Form factors and structure functions of heavy mesons and baryons

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Light Cone 2019, Ecole Polytechnique, Palaiseau, France,
16th of September 2019

Renormalization group procedure

Canonical Hamiltonian H_{can}

$\downarrow$

Regularization

$\downarrow$

$H^R = H^R_{\text{can}} + X$ $\xrightarrow{\text{RGPEP equations}}$ Renormalized Hamiltonian H_s

(s has an interpretation of the size of effective particles)

$\downarrow$

Are all matrix elements $\langle i|H_s|j\rangle$ UV-finite? $\xrightarrow{\text{No}}$ Add counterterms X $\uparrow$ (back to $H^R = H^R_{\text{can}} + X$)

$\downarrow$ Yes

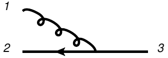
Compute observables and fix finite parts of counterterms

Choose proper renormalization group scale parameter s

Renormalized Hamiltonian

$$H_s = H_{s\,0} + g_s H_{s\,1} + g_s^2 H_{s\,2} + \dots$$

- $H_{s\,1}$, for example:

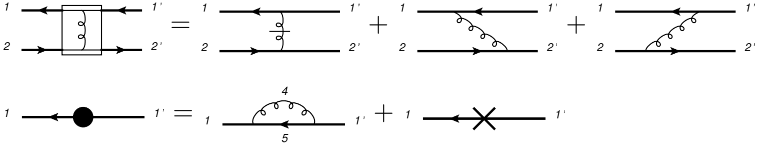


$$= r_{21.3}\; f_{21.3}\; \tilde{\delta}_{21.3}\; \bar{u}_2 \not{\epsilon}_1 u_3\; t^1_{23}$$

- Interaction form factors in vertices!

$$f_{21.3} = e^{-s^4(\mathcal{M}_{21} - m_3^2)^2}$$

- $H_{s\,2}$,



Heavy quarks allow for several simplifications

- We choose s such that

$$0.9 \text{ fm} \approx \frac{1}{\Lambda_{\text{QCD}}} \gg s \gtrsim \frac{1}{m_Q} \approx 0.05 \text{ fm (bottom quark)}$$

- For $s \ll 1/\Lambda_{\text{QCD}}$ effective coupling constant is small in accordance with asymptotic freedom, while choosing $s \gtrsim 1/m_Q$ we can neglect Fock sectors with extra $Q_s\bar{Q}_s$ pairs:

$$H_s|\psi_s\rangle = P^-|\psi_s\rangle$$

$$|\psi_s\rangle = \begin{bmatrix} \dots \\ \cancel{|Q_s\bar{Q}_s\, Q_s\bar{Q}_s\rangle} \\ \dots \\ |Q_s\bar{Q}_s\, 3G_s\rangle \\ |Q_s\bar{Q}_s\, 2G_s\rangle \\ |Q_s\bar{Q}_s\, G_s\rangle \\ |Q_s\bar{Q}_s\rangle \end{bmatrix}$$

- Gluons, however, still pose a problem, bacause they are massless.

Gluon mass ansatz

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & H_{s\,0} + g_s^2 H_{s\,2} & g_s H_{s\,1} \\ \cdot & \cdot & g_s H_{s\,1} & H_{s\,0} + g_s^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ |Q_s \bar{Q}_s\, G_s\rangle \\ |Q_s \bar{Q}_s\rangle \end{bmatrix} = P^- \begin{bmatrix} \cdot \\ \cdot \\ |Q_s \bar{Q}_s\, G_s\rangle \\ |Q_s \bar{Q}_s\rangle \end{bmatrix}$$

