# The Energy-Momentum Tensor for massive hadrons

in collaboration with:
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# QCD Energy-Momentum Tensor (EMT)

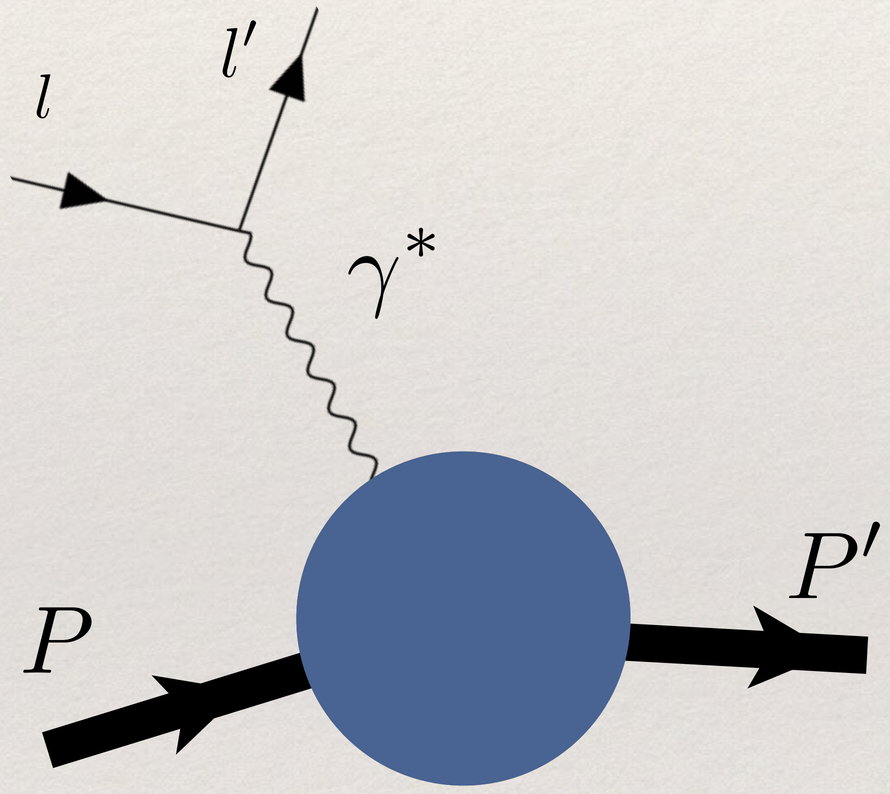
$$T^{\mu\nu} = \overline{\psi}\gamma^{\mu}\frac{i}{2}\overleftrightarrow{D}^{\nu}\psi - F^{a\mu\lambda}F^{a\nu}_{\lambda} + \frac{1}{4}g^{\mu\nu}F^{2}$$

- Fundamental object in every theory (Noether's conserved current related to space-time translations)
- Contains info on mass and angular momentum distribution
- Contains info on the mechanical properties of hadrons (momentum distribution, pressure, etc..)
- Asymmetric object in theories with spin, symmetric otherwise.

