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Gauge boson mass as regulator of front-form dynamics

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Small-x divergences of Abelian gauge theory in the front form of Hamiltonian dynamics are regulated using a mass parameter for gauge bosons, introduced through a mechanism analogous to the spontaneous breaking of global gauge symmetry [1], including the third polarization state for massive vector bosons. A corresponding family of ultraviolet and infrared finite, scale-dependent renormalized Hamiltonians, is calculable order-by-order using the renormalization group procedure for effective particles. This is illustrated on the second-order examples of: the electron mass-squared counter-term in QED and the resulting electron mass-squared Hamiltonian term in a finite effective theory. The orders of magnitude involved, given the current experimental upper limit on the photon mass, indicate the range of x that needs to be considered in resolving the small-x parton issues and the front-form vacuum and zero-mode problems.

[1] S. D. Głazek, Acta Physica Polonica B 50, 5 (2019).

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Outline:

1. Front-from Hamiltonian divergences

2. Gauge-boson mass κ as small-x regulator

3. Examples of mass^2 counter terms and effective mass^2 terms in $H_{QED\,\kappa}$

4. Parton model small-x and front-form vacuum - orders of magnitude

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Divergences in front-form Hamiltonians

$$x^0, x^3 \;\to\; x^\pm = x^0 \pm x^3 \qquad\qquad (x^1, x^2) = x^\perp$$

$$p^2 = p^+ p^- - p^{\perp\,2} = m^2 \qquad\qquad p^- = \frac{p^{\perp\,2} + m^2}{p^+}$$

$$A^\mu(x^-, x^\perp)_{x^+=0} = \sum_{\sigma=1}^{2} \int_0^\infty \frac{dp^+}{2p^+(2\pi)} \int_{-\infty}^{+\infty} \frac{d^2p^\perp}{(2\pi)^2} \left(\varepsilon^\mu_{p\sigma}\, a_{p\sigma}\, e^{-ipx} + ...\right)$$

$$A^+ = 0 \qquad\qquad \varepsilon^\mu_{p\sigma} = \left(\frac{2p^\perp \varepsilon^\perp_\sigma}{p^+},\; 0,\; \varepsilon^\perp_\sigma\right)$$

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$$A^\mu j_\mu \ : \qquad \varepsilon^-_\sigma = \frac{2p^\perp \varepsilon^\perp_\sigma}{p^+} \quad \text{contracts with} \quad j^+$$

$$\sum_{\sigma=1}^{2} |j^\mu \varepsilon_\mu|^2 \quad \to \quad \frac{p^{\perp\,2}}{p^{+2}}$$

parton model

$$p^+ = xP^+ \qquad p^\perp = xP^\perp + k^\perp \qquad \to \qquad \frac{k^{\perp\,2}}{x^2} \quad \text{or} \quad \left(\frac{\perp \ \ \infty}{x \ \ 0}\right)^2$$

zero-mode and vacuum triviality with cutoff ϵ^+ on p^+

$$p^+ > \epsilon^+ \ \to \ 0 \qquad\qquad x > \epsilon = x_{\min}$$

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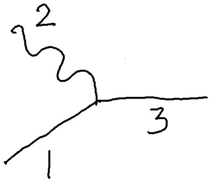
Regularization

$$\bar{\Psi} \not{A} \Psi \quad \longrightarrow \quad \bar{u}_1 \not{\varepsilon}_2^* u_3 \;\; b_1^\dagger \, a_2^\dagger \, b_3$$



$$r_{12.3} = e^{-t_r \left(\mathcal{M}_{12}^2 - m_3^2 \right)^2} \qquad \mathcal{M}_{12}^2 = (p_1 + p_2)^2$$

