On the light-front wavefunctions and related observables of quarkonium states

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QCD – Relativistic QFT

- Relativistic
- Nonperturbative
- Particle number not conserved
- Many-body system of fermions and bosons
  - positronium, mesons: fermion and anti-fermion plus ...
  - baryons: three quarks plus ...
  - interaction via exchange of photons, gluons
- Confinement
  - only color-singlets form asymptotic states

Light-Front Wave Functions

Light-Front Wave Function $\Psi(x_i; p_{\perp,i})$

- Solution of light-front Hamiltonian $\mathcal{M}^2 = P^+ P^- - \vec{P}_\perp^2$
- Fock space expansion
- Longitudinal momenta fraction x_i
  - Total LF momentum $\sum_i x_i = 1$
- Transverse momenta $\vec{p}_{\perp,i}$
  - Transverse CoM motion $\vec{P}_\perp = \sum_i \vec{p}_{\perp,i}$
- Light-cone gauge

Light-Front Hamiltonian

n	Sector	1 $q\bar{q}$	2 gg	3 $q\bar{q}\,g$	4 $q\bar{q}\,q\bar{q}$	5 $gg\,g$	6 $q\bar{q}\,gg$	7 $q\bar{q}\,q\bar{q}\,g$	8 $q\bar{q}\,q\bar{q}\,q\bar{q}$	9 $gg\,gg$	10 $q\bar{q}\,gg\,g$	11 $q\bar{q}\,q\bar{q}\,gg$	12 $q\bar{q}\,q\bar{q}\,q\bar{q}\,g$
1	$q\bar{q}$					•		•	•	•	•	•	•
2	gg				•			•	•		•	•	•
3	$q\bar{q}\,g$								•	•		•	•
4	$q\bar{q}\,q\bar{q}$		•			•				•	•		•
5	$gg\,g$	•			•			•	•			•	•
6	$q\bar{q}\,gg$								•				•
7	$q\bar{q}\,q\bar{q}\,g$	•	•			•				•			
8	$q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•		•	•			•	•		
9	$gg\,gg$	•		•	•			•	•			•	•
10	$q\bar{q}\,gg\,g$	•	•		•				•				•
11	$q\bar{q}\,q\bar{q}\,gg$	•	•	•		•				•			

Fock-space truncation

Quarkonium: fermion-antifermion bound states

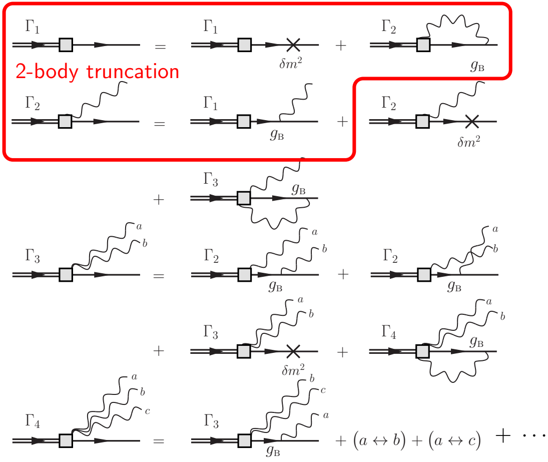
- Dominant Fock space $|q\bar{q}\rangle$
  - can obtain approximation to the mass
  - electromagnetic form factors in spacelike region

  however,
  ambiguities due to breaking of rotational symmetry
- Use models in dominant Fock space
  - extend Fock space as needed for specific observables
- Exotics require $|q\bar{q}g\rangle$ and/or $|q\bar{q}q\bar{q}\rangle$
- Strong decays of quarkonium require $|q\bar{q}q\bar{q}\rangle$
- Form factors in timelike region may require $|q\bar{q}q\bar{q}\rangle$
- Self-energies require at least one gluon $|q\bar{q}g\rangle$
- Infinite number of Fock spaces required for
  - Restoration of rotational symmetry
  - Dynamical chiral symmetry breaking
  - Confinement

Fock-space convergence in scalar Yukawa theory

$$|\chi_{\rm ph}\rangle = |\chi\rangle + |\chi\varphi\rangle + |\chi\varphi\varphi\rangle + |\chi\varphi\varphi\varphi\rangle + |\chi\varphi\varphi\varphi\varphi\rangle + \cdots$$

Diagrams in the one-body sector:



