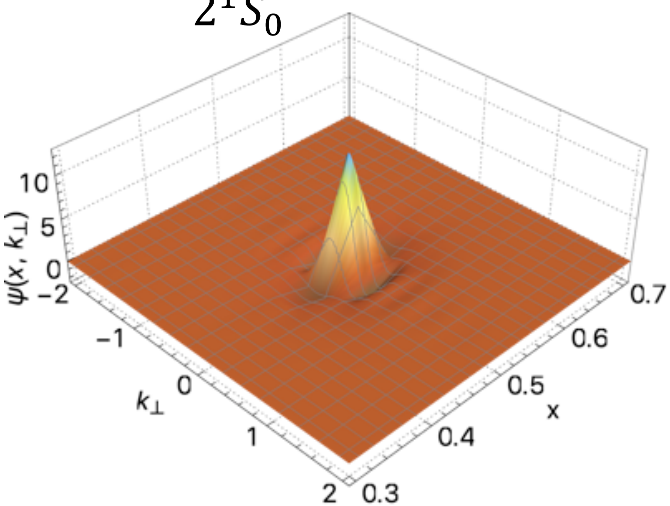
$N_{max} = 20, K = 21, M_J = 0$

[Kaiyu Fu et al, in preparation]

1^1S_0



2^1S_0

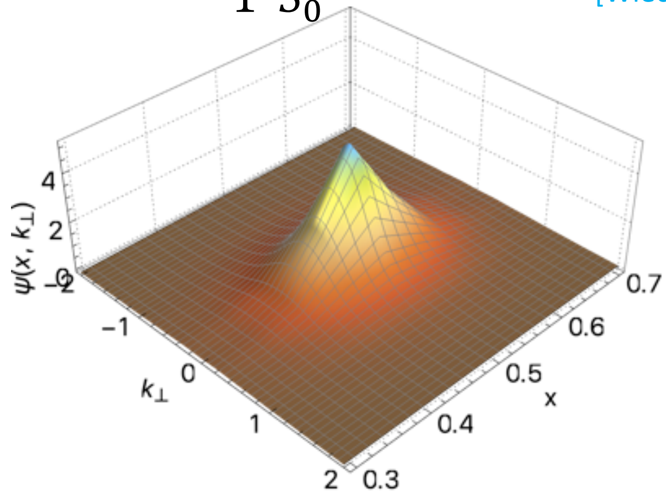


The effective one-photon-exchange

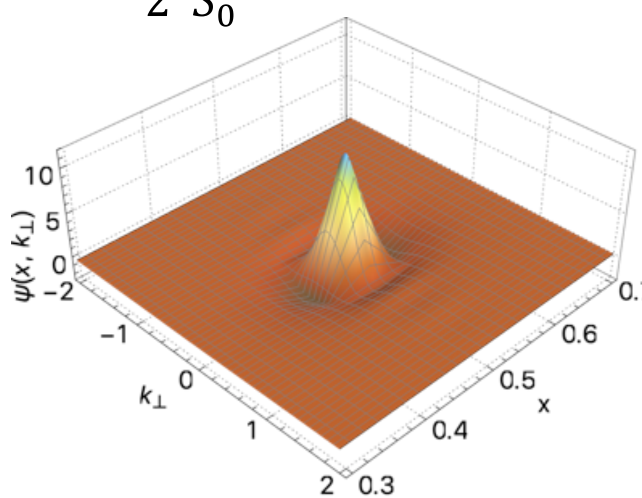
$N_{max} = 20, K = 19, M_J = 0$

[Wiecki, et al, 2015]

1^1S_0



2^1S_0

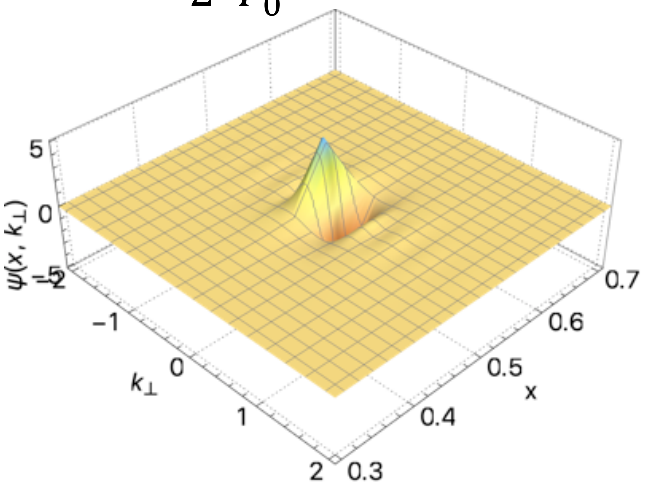


Nodal structure in radial direction

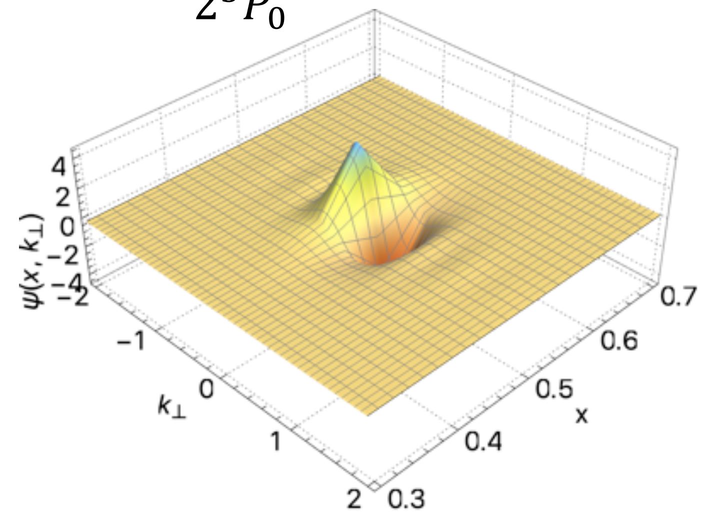
[Kaiyu Fu et al, in preparation]

