Probing Nuclear Structure at Short Distances

Frank Vera

FIU

September 17, 2019

In collaboration with Misak Sargsian.

Motivation

Important phenomena related with short inter-nucleonic distance:

- Nuclei stability – Repulsive nuclear core
- Neutron stars – How dense nuclear matter behave?

Fundamental questions in Physics:

- At what point does the Hadron picture breaks down?
  (How the transition between QCD dof and effective Hadrons occurs)

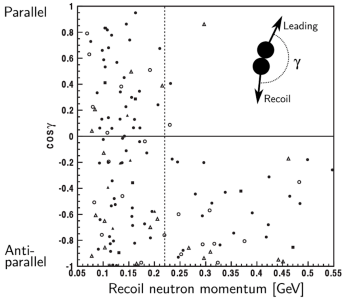
Introduction

(Semi)Inclusive experiments at quasi-elastic kinematics:

- Deeply bound nucleons emerge from Short-Range-Correlations
  High density ⇒ Large relative momentum ⇒ Relativistic desc.

Deuteron relevance:

- Pair correlations dominated by pn channel
  (compared to pp or nn)
- Indications that 3N-SRC could be a sequence of 2N-SRC



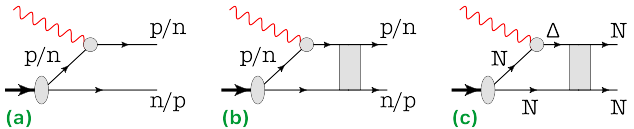
Piasetzky et.al. (2006)

Most important Diagrams:

$e + d \rightarrow e' + N_f + N_r$ (one photon exch.)

Assumption: $\Psi_d = \Psi_{pn}$

(pn Fock component dominance up to $p_i \sim 700$ (MeV/c))



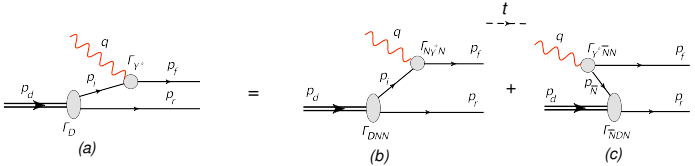
PWIA: Time ordering Feynman diagram

Short inter-nucleon distance ⇒ Large relative momentum $(\frac{m_N}{2} \lesssim p_i \simeq m_N)$

Needs:

- Deuteron in terms of its constituents (wave-function).
- The cross section for scattering of one of the constituents (off-shell CS).

Time-ordering Feynman diagram:



Identification of struck nucleon requires:

$$\mathbf{p}_f \sim \mathbf{q} \sim \text{a few(GeV/c)} \gg \mathbf{p}_r \sim \text{a few hundred (MeV/c)}$$
$$-q^2 = Q^2 \sim \text{a few (GeV/c)}^2$$

Instant Form (t-ordering):

- "Z-graph" contribution increases with Q^2 (can not be ignored).
- No wave function of deuteron in terms of *pn* component.

Light Front Dynamics

Instant Form (IF) and Light-Front Form (LF) Dynamics (Dirac1949):

- LF coordinates:

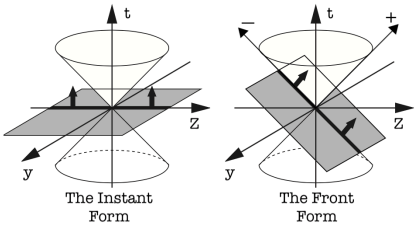
  $x^+ = t + z = \tau \; ; \;\; x^- = t - z$

  $\mathbf{x}_T = (x, y)$

- LF momentum (conserved at the interaction):

  $0 < p^+ = E + p_z \; ; \;\; \mathbf{p}_T = (p_x, p_y)$

- Momentum fraction: $\alpha_i = 2\frac{p_i^+}{P_T^+}$

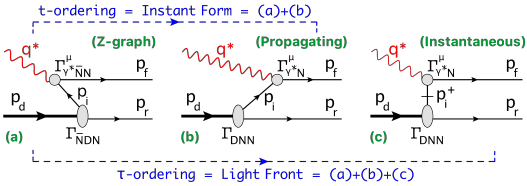


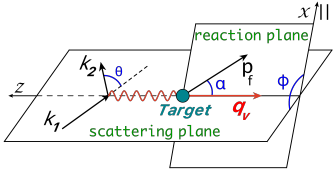
Advantages:

