Poincaré invariant UV regularization on the light-front and mass renormalization

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Earlier results on mass renormalization on the light-front

UV divergent one-loop corrections in QED and QCD on the light-front first calculated long ago
Mustaki, Pinsky, Shigemitsu and Wilson, PRD43 (1991)
Harindranath and Zhang, PRD48 (1993)

→ Longstanding fundamental issues for mass renormalization:

- Same result for the quark (or lepton) vertex mass correction as in covariant PT
- But different result for the quark (or lepton) kinetic mass correction (in energy denominators)
  ⇒ Do vertex mass and kinetic mass of fermions become different objects on the light front, with different counter-terms and different anomalous dimensions?
- Non-zero correction to the gluon and photon mass
  ⇒ Bare and renormalized gauge boson masses cannot be 0 simultaneously in light-front quantization?
UV regularization and one-particle phase-space

Measure for one-particle phase-space integration:

\[
\int d^D \mu_m(k) \left[ \ldots \right] \equiv \int \frac{d^D k}{(2\pi)^D} \theta(k^0) 2\pi \delta(k^2 - m^2) \left[ \ldots \right]
\]

Ex. of traditional UV regularization on the LF:

\[
\left[ \int d^4 \mu_m(k) \right] \mapsto \int \frac{d^{D-2} k}{(2\pi)^{D-2}} \int_{k_{\text{min}}^+}^{k_{\text{max}}^+} \frac{d k^+}{(2\pi)(2k^+)} = \frac{1}{4\pi} \int \frac{d^{D-2} k}{(2\pi)^{D-2}} \log \left( \frac{k_{\text{max}}^+}{k_{\text{min}}^+} \right) \to 0
\]

- Problem 1: 3 different UV regulators: transverse Dim. Reg., \( k_{\text{max}}^+ \) and \( k_{\text{min}}^+ \)
  \( \Rightarrow \) Violation of Poincaré symmetry

- Problem 2: information about the mass \( m \) lost
UV regularization and one-particle phase-space

Measure for one-particle phase-space integration:

\[ \int d^D \mu_m(k) \left[ \ldots \right] = \int \frac{d^D k}{(2\pi)^D} \theta(k^0) \ 2\pi \delta(k^2 - m^2) \left[ \ldots \right] \]

But using Dim. Reg. only:

\[ \int d^D \mu_m(k) = \int \frac{d^{D-1} \vec{k}}{(2\pi)^{D-1}} \frac{1}{2\sqrt{\vec{k}^2 + m^2}} = \frac{m^2}{(4\pi)^2} \Gamma \left(1 - \frac{D}{2} \right) \left( \frac{m^2}{4\pi} \right)^{\frac{D}{2} - 2} \]

Prescription for a better UV regularization in LFPT:

- Use a single Poincaré invariant UV regulator, for instance Dim. Reg.
- Delay performing the $k^-$ integrations and keep $d^D \mu_m(k)$ measure instead
- Make sure that any other regulator is applied only for non-UV divergences (IR, rapidity, . . . )
Ex 1: LFPT and mass renormalization for $\varphi^4$ theory

LF Hamiltonian density for $\varphi^4$ theory:

$$\hat{T}^{+-}(x) = \frac{1}{2} \partial_j \hat{\varphi}(x) \partial_j \hat{\varphi}(x) + \frac{m_0^2}{2} \hat{\varphi}(x)^2 + \frac{\lambda_0}{4!} \hat{\varphi}(x)^4 + \Lambda_0$$

From Wick theorem:

$$\hat{\varphi}(x)^2 =: \hat{\varphi}(x)^2 : + \langle 0 | \hat{\varphi}(x)^2 | 0 \rangle$$

$$\hat{\varphi}(x)^4 =: \hat{\varphi}(x)^4 : + 6 \langle 0 | \hat{\varphi}(x)^2 | 0 \rangle : \hat{\varphi}(x)^2 : + \langle 0 | \hat{\varphi}(x)^4 | 0 \rangle$$

$\Rightarrow$ Interaction operator for LFPT:

$$\hat{V}(x^+) = \int d^{D-2}x \int dx^- \left\{ \frac{\lambda_0}{4!} : \hat{\varphi}(x)^4 : + \frac{\lambda_0}{4} \langle 0 | \hat{\varphi}(x)^2 | 0 \rangle : \hat{\varphi}(x)^2 : \\
+ \frac{(m_0^2 - m^2)}{2} : \hat{\varphi}(x)^2 : \right\}$$
Ex 1: LFPT and mass renormalization for $\varphi^4$ theory

\[ \hat{V}(x^+) = \int d^{D-2}x \int dx^- \left\{ \frac{\lambda_0}{4!} : \varphi(x)^4 : + \frac{\lambda_0}{4} \langle 0 | \varphi(x)^2 | 0 \rangle : \varphi(x)^2 : + \frac{(m_0^2 - m^2)}{2} : \varphi(x)^2 : \right\} \]