[Wiecki, et al, 2015]

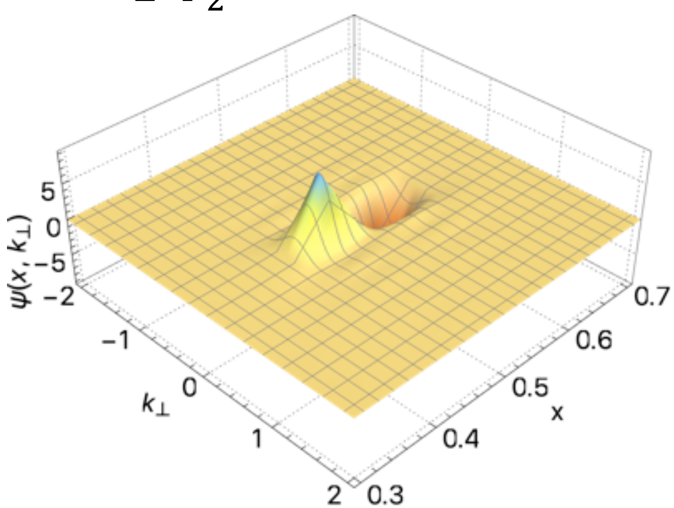
2^3P_0



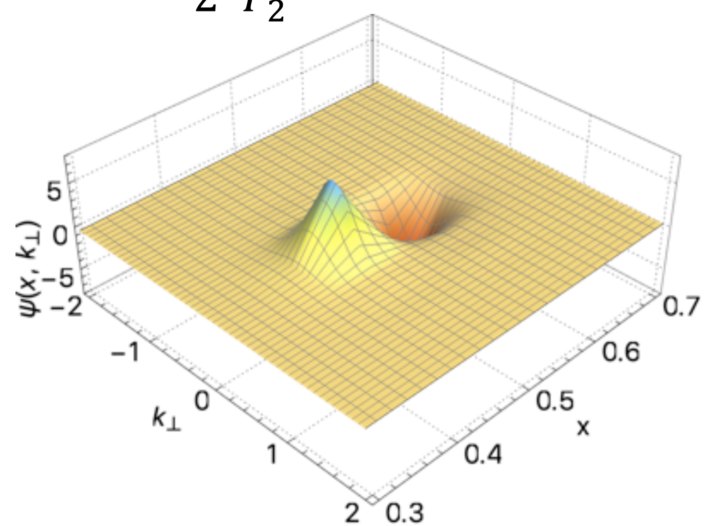
2^3P_0



2^3P_2



2^3P_2



Nodal structure in angular direction

Heavy quarkonium

Light-front QCD Hamiltonian + effective confining potential:

$$\mathrm{H} = P^{-} + V_{conf}$$

$m_q = m_{\bar{q}} = 1.5 GeV$

$\alpha = 0.9$

$\kappa_T = 0.2$

$\kappa_L = 0.3$

Confining potential:

Long-distance

$$V_{conf} = \underbrace{\kappa_T^4 x(1-x)\vec{r}_\perp^{\,2}}_{\text{LFHQCD}} - \underbrace{\frac{\kappa_L^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x)\partial_x)}_{\text{Longitudinal confinement}}$$

LFHQCD

Longitudinal confinement

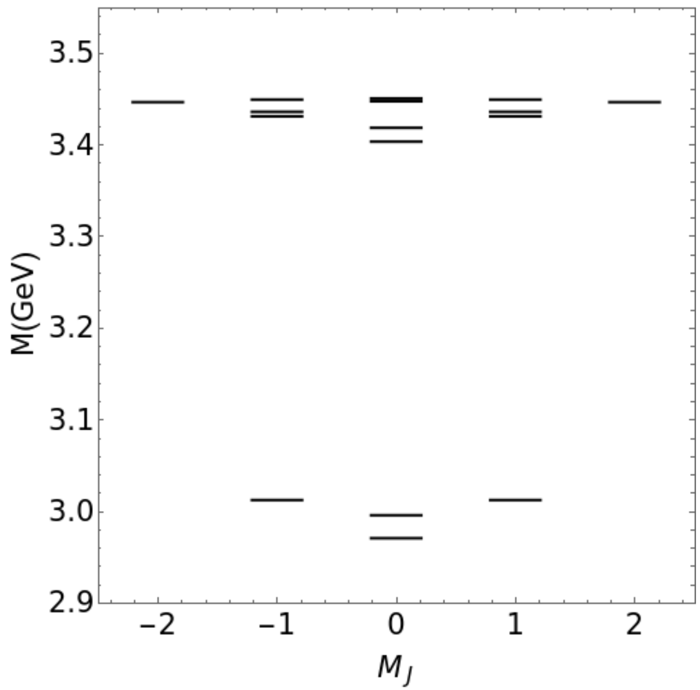
[Yang Li, et al, 2017]