- Absence of vacuum fluctuations & possibility of vanishing negative energy propagator.
- Independence of internal & center of mass variables: $\Psi_{(p_1,p_2)} = \Psi_{cm(P_{cm})}\psi_{(k)}$

PWIA: Time ordering Feynman diagram

Large relative momentum (SRCs): $\frac{m_N}{2} \lesssim p_i$





Identification of 2N-SRCs imposes:

$$-q^2 = Q^2 \sim \ a\ few\ (GeV/c) \qquad p_f \sim \ a\ few\ (GeV/c)^2$$

For LF:

- No "Z-Graph" when q_V is opposite to $\hat{z}$ ($q_V^+ < 0$).
- Deuteron LF-WF and scattering process treated relativistically (all τ-ordered diagrams are included)

Deuteron LF-WF

Light Front the wave function factorizes: $\Psi^d = \Psi^d_{cm(\mathbf{P}_{cm})} \Psi^d_{(\mathbf{k})}$

(All the dynamics is in $\Psi^d_{(\mathbf{k})}$)

In the on-shell (static) limit: $\psi_d^{s_d}(\mathbf{k}) \approx N\, \bar{u}\left[\gamma \cdot e^{s_d}(\Gamma_1) + (p_1 - p_2) \cdot e^{s_d}(\Gamma_2)\right] u \approx$ Non-Rel

$$\psi_d^{s_d} = N\, \bar{u}(\mathbf{k}, s) \left[\gamma \cdot e^{s_d} \left(u(k) - \frac{1}{\sqrt{2}} w(k)\right) + \frac{k \cdot e^{s_d}}{k^2} \left((E - m) u(k) + (2E + m)\right) \frac{1}{\sqrt{2}} w(k)\right] u(\mathbf{k}, s)$$

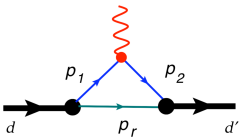
- $u(k)$ and $w(k)$ "known" (describe data) for, $k = p_1 - p_2 \lesssim 500(\mathrm{MeV/c})$
- k, is constructed from the LF variables of the nucleon.
- $E = \sqrt{m^2 + \mathbf{k}^2}$, $e^{s_d}_\mu$ are deuteron polarization vectors.

Normalization (N) is fixed relative to deuteron's elastic charge form-factor: $G_C(Q^2 = 0) = 1$

Deuteron EM current is given by 3 form factors:

$$\langle p'_d s'_d | J^\mu_d | p_d s_d \rangle = -\Big(G_1(Q^2)(e^*_{s'_d} \cdot e_{s_d}) - G_3(Q^2) \frac{(e^*_{s_d} \cdot q)(e_{s_d} \cdot q)}{2m_d^2} \Big) \left(p^\mu_d + p'^\mu_d\right)$$

$$+ G_M\left(Q^2\right) \left(e^\mu_{s_d} \left(e^*_{s'_d} \cdot q \right) - (e^\mu_{s'_d})^* (e_{s_d} \cdot q) \right)$$



It can be related to deuteron's WF
and nucleon EM current:

$$\langle p'_d s'_d | J^\mu_d | p_d s_d \rangle = \sum_{s_i} \int d^3 p_r \Psi^{s'_d \dagger}_d \ \tilde{J}^\mu_N \ \Psi^{s_d}_d$$

At $Q^2 = 0$, in the LF:

$$\frac{1}{3} \sum_{s_d} \langle p'_d s'_d | J^+ | p_d s_d \rangle = \left(p_d + p'_d\right)^+ G_1(0) = \left(p_d + p'_d\right)^+ G_C(0) = \left(p_d + p'_d\right)^+$$

Leading to:

$$\int \left|\Psi_d\left(\alpha, p_\perp\right)\right|^2 \frac{d\alpha}{\alpha} d^2 p_\perp = \frac{G_C(0)}{G_E(0)} = 1$$

where, $G_E(Q^2)$, is the Sachs electric form-factor of proton.