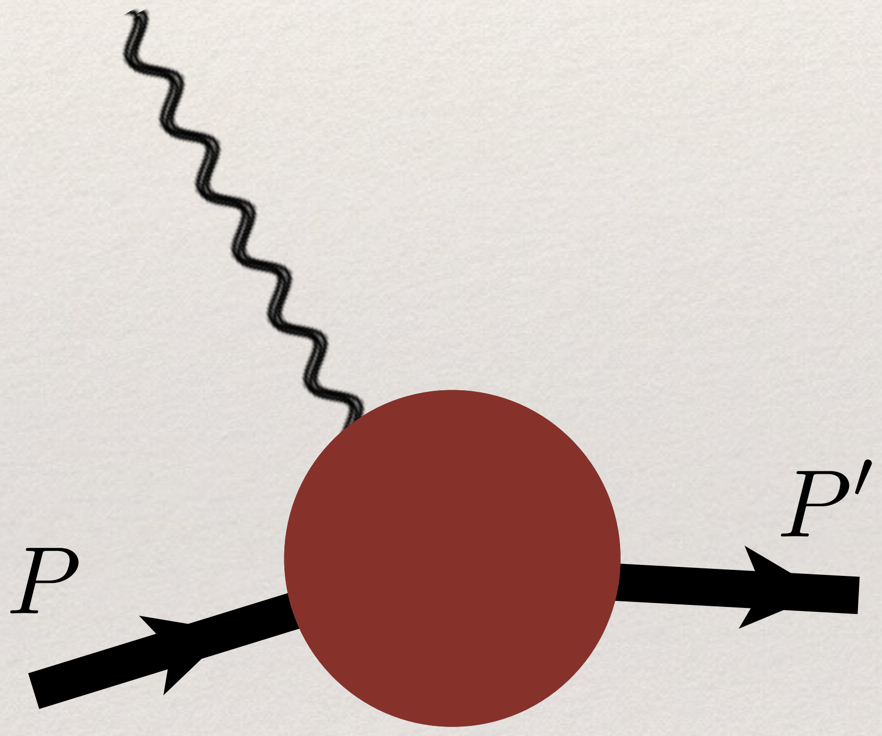
# Conserved currents

Local matrix elements, parametrized in terms of Form Factors

Electromagnetic interaction




Gravitational interaction



# Conserved currents

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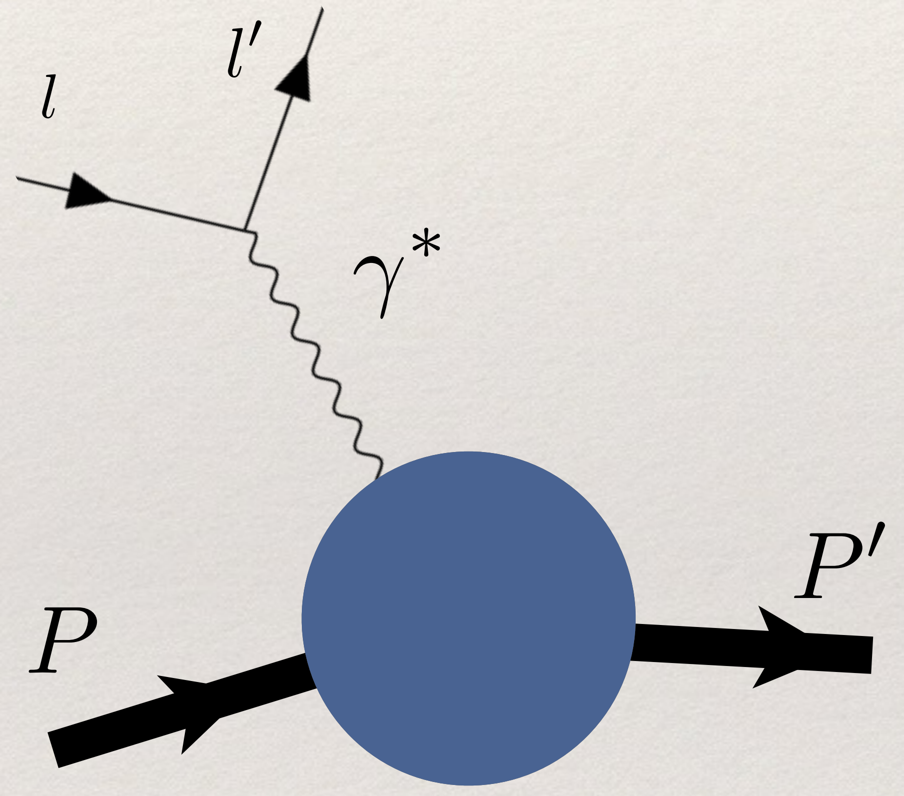
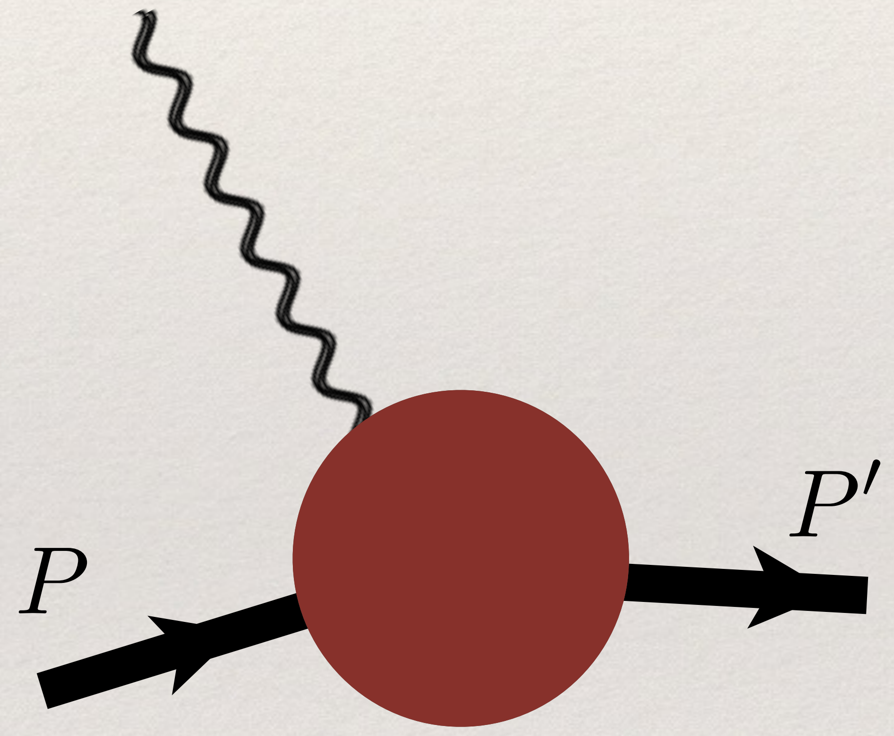



Electromagnetic probe, OK!