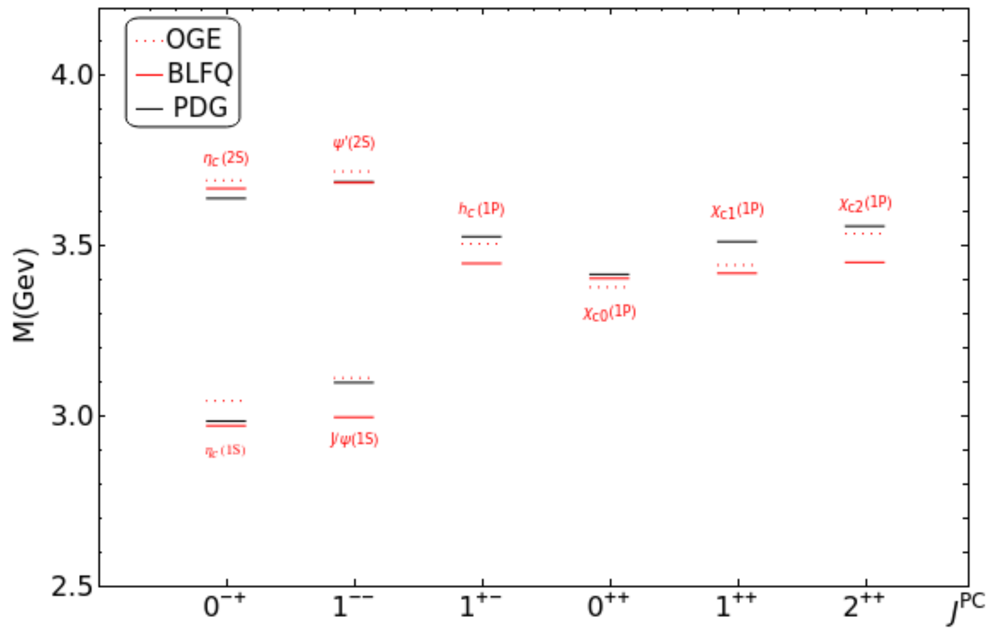
- Color factor as the coefficient of Hamiltonian
- effective confining potential including quark mass
- New parameters are introduced like κ_T, κ_L, transverse and longitudinal confining strength

Mass spectrum

$N_{max} = 12, K = 13$

Preliminary





Left : a representative charmonium mass spectrum.

Right : identified particle states in $M_J = 0$ sector compare with the OGE result and the PDG data.

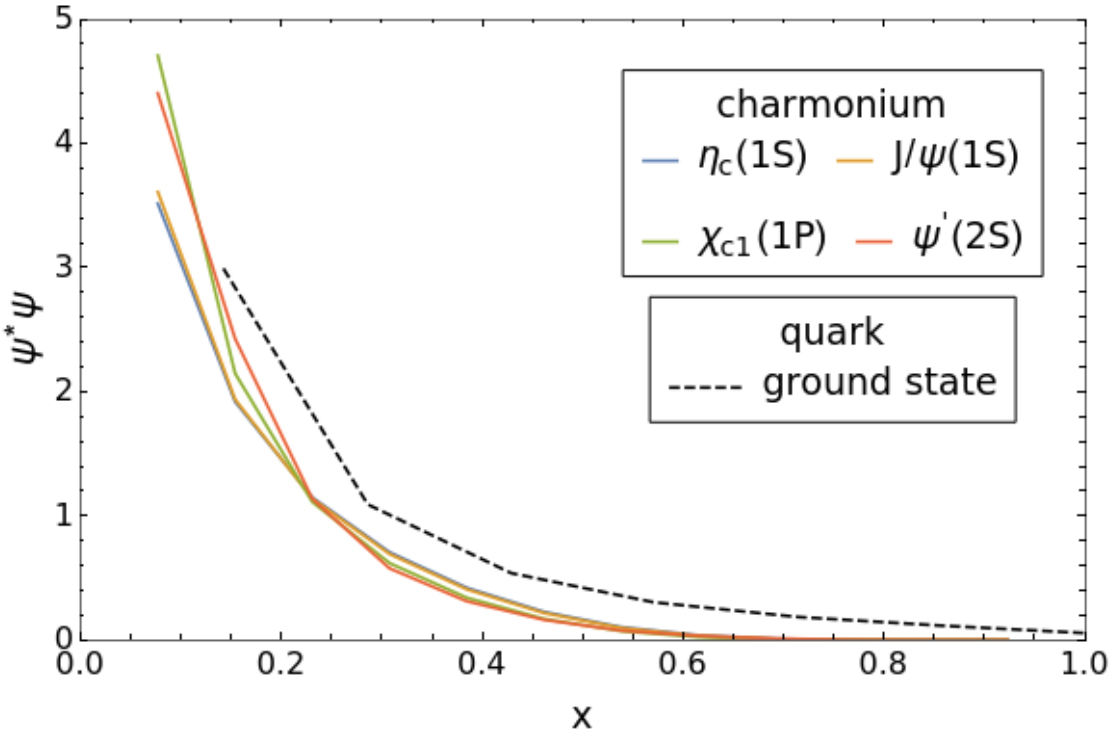
BLFQ:[Hengfei Zhao et al, in preparation]

OGE : [Yang Li, et al, 2017]