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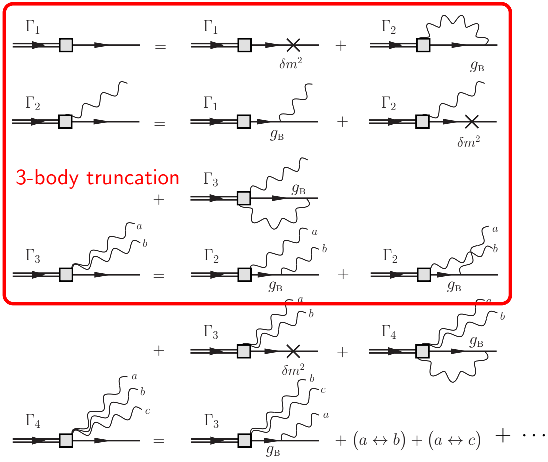

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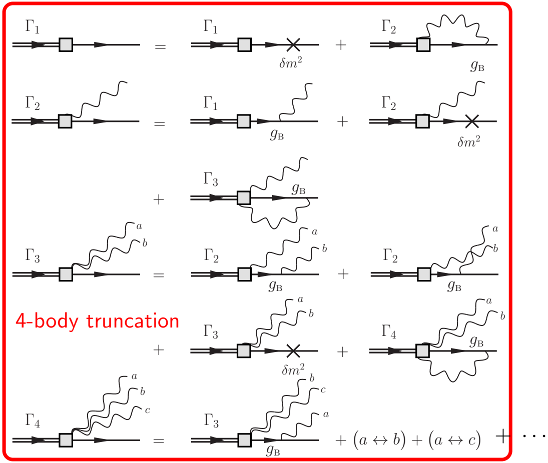


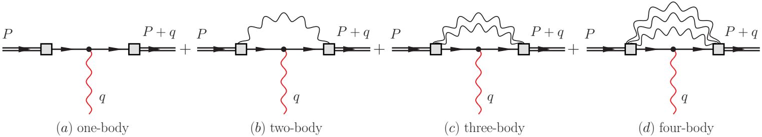
Convergence of form factor with Fock space expansion

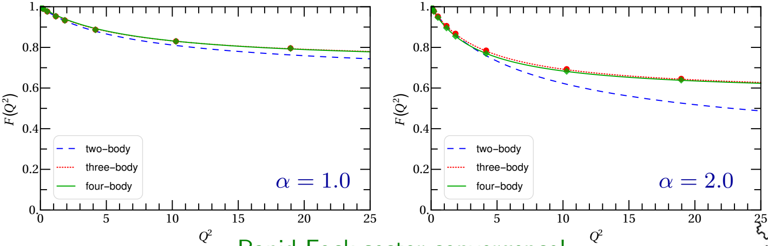
Yang Li, Karmanov, Maris, Vary, PLB748, 278 (2015)

Obtain solutions of the charge-one sector up to four-body: $\chi + \varphi\varphi\varphi$

Study the convergence of Fock sector expansion by comparing different Fock sector truncations

Fock sector convergence of the electromagnetic form factor:





Rapid Fock sector convergence!

Light-Front Holography and Confinement

- Holographic variable $\vec{\zeta}_\perp = \sqrt{x(1-x)}\ \vec{r}_\perp$
- Effective confining interaction in transverse direction

$$V_{\perp\ \mathrm{conf}} \quad = \quad \kappa^4 \zeta_\perp^2 = \kappa^4\ x(1-x)\ r_\perp^2$$

Brodsky, de Teramond, Dosch, Erlich, Phys. Rept. 584, 1 (2015)

- Effective longitudinal confinement

$$V_{x\ \mathrm{conf}} \quad = \quad -\frac{\kappa^4}{m_q + m_{\bar{q}}} \partial_x \left[x(1-x) \partial_x \right]$$

Yang Li, Maris, Zhao, Vary, PLB758, 118 (2016)

  - combines, in nonrelativistic limit, with transverse confinement into 3-D harmonic oscillator confinement
  - exactly solvable
  - distribution amplitudes match pQCD asymptotics

Basis Light-Front Quantization

see also Vary, Thursday

$$H_{\text{eff}}^0 = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 \vec{\zeta}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x \big(x(1-x)\partial_x\big)$$

- Effective Hamiltonian H_{eff}^0
  - LF kinetic energy
  - transverse and longitudinal confinement
- Add one-gluon exchange with running coupling V_{gluon}

$$V_{\text{gluon}} = -\frac{4}{3} \times \frac{4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{\sigma'} \gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$$

