Low-x_{Bj} DIS at NLO in light-cone perturbation theory with massive quarks

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Outline

- Motivation
- Dipole factorization framework for DIS
- DIS at NLO with massive quarks
  - One-loop QCD correction to the $\gamma_L^* \to q\bar{q}$ LCWF
  - Full DIS cross section for γ_L^* at NLO
- Conclusion

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DIS structure functions in the dipole factorization beyond LO

- Fast theoretical progress towards NLO accuracy:
  - Massless quarks: $F_{T/L}$ at NLO computed
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Charm and bottom quark contributions are sizable at HERA:

- Need to calculate $F_{T/L}$ with massive quarks at NLO (this talk)

Dipole factorization for DIS

q^μ $\bar{q}$

P^μ

Dipole picture: A relativistic projectile γ^* fluctuates into a $q\bar{q}$-pair which then **scatters** on a very dense **target**

- Kinematics in LC coordinates $(+, -, \perp)$

$$q^\mu = (q^+, \frac{\mathbf{q}^2 + q^2}{2q^+}, \mathbf{q}), \quad q^2 = -Q^2 > 0, \quad q^+ = \text{very large}$$

$$\text{Bjorken} \quad x_{B_j} \equiv \frac{Q^2}{2P \cdot q} \in [0, 1]$$

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- High energy limit: $x_{B_j} \to 0$ and Q^2 is "fixed"

$$F_{T,L}(x_{B_j}, Q^2) = \frac{Q^2}{(2\pi)^2 \alpha_{\rm em}} \sigma_{T,L}^{\gamma^*}, \quad F_2 = F_T + F_L, \quad 2x_{B_j} F_1 = F_T$$

DIS cross section at NLO

The total cross section for γ^* scattering from a classical gluon field (optical theorem)

$$\sigma^{\gamma^*}_{\lambda=T/L} = \frac{2}{2q^+(2\pi)\delta(q'^+ - q^+)} \Re e \left[{}_i\langle \gamma^*_\lambda(\vec{q}, Q^2) | 1 - \hat{S}_E | \, \gamma^*_\lambda(\vec{q}\,', Q^2) \rangle_i \right]$$

- Full perturbative Fock state decomposition for γ^*

$$|\gamma^*_\lambda(\vec{q}, Q^2)\rangle_i = |QED\rangle + \int PS_{(q\bar{q})} \psi^{\gamma^* \to q\bar{q}} |q\bar{q}\rangle + \int PS_{(q\bar{q}g)} \psi^{\gamma^* \to q\bar{q}g} |q\bar{q}g\rangle + \cdots$$

  where $\psi^{\gamma^* \to q\bar{q}/q\bar{q}g}, \ldots$ are the light cone wave-functions

- $\hat{S}_E$ describes the eikonal scattering on the gluon field

$$\hat{S}_E |q(..., \mathbf{x}_0, \alpha)\bar{q}(..., \mathbf{x}_1, \beta)\rangle = U_F(\mathbf{x}_0)_{\alpha\bar{\alpha}} U^\dagger_F(\mathbf{x}_1)_{\beta\bar{\beta}} |q(..., \bar{\alpha})\bar{q}(..., \bar{\beta})\rangle$$

DIS cross section at NLO

In mixed space $(k^+, \mathbf{x})$ factorization between $|\psi|^2$ and target

$$\sigma_\lambda^{\gamma^*} = 2N_c \int \widetilde{PS}_{q\bar{q}} |\widetilde{\psi}^{\gamma_\lambda^* \to q\bar{q}}|^2 \Re e[1 - \mathcal{S}_{01}] + 2N_c C_F \int \widetilde{PS}_{q\bar{q}g} |\widetilde{\psi}^{\gamma_\lambda^* \to q\bar{q}g}|^2 \Re e[1 - \mathcal{S}_{012}]$$

$k_0^+, \mathbf{x}_0$

$k_2^+, \mathbf{x}_2$

$k_1^+, \mathbf{x}_1$

- Dipole target: $\mathcal{S}_{01} \equiv \frac{1}{N_c} \mathrm{Tr} \left[U_F(\mathbf{x}_0) U_F^\dagger(\mathbf{x}_1) \right]$
- Dipole/g target: $\mathcal{S}_{012} \equiv \frac{1}{N_c C_F} \mathrm{Tr} \left[t^b U_F(\mathbf{x}_0) t^a U_F^\dagger(\mathbf{x}_1) \right] U_A(\mathbf{x}_2)_{ba}$