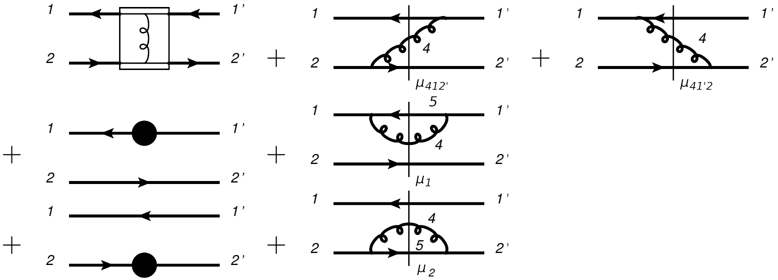
$$\downarrow$$

$$\begin{bmatrix} H_{s\,0} + \boxed{\mu_s^2} & g H_{s\,1} \\ g H_{s\,1} & H_{s\,0} + g^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} |Q_s \bar{Q}_s\, G_s\rangle \\ |Q_s \bar{Q}_s\rangle \end{bmatrix} = P^- \begin{bmatrix} |Q_s \bar{Q}_s\, G_s\rangle \\ |Q_s \bar{Q}_s\rangle \end{bmatrix}$$

$\downarrow$ integrate out perturbatively the higher sector $\downarrow$

$$H_{\mathrm{eff}\ s}|Q_s \bar{Q}_s\rangle = \frac{M^2 + P^{\perp 2}}{P^+}|Q_s \bar{Q}_s\rangle \ ,$$

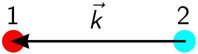
Effective interactions (mesons)



In the nonrelativistic limit we obtain potential between effective heavy quarks:

$$V(r) = \text{Smeared}\left(-\frac{C_F\alpha}{r}\right) + \frac{1}{2}\mu_{12}\omega^2 r^2$$

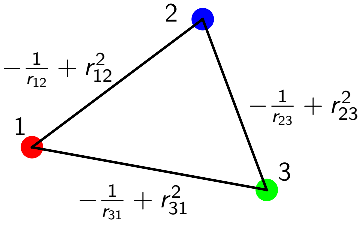
$$\omega = \sqrt{\frac{\alpha}{18\sqrt{\pi}\,\mu_{12}}\left(\frac{\lambda^2}{\sqrt{m_1^2+m_2^2}}\right)^3}$$



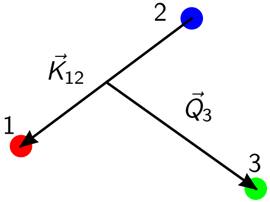
$$\mu_{12} = \frac{m_1 m_2}{m_1 + m_2}$$

Effective interactions (baryons)

Three Coulomb potentials and three harmonic oscillator potentials.



In relative variables

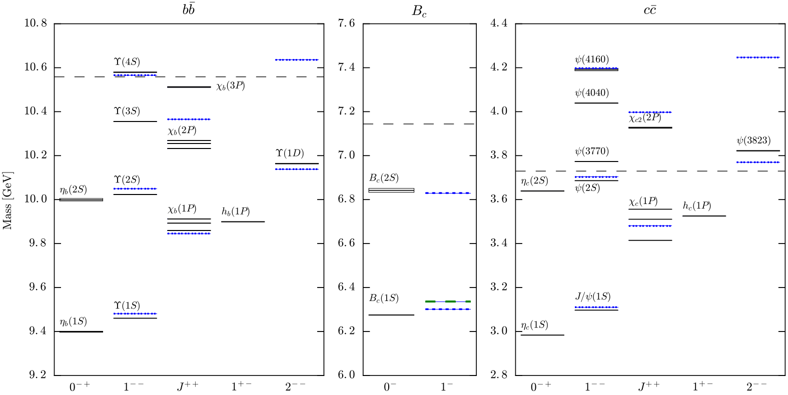


three Coulomb potentials and two collective harmonic oscillators with frequencies ω_{12} and $\omega_{3(12)}$.

$$\omega_{12}^2 = \frac{1}{m_1}\frac{\alpha\lambda^3}{18\sqrt{\pi}}\left[\left(\frac{\lambda^2}{2m_1^2}\right)^{3/2} + \frac{1}{2}\left(\frac{\lambda^2}{m_1^2+m_3^2}\right)^{3/2}\right], \quad \lambda = 1/s$$