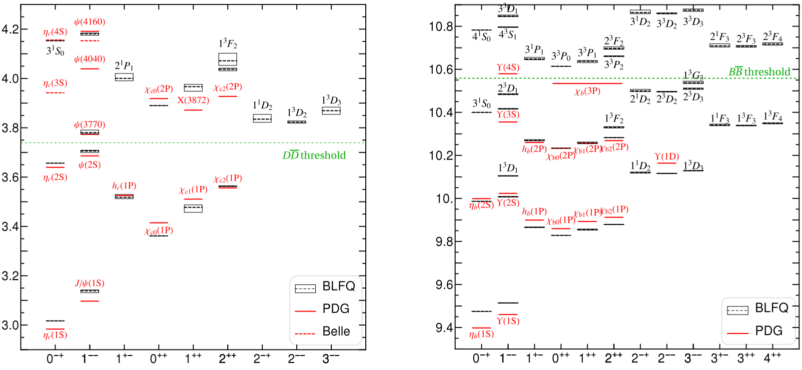
- Basis representation use eigenfunctions of H_{eff}^0

$$\psi(x, k_\perp) = \sum_{n,m,l} c_{nmlss'}\ \phi_{nm}\left(k_\perp/\sqrt{x(1-x)}\right)\ \chi_l(x)$$

  - transverse direction: 2-D harmonic oscillator functions
  - longitudinal dir: Jacobi polynomials weighted by $x^\alpha (1-x)^\beta$
- Truncation on number of HO quanta, N_{max}, and Jacobi polynomials, L_{max} (typically $N_{\text{max}} = L_{\text{max}}$)

Quarkonium Spectroscopy

Yang Li, Maris, Vary, PRD96, 016022 (2017)



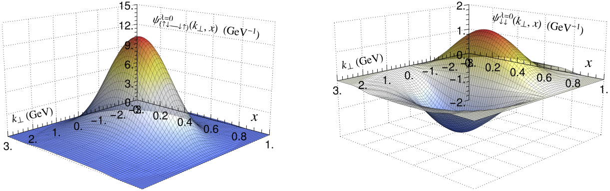
	κ (GeV)	m_q (GeV)	fitted states	rms dev. (MeV)	$\overline{\delta_J M}$ (MeV)	truncation $N_{\max}$	basis dim.
$c\bar{c}$	0.966	1.603	8	31	17	32	1812
$b\bar{b}$	1.389	4.902	14	38	8	32	1812

fitted value for κ follows expected trajectory $\kappa_h \propto \sqrt{M_h}$

LF Wave Functions

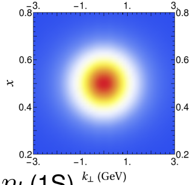
available at: Yang Li (2019), Mendeley Data, v2; DOI: 10.17632/cjs4ykv8cv.2

- Pseudoscalar mesons: two spin structures, $\psi(x, k_\perp)_{(\uparrow\downarrow-\downarrow\uparrow)}$ and $\psi(x, k_\perp)_{\downarrow\downarrow} = \psi(x, k_\perp)^*_{\uparrow\uparrow}$

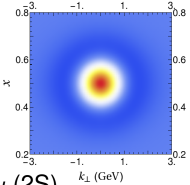


η_c

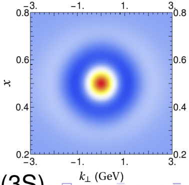
- Vector mesons: 6 different Dirac structures
- Heavy quarkonia: non-relativistic configurations dominate
- Radial excitations more spread out in coordinate space



$\eta_b(1S)$



$\eta_b(2S)$



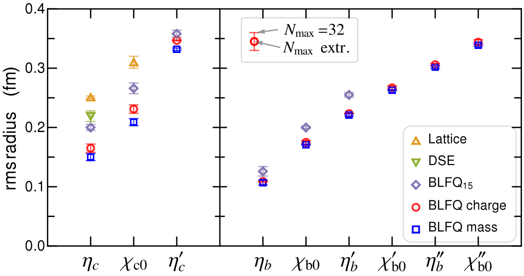
$\eta_b(3S)$

Electromagnetic and Gravitational radii

Scalar and pseudoscalar states

$$\langle r_c^2 \rangle = \frac{3}{2}\langle \vec{b}_\perp^2 \rangle \equiv \frac{3}{2}\sum_{s,\bar{s}}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp \,(1-x)^2\, \vec{r}_\perp^2\; \widetilde{\psi}^*_{s\bar{s}}(\vec{r}_\perp, x)\widetilde{\psi}_{s\bar{s}}(\vec{r}_\perp, x)$$

$$\langle r_m^2 \rangle = \frac{3}{2}\langle \vec{\zeta}_\perp^2 \rangle \equiv \frac{3}{2}\sum_{s,\bar{s}}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp \, x(1-x)\, \vec{r}_\perp^2\; \widetilde{\psi}^*_{s\bar{s}}(\vec{r}_\perp, x)\widetilde{\psi}_{s\bar{s}}(\vec{r}_\perp, x)$$