Deuteron Decay Vertex

On-Shell limit vs Realistic Off-Shell

Asymptotic NN scattering (on-shell limit):

- Only 2 independent momenta, $P_{cm} = p_1 + p_2$ $(\approx P_d)$, $p_{rel} = p_1 - p_2$.
- Only two parity even Dirac structures lead to: S and D partial waves.

Bound (Off-Shell) Decay function (projection operator):

- 3 independent momenta, $P_{cm} - P_d = \Delta \neq 0$
- Can have 6 parity even Dirac structures,

$$\Gamma_{DNN} \quad = \quad e_\mu^{s_d} \left[f_1 \gamma^\mu + f_2 k^\mu + f_3 \Delta^\mu + f_4 k^\mu \not{\Delta} + f_5 \Delta^\mu \not{\Delta} + \gamma_5 \epsilon_{\mu\nu\rho\sigma} P_{CM}^\nu P_d^\rho \Delta^\sigma \right]$$

$f_1 \to \Gamma_1$, and, $f_2 \to \Gamma_2$, when, $\Delta \to 0$ (on-shell limit).

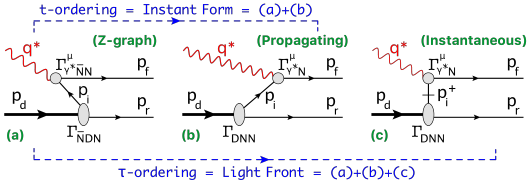
The structure is similar to Karmanov's one when written on the LF.

PWIA: Nuclear EM Current

LF perturbation rules on surviving diagrams ((**(b)** and **(c)**) give EM-current:

$$J^{\mu}_{N}(p_f s_f, p_i s_i) = \bar{u}(p_f s_f)\Gamma^{\mu}_{\gamma * N} u(p_i s_i)$$

$$+\ \bar{u}(p_f s_f)\Gamma^{\mu}_{\gamma * N}\frac{\gamma^{+}\left(p_i^{-} - p_{i\ on}^{-}\right)}{4m_N} u(p_i s_i)$$



Half off-shell electromagnetic vertex:

$$\Gamma^{\mu}_{\gamma^* N} = F_1(\gamma^{\mu} + q^{\mu}\frac{\not{q}}{Q^2}) + i\sigma^{\mu\nu}q_{\nu}F_2\frac{\kappa}{2m_N}$$

- $q_{\mu}J^{\mu}_{N} = 0$, as a consequence of a constrain at the interaction vertex (Ward-Takahashi)
- Nucleon's binding modifies Instant vertex: $\gamma^{+}\left(p_i^{-} - p_{i,on}^{-}\right) = 2\Delta\not{p}_i = \frac{\gamma^{+}_{i}}{p^{+}_{d}}\eta Q^2$
- Off-shell parameter: $\eta = \frac{1}{Q^2}\left(4\frac{(m_N^2 + \mathbf{p_T}^2)}{\alpha(2-\alpha)} - m_d^2\right)$

  Accounts for Off-Shell Effects.

PWIA: Cross Section

Within PWIA the cross section factorizes:

$$\frac{d\sigma}{d\epsilon_f d\Omega_{k_f} d^3p_f} = \sigma_{eN}\ S_d^{LF}(\alpha, \mathbf{p_T})$$

- Applies for any nuclei, *i.e.* the reaction mechanism is universal.
- $S_d^{LF}(\alpha, \mathbf{p_T})$, Spectral function.
- σ_{eN}, Electro bound-nucleon cross section:

$$\sigma_{eN} = \frac{1}{2p_d p_i}\ \sigma_{Mott}\ \frac{k_1}{E_f}\left(\eta_L\, V_L^N + \eta_{TL}\, V_{TL}^N \cos\phi + \eta_T V_T^N + \eta_{TT} V_{TT}^N \cos(2\phi)\right)$$

