Poincaré invariant UV regularization on the light-front and mass renormalization

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Earlier results on mass renormalization on the light-front

UV divergent one-loop corrections in QED and QCD on the light-front first calculated long ago
Mustaki, Pinsky, Shigemitsu and Wilson, PRD43 (1991)
Harindranath and Zhang, PRD48 (1993)

→ Longstanding fundamental issues for mass renormalization:

- Same result for the quark (or lepton) vertex mass correction as in covariant PT
- But different result for the quark (or lepton) kinetic mass correction (in energy denominators)

  ⇒ Do vertex mass and kinetic mass of fermions become different objects on the light front, with different counter-terms and different anomalous dimensions?

- Non-zero correction to the gluon and photon mass

  ⇒ Bare and renormalized gauge boson masses cannot be 0 simultaneously in light-front quantization?

UV regularization and one-particle phase-space

Measure for one-particle phase-space integration:

$$\int d^D\mu_m(k)\,\Big[\dots\Big] \equiv \int \frac{d^D k}{(2\pi)^D}\,\theta(k^0)\,2\pi\delta(k^2-m^2)\,\Big[\dots\Big]$$

Ex. of traditional UV regularization on the LF:

$$\left[\int d^4\mu_m(k)\right] \mapsto \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \int_{k^+_{\text{min}}}^{k^+_{\text{max}}} \frac{dk^+}{(2\pi)(2k^+)} = \frac{1}{4\pi}\int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}}\,\log\left(\frac{k^+_{\text{max}}}{k^+_{\text{min}}}\right) \to 0$$

- Problem 1: 3 different UV regulators: transverse Dim. Reg., k^+_{max} and k^+_{min}

  $\Rightarrow$ Violation of Poincaré symmetry
- Problem 2: information about the mass m lost

UV regularization and one-particle phase-space

Measure for one-particle phase-space integration:

$$\int d^D\mu_m(k)\,\Big[\dots\Big] \equiv \int \frac{d^Dk}{(2\pi)^D}\,\theta(k^0)\,2\pi\delta(k^2-m^2)\,\Big[\dots\Big]$$

But using Dim. Reg. only:

$$\int d^D\mu_m(k) = \int \frac{d^{D-1}\vec{k}}{(2\pi)^{D-1}}\,\frac{1}{2\sqrt{\vec{k}^2+m^2}} = \frac{m^2}{(4\pi)^2}\,\Gamma\Big(1-\frac{D}{2}\Big)\,\Big(\frac{m^2}{4\pi}\Big)^{\frac{D}{2}-2}$$

Prescription for a better UV regularization in LFPT:

- Use a single Poincaré invariant UV regulator, for instance Dim. Reg.
- Delay performing the k^- integrations and keep $d^D\mu_m(k)$ measure instead
- Make sure that any other regulator is applied only for non-UV divergences (IR, rapidity, ...)

Ex 1: LFPT and mass renormalization for φ^4 theory

LF Hamiltonian density for φ^4 theory:

$$\hat{T}^{+-}(x) = \frac{1}{2}\,\partial_j\hat{\varphi}(x)\,\partial_j\hat{\varphi}(x) + \frac{m_0^2}{2}\,\hat{\varphi}(x)^2 + \frac{\lambda_0}{4!}\,\hat{\varphi}(x)^4 + \Lambda_0$$

From Wick theorem:

$$\begin{aligned}\hat{\varphi}(x)^2 &=: \hat{\varphi}(x)^2 : + \langle 0|\hat{\varphi}(x)^2|0\rangle \\ \hat{\varphi}(x)^4 &=: \hat{\varphi}(x)^4 : + 6\langle 0|\hat{\varphi}(x)^2|0\rangle\ : \hat{\varphi}(x)^2 : + \langle 0|\hat{\varphi}(x)^4|0\rangle\end{aligned}$$

$\Rightarrow$ Interaction operator for LFPT:

$$\hat{V}(x^+) = \int d^{D-2}\mathbf{x} \int dx^- \left\{ \frac{\lambda_0}{4!} : \hat{\varphi}(x)^4 : + \frac{\lambda_0}{4}\langle 0|\hat{\varphi}(x)^2|0\rangle\ : \hat{\varphi}(x)^2 : + \frac{(m_0^2 - m^2)}{2} : \hat{\varphi}(x)^2 : \right\}$$