$H_{c.a}$ is regulated by $e^{-t_r(\mathcal{M}_c^2 - \mathcal{M}_a^2)^2}$

$t_r \;\; \rightarrow \;\; 0$ lifts regularization

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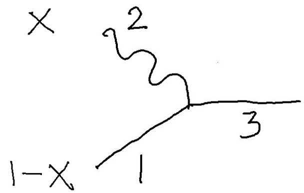
small-x problem for massless gauge bosons



$$p_2^\perp = x\, p_3^\perp + k^\perp$$

$$p_1^\perp = (1-x)\, p_3^\perp - k^\perp$$

$$\mathcal{M}_{12}^2 = (p_1 + p_2)^2 = \frac{k^{\perp\,2} + m^2}{1-x} + \frac{k^{\perp\,2}}{x}$$

small x is NOT regulated by $\mathcal{M}^2$ when $k^\perp \lesssim \sqrt{x}$

$$\frac{k^{\perp\,2}}{x^2} \sim \frac{1}{x} \qquad \int_\epsilon^1 \frac{dx}{x} = -\ln\epsilon \qquad \int_0^1 \frac{dx}{x}\, x^\delta = \frac{1}{\delta}$$

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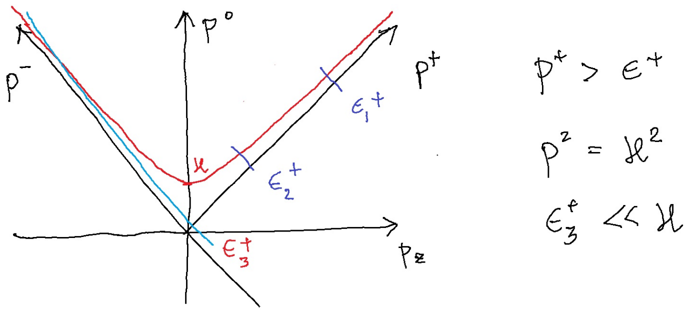
Small-x regularization by the boson mass



$$\mathcal{M}_{12}^2 = (p_1 + p_2)^2 = \frac{k^{\perp\,2} + m^2}{1 - x} + \frac{k^{\perp\,2} + \kappa^2}{x}$$

diverging increase of density of states with small p^+ or small x

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How to handle the situation?

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Proposal for handling the situation

1. Introduce mass as dictated by the gauge symmetry

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Example: masses of fermions and bosons in Abelian theory

orders of magnitude in the parton small-x region and FF vacuum

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Lagrangian

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_A + \mathcal{L}_{A\phi} - \mathcal{V}_\phi$$

$$\mathcal{L}_\psi = \bar{\psi}\left[\left(i\partial_\mu - gA_\mu\right)\gamma^\mu - m\right]\psi$$
$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$\mathcal{L}_{A\phi} = \left[(i\partial^\mu - g'A^\mu)\phi\right]^\dagger (i\partial_\mu - g'A_\mu)\phi$$
$$\mathcal{V}_\phi = -\mu^2\ \phi^\dagger\phi + \frac{\lambda^2}{2}\ (\phi^\dagger\phi)^2$$

gauge boson coupling to fermions is g and to extra (higgs-like) bosons is g'

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à la SSB of Higgs, Englert, Brout, and Kibble

$$\phi = \varphi \, e^{ig'\theta}/\sqrt{2}$$

For $\varphi = v + h$ and $h = 0$, minimum of $\mathcal{V}$ is $-\mu^4/(2\lambda^2)$ for $v = \sqrt{2}\,\mu/\lambda$.

v is a parameter of the theory.

The Lagrangian density depends on the gradient $\partial^\mu\theta$, not on θ.

$$\mathcal{L}_{A\phi} = \left[(i\partial^\mu - g' A^\mu)\phi\right]^\dagger (i\partial_\mu - g' A_\mu)\phi$$
$$= \frac{1}{2}\,(\partial^\mu\varphi)^2 + \frac{1}{2}\,g'^2(A^\mu + \partial^\mu\theta)^2\varphi^2$$

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"Massive limit"

$$g' \to 0 \qquad v \to \infty$$

$$g' v = \kappa \qquad \text{constant}$$

Parameter κ will be our gauge boson mass as a regulator.

The limit will lead to the theory of Soper (1971),

plus a free (decoupled) scalar field of arbitrary mass.