Gravitational interaction

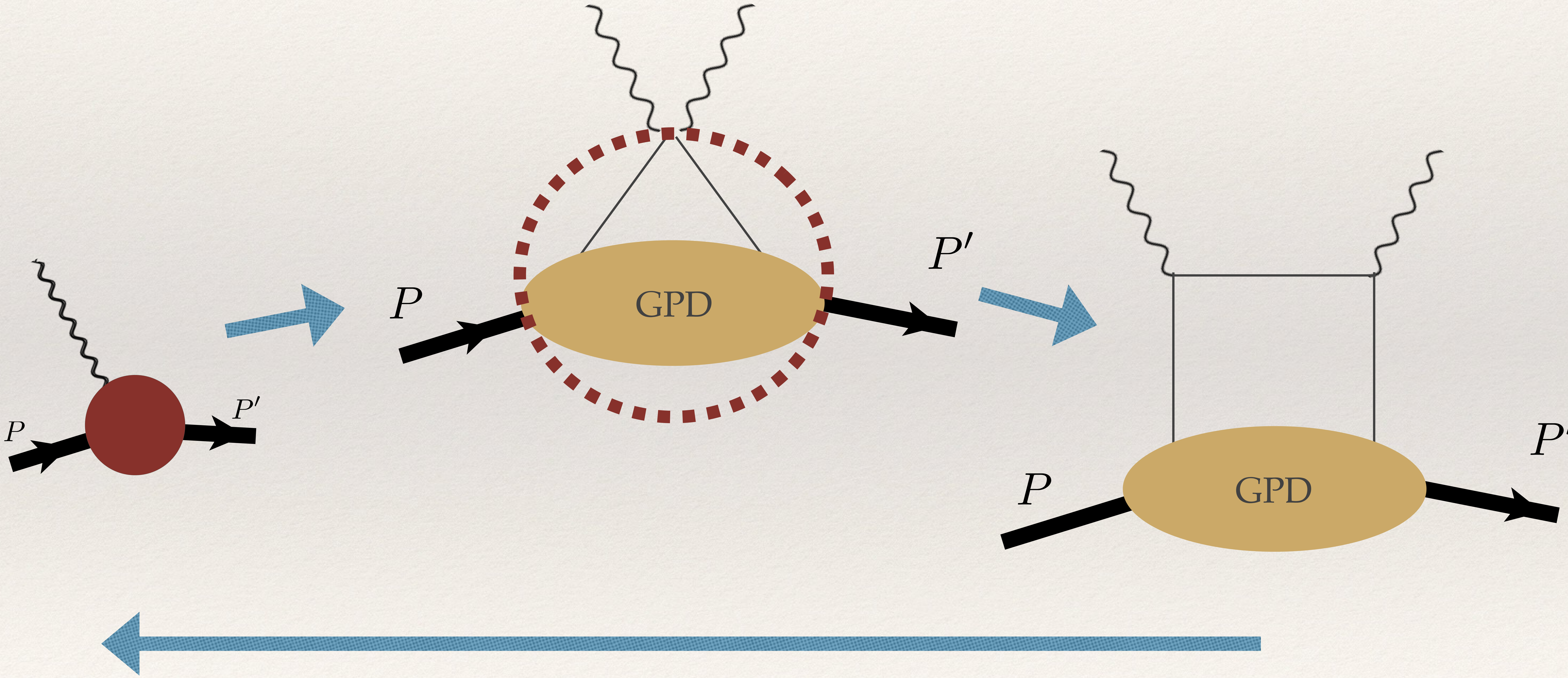



Impossible to probe directly through
"graviton scatterings"

The precise interaction with gravity is not known!

# EMT in QCD processes

The only way to (indirectly) access the FFs is through the relation between the EMT matrix element and the Mellin moments of the GPDs



# Parametrizations of local matrix elements

Redefine variables: $P = \frac{p' + p}{2}, \quad \Delta = p' - p, \quad t = \Delta^2 \qquad a = q, g$

$$\langle p', \lambda' \mid J_a^\mu(0) \mid p, \lambda \rangle = \bar{\eta}_{\lambda'}(p') \theta_a^\mu(P, \Delta)\, \eta_\lambda(p)$$

$$\langle p', \lambda' \mid T_a^{\mu\nu}(0) \mid p, \lambda \rangle = \bar{\eta}_{\lambda'}(p') \Gamma_a^{\mu\nu}(P, \Delta)\, \eta_\lambda(p)$$

Generalized Polarization Tensors (GPTs) **(More in Peter Lowdon's talk)**

| | Spin-0 | Spin-1/2 | Spin-1 | Spin-3/2 | Spin-2 | etc. |
|---|---|---|---|---|---|---|
| $\eta_\lambda(p) \propto$ | $e^{ipx}$ | $u(p, \lambda)$ | $\varepsilon_\alpha(p, \lambda)$ | $u_\alpha(p, \lambda)$ | $\varepsilon_{\alpha\beta}(p, \lambda)$ | $\cdots$ |

# EMT for hadrons

## Spin-0

$$\Gamma^{a\,\mu\nu}(P,\Delta) = 2M\left[\frac{P^\mu P^\nu}{M}\mathcal{A}^a(t) - \frac{1}{M}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\mathcal{C}^a(t) + g^{\mu\nu}M\bar{\mathcal{C}}^a(t)\right]$$

[Pagels (1966)]
[Donoghue, Leutwyler (1991)]
[Ji (1996)]

## Spin-1/2

$$\Gamma^{a\,\mu\nu} = \frac{P^\mu P^\nu}{M}\mathcal{A}^a(t) + \frac{1}{M}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\mathcal{C}^a(t) + g^{\mu\nu}M\bar{\mathcal{C}}^a(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{4M}\mathcal{G}^a(t)$$

[Kobzarev, Okun (1962)][Pagels (1966)]
[Ji (1996)][Bakker, Leader, Trueman (2004)]
[Leader, Lorcé (2014)]

Non-conserved terms

# EMT for hadrons

## Spin-1

$$
\begin{aligned}
\Gamma^{a\,\mu\nu;\alpha\beta} = & -2P^\mu P^\nu \left[g^{\alpha\beta}\mathcal{G}_1^a(t) - \frac{\Delta^\alpha\Delta^\beta}{2M^2}\mathcal{G}_2^a(t)\right] - \frac{1}{2}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\left[g^{\alpha\beta}\mathcal{G}_3^a(t) - \frac{\Delta^\alpha\Delta^\beta}{2M^2}\mathcal{G}_4^a(t)\right] \\
& + P^{\{\mu}\left(g^{\alpha\nu\}}\Delta^\beta - g^{\beta\nu\}}\Delta^\alpha\right)\mathcal{G}_5^a(t) \\
& + \frac{1}{2}\left[\Delta^{\{\mu}\left(g^{\alpha\nu\}}\Delta^\beta + g^{\beta\nu\}}\Delta^\alpha\right) - g^{\alpha\{\mu}g^{\nu\}\beta}\Delta^2 - g^{\mu\nu}\Delta^\alpha\Delta^\beta\right]\mathcal{G}_6^a(t) \\
& + g^{\alpha\{\mu}g^{\nu\}\beta}M^2\mathcal{G}_7^a(t) + g^{\mu\nu}g^{\alpha\beta}M^2\mathcal{G}_8^a(t) + \frac{1}{2}g^{\mu\nu}\Delta^\alpha\Delta^\beta\mathcal{G}_9^a(t)
\end{aligned}
$$