DIS computation at NLO: massless quark case

LCWF's for $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ computed by using the LCPT

$\gamma^* \to q\bar{q}$ **LCWF at NLO with** $m_q = 0$:

$$\psi^{\gamma^* \to q\bar{q}}_{1-loop} \sim \sqrt{\alpha_{em}}\alpha_s \int_0^{k^+_{max}} \frac{\mathrm{d}k^+}{k^+} \int \frac{\mathrm{d}^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \frac{num(\vec{k}, \vec{q}, Q^2, \dots)}{ED^-_{i,1} ED^-_{i,2} \cdots ED^-_{i,f}}$$

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- Cut-off $k^+ > \alpha q^+$ introduced to regulate the $k^+ \to 0$ div's
  - $\log(\alpha)$'s have to cancel at full cross section level

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- Cut-off $k^+ > \alpha q^+$ introduced to regulate the $k^+ \to 0$ div's
  - $\log(\alpha)$'s have to cancel at full cross section level
- UV divergences from transverse **k** integrals:
  - Transverse dimensional regularization in $D = 4 - 2\varepsilon_{uv}$

$$\frac{1}{\varepsilon_{uv}} + C_{scheme}, \qquad \frac{1}{\varepsilon_{uv}} \log(\alpha)$$

  - No UV renormalization at $\mathcal{O}(\alpha_{em}\alpha_s)$
    - UV divergences and finite regularization artifacts ($= C_{scheme}$) have to cancel at full cross section level

DIS computation at NLO: massive quark case

$\gamma^* \to q\bar{q}$ **LCWF at NLO with** $m_q \neq 0$:

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New features and technical compilications appearing in the massive case:

DIS computation at NLO: massive quark case

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- New spinor structures induced by quark LC helicity flip vertices
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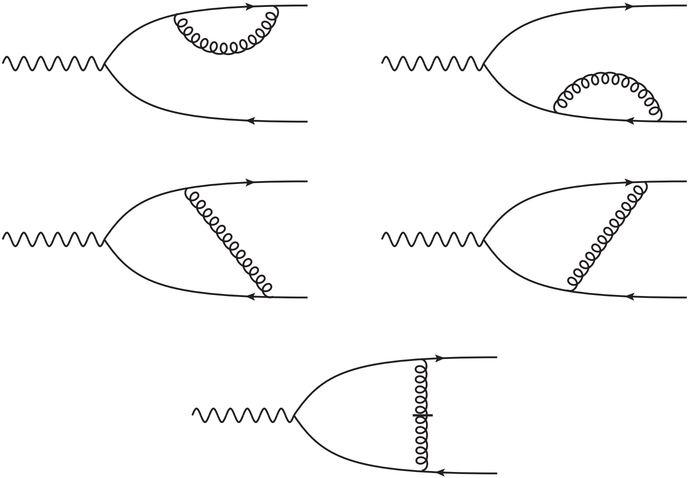
New features and technical compilications appearing in the massive case:

- New spinor structures induced by quark LC helicity flip vertices
- Quark mass renormalization has to be performed (LO result depends on m_q)
- Loop/Fourier transform integrals cannot fully be done analytically anymore:
  - Extract the **divergent** parts from the full expression (can be done analytically)
  - Some of the remaining **finite** parts given as integrals over Feynman/Schwinger parameters (can be done numerically)

One-loop LC diagrams for the $\gamma_L^* \to q\bar{q}$

At one-loop the following 5 LC diagrams must be calculated



Results: NLO $\gamma_L^* \to q\bar{q}$ LCWF in momentum space

$$\psi_{NLO}^{\gamma_L^* \to q\bar{q}} = \psi_{LO}^{\gamma_L^* \to q\bar{q}} \left(\frac{\alpha_s C_F}{2\pi} \right) \mathcal{K}^L + \psi_{hf}^{\gamma_L^* \to q\bar{q}} + \mathcal{O}(\alpha_{em}\alpha_s^2)$$