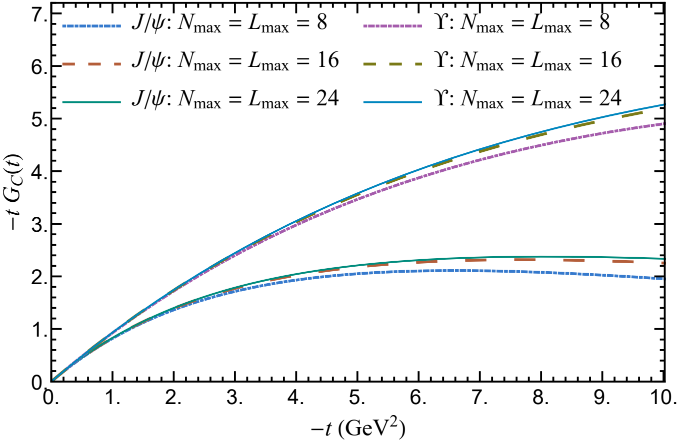
Yang Li, Maris, Vary, PRD96, 016022 (2017)

- 'Charge' radii slightly larger than mass radii for charmonia, but nearly equal for bottomonia
  - splitting is relativistic effect

Electromagnetic form factors

see also Li, Friday

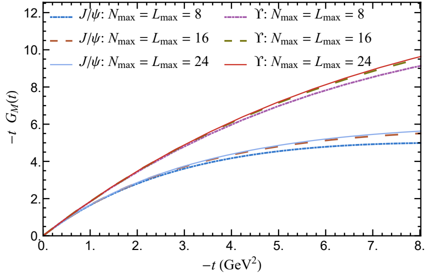
Adhikari, Yang Li, Meijian Li, Vary, PRC99, 035208 (2019)

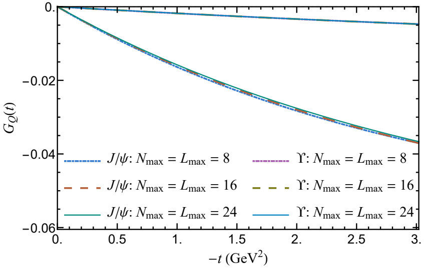


- Reasonably well converged with basis expansion
- Υ form factor is larger than that of J/ψ in spacelike region, and correspondingly smaller radius, as expected

Magnetic and Quadrupole form factors

Adhikari, Yang Li, Meijian Li, Vary, PRC99, 035208 (2019)





μ	this work	lat	DSE
J/ψ	1.952(3)	2.10(3)	2.13(4)
ψ'	2.05(2)		
Υ	1.985(1)		
Υ'	1.992(1)		
Q	this work	lat	DSE
J/ψ	-0.78(2)	-0.23(2)	-0.28(1)
ψ'	0.2(2)		
Υ	-0.73(1)		
Υ'	0.1(1)		

lat: Dudek, Edwards, Richards, PRD73,074507 (2006)

DSE: Bhagwat and Maris, PRC77, 025203 (2008)

Covariant Bethe–Salpeter Equation

Bound state momentum P, and relative momenta p, k

$$\Gamma_{\rm BS}(p,P) \;=\; i\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,K(p,k;P)\,S(k_1)\,\Gamma_{\rm BS}(k,P)\,S(k_2)$$

normalized by canonical normalization condition

- Conventionally
  - Wick rotate from Minkowski metric to Euclidean metric
  - Solve for Euclidean (spacelike) variable p_E^2 and angle α between p_E and P_E at bound state pole $P_E^2 = -M^2$
  - Can be done with bare constituents and with nonperturbatively dressed propagators
  - Straightforward to obtain e.g. form factors over a limited spacelike and timelike region
  - Question: Extract Light-Front Wave Function from Γ ?
- Solve in Minkowski metric for spacelike and timelike p^2?
  - Covariant Spectator Theory
  - Nakanishi representation
  - Un-Wick rotate p_0 from Euclidean to Minkowski metric

Covariant Spectator Theory

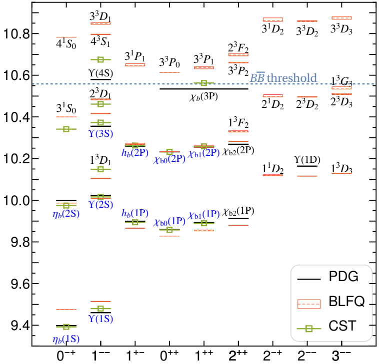
see also Biernat, Wednesday

- Keep only the contribution from the positive-energy pole of one quark in the k_0 contour integration

  Gross, PR186, 1448 (1969)

- Can be extended to confined constituents

- Has correct one-body and nonrelativistic limits
- Use linear confining potential plus one-gluon exchange kernel
- Use Pauli–Villars regularization
- Solve numerically in Minkowski space rest-frame



Leitão *et al*, EPJ C77:696 (2017)