η's, V^N's leptonic and nucleonic structure functions (SF),

$$V_L^N = \frac{\mathbf{q}^2}{4Q^2}\left(H_N^{++}\frac{Q^2}{(q^+)^2} + 2H_N^{+-} + \frac{(q^+)^2}{Q^2}H_N^{--}\right) \qquad V_T^N = H_N^{\parallel\parallel} + H_N^{\perp\perp}$$

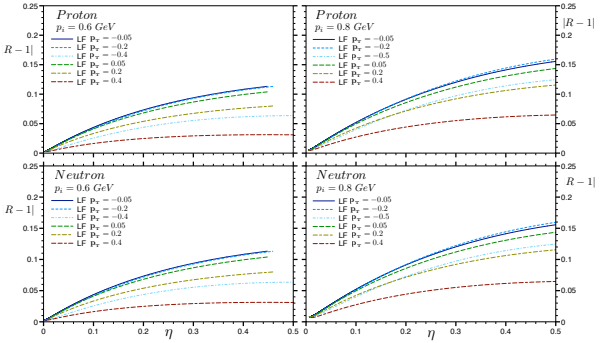
$$V_{TL}^N = \frac{|\mathbf{q}|}{q^+}\left(H_N^{+\parallel} + H_N^{-\parallel}\frac{(q^+)^2}{Q^2}\right) \qquad V_{TT}^N = H_N^{\parallel\parallel} - H_N^{\perp\perp}$$

- Explicit $H_N^{\mu\nu}$ matrix elements, no need to force current conservation: $J_N^- = J_N^+ q^-/q^+$.
- SF are polynomials in powers of: $\eta = \frac{1}{Q^2}\left(4\frac{(m_N^2 + \mathbf{p_T}^2)}{\alpha(2-\alpha)} - m_d^2\right) \implies$ Controls Off-Shell Effects.

Results of PWIA

Variable controlling off-shell effects: $\eta = \frac{1}{Q^2}\left(4\frac{(m_N^2+\mathbf{pT}^2)}{\alpha(2-\alpha)} - m_d^2\right)$

- σ_{eN} depends on nucleon's binding effects through powers of η
- $\eta < 0.1$ constrains off-shell effects to less than 5%



$R = \sigma_{eN}/\sigma_{eN}^{free}$ (binding effects), σ_{eN}^{free} (free cross sect.), σ_{eN} (bound cs)

Results

Kinematics relevant for experiment JLab E12-10-003 (Boeglin *et al.*, arXiv:1410.6770)

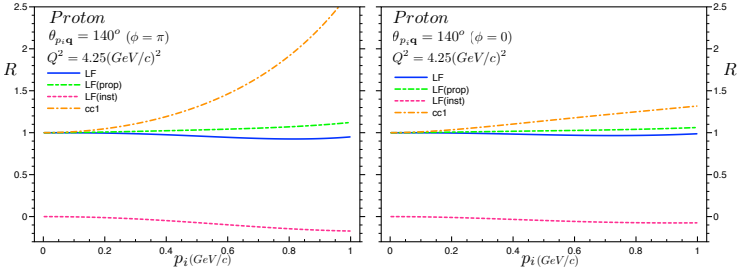


Figure 1: p_i distribution. $R = \frac{\sigma_{eN}}{\sigma_{eN}^{free}}$

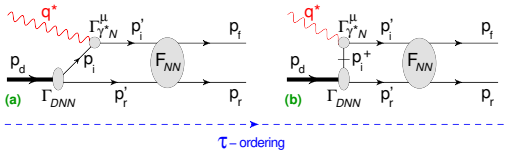
Comparison with de Forest cc1 prescription

Shows dramatic disagreement with increasing momentum of the bound nucleon.