At one loop: only tadpole insertion contributes to mass renormalization

In on-shell mass renormalization scheme:

\[ m_0^2 - m^2 = -\frac{\lambda_0}{2} \langle 0 | \varphi(x)^2 | 0 \rangle + O(\lambda_0^2) = -\frac{\lambda}{2} (\mu^2)^{2-D/2} \int d^D \mu m(k) + O(\lambda^2) \]

\[ = -\frac{m^2 \lambda}{2(4\pi)^2} \Gamma \left( 1 - \frac{D}{2} \right) \left( \frac{m^2}{4\pi \mu^2} \right)^{\frac{D}{2}-2} + O(\lambda^2) \]

⇒ Same result as in covariant perturbation theory!

By contrast, tadpole vanishes with transverse Dim. Reg. + cutoffs

⇒ $m_0^2 - m^2 = 0 + O(\lambda^2)$
\[ \hat{P}^+ |0\rangle = \left( \int d^D x \, \delta(x^+) \langle 0 | \hat{T}^{++}(x) |0\rangle \right) |0\rangle \]

In a theory with \(N_g\) gauge fields, \(N_d\) Dirac spinors and \(N_s\) scalars:

\[ \langle 0 | \hat{T}^{++}(x) |0\rangle = N_g \left( D_s - 2 \right) \int d^D \mu_0(k) \, (k^+)^2 - 4N_d \int d^D \mu_m(k) \, (k^+)^2 \]
\[ + N_s \int d^D \mu_m(k) \, (k^+)^2 \]

\[ \int d^D \mu_m(k) \, (k^+)^2 = \int \frac{d^{D-1} \vec{k}}{(2\pi)^{D-1}} \frac{1}{2\sqrt{k^2 + m^2}} \left[ \sqrt{k^2 + m^2 + \vec{k} \cdot \vec{n}} \right]^2 \]
\[ = \frac{\left( m^2 \right)^{D/2}}{4(4\pi)^{D-1} \Gamma \left( \frac{D-1}{2} \right)} \left[ \frac{D}{(D-1)} \frac{\Gamma \left( \frac{D+1}{2} \right) \Gamma \left( -\frac{D}{2} \right)}{\Gamma \left( \frac{1}{2} \right)} + \frac{\Gamma \left( \frac{D-1}{2} \right) \Gamma \left( 1 - \frac{D}{2} \right)}{\Gamma \left( \frac{1}{2} \right)} \right] = 0 \]

So that \( \hat{P}^+ |0\rangle = 0 \) unambiguously (without normal ordering imposed)
Constraint equation for spinor in QCD

Good and bad components of spinors:

\[ \Psi_G(x) \equiv \frac{\gamma^- \gamma^+}{2} \Psi(x) \quad \Psi_B(x) \equiv \frac{\gamma^+ \gamma^-}{2} \Psi(x) \]

Multiplying QCD Dirac equation by \( \gamma^+ \) (in LC gauge):

\[ 2i\partial_- \Psi_B(x) + \gamma^+ (i\gamma^j D_j - m_0) \Psi_G(x) = 0 \]

\( \rightarrow \) Constraint equation determining \( \Psi_B \).

Usual solution on the LF (with anti-periodic B.C. in \( x^- \)):

\[ \Psi_B(x) = \left( \frac{i}{4} \right) \int d^D y \delta(x^+ - y^+) \delta^{(D-2)}(x - y) \text{sgn}(x^- - y^-) \gamma^+ \left( i\gamma^j D_{ij} - m_0 \right) \Psi_G(y) \]
Suggestion: Rewrite this solution as

\[ \Psi_B(x) = -\int d^Dy \, \delta(x^+ - y^+) \, G_\Box(x - y; m) \gamma^+ \left( i\gamma^j D_j - m_0 \right) \Psi_G(y) \]

with the scalar antisymmetric propagator with mass \(m\)

\[ G_\Box(x - y; m) \equiv \int d^D\mu_m(k) \\left\{ e^{-ik \cdot (x-y)} - e^{ik \cdot (x-y)} \right\} \]

Formally equivalent expressions, because

\[ G_\Box(x - y; m) \bigg|_{x^+ = y^+} = -\frac{i}{4} \, \text{sgn}(x^- - y^-) \, \delta^{(D-2)}(x - y) \]

but more convenient for LFPT, by keeping track of \(m\).
Constraint equation for gluon in QCD

\[ \nu = + \text{ component of Yang-Mills equations (in LC gauge):} \]

\[ -\partial_-^2 A_a^-(x) - \partial_- \partial_j A_a^j(x) = g_0 J_a^+(x) \]

→ Constraint equation determining \( A_a^-(x) \).