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where the NLO kernel

$$\mathcal{K}^L = 2\left[\log\left(\frac{\alpha}{\sqrt{z(1-z)}}\right) + \frac{3}{4}\right]\left\{\frac{1}{\varepsilon_{\overline{MS}}} + \log\left(\frac{\mu^2}{M^2}\right) - 2\log\left(\frac{\mathbf{P}^2 + M^2}{M^2}\right)\right\} + \log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{3} + \frac{5}{2} + C_{scheme} + \mathcal{G}^L(m_q;z) + \mathcal{J}^L(m_q;z) + \mathcal{O}(\varepsilon)$$

- Notations: $z = k_0^+/q^+$, $z \in [0,1]$ and $\xi = k^+/k_0^+$, $\xi \in [0,1]$

$$\overline{Q}^2 = z(1-z)Q^2, \quad \mathbf{P} = \mathbf{k}_0 - z\mathbf{q}, \quad M^2 = \overline{Q}^2 + m_q^2$$

- Regularization artifact: $C_{scheme} = 1/2$ in CDR and 0 in FDH
- At NLO the UV finite hf-term vanishes on the cross section level

$$\psi_{hf}^{\gamma_L^*\to q\bar{q}}(\psi_{LO}^{\gamma_L^*\to q\bar{q}})^* \to 0$$

The finite functions $\mathcal{G}^L$ and $\mathcal{J}^L$ simplifes to ($\gamma \equiv \sqrt{1+4m_q^2/Q^2}$):

$$\begin{aligned}\mathcal{G}^L = &\sum_{k=\pm1}\left[\mathrm{Li}_2\left(\frac{1}{1-\frac{1}{2z}(1+k\gamma)}\right)+\mathrm{Li}_2\left(\frac{1}{1-\frac{1}{2(1-z)}(1+k\gamma)}\right)\right]\\ &+\frac{1}{2z}\left[\log(1-z)+\gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right.\\ &+\frac{1}{2(1-z)}\left[\log(z)+\gamma\log\left(\frac{1+\gamma}{1+\gamma-2(1-z)}\right)\right.\\ &+\frac{\gamma-1}{4z(1-z)}\log\left(\frac{\overline{Q}^2+m_q^2}{m_q^2}\right)+\frac{m_q^2}{2\overline{Q}^2}\log\left(\frac{\overline{Q}^2+m_q^2}{m_q^2}\right)\end{aligned}$$

$$\begin{aligned}\mathcal{J}^L = &\int_0^1\frac{\mathrm{d}\xi}{\xi}\left(-\frac{2\log(\xi)}{1-\xi}+\frac{1+\xi}{2}\right)\left[1-\frac{\mathbf{P}^2+M^2}{\mathbf{P}^2+M^2+\frac{z\xi}{1-\xi}m_q^2}\right]\\ &+m_q^2\int_0^1\mathrm{d}\xi\int_0^1\mathrm{d}x\frac{C^L(x,\xi,z)}{x(1-x)(1-\xi)^2\mathbf{P}^2+(1-x)\Delta_1+x\Delta_2}+[z\to1-z]\end{aligned}$$

$$\Delta_1=\frac{\xi(1-\xi)}{1-z}[\mathbf{P}^2+M^2]+\xi^2m_q^2,\quad \Delta_2=(1-\xi)[1+\frac{z\xi}{1-z}]\overline{Q}^2+m_q^2$$

Results: full $q\bar{q}$ contribution to $\sigma_L^{\gamma^*}$ at NLO

Fourier transformation to mixed space:

$$\widetilde{\psi}^{\gamma_L^* \to q\bar{q}} = \int \frac{\mathrm{d}^{D-2}\mathbf{P}}{(2\pi)^{D-2}} e^{+i\mathbf{P}\cdot\mathbf{x}_{01}} \psi^{\gamma_L^* \to q\bar{q}}$$

where $\mathbf{x}_{01} \equiv \mathbf{x}_0 - \mathbf{x}_1$

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}} \equiv 2N_c \int \widetilde{PS}_{q\bar{q}} |\widetilde{\psi}^{\gamma_L^* \to q\bar{q}}|^2 \Re e[1 - \mathcal{S}_{01}]$$

gives

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0,\mathbf{x}_1} \int_0^1 \mathrm{d}z [z(1-z)]^2 \left(\frac{M}{2\pi|\mathbf{x}_{01}|}\right)^{-\varepsilon} K_\varepsilon\left(|\mathbf{x}_{01}|M\right)$$
$$\times \left\{ \left(\frac{M}{2\pi|\mathbf{x}_{01}|}\right)^{-\varepsilon} K_\varepsilon\left(|\mathbf{x}_{01}|M\right) + \left(\frac{\alpha_s C_F}{\pi}\right) \widetilde{\mathcal{K}}^L \right\} \Re e[1 - \mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2)$$