Non-conserved terms

[Holstein (2006)]
[Abidin, Carlson (2008)]
[Taneja, Kathuria, Liuti, Goldstein (2012)]
[Cosyn, SC, Freese, Lorcé (2019)]
[Polyakov, Sun, (2019)]

# Arbitrary spin

## Vector current [Lorcé 2009]

$$\langle p', \lambda' \mid J_a^\mu(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\theta_a^\mu(P, \Delta)\,\eta_\lambda(p)$$

## Tensor current [in preparation]

$$\langle p', \lambda' \mid T_a^{\mu\nu}(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\Gamma_a^{\mu\nu}(P, \Delta)\,\eta_\lambda(p)$$

# Arbitrary spin

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| | |
|---|---|
| Integer spin | $j$ |

## Tensor current [in preparation]

$$\langle p', \lambda' \mid T_a^{\mu\nu}(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\Gamma_a^{\mu\nu}(P,\Delta)\,\eta_\lambda(p)$$

Polarization tensors $\varepsilon_{\alpha'_1\cdots\alpha'_j}(p,\lambda)$

# Arbitrary spin

## Vector current [Lorcé 2009]

$$\langle p', \lambda' \mid J^\mu_a(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\theta^\mu_a(P,\Delta)\,\eta_\lambda(p)$$

Integer spin $j$

$$\theta^\mu_a(P,\Delta) = P^\mu \sum_{(k,j)} F_{2k+1}(t) + \left(g^{\mu\alpha_j}\Delta^{\alpha'_j} - g^{\mu\alpha'_j}\Delta^{\alpha_j}\right) \sum_{(k,j-1)} F_{2k+2}(t)$$

Sum:

$$\sum_{(k,j)} \equiv \sum_{k=0}^{j}\left[\prod_{i=1}^{k}\Delta^{\alpha'_i}\Delta^{\alpha_i}\prod_{i=k+1}^{j}g^{\alpha'_i\alpha_i}\right]$$

## Tensor current [in preparation]

$$\langle p', \lambda' \mid T^{\mu\nu}_a(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\Gamma^{\mu\nu}_a(P,\Delta)\,\eta_\lambda(p)$$

Polarization tensors $\varepsilon_{\alpha'_1\cdots\alpha'_j}(p,\lambda)$

$$\begin{aligned}\Gamma^\mu_a(P,\Delta) =& P^{\{\mu}P^{\nu\}} \sum_{(k,j)} F_{2k+1}(t)\\ &+ \left(\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}\right)\sum_{(k,j)} F_{2k+2}(t)\\ &+ P^{\{\mu}\left(g^{\nu\}\alpha_j}\Delta^{\alpha'_j} - g^{\nu\}\alpha'_j}\Delta^{\alpha_j}\right)\sum_{(k,j-1)} F_{2k+3}(t)\\ &\left[\Delta^{\{\mu}(g^{\nu\}\alpha_j}\Delta^{\alpha'_j} + g^{\nu\}\alpha'_j}\Delta^{\alpha_j} - \Delta^2 g^{\{\mu\alpha_j}g^{\nu\}\alpha'_j}\right.\\ &\left.- g^{\mu\nu}\Delta^{\alpha_j}\Delta^{\alpha'_j})\right]\sum_{(k,j-1)} F_{2k+4}(t)\end{aligned}$$

+ nonconserved terms

# Arbitrary spin

## Vector current [Lorcé 2009]

$$\langle p', \lambda' \mid J^\mu_a(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\theta^\mu_a(P,\Delta)\,\eta_\lambda(p)$$

Half-integer spin $j = n + \frac{1}{2}$

n integer

$$\theta^\mu_a(P,\Delta) = P^\mu \sum_{(k,n)} F_{2k+1}(t) + i\sigma^{\mu\rho}\Delta_\rho \sum_{(k,n)} F_{2k+2}(t)$$

Sum:

$$\sum_{(k,j)} \equiv \sum_{k=0}^{j}\left[\prod_{i=1}^{k}\Delta^{\alpha'_i}\Delta^{\alpha_i}\prod_{i=k+1}^{j} g^{\alpha'_i\alpha_i}\right]$$

## Tensor current [in preparation]

$$\langle p', \lambda' \mid T^{\mu\nu}_a(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p')\Gamma^{\mu\nu}_a(P,\Delta)\,\eta_\lambda(p)$$

Generalized polarization tensor $u_{\alpha'_1\cdots\alpha'_n}(p,\lambda)$

$$\begin{aligned}\Gamma^\mu_a(P,\Delta) = & P^{\{\mu}P^{\nu\}} \sum_{(k,n)} F_{2k+1}(t) \\ & + (\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}) \sum_{(k,n)} F_{2k+2}(t) \\ & + P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho \sum_{(k,n)} F_{2k+3}(t) \\ & \Big[\Delta^{\{\mu}(g^{\nu\}\alpha_n}\Delta^{\alpha'_n} + g^{\nu\}\alpha'_n}\Delta^{\alpha_n} - \Delta^2 g^{\{\mu\alpha_n}g^{\nu\}\alpha'_n} \\ & - g^{\mu\nu}\Delta^{\alpha_n}\Delta^{\alpha'_n})\Big] \sum_{(k,n-1)} F_{2k+4}(t)\end{aligned}$$