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Gauge symmetry in terms of ψ, A^μ, φ and θ

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_A + \mathcal{L}_{A\phi} - \mathcal{V}_\phi,$$

$$\begin{aligned}
\mathcal{L}_\psi &= \bar{\psi} \left[\left(i\partial_\mu - g A_\mu \right) \gamma^\mu - m \right] \psi \\
\mathcal{L}_A &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
\mathcal{L}_{A\phi} &= \frac{1}{2} \left(\partial^\mu \varphi \right)^2 + \frac{1}{2} g'^2 (A^\mu + \partial^\mu \theta)^2 \varphi^2 \\
\mathcal{V}_\phi &= \mathcal{V}(\varphi/\sqrt{2})
\end{aligned}$$

Gauge transformations are realized by substitutions

$$\psi = e^{-igf} \tilde{\psi} \qquad A^\mu = \tilde{A}^\mu + \partial^\mu f \qquad \varphi = \tilde{\varphi} \qquad \theta = \tilde{\theta} - f$$

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Gauge choice $f = -\theta$

$$\psi = e^{ig\theta}\tilde{\psi} \qquad A^\mu = \tilde{A}^\mu - \partial^\mu\theta \qquad \varphi = \tilde{\varphi} \qquad \theta = \tilde{\theta} + \theta$$

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_A + \mathcal{L}_{A\phi} - \mathcal{V}_\phi$$

$$\mathcal{L}_\psi = \bar{\tilde{\psi}}\left[\left(i\partial_\mu - g\tilde{A}_\mu\right)\gamma^\mu - m\right]\tilde{\psi}$$

$$\mathcal{L}_A = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\mathcal{L}_{A\phi} = \frac{1}{2}\left(\partial^\mu\tilde{\varphi}\right)^2 + \frac{1}{2}g'^2\tilde{A}^{\mu\,2}\tilde{\varphi}^2$$

$$\mathcal{V}_\phi = \mathcal{V}(\tilde{\varphi}/\sqrt{2})$$

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$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_A + \mathcal{L}_{A\phi} - \mathcal{V}_\phi$ in the massive limit

$$\begin{aligned}
\mathcal{L}_\psi &= \bar{\tilde{\psi}} \left[\left(i\partial_\mu - g\tilde{A}_\mu \right) \gamma^\mu - m \right] \tilde{\psi} \\
\mathcal{L}_A &= -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \\
\mathcal{L}_{A\phi} &= \frac{1}{2} (\partial^\mu \tilde{h})^2 + \frac{1}{2} \kappa^2 \tilde{A}^2 \\
\mathcal{V}_\phi &= -\frac{\mu^2}{2\lambda^2} + \frac{1}{2} (\sqrt{2}\,\mu)^2 \, \tilde{h}^2
\end{aligned}$$

a massive vector field $\tilde{A}$ minimally coupled to the fermion field $\tilde{\psi}$

a free field $\tilde{h}$ of arbitrary mass $\sqrt{2}\,\mu$

gauge symmetry as natural phenomenon $\quad 0 < m_\gamma = \kappa < 10^{-25} m_e \;\; \rightarrow \;\;$ extra scalar (DM?)

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Gauge choice $\tilde{A}^+ = 0$

$$\partial^+ f = A^+$$

$$f(x) = \frac{1}{4}\left(\int_{-\infty}^{x^-} - \int_{x^-}^{\infty}\right) dy^- A^+(x^+, y^-, x^\perp)$$

In the massive limit, $\tilde{A}^+ = 0$ and

$$\mathcal{L}_\psi = \bar{\tilde{\psi}}\left[\left(i\partial_\mu - g\tilde{A}_\mu\right)\gamma^\mu - m\right]\tilde{\psi}$$

$$\mathcal{L}_A = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\mathcal{L}_{A\phi} = \frac{1}{2}(\partial^\mu\tilde{h})^2 + \frac{1}{2}\kappa^2(\tilde{A}^\mu + \partial^\mu\tilde{\theta})^2$$

$$\mathcal{V}_\phi = -\frac{\mu^2}{2\lambda^2} + \frac{1}{2}(\sqrt{2}\,\mu)^2\,\tilde{h}^2$$

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FF Hamiltonian density

$\tilde{A}^+ = 0$ leads to soluble constraints on $\tilde{A}^-_f$ and $\tilde{\psi}_{f-} = \frac{1}{2}\gamma^0\gamma^-\tilde{\psi}$