The Fourier transformed $\widetilde{\mathcal{K}}^L$ simplifies to:

$$\widetilde{\mathcal{K}}^L = \Bigg\{2\left[\log\left(\frac{\alpha}{\sqrt{z(1-z)}}\right)+\frac{3}{4}\right]\left\{\frac{1}{\varepsilon_{\overline{MS}}}+\log\left(\frac{\mathbf{x}_{01}^2\mu^2}{4}\right)-2\Psi_0(1)\right\}+\log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{3}+\frac{5}{2}+C_{scheme}+\mathcal{G}^L\Bigg\}\left(\frac{M}{2\pi|\mathbf{x}_{01}|}\right)^{-\varepsilon}K_\varepsilon\left(|\mathbf{x}_{01}|M\right)+\widetilde{\mathcal{J}}^L$$

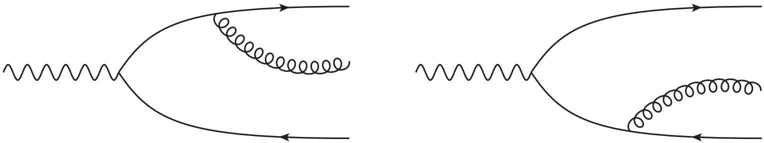
where

$$\widetilde{\mathcal{J}}^L = \int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[\frac{-2\log(\xi)}{(1-\xi)}+\frac{(1+\xi)}{2}\right]\left\{K_0\left(|\mathbf{x}_{01}|M\right)-K_0\left(|\mathbf{x}_{01}|\sqrt{M^2+\frac{\xi(1-z)m_q^2}{(1-\xi)}}\right)\right\}$$
$$+m_q^2\int_0^1\int_0^1 \mathrm{d}x\,\mathrm{d}\xi\left\{K_0\left(|\mathbf{x}_{01}|M\right)-K_0\left(|\mathbf{x}_{01}|\sqrt{\frac{M^2}{(1-x)}+\kappa}\right)\right\}$$
$$\times\frac{(1-\xi)^{-1}(1-x)^{-1}C^L}{\left[x(1-\xi)+\frac{\xi}{(1-z)}\right]\left[\frac{xM}{(1-x)}+\kappa\right]}+[z\to 1-z]$$

with $\kappa = m_q^2\tilde{\kappa}(z,\xi,x)$

Tree-level $\gamma_L^* \to q\bar{q}g$ contribution

For the full NLO $\sigma_L^{\gamma^*}$ we need to compute the $\gamma_L^* \to q\bar{q}g$ LCWF



3 Steps:

- Compute the corresponding $\psi^{\gamma_L^* \to q\bar{q}g}$
- Fourier transform $\psi^{\gamma_L^* \to q\bar{q}g}$ into mixed space

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}g} = 2N_c C_F \int \widetilde{PS}_{q\bar{q}g} |\widetilde{\psi}^{\gamma_\lambda^* \to q\bar{q}g}|^2 \Re e[1 - \mathcal{S}_{012}]$$

- UV subtraction
  - Add the $q\bar{q}$ and $q\bar{q}g$ terms together: cancellation of UV div's and C_{scheme}

Full NLO result for $\sigma_L^{\gamma^*}$

$$\sigma_L^{\gamma^*} = \sigma_L^{\gamma^*}\Big|_{q\bar{q},fin} + \sigma_L^{\gamma^*}\Big|_{q\bar{q}g,fin}$$

where the UV finite dipole contribution

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q},fin} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0,\mathbf{x}_1} \int_0^1 \mathrm{d}z [z(1-z)]^2 \Bigg\{ [K_0(|\mathbf{x}_{01}|M)]^2$$
$$\times \left[1 + \left(\frac{\alpha_s C_F}{\pi}\right) \left\{ \log^2\left(\frac{z}{1-z}\right) - \frac{\pi^2}{6} + \frac{5}{2} + \mathcal{G}^L \right\} \right]$$
$$+ K_0(|\mathbf{x}_{01}|M) \left(\widetilde{\mathcal{J}}^L + [z \to 1-z] \right) \Bigg\} \Re e[1 - \mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2)$$