+ nonconserved terms

# EMT to order O(Δ)

Terms with a dependence at most linear in Δ…

$$\Gamma^{\mu}_{a}(P,\Delta) = P^{\{\mu}P^{\nu\}} \sum_{(k,n)} F_{2k+1}(t) + (\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}) \sum_{(k,n)} F_{2k+2}(t) + P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_{\rho} \sum_{(k,n)} F_{2k+3}(t) \left[\Delta^{\{\mu}(g^{\nu\}\alpha_n}\Delta^{\alpha'_n} + g^{\nu\}\alpha'_n}\Delta^{\alpha_n} - \Delta^2 g^{\{\mu\alpha_n}g^{\nu\}\alpha'_n} - g^{\mu\nu}\Delta^{\alpha_n}\Delta^{\alpha'_n})\right] \sum_{(k,n-1)} F_{2k+4}(t)$$

$$\Gamma^{\mu}_{a}(P,\Delta) = P^{\{\mu}P^{\nu\}} \sum_{(k,j)} F_{2k+1}(t) + (\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}) \sum_{(k,j)} F_{2k+2}(t) + P^{\{\mu}(g^{\nu\}\alpha_j}\Delta^{\alpha'_j} - g^{\nu\}\alpha'_j}\Delta^{\alpha_j}) \sum_{(k,j-1)} F_{2k+3}(t) \left[\Delta^{\{\mu}(g^{\nu\}\alpha_j}\Delta^{\alpha'_j} + g^{\nu\}\alpha'_j}\Delta^{\alpha_j} - \Delta^2 g^{\{\mu\alpha_j}g^{\nu\}\alpha'_j} - g^{\mu\nu}\Delta^{\alpha_j}\Delta^{\alpha'_j})\right] \sum_{(k,j-1)} F_{2k+4}(t)$$

# EMT to order O(Δ)

Terms with a dependence at most linear in Δ…

$$\Gamma^{\mu}_{a}(P,\Delta) = P^{\{\mu}P^{\nu\}}\sum_{(k,n)}F_{2k+1}(t)$$
$$+\left(\Delta^{\mu}\Delta^{\nu} - \Delta^{2}g^{\mu\nu}\right)\sum_{(k,n)}F_{2k+2}(t)$$
$$+P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_{\rho}\sum_{(k,n)}F_{2k+3}(t)$$
$$\left[\Delta^{\{\mu}(g^{\nu\}\alpha_n}\Delta^{\alpha'_n} + g^{\nu\}\alpha'_n}\Delta^{\alpha_n} - \Delta^{2}g^{\{\mu\alpha_n}g^{\nu\}\alpha'_n} - g^{\mu\nu}\Delta^{\alpha_n}\Delta^{\alpha'_n})\right]\sum_{(k,n-1)}F_{2k+4}(t)$$

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# EMT to order O(Δ)

Terms with a dependence at most linear in Δ…

$$
\begin{aligned}
\Gamma^{\mu}_{a}(P,\Delta) =\, & P^{\{\mu}P^{\nu\}} \sum_{(k,n)} F_{2k+1}(t) \\
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& \Big[\Delta^{\{\mu}(g^{\nu\}\alpha_j}\Delta^{\alpha'_j} + g^{\nu\}\alpha'_j}\Delta^{\alpha_j} - \Delta^{2} g^{\{\mu\alpha_j} g^{\nu\}\alpha'_j} \\
& - g^{\mu\nu}\Delta^{\alpha_j}\Delta^{\alpha'_j})\Big] \sum_{(k,j-1)} F_{2k+4}(t)
\end{aligned}
$$

Always related to the Lorentz generators in the chosen spin representation!

# Universal EMT decomposition

If we restrict ourselves to the terms O(Δ), the EMT for ANY spin reads:

[Boulware, Deser (1975)]
[SC, Lorcé, Lowdon, (2019)]
[Lorcé,Lowdon arXiv:1908.02567}

$$\langle p', \lambda' \mid T^{a\,\mu\nu}(0) \mid p, \lambda\rangle = \bar{\eta}_{\lambda'}(p') \left( P^{\{\mu}P^{\nu\}} \mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho \mathcal{G}^a(t) + \cdots \right) \eta_\lambda(p)$$

Lorentz generator in the given spin representation, e.g.:

| | Spin-0 | Spin-1/2 | Spin-1 | Spin-3/2 | Spin-2 |
|---|---|---|---|---|---|
| $\Sigma^{\nu\rho}$ | $0$ | $\frac{i}{4}[\gamma^\nu, \gamma^\rho]$ | $ig^{[\nu\alpha}g^{\rho]\beta}$ | $ig^{[\nu\alpha}g^{\rho]\beta} + \sigma^{\nu\rho}g^{\alpha\beta}$ | $ig^{[\nu\alpha_1}g^{\rho]\beta_2}g^{\alpha_2\beta_2} + (1\to 2)$ |

# Constraints from Poincaré symmetry

**See Peter Lowdon's talk**

**[Lowdon, Chiu, Brodsky (2017)]**

**[SC, Lorcé, Lowdon,(2019)]**

$$J^i \propto \epsilon^{ijk} \int d^4x \, \left[x^j T^{0k}(x) - x^k T^{0j}(x)\right]$$