Solution in the absence of \( J_a^+(x) \):

\[ \tilde{A}_a^-(x) \equiv -2i \int d^D y \delta(x^+ - y^+) \, G_{[\cdot]}(x - y; 0) \partial_{yj} A_a^j(y) \]

Complete solution of the constraint:

\[ A_a^\mu(x) = \tilde{A}_a^\mu(x) + g^\mu - 4g_0 \int d^D y \delta(x^+ - y^+) \]

\[ \times \left[ \int d^D z \delta(x^+ - z^+) \, G_{[\cdot]}(x - z; 0) \, G_{[\cdot]}(z - y; 0) \right] J_a^+(y) \]
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

\[ P^- = \mathcal{K} + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. } q} + \mathcal{V}_{\text{inst. } g} + \mathcal{V}_{\text{kin. m c.t.}} + \mathcal{V}_{\text{vertex m c.t.}} + \text{Const.} \]

- Free kinetic term \( \mathcal{K} \)
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

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- Free kinetic term \( \mathcal{K} \)
- Standard \( q\bar{q}g \) and \( ggg \) interaction in QCD

\[ \mathcal{V}_{AJ} \equiv g_0 \int d^D x \delta(x^+) \tilde{A}_a^\mu(x) \tilde{J}_a^\mu(x) \]
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

\[ P^- = \mathcal{K} + \mathcal{N}_{AJ} + \mathcal{N}_{4g} + \mathcal{N}_{\text{inst. } q} + \mathcal{N}_{\text{inst. } g} + \mathcal{N}_{\text{kin. m c.t.}} + \mathcal{N}_{\text{vertex m c.t.}} + \text{Const.} \]

- Free kinetic term \( \mathcal{K} \)
- Standard \( q\bar{q}g \) and \( ggg \) interaction in QCD

\[ \mathcal{N}_{AJ} \equiv g_0 \int d^D x \delta(x^+) \tilde{A}_a^\mu(x) \tilde{J}_a^\mu(x) \]

- Standard \( gggg \) interaction in QCD

\[ \mathcal{N}_{4g} \equiv \frac{g_0^2}{4} f^{abc} f^{cde} \int d^D x \delta(x^+) A^{i}_a(x) A^{i}_b(x) A^{i}_c(x) A^{i}_d(x) \]
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

\[ P^- = K + V_{AJ} + V_{4g} + V_{\text{inst. } q} + V_{\text{inst. } g} + V_{\text{kin. m c.t.}} + V_{\text{vertex m c.t.}} + \text{Const.} \]

- Non-local \( q\bar{q}gg \) interaction by instantaneous quark exchange induced by the constraint on \( \Psi_B(x) \)

\[
V_{\text{inst. } q} \equiv g_0^2 \int d^Dx \, \delta(x^+) \int d^Dy \, \delta(y^+) \, \overline{\Psi}_G(x) \left[ \gamma^i t^a A^i_a(x) \right] \\
\times G_{[.,]}(x-y; m) \, \gamma^+ \left[ \gamma^j t^b A^j_b(y) \right] \Psi_G(y)
\]
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

\[ P^- = \mathcal{K} + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. } q} + \mathcal{V}_{\text{inst. } g} + \mathcal{V}_{\text{kin. m c.t.}} + \mathcal{V}_{\text{vertex m c.t.}} + \text{Const.} \]

- Non-local \( q\bar{q}gg \) interaction by instantaneous quark exchange induced by the constraint on \( \Psi_B(x) \)

\[
\mathcal{V}_{\text{inst. } q} \equiv g_0^2 \int d^D x \delta(x^+) \int d^D y \delta(y^+) \overline{\Psi}(x) \left[ \gamma^i t^a A^i_a(x) \right] \\
\times G_{[i]}(x-y; m) \gamma^+ \left[ \gamma^j t^b A^j_b(y) \right] \Psi_G(y)
\]