Gluon Distribution In Charmonium

$N_{max} = 12, K = 13$

Preliminary



Normalization:

$$\int_0^1 f(x)dx = norm2$$

$$norm2_{\text{charm}} = 0.753$$
$$norm2_{\text{q}} = 0.247$$

$\alpha = 0.9$

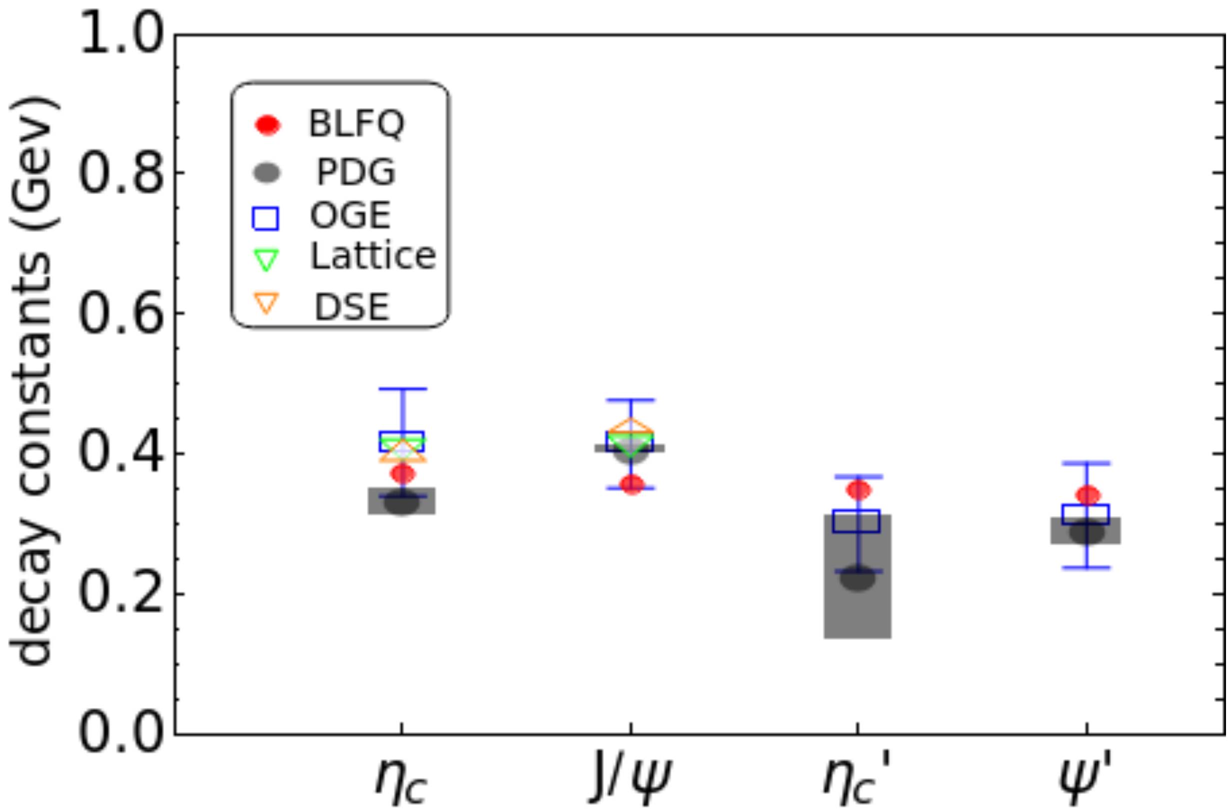
- P-wave gluons have larger probability at small-x region
- Since gluon is massless, peak is at small-x region

[Hengfei Zhao et al, in preparation]

Decay constants

Wave function at the origin – probe short-distance physics in LFWF representation :

$$\frac{f_{P,V}}{2\sqrt{2N_c}}\phi_{P,V}(x;\mu) = \lim_{\Lambda_{UV}\to\infty} Z_2(\mu,\Lambda_{UV})\int_0^1 \frac{dx}{\sqrt{x(1-x)}}\int^{\Lambda_{UV}} \frac{d^2k_\perp}{2(2\pi)^3}\Psi^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(x,\vec{k}_\perp)$$



Preliminary

OGE : [Yang Li, et al, 2017]

Lattice : [C. McNeile et al, 2012]

Results near the PDG's data

Dyson-Schwinger equations : [M. Blank, et al, 2011]

Distribution amplitudes

$\alpha = 0.9$

DAs Control exclusive process at large momentum transfer

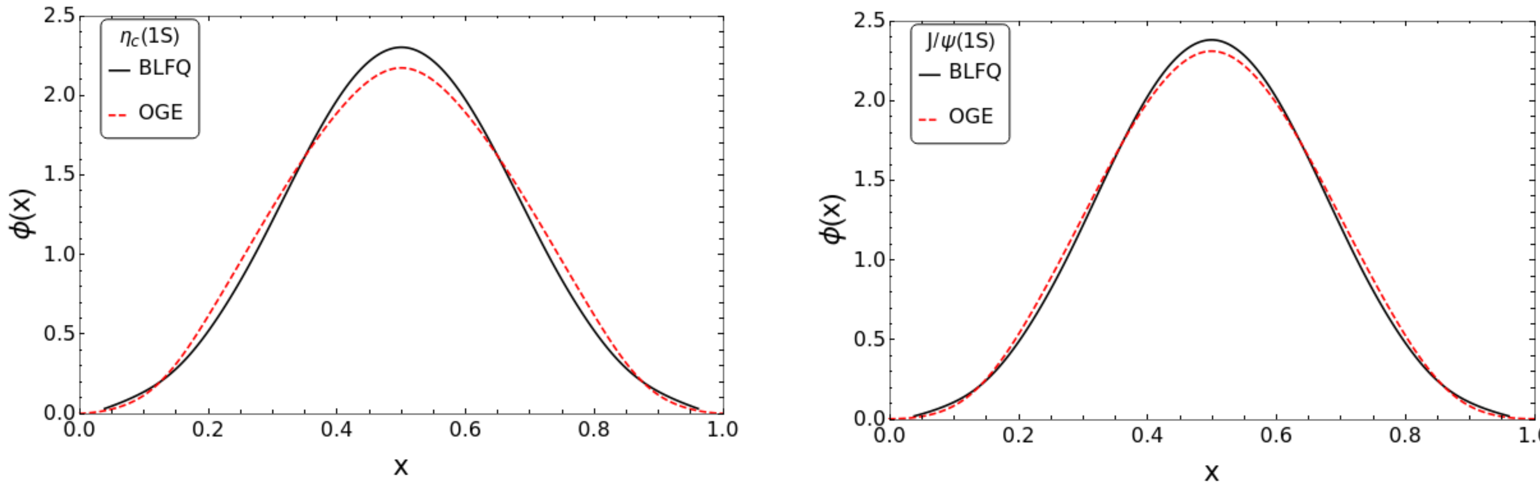
[e.g. Lepage '80]