Ex 1: LFPT and mass renormalization for φ^4 theory

$$\hat{V}(x^+) = \int d^{D-2}\mathbf{x} \int dx^- \left\{ \frac{\lambda_0}{4!} :\hat{\varphi}(x)^4: + \frac{\lambda_0}{4} \langle 0|\hat{\varphi}(x)^2|0\rangle \; :\hat{\varphi}(x)^2: + \frac{(m_0^2 - m^2)}{2} :\hat{\varphi}(x)^2: \right\}$$

At one loop: only tadpole insertion contributes to mass renormalization

In on-shell mass renormalization scheme:

$$m_0^2 - m^2 = -\frac{\lambda_0}{2} \langle 0|\hat{\varphi}(x)^2|0\rangle + O(\lambda_0^2) = -\frac{\lambda}{2} (\mu^2)^{2-\frac{D}{2}} \int d^D\mu_m(k) + O(\lambda^2)$$

$$= -\frac{m^2\,\lambda}{2(4\pi)^2} \Gamma\left(1-\frac{D}{2}\right) \left(\frac{m^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2} + O(\lambda^2)$$

$\Rightarrow$ Same result as in covariant perturbation theory!

By contrast, tadpole vanishes with transverse Dim. Reg. + cutoffs
$\Rightarrow m_0^2 - m^2 = 0 + O(\lambda^2)$

Ex 2: Poincaré invariance of the vacuum in LFPT

$$\hat{P}^+|0\rangle = \left(\int d^Dx\, \delta(x^+)\langle 0|\,\hat{T}^{++}(x)|0\rangle\right)\; |0\rangle$$

In a theory with N_g gauge fields, N_d Dirac spinors and N_s scalars:

$$\langle 0|\,\hat{T}^{++}(x)|0\rangle = N_g\,(D_s - 2)\int d^D\mu_0(k)\;\; (k^+)^2 - 4N_d\int d^D\mu_{m_d}(k)\;\; (k^+)^2 + N_s\int d^D\mu_{m_s}(k)\;\; (k^+)^2$$

$$\int d^D\mu_m(k)\;\; (k^+)^2 = \int \frac{d^{D-1}\vec{k}}{(2\pi)^{D-1}}\,\frac{1}{2\sqrt{\vec{k}^2+m^2}}\;\left[\frac{\sqrt{\vec{k}^2+m^2}+\vec{k}\cdot\vec{n}}{\sqrt{2}}\right]^2$$

$$= \frac{(m^2)^{\frac{D}{2}}}{4(4\pi)^{\frac{D-1}{2}}\,\Gamma\left(\frac{D-1}{2}\right)}\;\left[\frac{D}{(D-1)}\,\frac{\Gamma\left(\frac{D+1}{2}\right)\Gamma\left(-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} + \frac{\Gamma\left(\frac{D-1}{2}\right)\Gamma\left(1-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}\right] = 0$$

So that $\hat{P}^+|0\rangle = 0$ unambiguously (without normal ordering imposed)

Constraint equation for spinor in QCD

Good and bad components of spinors:

$$\Psi_G(x) \equiv \frac{\gamma^- \gamma^+}{2}\Psi(x) \qquad\qquad \Psi_B(x) \equiv \frac{\gamma^+ \gamma^-}{2}\Psi(x)$$

Multiplying QCD Dirac equation by γ^+ (in LC gauge):

$$2i\partial_- \Psi_B(x) + \gamma^+ \left(i\gamma^j D_j - m_0\right) \Psi_G(x) = 0$$

$\rightarrow$ Constraint equation determining Ψ_B.

Usual solution on the LF (with anti-periodic B.C. in x^-):

$$\Psi_B(x) = \left(\frac{i}{4}\right) \int d^D y \delta(x^+ - y^+) \delta^{(D-2)}(\mathbf{x} - \mathbf{y}) \mathrm{sgn}(x^- - y^-)\, \gamma^+ \left(i\gamma^j D_{\mathbf{y}^j} - m_0\right) \Psi_G(y)$$

Constraint equation for spinor in QCD

Suggestion: Rewrite this solution as

$$\Psi_B(x) = -\int d^D y\, \delta(x^+ - y^+)\, G_{[,]}(x-y;m)\, \gamma^+ \Big(i\gamma^j D_j - m_0\Big) \Psi_G(y)$$

with the scalar antisymmetric propagator with mass m

$$G_{[,]}(x-y;m) \equiv \int d^D\mu_m(k)\ \Big\{e^{-ik\cdot(x-y)} - e^{ik\cdot(x-y)}\Big\}$$

Formally equivalent expressions, because

$$G_{[,]}(x-y;m)\Big|_{x^+=y^+} = -\frac{i}{4}\, \mathrm{sgn}(x^- - y^-)\, \delta^{(D-2)}(\mathbf{x}-\mathbf{y})$$

but more convenient for LFPT, by keeping track of m.