- Non-local Coulomb interaction between color currents induced by the constraint on \( A_a^-(x) \)

\[
\mathcal{V}_{\text{inst. } g} \equiv 2g_0^2 \int d^D x \delta(x^+) \int d^D y \delta(y^+) \mathcal{J}_a^+(x) \\
\times \left[ \int d^D z \delta(z^+) G_{[i]}(x-z; 0) G_{[i]}(z-y; 0) \right] \mathcal{J}_a^+(y)
\]
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

\[ P^− = K + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. } q} + \mathcal{V}_{\text{inst. } g} + \mathcal{V}_{\text{kin. } m \text{ c.t.}} + \mathcal{V}_{\text{vertex } m \text{ c.t.}} + \text{Const.} \]

- Mass counterterm induced by the kinetic term:
  - takes care of quark mass renormalization in Energy Denominators

\[ \mathcal{V}_{\text{kin. } m \text{ c.t.}} \equiv (m_0^2 - m^2) \int d^Dx \delta(x^+) \int d^Dy \delta(y^+) G_{[i]}(x−y; m) \overline{\psi}_G(x) \gamma^+ \psi_G(y) \]
Using these solutions for the constraints, the LF QCD Hamiltonian writes (before expansion in normal-ordered operators)

$$P^- = \mathcal{K} + \mathcal{V}_{AJ} + \mathcal{V}_{4g} + \mathcal{V}_{\text{inst. } q} + \mathcal{V}_{\text{inst. } g} + \mathcal{V}_{\text{kin. m c.t.}} + \mathcal{V}_{\text{vertex m c.t.}} + \text{Const.}$$

- Mass counterterm induced by the kinetic term:
  takes care of quark mass renormalization in Energy Denominators

$$\mathcal{V}_{\text{kin. m c.t.}} \equiv (m_0^2 - m^2) \int d^D x \delta(x^+) \int d^D y \delta(y^+) G_{[i]}(x-y; m) \overline{\Psi}_G(x) \gamma^+ \Psi_G(y)$$

- Mass counterterm induced by the mass dependence of the $q\bar{q}g$ vertex:
  takes care of quark mass renormalization in the numerator

$$\mathcal{V}_{\text{vertex m c.t.}} \equiv (m_0 - m) g_0 \int d^D x \delta(x^+) \int d^D y \delta(y^+) G_{[i]}(x-y; m) \times \left[ A^j_a(x) - A^j_a(y) \right] \overline{\Psi}_G(x) \gamma^+ \gamma^j t^a \Psi_G(y)$$
QCD: mass renormalization

Quark loop insertion on a gluon line

- Standard 1-loop graph:

\[ = 4g_0^2 T_F \int d^D \mu m(k) \theta(p^+ - k^+) \left[ \frac{2k^+ K^2}{(D-2)p^+(K^2 + m^2)} - 1 \right] \]

Parent gluon with momentum \( p \)

Relative transverse momentum \( K = k - (k^+ / p^+)p \)

- Instantaneous 1-loop graphs from instant. quark exchange:

\[ + \quad = 4g_0^2 T_F \int d^D \mu m(k) \left[ \theta(p^+ - k^+) - 1 \right] \]
Quark loop insertion on a gluon line

Total:

\[
= 4g_F^2 T_F \int d^D \mu_m(k) \left[ \frac{2k^+K^2}{(D-2)p^+(K^2 + m^2)} \theta(p^+ - k^+) - 1 \right]
\]

\[
= 4g^2 T_F \frac{m^2}{(4\pi)^2} \left( \frac{m^2}{4\pi \mu^2} \right)^{\frac{D}{2} - 2} \left[ -\frac{2}{(D-2)} \Gamma \left( 2 - \frac{D}{2} \right) - \Gamma \left( 1 - \frac{D}{2} \right) \right]
\]

\[
= 0
\]

⇒ No contribution to gluon mass renormalization from quark loops!

Exact cancellation of standard graph by tadpole graph:
require Poincaré invariant UV regularization
Gluon loop insertion on a gluon line

- Standard 1-loop graph:

\[
= g_0^2 C_A \int d^D\mu_0(k) \theta(p^+ - k^+) \left[ -\frac{4p^+}{k^+} - 2 \frac{(D_s-2)}{(D-2)} \frac{k^+}{p^+} \right]
\]

Parent gluon with momentum \( p \)

- Instantaneous 1-loop graphs from instant. Coulomb interaction:

\[
= g_0^2 C_A \left[ \frac{4p^+}{k^+} \theta(p^+ - k^+) + 1 \right]
\]

- Instantaneous 1-loop graph from the standard 4-gluons vertex:

\[
= g_0^2 C_A (D_s - 3) \int d^D\mu_0(k)
\]
Gluon loop insertion on a gluon line

Total:

\[ -g_0^2 C_A (D_s - 2) \int d^D \mu_0(k) \left[ \frac{2k^+}{(D - 2)p^+} \theta(p^+ - k^+) - 1 \right] \]

\[ = 0 \]

Same expression as for quark loop but in the massless limit

⇒ No contribution to gluon mass renormalization from gluon loops!