$$< 0|\bar{\psi}(z)\gamma^+\gamma^5\psi(-z)|P(p) >_\mu = ip^+ f_p \int_0^1 dx e^{ip^+z^-(x-\frac{1}{2})}\phi_P(x;\mu)\,|_{z^+,\vec{z}_\perp=0,}$$

Preliminary

In LFWF representation:

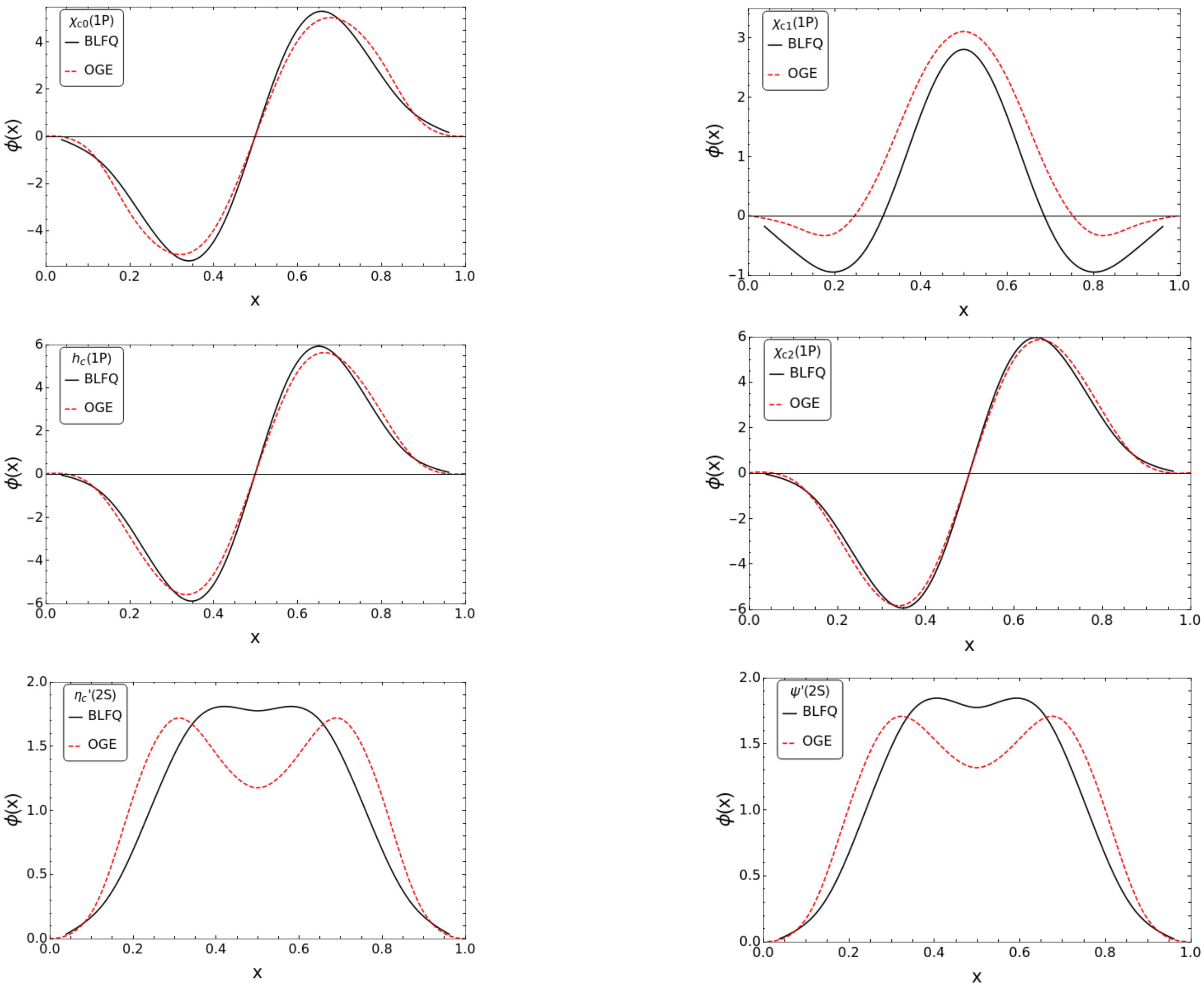
$$\frac{f_{P,V}}{2\sqrt{2N_c}}\phi_{P,V}(x;\mu) = \frac{1}{\sqrt{x(1-x)}}\int^{\leq\mu^2}\frac{d^2k_\perp}{2(2\pi)^3}\Psi^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(x,\vec{k}_\perp) = \frac{1}{4\pi}\Psi^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(x,\vec{r}=0_\perp)$$



DAs agree with the OGE results. Especially for low lying state, like J/ψ and η_c

BLFQ: [Hengfei Zhao et al, in preparation]

OGE : [Yang Li, et al, 2017]