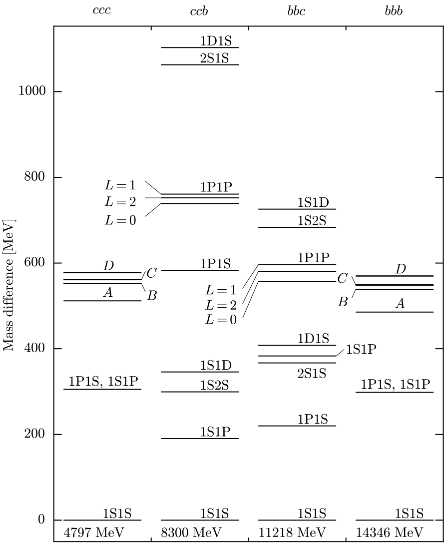
$$\omega_{3(12)}^2 = \frac{2m_1+m_3}{2m_1 m_3}\frac{\alpha\lambda^3}{18\sqrt{\pi}}\left(\frac{\lambda^2}{m_1^2+m_3^2}\right)^{3/2}.$$

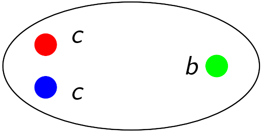
Meson spectra used for fitting, Coulomb as a perturbation

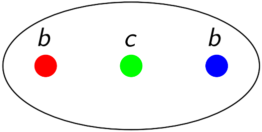


Dotted blue: our masses (K. Serafin, M. Gómez-Rocha, J. More, S. Głazek, EPJC 78, 964).
Solid black: PDG masses.
Dashed green: Gómez-Rocha, Hilger, Krassnigg, PRD93 074010 (2016)

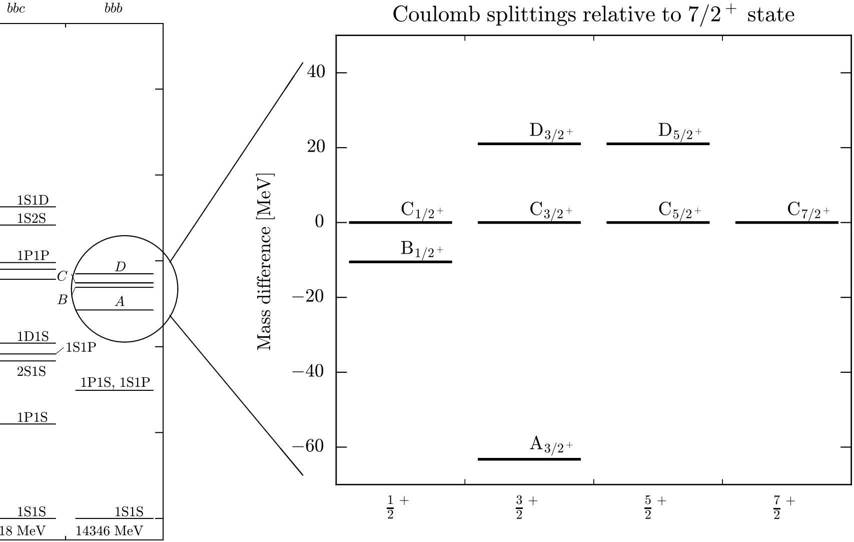
Baryon spectra







Splittings of the second band of harmonic oscillator

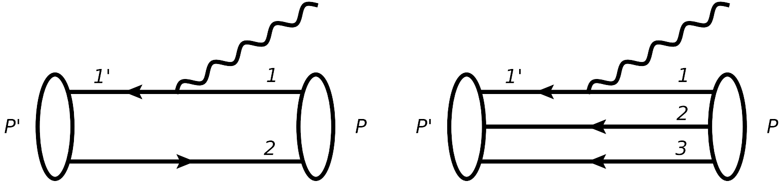


Summary of spectra

- Ground states of baryons are in the ballpark of expectations except for Ω_{ccb}.
- Qualitative agreement with Lattice splittings (S. Meinel, PRD85, 114510).
- Problem with Σ term.
- Mixed-flavor systems may need explicit inclusion of two scales.