Vanishing total 1-loop correction to the gluon mass, using a Poincaré invariant UV regularization in LFPT
One-loop correction to a quark line

- Standard 1-loop self-energy graph:

\[ = g_0^2 C_F \int d^D \mu_0(q) \theta(p^+ - q^+) \left[ -\frac{4p^+}{q^+} + \frac{4q^+}{p^+} \frac{m^2}{K^2 + (q^+/p^+)^2 m^2} \right] \]

\[- g_0^2 C_F (D_s - 2) \int d^D \mu_m(k) \theta(p^+ - k^+) \]

- Instantaneous 1-loop graphs from instant. Coulomb interaction:

\[ = g_0^2 C_F \left\{ \int d^D \mu_0(q) + \int d^D \mu_m(k) \left[ \theta(p^+ - k^+) - 1 \right] \right\} \]

\[ + \]

- Instantaneous 1-loop graph from instant. quark exchange:

\[ = g_0^2 C_F \int d^D \mu_0(q) \theta(p^+ - q^+) \frac{4p^+}{q^+} \]
One-loop correction to a quark line

Sum of one-loop diagrams:

\[ = g_0^2 C_F \int d^D \mu_0(q) \theta(p^+ - q^+) \frac{4q^+}{p^+} \left[ K^2 + \left( q^+ / p^+ \right)^2 m^2 \right] \]

\[ + g_0^2 C_F (D_s - 2) \left[ \int d^D \mu_0(q) - \int d^D \mu_m(k) \right] \]

\[ = \frac{\alpha_s C_F}{\pi} m^2 \Gamma \left( 2 - \frac{D}{2} \right) \left( \frac{m^2}{4\pi} \right) \frac{D^2 - 2}{2} \left[ \frac{1}{(D-3)} + \frac{(D_s - 2)}{2(D-2)} \right] \]

On-shell mass renormalization scheme:
1-loop contributions exactly cancelled by the (kinetic) mass counterterm

\[ m_0^2 - m^2 = m^2(Z_m^2 - 1) = 2m^2(Z_m - 1) + O(\alpha_s^2) \]
Taking $D_s = D$ in CDR or HV, or $D_s = 4$ in DRED or FDH, and then expanding in $\epsilon = 2 - \frac{D}{2}$:

\[
(Z_{m}^{OS} - 1)_{|_{CDR/HV}} = - \frac{\alpha_s C_F}{\pi} \left\{ \frac{3}{4} \left[ \frac{1}{\epsilon} - \log \left( \frac{m^2}{\mu^2} \right) \right] + 1 + O(\epsilon) \right\}
\]

\[
(Z_{m}^{OS} - 1)_{|_{DRED/FDH}} = - \frac{\alpha_s C_F}{\pi} \left\{ \frac{3}{4} \left[ \frac{1}{\epsilon} - \log \left( \frac{m^2}{\mu^2} \right) \right] + \frac{5}{4} + O(\epsilon) \right\}
\]

- Consistent with the vertex mass renormalization
  \[ \Rightarrow \] Only one quark mass parameter
- Consistent with the results in covariant PT, including the finite term for any variant of Dim. Reg.
Conclusion

- Slight reformulation of LFPT with Poincaré invariant UV regularization
  - Delay $k^-$ integration and keep $D$-dimensional momentum integration measure
  - Parameterize instantaneous exchanges with antisymmetric propagator instead of sign function

- Longstanding issues with mass renormalization in LFPT finally resolved
  - Expected results finally obtained:
    - The gluon mass stays zero at 1-loop without needing a counter-term
    - Same quark mass correction in the energy denominators and in the numerator (vertices)
    - Same quark mass renormalization constant as in covariant PT, including the finite terms in the on-shell scheme, in all variants of Dim. Reg.
    - Same results also valid in QED

- New UV prescription relevant for mass renormalization and for vacuum properties, but should have no effect in other types of LFPT calculations