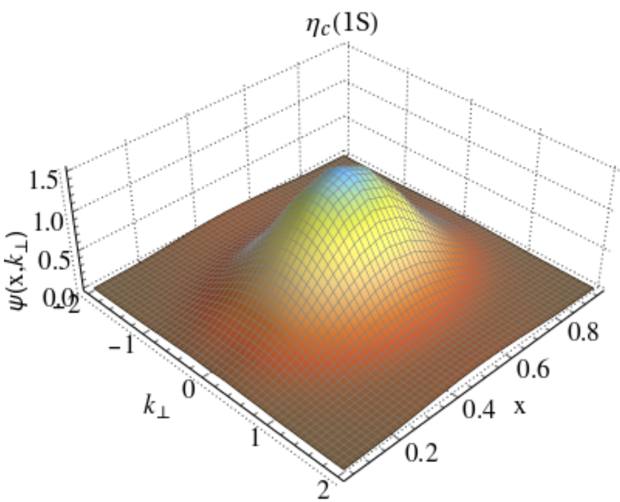


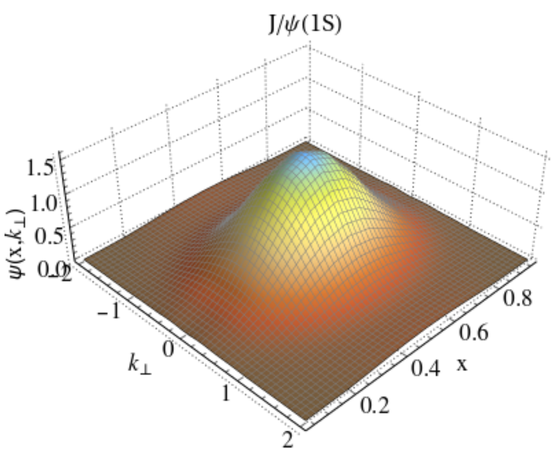
χ_{c0}(1P)
BLFQ
OGE
χ_{c1}(1P)
BLFQ
OGE
h_c(1P)
BLFQ
OGE
χ_{c2}(1P)
BLFQ
OGE
η_c'(2S)
BLFQ
OGE
ψ'(2S)
BLFQ
OGE
$\phi(x)$
x

Preliminary Wavefunction

This work

$N_{max} = 12, K = 13, M_J = 0$

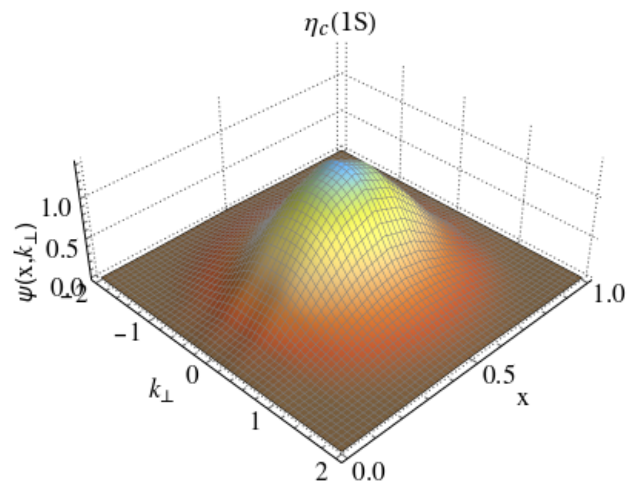


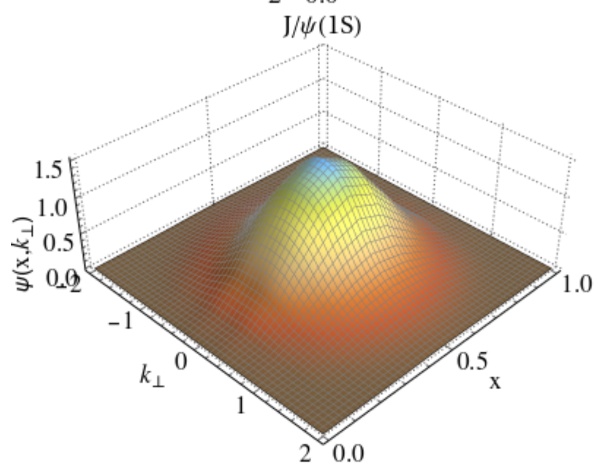


[Hengfei Zhao et al, in preparation]