Light-Front Wave Functions from CST

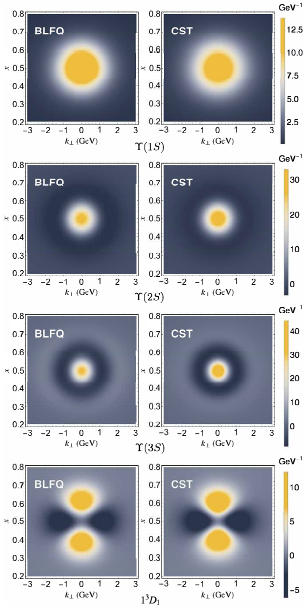
CST amplitudes with helicity $\rho = \pm 1$

$$\Psi^{+\rho}_{\lambda_1\lambda_2}(\vec{k}) \equiv \frac{m}{E_k}\frac{\rho}{(1-\rho)E_k + \rho M}$$
$$\bar{u}^{+}_{1}(\vec{k},\lambda_1)\,\Gamma(k)\,u^{\rho}_{2}(\vec{k},\lambda_2)$$

can be interpreted as LFWF

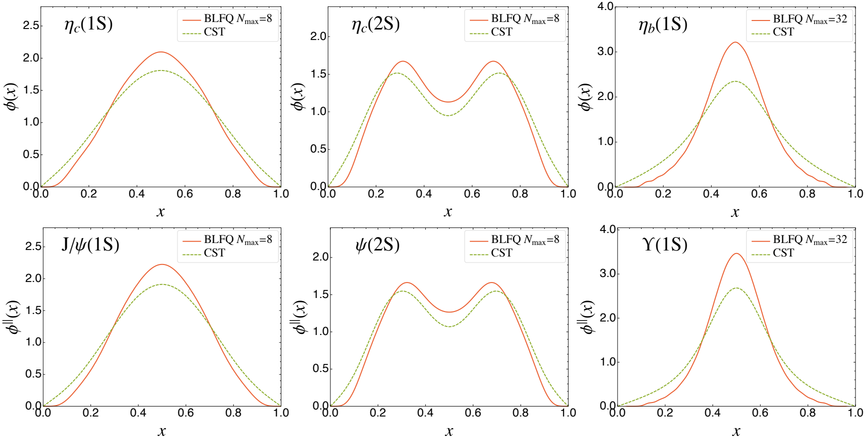
with suitable definition of $x = \frac{k_1^+}{P^+}$

- in principle:
  $P^+ = M$ and $k_1^+ = E_k + k_3$,
  so $x = (E_k + k_3)/M$, but it is not guaranteed that $0 < x < 1$
- use Brodsky–Huang–Lepage prescription $x = (E_k + k_3)/(2E_k)$, which satisfies $0 < x < 1$



Comparison of Distribution Amplitudes

Leitão, Yang Li, Maris, Peña, Stadler, Vary, Biernat, EPJ C77:696 (2017)



- Different models give very similar distribution amplitudes
- Radial excitations show characteristic 'double hump'
- Bottomonium DA narrower than charmonium

Nakanishi integral representation

Nakanishi, Phys.Rev. 130, 1230 (1963); Prog.Theor.Phys.Suppl. 43, 1 (1969)

- For propagators (2-point functions) of asymptotic states

$$S(p) \;=\; -i\int_0^\infty d\gamma \frac{\rho(\gamma)}{(\gamma + m^2 - p^2 - i\epsilon)^n}$$

  $n = 1$ gives usual Källen–Lehmann representation
- Applicability to confined states unclear
- For two-body BSA for bound state with mass $M^2 = P^2$

$$\Gamma(p;P) \;=\; -i\int_{-1}^1 dz \int_0^\infty d\gamma \frac{g(\gamma,z)}{(\gamma + m^2 - M^2/4 - p^2 - p\cdot P\,z - i\epsilon)^n}$$

- Used for
  - 2-body scalar BSE — Kusaka *et al*, PRD56, 5071 (1997)
  - fermion DSE and BSE — Sauli, JHEP 0303, 1 (2003)
  - recent work: Carbonell, Karmanov, Frederico, Salmè, …

Un-Wick rotating from Euclidean to Minkowski metric

$$\Gamma(p;P) \quad = \quad g^2 \int_{-\infty}^{\infty} dk_0 \int \frac{d^3\vec{k}}{(2\pi)^4} \, K(p,k;P)\, S(k+p/2)\, \Gamma(k;P)\, S(k-p/2)$$

- Un-Wick rotate p_0 and k_0 from Euclidean metric in decrements θ starting from $\theta = \pi/2$

$$\begin{aligned} p_4 &\rightarrow \exp\left(-i(\pi/2-\theta)\right)\, p_4 = \exp(i\theta)\, p_0 \\ k_4 &\rightarrow \exp\left(-i(\pi/2-\theta)\right)\, k_4 = \exp(i\theta)\, k_0 \end{aligned}$$