$B = -\kappa\theta$ omitting tilde

$$H = \frac{1}{2}\int d^2x^\perp dx^- \ \left(\mathcal{H} = T^{+-}_{\rm can}\right)$$

$$\mathcal{H} = \frac{1}{2}\bar{\psi}_f\gamma^+\frac{-\partial^{\perp\,2}+m^2}{i\partial^+}\psi_f + \frac{1}{2}A^\perp(-\partial^{\perp\,2}+\kappa^2)A^\perp + \frac{1}{2}B(-\partial^{\perp\,2}+\kappa^2)B$$
$$+ g\bar{\psi}_f \not{A}_f\psi_f + g\bar{\psi}_f\gamma^+\psi_f\frac{1}{i\partial^+}(-i\kappa B)$$
$$+ \frac{1}{2}g^2\bar{\psi}_f\gamma^+\psi_f\frac{1}{(i\partial^+)^2}\bar{\psi}_f\gamma^+\psi_f + g^2\bar{\psi}_f\not{A}_f\frac{\gamma^+}{2i\partial^+}\not{A}_f\psi_f$$
$$+ \frac{1}{2}h(-\partial^{\perp\,2}+2\mu^2)h - \frac{\mu^4}{2\lambda^2}$$

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Quantization

$$\begin{aligned}
\hat{\psi}_f &= \sum_{\sigma=1}^{2} \int [p] \left[u_{p\sigma} \hat{b}_{p\sigma} e^{-ipx} + v_{p\sigma} \hat{d}^\dagger_{p\sigma} e^{ipx} \right]_{x^+=0} \\
A^\mu_f &= \sum_{\sigma=1}^{2} \int [p] \left[\varepsilon^\mu_{p\sigma} \hat{a}_{p\sigma} e^{-ipx} + \varepsilon^{\mu *}_{p\sigma} \hat{a}^\dagger_{p\sigma} e^{ipx} \right]_{x^+=0} \\
\hat{B} &= \int [p] \left[-i \hat{a}_{p3} e^{-ipx} + i \hat{a}^\dagger_{p3} e^{ipx} \right]_{x^+=0} \\
\hat{h} &= \int [p] \left[\hat{a}_{ph} e^{-ipx} + \hat{a}^\dagger_{ph} e^{ipx} \right]_{x^+=0} \quad ,
\end{aligned} \tag{1}$$

$$[p] = dp^+ \theta(p^+) d^2 p^\perp / [2p^+ (2\pi)^3]$$

$$[\hat{a}_{p\lambda}, \hat{a}^\dagger_{q\sigma}] = 2p^+ (2\pi)^3 \delta(p^+ - q^+) \delta^2(p^\perp - q^\perp) \delta_{\lambda\sigma}$$

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Quantum Hamiltonian

drop decoupled h

$$H = H_{\psi^2} + H_{A^2} + H_{B^2} + H_{\psi A \psi} + H_{\psi B \psi} + H_{\psi A A \psi} + H_{(\psi\psi)^2} + H_{CT}$$

$$H_{\psi^2} = \sum_{\sigma=1}^{2} \int [p] \frac{p^{\perp\,2} + m^2}{p^+} \left[b^\dagger_{p\sigma} b_{p\sigma} + d^\dagger_{p\sigma} d_{p\sigma} \right]$$

$$H_{A^2} = \sum_{\sigma=1}^{2} \int [p] \frac{p^{\perp\,2} + \kappa^2}{p^+} a^\dagger_{p\sigma} a_{p\sigma}$$

$$H_{B^2} = \int [p] \frac{p^{\perp\,2} + \kappa^2}{p^+} c^\dagger_p c_p$$

$$H_{\psi^2} + H_{A^2} + H_{B^2} = H_f$$

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Terms of first order including regularization

$$H_{\psi A\psi} = g \sum_{123} \int [123]\ 2(2\pi)^3 \delta_{12.3}\ e^{-t_r(\mathcal{M}_{12}^2 - m_3^2)^2}$$

$$\times \left[\bar{u}_2 \not\varepsilon_1^* u_3\ b_2^\dagger a_1^\dagger b_3 - \bar{v}_3 \not\varepsilon_1^* v_2\ d_2^\dagger a_1^\dagger d_3 + \bar{u}_1 \not\varepsilon_3 v_2\ b_1^\dagger d_2^\dagger a_3\ +\ h.c. \right]$$

$$H_{\psi B\psi} = -g \sum_{12} \int [123]\ 2(2\pi)^3 \delta_{12.3}\ e^{-t_r(\mathcal{M}_{12}^2 - m_3^2)^2}$$