Form factors, just the very basics

$$J^{\mu}_{\sigma'\sigma}(P',P) \quad = \quad \langle P',\sigma'|\ \hat{J}^{\mu}(0)\ |P,\sigma\rangle\ ,$$



$$\begin{aligned}
\text{Spin 0} \quad J^{\mu} &= (P^{\mu}+P'^{\mu})\ F(Q^2)\ , \\
\text{Spin } \frac{1}{2} \quad J^{\mu}_{\sigma'\sigma} &= \bar{u}_{\sigma'}(P')\left[\gamma^{\mu}F_1(Q^2)+\frac{i\sigma^{\mu\nu}q_{\nu}}{2M}F_2(Q^2)\right]u_{\sigma}(P)\ , \\
\text{Spin 1} \quad J^{\mu}_{\sigma'\sigma} &= -(P^{\mu}+P'^{\mu})(\varepsilon'^{*}\cdot\varepsilon)F_1(Q^2) \\
&+ \left[\varepsilon'^{\mu *}(\varepsilon\cdot q)-\varepsilon^{\mu}(\varepsilon'^{*}\cdot q)\right]F_2(Q^2) \\
&+ (P^{\mu}+P'^{\mu})\frac{(\varepsilon\cdot q)(\varepsilon'^{*}\cdot q)}{2M^2}F_3(Q^2)\ ,
\end{aligned}$$

Relativistic corrections to relative motion of quarks

Mesons

$$\eta(nS): \quad \psi_{\sigma_1\sigma_2}(\vec{k}) = \mathcal{N}_{PS}\ \bar{u}_1\gamma^5 v_2\ \psi_{nS}(\vec{k})$$
$$= \psi_{nS}(\vec{k})\left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} - \frac{k^1 - ik^2}{2\sqrt{2}\mu_{12}}|\uparrow\uparrow\rangle - \frac{k^1 + ik^2}{2\sqrt{2}\mu_{12}}|\downarrow\downarrow\rangle + \ldots\right.$$
$$\chi_0(1P): \quad \psi^{\sigma}_{\sigma_1\sigma_2}(\vec{k}) = \mathcal{N}_S \bar{u}_1 v_2\ e^{-\nu\vec{k}^2}\ ,$$
$$\Upsilon(nS): \quad \psi^{\sigma}_{\sigma_1\sigma_2}(\vec{k}) = \mathcal{N}_V\ \bar{u}_1\gamma^\mu v_2\varepsilon_{\sigma\mu}\ \psi_{nS}(\vec{k})\ ,$$

Ground states of baryons, $\Omega_{QQQ'}(1S1S)$,

$$\psi^{\sigma}_{\sigma_1\sigma_2\sigma_3}(\vec{K}_{12},\vec{Q}_3) = \mathcal{N}\left(\bar{u}_1\gamma^\mu C\bar{u}_2^T\right)\ \bar{u}_3\gamma_\mu\gamma^5 u_{M_{123}}(P,\sigma)\ \psi_{1S1S}(\vec{K}_{12},\vec{Q}_3)\ ,$$

Summary of charge radii (mesons)

$c\bar{c}$					
	$\eta_c(1S)$	$\chi_{c0}(1P)$	$\eta_c(2S)$	J/ψ	$\psi(2S)$
$\sqrt{r_1^2}$ [fm]	0.249	0.322	0.381	0.257	0.385
BLFQ, Adhikari 2018	0.207	0.265	0.386	0.212	0.387
CI, Raya 2017	0.210			0.261	
Lattice, Dudek 2006	0.251	0.308		0.257	
DSE, Bhagwat 2006	0.219			0.228	
$b\bar{b}$					
	$\eta_b(1S)$	$\chi_{b0}(1P)$	$\eta_b(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$
$\sqrt{r_1^2}$ [fm]	0.1521	0.1963	0.2323	0.1535	0.2331
BLFQ, Adhikari 2018	0.126	0.192	0.237	0.126	0.239
CI, Raya 2017	0.110			0.195	
$c\bar{b}$					
	$B_c(1S)$	$\chi_{bc0}(1P)$	$B_c(2S)$	$B_c^*(1S)$	$B_c^*(2S)$
$\sqrt{r_c^2}$ [fm]	0.337	0.435	0.515	0.342	0.516
$\sqrt{r_b^2}$ [fm]	0.105	0.136	0.160	0.106	0.161
$\sqrt{r^2}$ [fm]	0.282	0.364	0.430	0.286	0.433