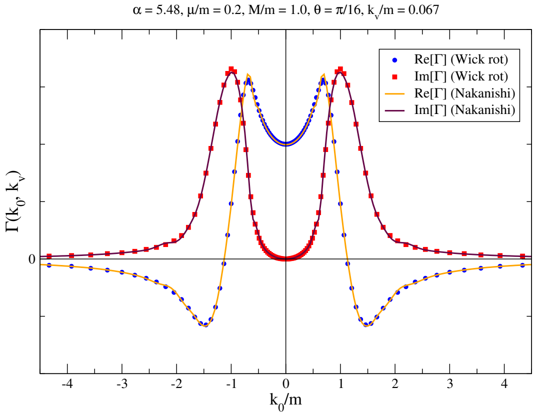
- Solve BSE iteratively as function of p_0 and $\vec{p}^2$ along rotated p_0 axis, starting with solution at previous value of θ, to obtain Green's functions as function of $p_0\, \mathrm{e}^{i\theta}$ and $\vec{p}^2$, instead of as function of Lorentz scalar p^2
- Use Pauli–Villars regulator to remove UV divergences
- Approach Minkowski space for $\theta \rightarrow 0$
  - space-like region $p_0^2 = 0$ with $\vec{p}^2 > 0$
  - time-like region $p_0^2 > 0$ with $\vec{p}^2 = 0$
- Manifestly covariant BSA for space- and time-like momenta

Example: scalar model in ladder truncation

Castro *et al*, JPCS 1291, 012006 (2019)

Use Nakanishi representation for $\chi(k;P)$ at $P^2 = M^2$

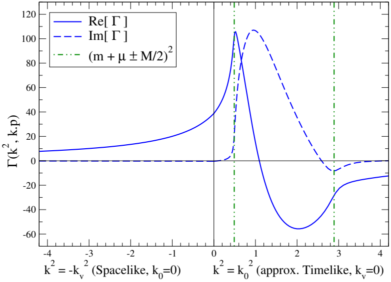
$$\begin{aligned}\chi(k;P) &\equiv \Delta(k+P/2)\,\Gamma(k;P)\,\Delta(k-P/2)\\ &= -i\int_{-1}^{1} dz \int_0^\infty d\gamma \frac{g(\gamma,z)}{(\gamma+m^2-P^2/4-k^2-k\cdot P\,z-i\epsilon)^3}\end{aligned}$$

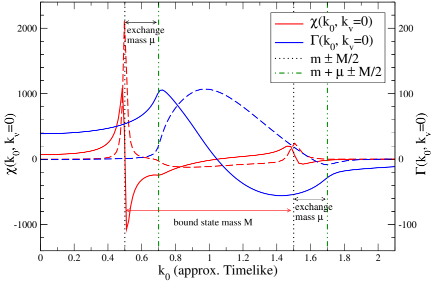


- Calculate Γ using $\Delta^{-1}\,\chi\,\Delta^{-1}$
- Γ has singularities at $k_0^{\pm} = \sqrt{(m+\mu)^2+\vec{k}^2} \pm \frac{M}{2}$
- $\chi(k;P)$ contains constituent poles at $k\cdot p = \pm(k^2 - m^2 + M^2/4)$, as well as above singularities

Spacelike and (almost) timelike BS Amplitudes

Castro *et al*, JPCS 1291, 012006 (2019)





- $\Gamma(k_0, \vec{k})$ has singularities at $k_0^{\pm} = \sqrt{(m+\mu)^2 + \vec{k}^2} \pm \frac{M}{2}$
- $\chi = \Delta\,\Gamma\,\Delta$ has additional singularities
  due to the mass poles in the constituents $\Delta(P/2 \pm k)$

LFWF from Covariant Bethe–Salpeter Amplitude

work in progress, w. Ydrefors, de Paula, Frederico, Jia, ...

Project BSA $\chi(k;P) = \Delta(k+P/2)\,\Gamma(k;P)\,\Delta(-k+P/2)$ onto the light-front to obtain the LFWF $\psi(x,k_\perp)$

$$\psi(x,k_\perp) = i\,P^+\,x\,(1-x)\int_{-\infty}^{\infty}\frac{dk^-}{2\pi}\,\chi(k;P)$$

- Can be done with Nakanishi representation for χ
- Can be approximated by un-Wick rotating the BSE from the spacelike region and project