$$\times \left[\bar{u}_2 \frac{\kappa\gamma^+}{p_1^+} u_3\ b_2^\dagger c_1^\dagger b_3 - \bar{v}_3 \frac{\kappa\gamma^+}{p_1^+} v_2\ d_2^\dagger c_1^\dagger d_3 + \bar{u}_1 \frac{\kappa\gamma^+}{p_3^+} v_2\ b_1^\dagger d_2^\dagger c_3\ +\ h.c. \right]$$

$$\mathcal{M}_{12}^2 = (p_1 + p_2)^2 = \frac{k^{\perp\,2} + m^2}{1-x} + \frac{k^{\perp\,2} + \kappa^2}{x} \qquad \text{for} \quad 1 = f \ \text{ and } \ 2 = b$$

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Application of the RGPEP

mass^2 terms

$$H_r = H_f + gH_{r1} + g^2 H_{r2} + CT_r$$

effective quanta $(b_t, d_t, a_t, c_t) = \mathcal{U}_t \; (b, d, a, c)_{\rm can} \; \mathcal{U}_t^\dagger$

$t = s^4$ s = size of effective quanta $s = 0$ in the canonical theory

regulated canonical H_r is the initial condition at $t = 0$

$$\frac{dH_t}{dt} = [[H_f, \tilde{\mathcal{H}}_t], \mathcal{H}_t]$$

history: Glazek-Wilson (1993), Wegner (1994) . . . for 4th order see Glazek (2012)

RGPEP contributors: S. Dawid, P. Kubiczek, T. Masłowski, J. Młynik, J. More, J. Narębski, **M. Gómez-Rocha, K. Serafin, A. Trawiński**, M. Więckowski

Perturbative RGPEP 2nd order $g_t = g$

$$\mathcal{H}_t = H_f + g\mathcal{H}_{t1} + g^2\mathcal{H}_{t2} + O(g_t^3)$$

equating coefficients of powers of g on both sides of $\mathcal{H}'_t = [[H_f, \tilde{\mathcal{H}}_t], \mathcal{H}_t]$

$$H'_f = 0$$

$$\mathcal{H}'_{t\,1} = \left[[H_f, \tilde{\mathcal{H}}_{t\,1}], H_f\right]$$

$$\mathcal{H}'_{t\,2} = \left[[H_f, \tilde{\mathcal{H}}_{t\,2}], H_f\right] + \left[[H_f, \tilde{\mathcal{H}}_{t\,1}], \mathcal{H}_{t\,1}\right]$$

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"Free" Hamiltonian terms

$$H_{t\,\psi^2} = \sum_{\sigma=1}^{2} \int [p] \, \frac{p^{\perp\,2} + m^2}{p^+} \left[b^\dagger_{t\,p\sigma} b_{t\,p\sigma} + d^\dagger_{t\,p\sigma} d_{t\,p\sigma} \right]$$

$$H_{t\,A^2} = \sum_{\sigma=1}^{2} \int [p] \, \frac{p^{\perp\,2} + \kappa^2}{p^+} \, a^\dagger_{t\,p\sigma} a_{t\,p\sigma}$$

$$H_{t\,B^2} = \int [p] \, \frac{p^{\perp\,2} + \kappa^2}{p^+} \, c^\dagger_{t\,p} c_{t\,p}$$

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First-order interaction terms

$$H_{t\,\psi A\psi} \;=\; g\sum_{123}\int[123]\;2(2\pi)^3\delta_{12.3}\;e^{-(t+t_r)(\mathcal{M}_{12}^2-m_3^2)^2}$$

$$\times\;\left[\bar u_2\,\not\varepsilon_1^*u_3\;b_{t\,2}^\dagger a_{t\,1}^\dagger b_{t\,3}-\bar v_3\,\not\varepsilon_1^*v_2\;d_{t\,2}^\dagger a_{t\,1}^\dagger d_{t\,3}+\bar u_1\,\not\varepsilon_3 v_2\;b_{t\,1}^\dagger d_{t\,2}^\dagger a_{t\,3}\;+\;h.c.\right]$$