Summary of charge radii (baryons)

	Ω_{ccc}	Ω_{ccb}	Ω_{bbc}	Ω_{bbb}
$\sqrt{r_c^2}$ [fm]	0.31	0.35	0.32	
$\sqrt{r_b^2}$ [fm]		0.18	0.20	0.19
$\sqrt{r^2}$ [fm]	0.31	0.39	0.20	0.19
Lattice, Can 2015, $\sqrt{\langle r_E^2 \rangle_c}$ [fm]	0.29			

Table: Summary of charge radii of baryons.

	μ_1	BLFQ, Adhikari 2018	CI, Raya 2017	Lattice, Dudek 2006	DSE, Bhagwat 2006
J/ψ	2 ± 0.13	1.952(3)	2.047	2.10(3)	2.13(4)
$\psi(2S)$	2 ± 0.54	2.05(2)			
$\Upsilon(1S)$	2 ± 0.02	1.985(1)	2.012		
$\Upsilon(2S)$	2 ± 0.14	1.992(1)			

Table: Summary of magnetic dipole moments of charmonia and bottomonia and comparison with some results available in literature. My estimation of error is $\mu_1 \cdot (M - M_{12})/M_{12}$.

	μ	Lahde 2003	Dhir 2013	Faessler 2006	Simonis 2016, 2018
$B_c^{+*}(1S)$	3.25 ± 0.07	2.88			2.57
$B_c^{+*}(2S)$	3.24 ± 0.35	2.65			
Ω_{ccb}	5.16 ± 0.46		4.49, 4.62	4.69	4.03
Ω_{bbc}	-2.77 ± 0.09		-2.45, -2.39	-2.39	-2.24

Table: Summary of magnetic dipole moments of $c\bar{b}$ particles and baryons and comparison with some results found in literature. My estimation of error is $\mu \cdot (M - M_{12})/M_{12}$ for mesons and $\mu \cdot (M - M_{123})/M_{123}$ for baryons.

Structure functions

Hadronic tensor is,

$$W^{\mu\nu} = \frac{1}{2s+1}\sum_{\sigma=-s}^{s}\frac{1}{4\pi}\int d^4 z e^{iqz}\langle P,\sigma|\left[\hat{J}^{\mu}(z),\hat{J}^{\nu}(0)\right]|P,\sigma\rangle \ ,$$

where s is the spin of the hadron.

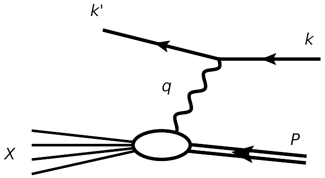
$$W^{\mu\nu} = -\left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^2}\right)W_1 + \left(P^{\mu} - \frac{P\cdot q}{q^2}q^{\mu}\right)\left(P^{\nu} - \frac{P\cdot q}{q^2}q^{\nu}\right)\frac{W_2}{M^2}$$

In the Bjorken limit,

$$F_1(x) = \lim_{Q^2\to\infty,\ \nu\to\infty} W_1 \ ,$$

$$F_2(x) = \lim_{Q^2\to\infty,\ \nu\to\infty} \frac{\nu}{M}W_2 \ ,$$

where $x = Q^2/2P\cdot q$, $\nu = P\cdot q/M$.