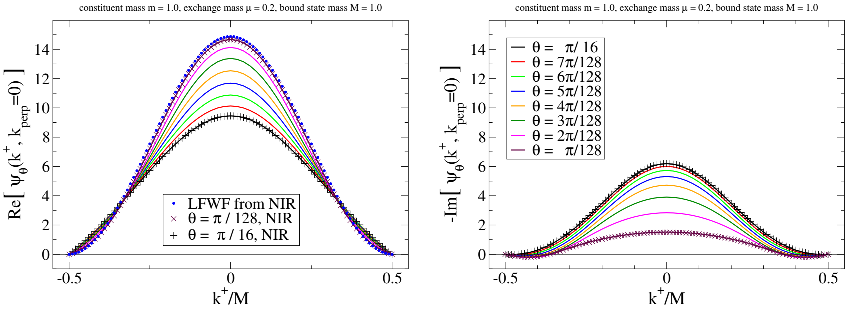
$$\psi_\theta(k^+,k_\perp) = i\,M\left(\frac{1}{2}+\frac{k^+}{M}\right)\left(\frac{1}{2}-\frac{k^+}{M}\right)\int\frac{dk^-}{2\pi}\,\chi(k_\theta;p)$$

where $k_\theta = (k_0\exp(i\theta),\vec{k})$, and $k^\pm = k_0 \pm k_3$

- In the limit $\theta \to 0$, the 'quasi' LFWF $\psi_\theta(k^+,k_\perp)$ becomes the LFWF $\psi(x,k_\perp)$ with $x = \frac{1}{2}+\frac{k^+}{M}$

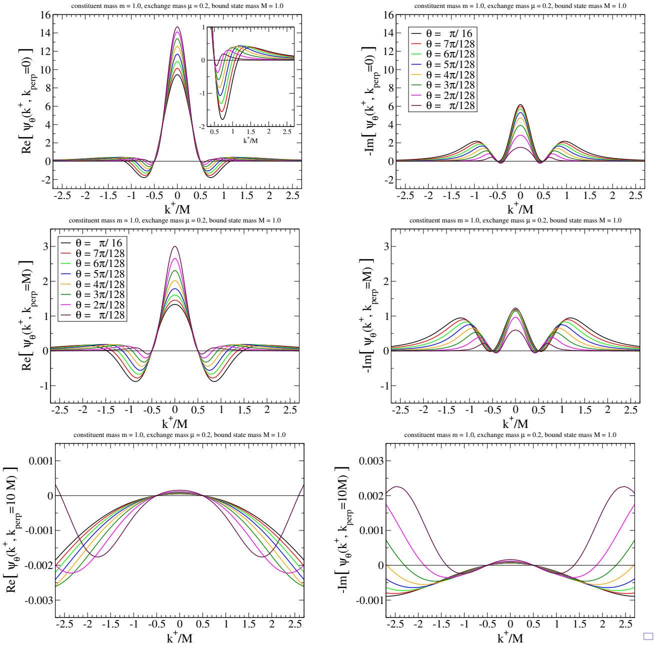
Example: LFWF for scalar model

work in progress, w. Ydrefors, de Paula, Frederico, Jia, . . .



- Perfect agreement for 'quasi' LFWF $\psi_\theta(k^+, k_\perp)$ at $\theta > 0$ between independent calculations using the Nakanishi Intergral Representation and by un-Wick rotating the BSE from the spacelike region

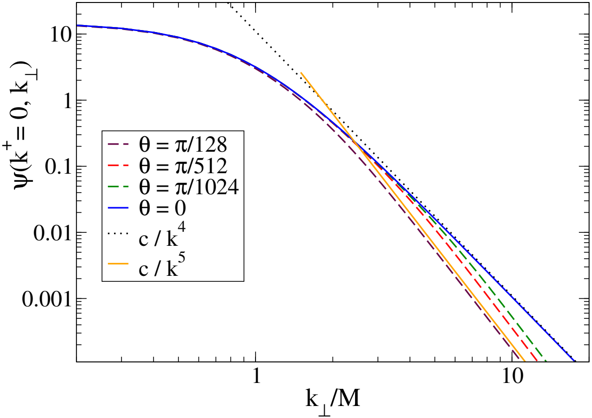
LFWF from covariant Bethe–Salpeter Eqn



- Finite domain $0 < x < 1$ arises naturally as θ decreases
- No need to constrain range on k^+
- However, as $k_\perp$ increases, one needs very small values of θ
- Can take the limit $\theta \to 0$ with NIR

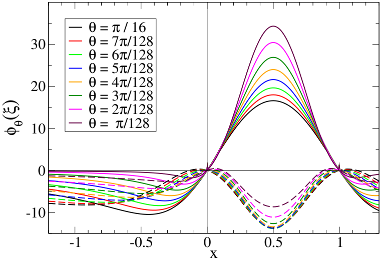
Asymptotic behavior of LFWF

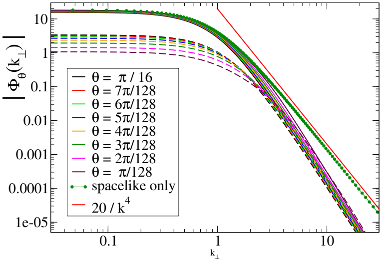
Work in progress Ydrefors, de Paula, Frederico, Jia, . . .