Constraint equation for gluon in QCD

$\nu = +$ component of Yang-Mills equations (in LC gauge):

$$-\partial_-{}^2 A_a^-(x) - \partial_-\partial_j A_a^j(x) = g_0 J_a^+(x)$$

$\rightarrow$ Constraint equation determining $A_a^-(x)$.

Solution in the absence of $J_a^+(x)$:

$$\widetilde{A}_a^-(x) \equiv -2i \int d^D y\, \delta(x^+ - y^+)\, G_{[,]}(x-y;0)\, \partial_{y^j} A_a^j(y)$$

Complete solution of the constraint:

$$A_a^\mu(x) = \widetilde{A}_a^\mu(x) + g^\mu{}_-\, 4g_0 \int d^D y\, \delta(x^+ - y^+) \times \left[\int d^D z\, \delta(x^+ - z^+)\, G_{[,]}(x-z;0)\, G_{[,]}(z-y;0)\right] J_a^+(y)$$

LF QCD Hamiltonian

Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

$$P^- = \mathcal{K} + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. }q} + \mathcal{V}_{\text{inst. }g} + \mathcal{V}_{\text{kin. m c.t.}} + \mathcal{V}_{\text{vertex m c.t.}} + \text{Const.}$$

- Free kinetic term $\mathcal{K}$

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- Free kinetic term $\mathcal{K}$
- Standard $q\bar{q}g$ and ggg interaction in QCD

$$\mathcal{V}_{AJ} \equiv g_0 \int d^D x \, \delta(x^+) \, \widetilde{A}_a^\mu(x) \, \widetilde{J}_a^\mu(x)$$

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- Free kinetic term $\mathcal{K}$
- Standard $q\bar{q}g$ and ggg interaction in QCD

$$\mathcal{V}_{AJ} \equiv g_0 \int d^D x \, \delta(x^+) \, \widetilde{A}^\mu_a(x) \, \widetilde{J}^\mu_a(x)$$

- Standard $gggg$ interaction in QCD

$$\mathcal{V}_{4g} \equiv \frac{g_0^2}{4} f^{abe} f^{cde} \int d^D x \, \delta(x^+) \, A^i_a(x) A^j_b(x) A^i_c(x) A^j_d(x)$$

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$$P^- = \mathcal{K} + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. } q} + \mathcal{V}_{\text{inst. } g} + \mathcal{V}_{\text{kin. m c.t.}} + \mathcal{V}_{\text{vertex m c.t.}} + \text{Const.}$$

- Non-local $q\bar{q}gg$ interaction by instantaneous quark exchange induced by the constraint on $\Psi_B(x)$

$$\mathcal{V}_{\text{inst. } q} \equiv g_0^2 \int d^D x\, \delta(x^+) \int d^D y\, \delta(y^+)\, \overline{\Psi_G}(x) \left[\gamma^i t^a A_a^i(x)\right] \times\; G_{[,]}(x-y; m)\, \gamma^+ \left[\gamma^j t^b A_b^j(y)\right] \Psi_G(y)$$

LF QCD Hamiltonian

Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

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- Non-local Coulomb interaction between color currents induced by the constraint on $A_a^-(x)$

$$\mathcal{V}_{\text{inst. } g} \equiv 2g_0^2 \int d^D x\, \delta(x^+) \int d^D y\, \delta(y^+)\, J_a^+(x) \times \left[\int d^D z\, \delta(z^+)\, G_{[,]}(x-z;0)\, G_{[,]}(z-y;0)\right] J_a^+(y)$$

LF QCD Hamiltonian

Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

$$P^- = \mathcal{K} + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. }q} + \mathcal{V}_{\text{inst. }g} + \mathcal{V}_{\text{kin. m c.t.}} + \mathcal{V}_{\text{vertex m c.t.}} + \text{Const.}$$