The effective one-gluon-exchange

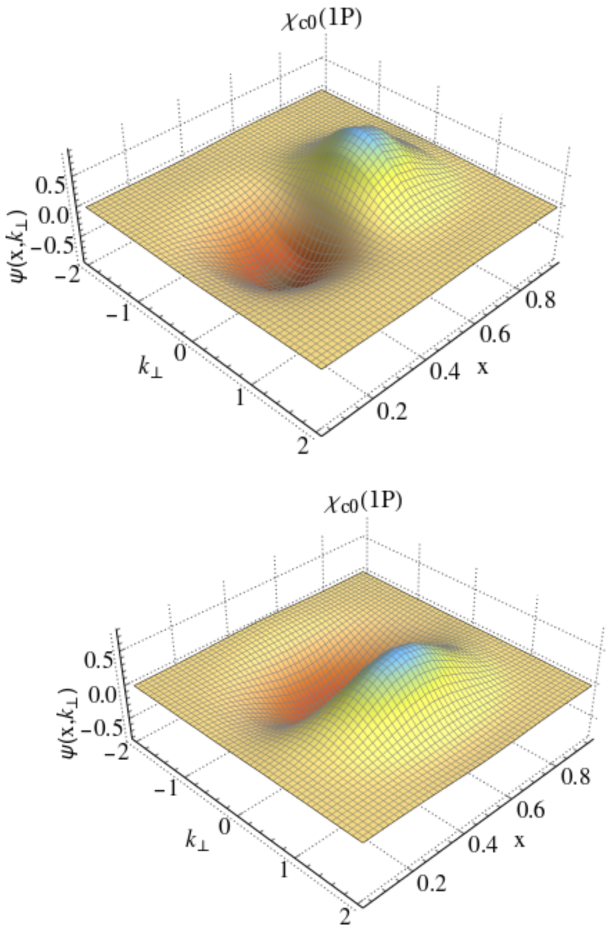
$N_{max} = 8, L = 8, M_J = 0$



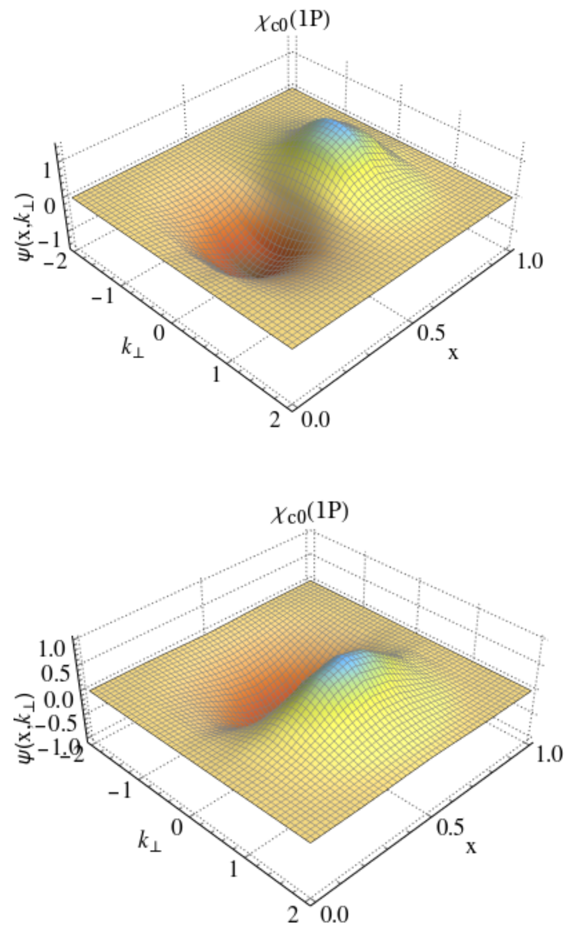


[Yang Li, et al, 2017]

Preliminary

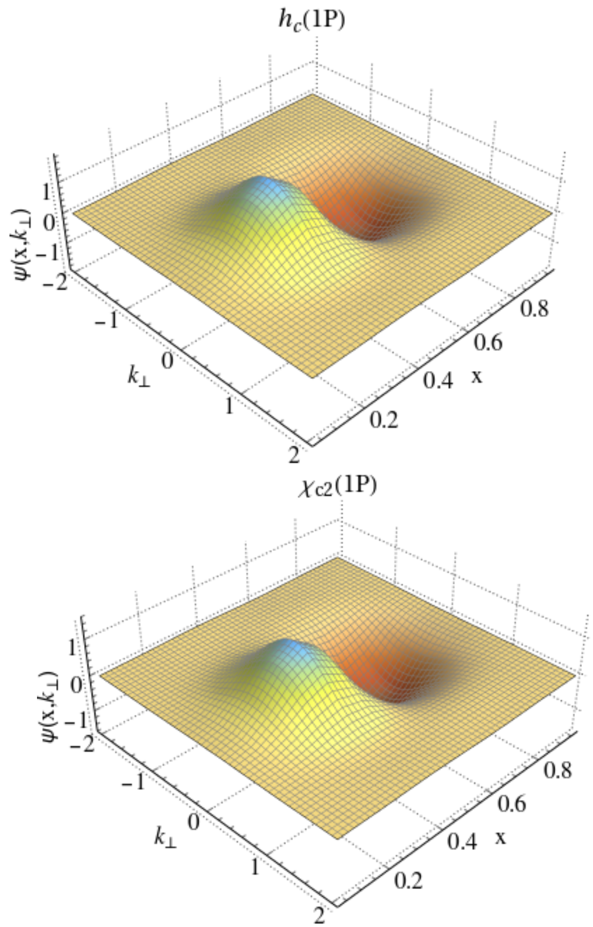


[Hengfei Zhao et al, in preparation]

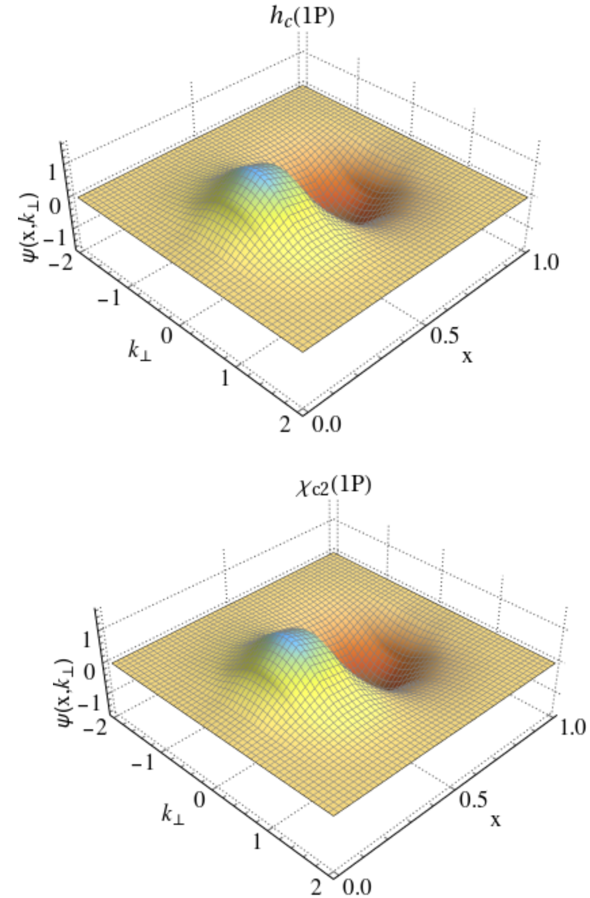


[Yang Li, et al, 2017]