$$H_{t\,\psi B\psi} \;=\; -g\sum_{23}\int[123]\;2(2\pi)^3\delta_{12.3}\;e^{-(t+t_r)(\mathcal{M}_{12}^2-m_3^2)^2}$$

$$\times\;\left[\bar u_2\frac{\kappa\gamma^+}{p_1^+}u_3\;b_{t\,2}^\dagger c_{t\,1}^\dagger b_{t\,3}-\bar v_3\frac{\kappa\gamma^+}{p_1^+}v_2\;d_{t\,2}^\dagger c_{t\,1}^\dagger d_{t\,3}+\bar u_1\frac{\kappa\gamma^+}{p_3^+}v_2\;b_{t\,1}^\dagger d_{t\,2}^\dagger c_{t\,3}\;+\;h.c.\right]$$

The meaning of local gauge theory in the RGPEP

$$H_{t\,\psi A\psi}\;=\;H_{\psi_t A_t\psi_t} \qquad\qquad H_{t\,\psi B\psi}\;=\;H_{\psi_t B_t\psi_t}$$

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Mass^2 terms order g^2 $\qquad H_f + g^2 \delta \mathrm{mass}^2$

$$\sum_{\sigma=1}^{2} \int [p] \, \frac{m^2 + g^2 \delta m^2(t)}{p^+} \, \left(b^\dagger_{t\,\sigma p} b_{t\,\sigma p} + d^\dagger_{t\,\sigma p} d_{t\,\sigma p} \right)$$

$$\sum_{\sigma=1}^{2} \int [p] \, \frac{\kappa^2 + g^2 \delta \kappa_A^2(t)}{p^+} \, a^\dagger_{t\,\sigma p} a_{t\,\sigma p}$$

$$\int [p] \, \frac{\kappa^2 + g^2 \delta \kappa_B^2(t)}{p^+} \, c^\dagger_{t\,p} c_{t\,p}$$

counter terms were adjusted to produce physical eigenvalues m^2 and κ^2

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$$\delta m^2(t) = \frac{\alpha}{4\pi} \int_0^1 dx \left[\frac{1+(1-x)^2}{x} \sqrt{\frac{\pi}{2t}} \,\mathrm{erfc}(\mathcal{M}_f^2 \sqrt{2t}) - \left(2m^2 + \kappa^2\right) \Gamma(0, 2t\mathcal{M}_f^4) \right]$$

$$\mathcal{M}_f^2 = \kappa^2/x + m^2/(1-x) - m^2$$

$$\delta\kappa_A^2(t) = \frac{\alpha}{4\pi} \int_0^1 dx \left\{ \left[x^2 + (1-x)^2\right] \sqrt{\frac{\pi}{2t}} \,\mathrm{erfc}(\mathcal{M}_b^2 \sqrt{2t}) + \left[2m^2 + \kappa^2 - 2\kappa^2 x(1-x)\right] \Gamma(0, 2t\mathcal{M}_b^4) \right\}$$

$$\delta\kappa_B^2(t) = \kappa^2 \frac{\alpha}{\pi} \int_0^1 dx\, x(1-x)\, \Gamma(0, 2t\mathcal{M}_b^4)$$

$$\mathcal{M}_b^2 = m^2/x + m^2/(1-x) - \kappa^2$$

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Orders of magnitude QED_κ $\alpha \sim 1/137$

1. we are used to "think" in terms of $\kappa \sim m \sim 1/s$ and $x \sim 1/2$

$$\frac{\delta m^2}{m^2} = 2.79\ 10^{-13} \quad \delta\kappa_A^2/\kappa^2 = 2.32\ 10^{-13} \quad \delta\kappa_B^2/\kappa^2 = 5.64\ 10^{-14}$$

2. experimentally $m_\gamma < 10^{-18}$ eV PDG 2019

if $\kappa = 10^{-25}\ m$ and $s = 1/(10\ \mathrm{TeV})$, then $x_{min} \sim 10^{-50}$, resolution in x

$$\frac{\delta m^2}{m^2} = 2\ 10^{+13} \quad \delta\kappa_A^2/\kappa^2 = 5\ 10^{+60} \quad \delta\kappa_B^2/\kappa^2 = 6\ 10^{-2} \qquad \text{errors} \sim 20\%$$

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Conclusion

The front-form RGPEP appears to bring us close

to addressing puzzles of particle theory.

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