- Mass counterterm induced by the kinetic term:
  takes care of quark mass renormalization in Energy Denominators

$$\mathcal{V}_{\text{kin. m c.t.}} \equiv \left(m_0^2 - m^2\right) \int d^D x\, \delta(x^+) \int d^D y\, \delta(y^+)\, G_{[,]}(x-y; m)\, \overline{\Psi_G}(x)\, \gamma^+ \Psi_G(y)$$

LF QCD Hamiltonian

Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

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  takes care of quark mass renormalization in Energy Denominators

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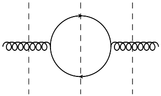
- Mass counterterm induced by the mass dependence of the $q\bar{q}g$ vertex:
  takes care of quark mass renormalization in the numerator

$$\mathcal{V}_{\text{vertex m c.t.}} \equiv (m_0 - m) g_0 \int d^D x\, \delta(x^+) \int d^D y\, \delta(y^+)\, G_{[,]}(x-y; m) \times \left[A_a^j(x) - A_a^j(y)\right] \overline{\Psi_G}(x)\, \gamma^+ \gamma^j t^a \Psi_G(y)$$

Quark loop insertion on a gluon line

- Standard 1-loop graph:

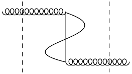


$$=4g_0^2\,T_F \int d^D\mu_m(k)\;\theta(p^+ - k^+)\left[\frac{2k^+\mathbf{K}^2}{(D-2)p^+(\mathbf{K}^2+m^2)} - 1\right]$$

Parent gluon with momentum p

Relative transverse momentum $\mathbf{K} = \mathbf{k} - (k^+/p^+)\mathbf{p}$

- Instantaneous 1-loop graphs from instant. quark exchange:



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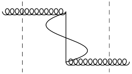
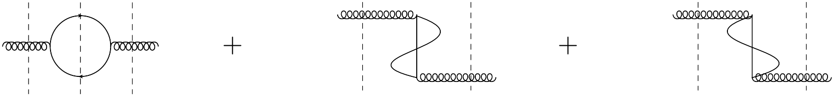


$$=4g_0^2\,T_F \int d^D\mu_m(k)\;\left[\theta(p^+ - k^+) - 1\right]$$

Quark loop insertion on a gluon line

Total:



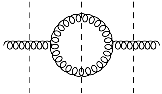
$$
\begin{aligned}
=& 4g_0^2\, T_F \int d^D\mu_m(k)\ \left[\frac{2k^+\mathbf{K}^2}{(D-2)p^+(\mathbf{K}^2+m^2)}\,\theta(p^+-k^+)-1\right]\\
=& 4g^2\, T_F\, \frac{m^2}{(4\pi)^2}\left(\frac{m^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2}\left[-\frac{2}{(D-2)}\Gamma\left(2-\frac{D}{2}\right)-\Gamma\left(1-\frac{D}{2}\right)\right]\\
=& 0
\end{aligned}
$$

$\Rightarrow$ No contribution to gluon mass renormalization from quark loops!

Exact cancellation of standard graph by tadpole graph:
require Poincaré invariant UV regularization

Gluon loop insertion on a gluon line

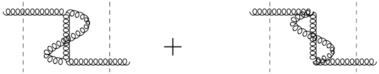
- Standard 1-loop graph:



$$= g_0^2 C_A \int d^D\mu_0(k)\, \theta(p^+ - k^+) \left[-\frac{4p^+}{k^+} - 2\frac{(D_s-2)}{(D-2)}\,\frac{k^+}{p^+} \right]$$

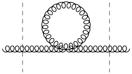
Parent gluon with momentum p

- Instantaneous 1-loop graphs from instant. Coulomb interaction :



$$= g_0^2 C_A \int d^D\mu_0(k) \left[\frac{4p^+}{k^+} \theta(p^+ - k^+) + 1 \right]$$

- Instantaneous 1-loop graph from the standard 4-gluons vertex :



$$= g_0^2 C_A (D_s - 3) \int d^D\mu_0(k)$$

Gluon loop insertion on a gluon line

Total:

$$= -\, g_0^2\, C_A(D_s - 2) \int d^D\mu_0(k)\, \left[\frac{2k^+}{(D-2)p^+}\,\theta(p^+ - k^+) - 1\right]$$

$$= 0$$

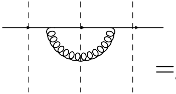
Same expression as for quark loop but in the massless limit

$\Rightarrow$ No contribution to gluon mass renormalization from gluon loops!