Angular momentum

$$K^i \propto \int d^4x \, \left[x^0 T^{0i}(x) - x^i T^{00}(x)\right]$$

Boost

$$P^\mu \propto \int d^4x \, T^{0\mu}(x)$$

Momentum

# Constraints from Poincaré symmetry

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$$J^i \propto \epsilon^{ijk} \int d^4x \, \left[x^j T^{0k}(x) - x^k T^{0j}(x)\right]$$ Angular momentum

$$K^i \propto \int d^4x \, \left[x^0 T^{0i}(x) - x^i T^{00}(x)\right]$$ Boost

$$P^\mu \propto \int d^4x \, T^{0\mu}(x)$$ Momentum

Poincaré symmetry puts univocally constraints on: $\left(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho \mathcal{G}^a(t) + \cdots\right)$

$$\mathcal{G}(t) = A(t) + B(t)$$

$$\mathcal{A}(0) = \mathcal{G}(0) = 1$$

Massive on-shell states (**SC,Lorcé,Lowdon, arXiv:1905.11969**)
All on-shell states (**Lorcé,Lowdon arXiv:1908.02567**)

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$$P^\mu \propto \int d^4x \, T^{0\mu}(x)$$ Momentum

Poincaré symmetry puts univocally constraints on: $\left(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho \mathcal{G}^a(t) + \cdots\right)$

$\mathcal{G}(t) = A(t) + B(t)$

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Independent on the spin!

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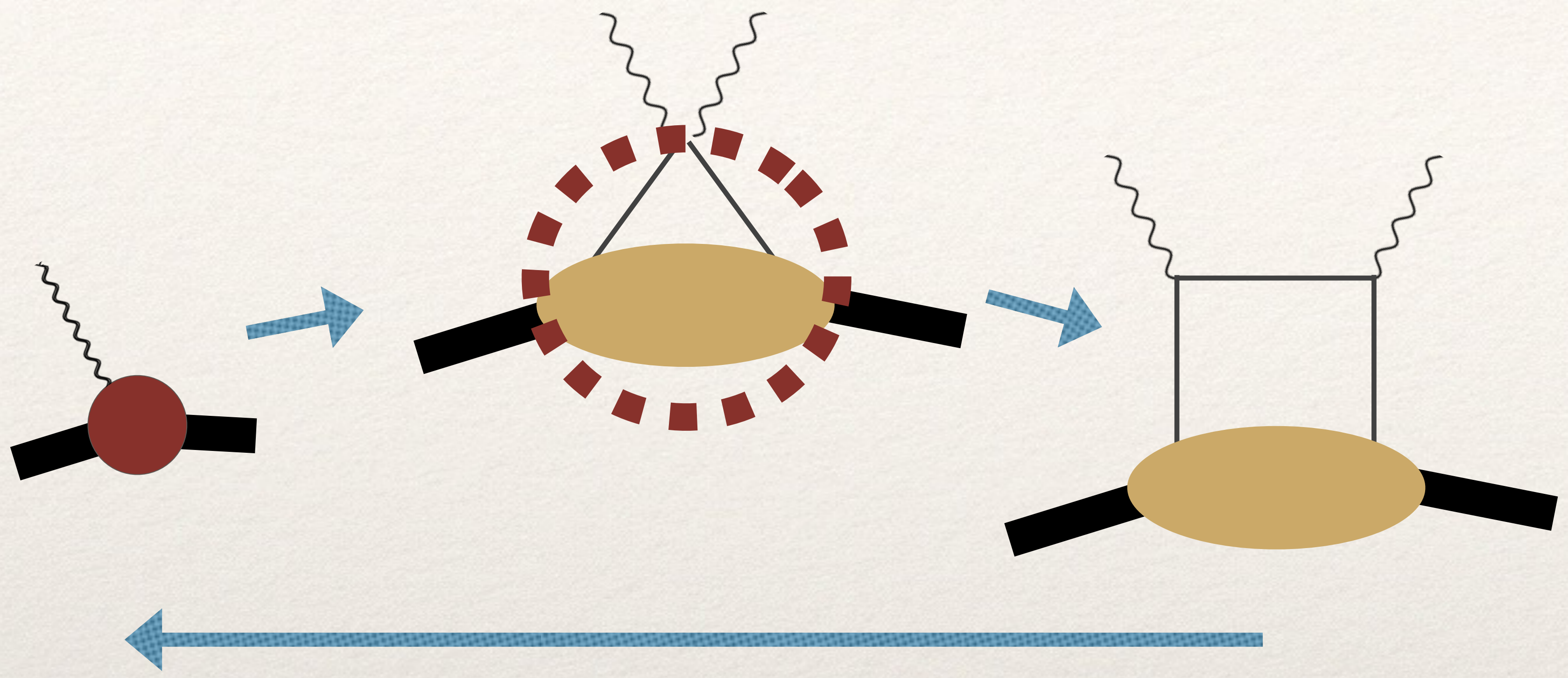
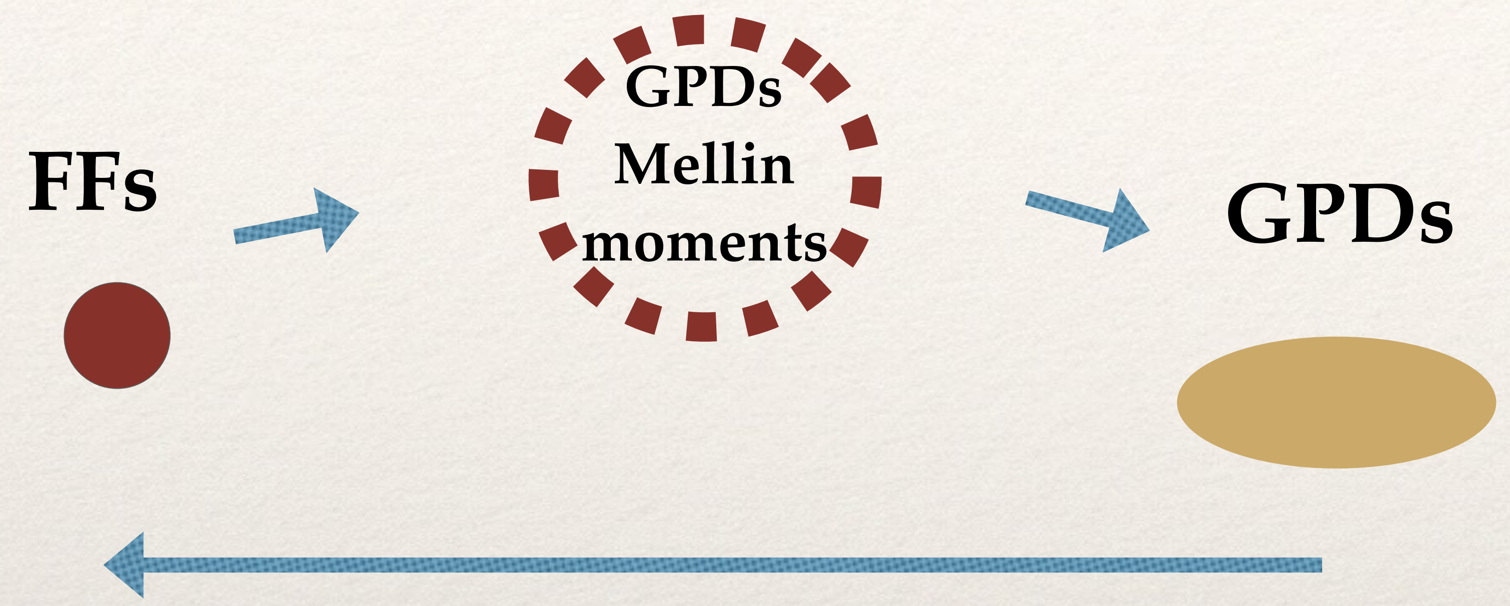
# Gravitational Form Factors and GPDs