and the UV finite $q\bar{q}g$ contribution

$$\sigma_L^{\gamma^*}\Big|_{q\bar{q}g,fin} = 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^3}\left(\frac{\alpha_s C_F}{\pi}\right)\int_{\mathbf{x}_0\mathbf{x}_1\mathbf{x}_2}\int_0^\infty \mathrm{d}k_0^+\,\mathrm{d}k_1^+\int_0^\infty \frac{\mathrm{d}k_2^+}{k_2^+}\frac{\delta(q^+ - \sum_{i=0}^2 k_i^+)}{(q^+)^5}$$

$$\left\{F_{(1)}\left[\frac{\mathbf{x}_{20}^2}{64}G_{(1)}^2 \Re e[1-\mathcal{S}_{012}] - \left[K_0(|\mathbf{r}_{01}|M_1)\right]^2 \frac{e^{-\frac{\mathbf{x}_{20}^2}{(\mathbf{x}_{01}^2 e^{\gamma_E})}}}{\mathbf{x}_{20}^2}\Re e[1-\mathcal{S}_{01}]\right] + [k_0^+ \leftrightarrow k_1^+, \mathbf{x}_0 \leftrightarrow \mathbf{x}_1]\right.$$

$$-\frac{k_0^+ k_1^+}{32}\left[2\left[(k_0^+ + k_2^+)k_0^+ + (k_2^+ + k_1^+)k_1^+\right](\mathbf{x}_{20}\cdot\mathbf{x}_{21})G_{(1)}G_{(2)}\right.$$

$$\left.+\frac{m^2(k_2^+)^4}{16}\left[\frac{(k_1^+)^2}{(k_0^+ + k_2^+)^2}\overline{G}_{(1)}^2 + \frac{(k_0^+)^2}{(k_1^+ + k_2^+)^2}\overline{G}_{(2)}^2 - 2\frac{k_0^+ k_1^+}{(k_0^+ + k_2^+)(k_2^+ + k_1^+)}\overline{G}_{(1)}\overline{G}_{(2)}\right]\Re e[1-\mathcal{S}_{012}]\right\}$$

where

$$G_{(i)} = \int_0^\infty \frac{\mathrm{d}u}{u} e^{-uM_i^2} e^{-\mathbf{x}_{102}^2/(4u)} \int_0^{u/\omega_i} \frac{\mathrm{d}t}{t^2} e^{-t\omega_i\lambda_i m_q^2} e^{-\mathbf{x}_{20}^2/(4t)}$$

$$\overline{G}_{(i)} = \int_0^\infty \frac{\mathrm{d}u}{u} e^{-uM_i^2} e^{-\mathbf{x}_{102}^2/(4u)} \int_0^{u/\omega_i} \frac{\mathrm{d}t}{t} e^{-t\omega_i\lambda_i m_q^2} e^{-\mathbf{x}_{20}^2/(4t)}$$

with

$$F_{(i)} = F_{(1)}(k_j^+), \quad M_i^2 = \overline{Q}_i^2 + m_q^2, \quad \lambda_i = \lambda_i(k_j^+, Q^2), \quad \omega_i = \omega_i(k_j^+),$$

- The $m_q \to 0$ limit fully recovers the massless result
- Remaining UV/IR finite integrals computed numerically

Conclusion

This talk:

- One-loop computation of $\gamma_L^* \to q\bar{q}$ LCWF with massive quarks
- Full NLO correction to DIS F_L: combination of massive $q\bar{q}$ and $q\bar{q}g$ contributions

Theory/Phenomenology outlook:

- Full NLO correction for F_T (**in preparation**)
  - Understand the LCPT mass renormalization in QCD (see G. Beuf talk)
- Application of the NLO $\gamma_{T/L}^* \to q\bar{q}$ LCWFs to calculate other DIS observables at NLO (vector meason production, ...)
- Numerical implementation of $F_{L/T}$: Fits to HERA data at NLO+LL/NLL accuracy