- Asymptotic behavior LFWF $1/k_\perp^4$ recovered in limit $\theta = 0$
- For $\theta > 0$, asymptotic behavior 'quasi' LFWF $1/k_\perp^5$
- As θ decreases, $\psi_\theta(k^+, k_\perp)$ approaches $\psi_0(k^+, k_\perp)$ even for large $k_\perp$, but one may need very small θ

Valence distribution amplitudes





- Distribution amplitudes

$$\phi(x) \quad = \quad \int \frac{d^2 k_\perp}{(2\pi)^2} \psi(x, k_\perp)$$

- Finite range in x emerges automatically in limit $\theta \to 0$

- Analogously,
transverse 'distribution amplitudes'

$$\Phi(k_\perp) \quad = \quad \int_0^1 \frac{dx}{2x(1-x)} \psi(x, k_\perp)$$

- Correct asymptotics only in limit $\theta \to 0$
- Can be computed directly in Euclidean metric with correct asymptotics

$$\Phi(k_1, k_2) \quad = \quad \int \frac{dk_3 dk_4}{(2\pi)} \chi(k; P)$$

Valence probability

- Valence probability from projected BSA

$$\mathcal{P} = \int_0^1 \frac{dx}{x(1-x)} \int \frac{d^2 k_\perp}{2(2\pi)^3} \, |\psi(x, k_\perp)|^2$$

  - $\mathcal{P} \sim 0.65$ to 0.8 for moderate and strong binding
  - $\mathcal{P} \to 1$ in the limit of zero binding

Frederico, Salmè, Viviani, PRD89 016010 (2014)

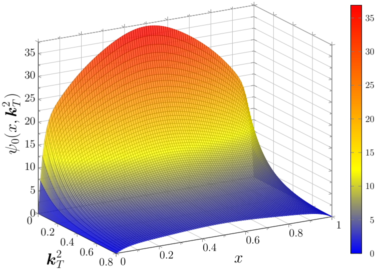
- BSA also contains contributions from $|q\bar{q}g\rangle$ Fock sectors as is also evident from the singularities at $k_0^\pm = \sqrt{(m+\mu)^2 + \vec{k}^2} \pm \frac{M}{2}$

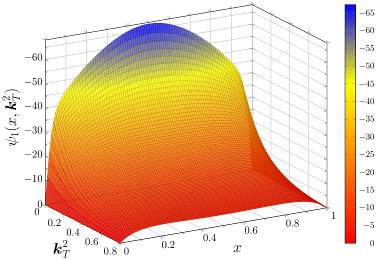
Castro *et al*, JPCS 1291, 012006 (2019)

- Calculate (light-front) observables directly from BSA instead of projecting BSA on the LFWF, and computing observables from LFWF

Application to QCD: pion LFWF from BSE

Chao Shi, Cloet, PRL 122, 082301 (2019)





$$\psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x, \vec{k}_\perp^2) = \sqrt{3}\, i \int \frac{dk^+ dk^-}{2\pi} \mathrm{Tr}\left[\gamma^+ \gamma_5 \chi(k,p)\right] \delta\left(x\, p^+ - k^+\right)$$

$$\psi_{\uparrow\uparrow}(x, \vec{k}_\perp^2) = -\sqrt{3}\, i \int \frac{dk^+ dk^-}{2\pi} \frac{1}{\vec{k}_\perp^2} \mathrm{Tr}\left[i\sigma_{+i} \vec{k}_\perp^i\, \gamma_5\, \chi(k,p)\right] \delta\left(x\, p^+ - k^+\right)$$

- Quark propagators parametrized by two pairs of c.c. poles
- Use dominant BS amplidudes proportional to γ_5 and $P\!\!\!/\gamma_5$
- Note: LFWF in terms of dressed quark propagators which evolve from constituent quarks at small momenta to current quarks at large momenta

Conclusions and Outlook

- Quarkonium forms an ideal system to develop and validate methods to compute Light-Front Wave Functions
  - even simpler: scalar Yukawa model
- Conventionally
  - obtain LFWF as eigenfunctions of effective LF Hamiltonian
  - typically limited to minimal Fock space
- Fock space convergence
  - can achieve convergence in scalar Yukawa model but have to go beyond minimal Fock space
- LFWF can be obtained from Bethe–Salpeter Equation
  - qualitatively similar quarkonia results using confining effective LF Hamiltonian and confining CST model
  - use Nakanishi representation or explicit rotation of p_0 from Euclidean to Minkowski axis
  - BSA contains more information than minimal Fock space
- Outlook
  - project meson BSA onto LF wavefunction
  - use BSA directly to calculate LF observables