The gravitational Form Factors are related to the Mellin moments of the GPD matrix element!

# Gravitational Form Factors and GPDs




The gravitational Form Factors are related to the Mellin moments of the GPD matrix element!

- GPD matrix element for quarks (for simplicity)

$$\langle p';\lambda'|\mathcal{O}^{\mu}_{qV}|p;\lambda\rangle = \frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{2\pi}e^{i(Pn)zx}\left\langle p';\lambda'\middle|\overline{\psi}\left(-\frac{nz}{2}\right)\gamma^{\mu}\mathcal{W}_{[-\frac{nz}{2},\frac{nz}{2}]}\psi\left(\frac{nz}{2}\right)\middle|p;\lambda\right\rangle$$

- Second Mellin moments of GPDs

$$\int_{-1}^{1}\mathrm{d}x\,x\,\mathcal{O}^{\mu}_{qV} = \frac{1}{4(Pn)^2}\overline{\psi}(0)\gamma^{\mu}(i\overleftrightarrow{D}\,n)\psi(0)$$

- Relation with the EMT

$$\int_{-1}^{1}\mathrm{d}x\,x\,\mathcal{O}^{\mu}_{qV} = \frac{T^{\mu n}_{q}}{2(Pn)^2}$$

# Ji's sum rule

Twist-2 GPD parametrization for any spin up to order O(Δ) **[SC, Lorcé, Lowdon,(2019)]**

$$\langle p', \lambda' | \mathcal{O}^n | p, \lambda \rangle = \bar{\eta}_{\lambda'}(p') \left[ H_1(x,\xi,t) + i \frac{\Sigma^{n\rho} \Delta_\rho}{\bar{p}n} H_2(x,\xi,t) + \cdots \right] \eta_\lambda(p)$$

$$\int_{-1}^{1} dx\, x H_1(x,\xi,t) = A(t) + \cdots\,,$$

$$\int_{-1}^{1} dx\, x H_2(x,\xi,t) = G(t) + \cdots\,.$$

terms depending on higher powers of Δ

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$$\int_{-1}^{1} dx\, x H_1(x,\xi,t) = A(t) + \cdots ,$$

$$\int_{-1}^{1} dx\, x H_2(x,\xi,t) = G(t) + \cdots .$$

terms depending on higher powers of Δ

$$P^z = \sum_{a=q,g} \int_{-1}^{1} dx\, x H_1^a(x,0,0) = A(0) = 1,$$

$$J^z = \sum_{a=q,g} \int_{-1}^{1} dx\, x H_2^a(x,0,0) = G(0) = 1,$$

- The FFs $\mathcal{A}(0)$, $\mathcal{G}(0)$ are ultimately related to the mass and the spin;
- No need for explicitly treating the EMT spin by spin to obtain the Ji's sum rule for all spin

Independent on the spin!

# Spin-multipoles expansion

Let's complete the expansion…

$$\Big(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho\mathcal{G}^a(t) + \cdots\Big)$$

This can be seen as an expansion in spin-multipoles, built from the Lorentz generators.