Nodal structure in angular direction



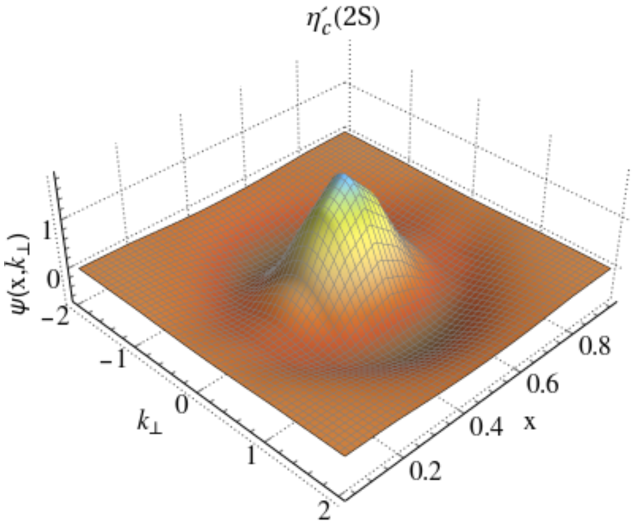
[Hengfei Zhao et al, in preparation]

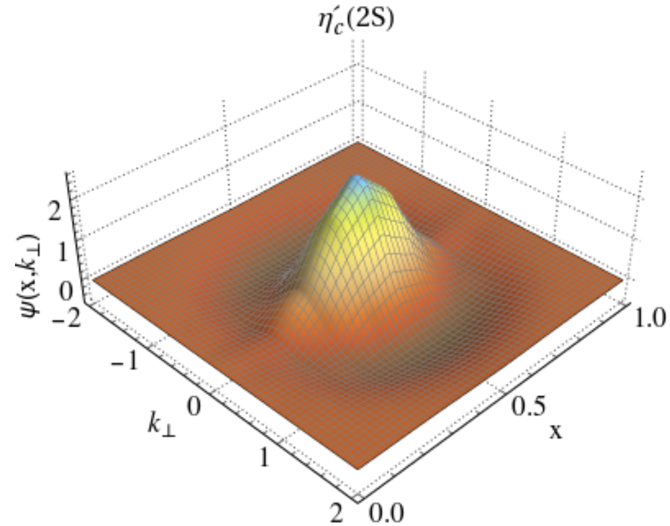


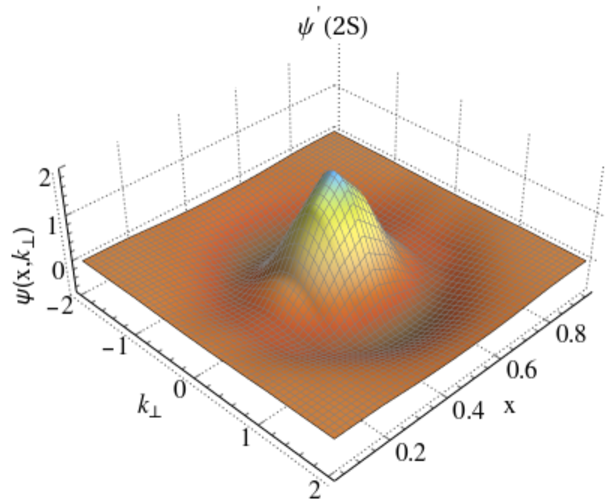
[Yang Li, et al, 2017]

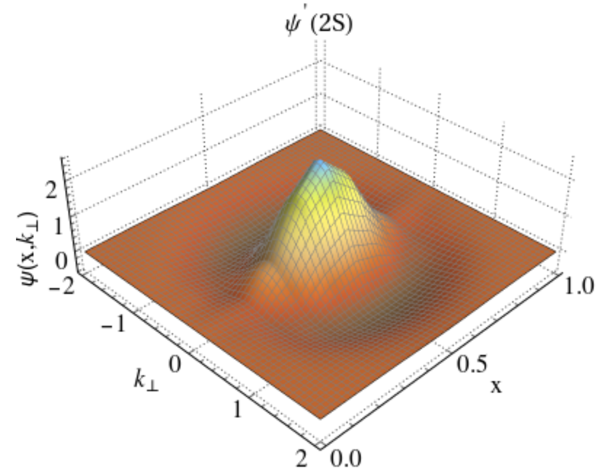
Nodal structure in angular direction

Preliminary









[Hengfei Zhao et al, in preparation]

[Yang Li, et al, 2017]

Nodal structure in radial direction

Conclusions

- Calculation based on first-principle (additional effective potential for quarkonium)
- Direct access to photon(gluon) content
- Rotation Symmetry is restoring as basis size increase
- Mass renormalization is performed on the level of electron
- Wave function and energy spectrum for low-lying states reasonably agree with those from the effective one-photon(gluon)-exchange approach
- The convergence of positronium results looks promising

Outlook

- Further convergence study for both systems

- More observables: PDF, GPD, TMD, GTMD,

Wigner distribution, double parton distribution function…

- Light meson systems

- Exotic hadron states