Vanishing total 1-loop correction to the gluon mass, using a Poincaré invariant UV regularization in LFPT

One-loop correction to a quark line

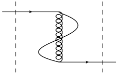
- Standard 1-loop self-energy graph:



$$=g_0^2\, C_F \int d^D\mu_0(q)\; \theta(p^+ - q^+)\left[-\frac{4p^+}{q^+} + \frac{4q^+}{p^+}\, \frac{m^2}{\left[\mathbf{K}^2 + (q^+/p^+)^2\, m^2\right]}\right]$$

$$-\, g_0^2\, C_F (D_s - 2) \int d^D\mu_m(k)\; \theta(p^+ - k^+)$$

- Instantaneous 1-loop graphs from instant. Coulomb interaction :



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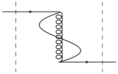
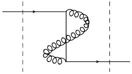


$$=g_0^2\, C_F \int d^D\mu_0(q)\; \theta(p^+ - q^+)\, \frac{4p^+}{q^+}$$

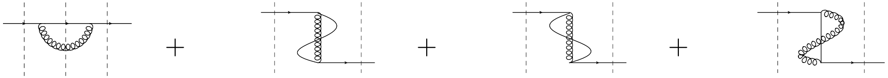
- Instantaneous 1-loop graph from instant. quark exchange :



$$=g_0^2\, C_F (D_s - 2)\left\{\int d^D\mu_0(q) + \int d^D\mu_m(k)\left[\theta(p^+ - k^+) - 1\right]\right\}$$

One-loop correction to a quark line

Sum of one-loop diagrams:



$$
\begin{aligned}
=& g_0^2\, C_F \int d^D\mu_0(q)\; \theta(p^+ - q^+)\, \frac{4q^+}{p^+}\, \frac{m^2}{\left[\mathbf{K}^2 + \left(q^+/p^+\right)^2 m^2\right]}\\
&+ g_0^2\, C_F (D_s - 2) \left[\int d^D\mu_0(q) - \int d^D\mu_m(k)\right]\\
=& \frac{\alpha_s C_F}{\pi}\, m^2\, \Gamma\!\left(2 - \frac{D}{2}\right) \left(\frac{m^2}{4\pi}\right)^{\frac{D}{2}-2} \left[\frac{1}{(D-3)} + \frac{(D_s-2)}{2(D-2)}\right]
\end{aligned}
$$

On-shell mass renormalization scheme:
1-loop contributions exactly cancelled by the (kinetic) mass counterterm

$$
m_0^2 - m^2 = m^2 ({Z_m}^2 - 1) = 2m^2 (Z_m - 1) + O(\alpha_s^2)
$$

Quark mass counter-term

Taking $D_s = D$ in CDR or HV, or $D_s = 4$ in DRED or FDH, and then expanding in $\epsilon = 2 - \frac{D}{2}$:

$$\left(Z_m^{OS} - 1\right)\Big|_{CDR/HV} = -\frac{\alpha_s C_F}{\pi}\left\{\frac{3}{4}\left[\frac{1}{\overline{\epsilon}} - \log\left(\frac{m^2}{\mu^2}\right)\right] + 1 + O(\epsilon)\right\}$$
$$\left(Z_m^{OS} - 1\right)\Big|_{DRED/FDH} = -\frac{\alpha_s C_F}{\pi}\left\{\frac{3}{4}\left[\frac{1}{\overline{\epsilon}} - \log\left(\frac{m^2}{\mu^2}\right)\right] + \frac{5}{4} + O(\epsilon)\right\}$$

- Consistent with the vertex mass renormalization
  $\Rightarrow$ Only one quark mass parameter
- Consistent with the results in covariant PT, including the finite term for any variant of Dim. Reg.

Conclusion

- Slight reformulation of LFPT with Poincaré invariant UV regularization
  - Delay k^- integration and keep D-dimensional momentum integration measure
  - Parameterize instantaneous exchanges with antisymmetric propagator instead of sign function

- Longstanding issues with mass renormalization in LFPT finally resolved

  $\rightarrow$ Expected results finally obtained:

  - The gluon mass stays zero at 1-loop without needing a counter-term
  - Same quark mass correction in the energy denominators and in the numerator (vertices)
  - Same quark mass renormalization constant as in covariant PT, including the finite terms in the on-shell scheme, in all variants of Dim. Reg.
  - Same results also valid in QED

- New UV prescription relevant for mass renormalization and for vacuum properties, but should have no effect in other types of LFPT calculations