# Spin-multipoles expansion

Let's complete the expansion…

$$\left( P^{\{\mu} P^{\nu\}} \mathcal{A}^a(t) + P^{\{\mu} \Sigma^{\nu\}\rho} \Delta_\rho \mathcal{G}^a(t) + \cdots \right)$$

This can be seen as an expansion in spin-multipoles, built from the Lorentz generators.

$\mathcal{I}^{\mu\nu} = g^{\mu\nu}$ **Monopole**

$\mathcal{D}^{\mu\nu} = \Sigma^{\mu\nu}$ **Antisymm Dipole**

$\mathcal{Q}^{\mu\nu,\rho\sigma} = \{\Sigma^{\mu\nu}, \Sigma^{\rho\sigma}\} - \text{Trace}$ **Symm Traceless Quadrupole**

$\ldots$

$$D^\mu = \mathcal{D}^{\mu\nu}\Delta_\nu \qquad Q^{\mu\nu} = g_{\lambda\tau}\mathcal{Q}^{\mu\lambda,\tau\nu} \qquad Q^{\mu\nu}_\Delta = \Delta_\lambda \Delta_\tau \mathcal{Q}^{\mu\lambda,\tau\nu}$$

# Spin-1 revisited

[Cosyn, SC, Freese, Lorcé (2019)]

$$I^{\mu\nu} = \frac{1}{4} g^{\mu\nu} g^{\alpha\beta}$$

$$D^{\mu} = \frac{1}{2}(g^{\mu\alpha}\Delta^{\beta} - \Delta^{\alpha} g^{\mu\beta})$$

$$Q^{\mu\nu} = \frac{1}{2}(g^{\mu\alpha} g^{\nu\beta} + g^{\nu\alpha} g^{\mu\beta}) - \frac{1}{4} g^{\mu\nu} g^{\alpha\beta}$$

The 9 FF of spin-1 corresponds to the structures:

$$\left.\begin{array}{l} I^{\mu\nu},\ P^{\mu} I^{\nu P}, \Delta^{\mu} I^{\nu\Delta} \\ P^{\{\mu} D^{\nu\}} \\ Q^{\mu\nu},\ \Delta^{\{\mu} Q^{\nu\}\Delta},\ \eta^{\mu\nu} Q^{\Delta\Delta},\ P^{\mu} P^{\nu} Q^{\Delta\Delta},\ \Delta^{\mu}\Delta^{\nu} Q^{\Delta\Delta} \end{array}\right\} \text{ linear combination of } \begin{array}{c} \mathcal{G}_1(t) \\ \vdots \\ \mathcal{G}_9(t) \end{array}$$

All the other possible multipoles are not independent on the previous ones

# Higher spins - Ongoing…

[SC, Lorcé, Lowdon, Morales, in preparation]

The procedure can be generalized for higher spin and (in principle) higher-rank operators

Multipoles $\Sigma\Sigma\Sigma\cdots$

$\#$ **of (conserved) FFs**

**Monopole** **Dipole** **Quadrupole** **Octupole** …

Rank of the operator $\mathcal{O}^{\mu\nu\cdots}$

$\mathcal{O}^{\mu}$

$\mathcal{O}^{\mu\nu}$

$\vdots$

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Multipoles $\Sigma\Sigma\Sigma\cdots$

**# of (conserved) FFs**

Rank of the operator $\mathcal{O}^{\mu\nu\cdots}$

| | Monopole | Dipole | Quadrupole | Octupole | … |
|---|---|---|---|---|---|
| $\mathcal{O}^{\mu}$ | | | | | |
| $\mathcal{O}^{\mu\nu}$ | Spin-0 | Spin-1/2 | Spin-1 | Spin-3/2 | etc… |
| $\vdots$ | | | | | |

# Higher spins - Ongoing…

[SC, Lorcé, Lowdon, Morales, in preparation]

The procedure can be generalized for higher spin and (in principle) higher-rank operators

Multipoles $\Sigma\Sigma\Sigma\cdots$

# of (conserved) FFs

Rank of the operator $\mathcal{O}^{\mu\nu\cdots}$

| | Monopole | Dipole | Quadrupole | Octupole | … | # of (conserved) FFs |
|---|---|---|---|---|---|---|
| $\mathcal{O}^{\mu}$ | | | | | | $2j+1$ |
| $\mathcal{O}^{\mu\nu}$ | Spin-0 | Spin-1/2 | Spin-1 | Spin-3/2 | etc… | ✔ * |
| $\vdots$ | | | | | | |


*same counting for integer spins as obtained by Polyakov et al.

# Summary and Outlook

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- Exclusive processes give (indirect) access to gravitational FFs
- Some FFs in the parametrization of the EMT are constrained by Poincaré symmetry in a spin-universal way for all on-shell states
- Characterizing the EMT for higher spins is useful to understand how the number of structures grow with spin.
- The parametrization of the EMT for arbitrary spin allows to derive relations and sum rules that are valid independently of the spin
- Such universal relations can be tested in experiments involving different hadrons

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## Thank you!

# Comments

- Universal, nonperturbative procedure to derive momentum and angular momentum sum rules for any state (massive states in **SC,Lorcé,Lowdon, arXiv:1905.11969**, extension to all on-shell states in **Lorcé,Lowdon arXiv: 1908.02567**)
- No need for explicitly treating the EMT spin by spin to obtain the Ji's sum rule for all spin
- The relation $\mathcal{A}(0) = \mathcal{G}(0) = 1$ automatically implies that, when defining $\mathcal{G}(t) = A(t) + B(t)$, then B(0) identically vanishes for all spins.
- Poincaré generators are related to the first moment of the EMT, thus they constraint structures at most linear in Δ.

**More in Peter Lowdon's talk**