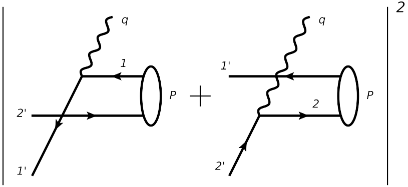
Structure functions

Hadronic tensor rewritten:

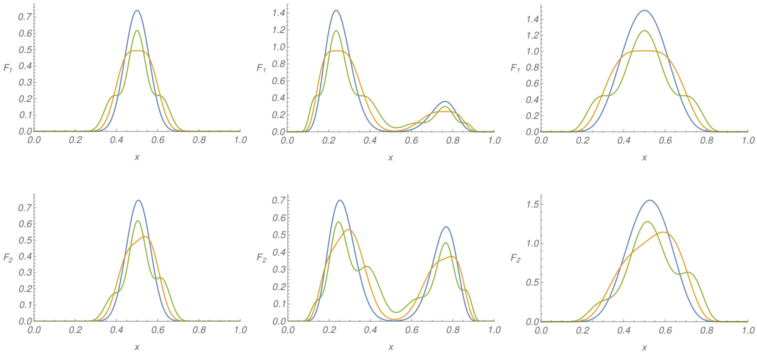
$$W^{\mu\nu} = \frac{1}{2s+1}\sum_{\sigma=-s}^{s}\frac{1}{4\pi} \times \sum_X (2\pi)^4\delta^{(4)}(P+q-P_X)\langle P,\sigma|\hat{J}^\mu(0)|X\rangle\langle X|\hat{J}^\nu(0)|P,\sigma\rangle$$

Sum over hadronic final states is replaced with the sum over quark final states,

$$\sum_X |X\rangle\langle X|\ \delta^{(4)}(P+q-P_X) \to \int_{1'2'} b^\dagger_{1'}d^\dagger_{2'}|0\rangle\langle 0|d_{2'}b_{1'}\ \delta^{(4)}(P+q-p_{1'}-p_{2'})$$

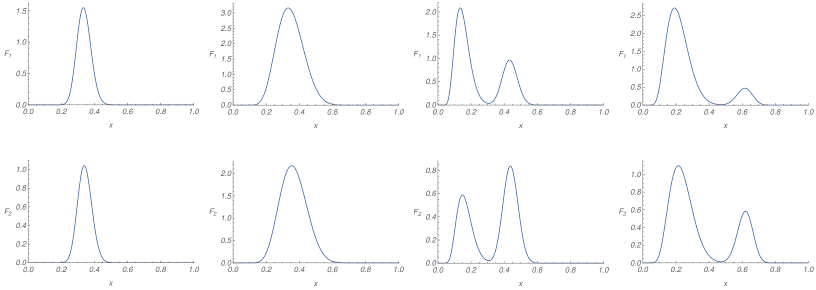


For W^{++}, W^{+i}, W^{i+}, and W^{ij} components the desired relativistic form of W follows from the calculation with structure functions F_1 and F_2 that fulfill Callan-Gross relation.



(a) (b) (c)

Figure: Structure functions $F_1(x)$ and $F_2(x)$ for (a) bottomonia, (b) B_c particles, (c) charmonia. On each plot, the highest curve (blue) represents the $1S$ state, the curve with the widest top (orange) represents the $1P$ state, and the curve with steps (green) represents the $2S$ state.

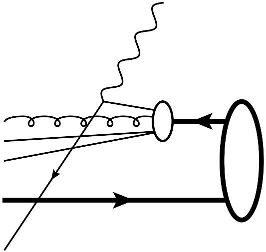


(a) (b) (c) (d)

Figure: Structure functions $F_1(x)$ and $F_2(x)$ for (a) *bbb*, (b) *ccc*, (c) *bbc*, and (d) *ccb* particles.

Scaling violations

Two vastly different momentum scales – scale of binding and scale of the probing photon. Particles of different sizes in RGPEP are related through a unitary operator. So far I appoximated this operator by a unity. Extension beyond that approximation should give evolution in Q^2.



Outlook

- Fourth-order effective Hamiltonians.
  - Spin-dependents interactions.
  - Test for gluon mass ansatz.
  - Better form factors.
- Nonperturbative diagonalization of eigenproblems with gluon sectors.
- Evolution of structure functions using unitary relation between particles of different sizes.

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