Published: Phys. Rev. C **98**, no. 3, 035202. (arXiv:1805.04639)

Final State Interactions (FSI)

Generalized Eikonal Approximation:



- Provides perturbation calculus.
- Specially reliable when, $|p'_i| \sim |p_f|$, $|p_f| \sim |\mathbf{q}_V| > 1$
- In LF variables, $p_f^+ \sim p_i'^+ \sim 0$, $p_f^z \sim |\mathbf{q}_V|$
- Re-scattering mostly transverse to $\mathbf{q}_V$ (strong angular anisotropy)
- Current calculation (VNA). Reliable up to $\sim$ 500MeV/c
- Same EM-transition-current to PWIA (Half-off-shell).
  (Main contribution from the pole term)
- Expected reliability improvement up to 700 MeV/c.

Summary and Outlook

- Reaction mechanism and nuclear wave function description based on fully relativistic approach (Light Front dynamics).
- Identification of the parameter η that universally characterizes the off-shellness involved in the reaction mechanism.
- Expression for EM-current (J_N) can be used to evaluate main contribution to FSI within the generalized eikonal approximation.
- Expand a reliable description of data beyond $k = 500$ (MeV/c)

End

Acknowledgments:

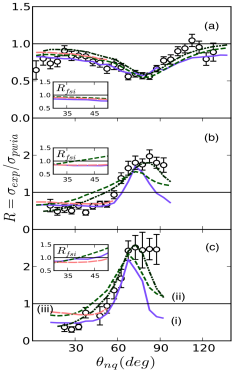
Chris Leon

Thanks!

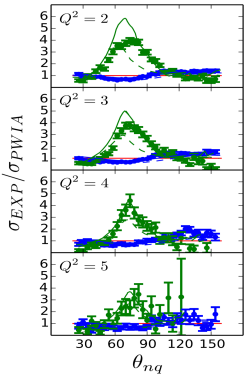
Backup Slides

Backup - Summary and Outlook

- Disagreement beyond $\theta_{nq} > 70^0$ likely related with intermediate resonance (Δ)
- Constrained to $x_{Bj} = \frac{Q^2}{2p_d \cdot q} > 1$, intermediate isobar state is suppressed.
  PWIA and FSI expected to provide very reliable description.



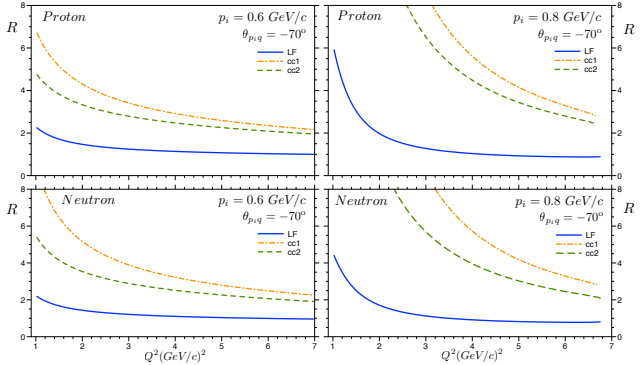
(I)



(II)

Backup - PWIA Results

Nucleon's off-shell effects reduces with Q^2:



$R = \sigma_{eN}/\sigma_{eN}^{free}$ (shows the binding effects)

Cross Sections for nucleon: σ_{eN} (bound)

σ_{eN}^{free} (free)

Backup - Final State Interactions (FSI)

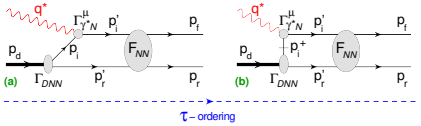
FSI in LF approach

Rescattering amplitude has two contributions:

- The pole of the propagator
  (main contribution)
- Principal value contribution

Within GEA:

- EM-transition-current the same as PWIA (Half-off-shell).
  (Main contribution from the pole term)



$$A_1^{\mu} = \sqrt{2(2\pi)^3 2m_N} \int \frac{d\alpha'_i d^2 p_{i\mathbf{T}}}{2\alpha'_i (2\pi)^3} \frac{\sqrt{s(s-4m_N^2)}}{2m_N \mathbf{q}} \frac{f_{pn}(p_{f\mathbf{T}} - p'_{i\mathbf{T}})}{\alpha'_i - \alpha_f - \frac{Q^2}{2\mathbf{q}^2}\frac{E_n - m_N}{m_N} + i\epsilon}$$
$$J_N^{\mu}(\alpha'_i, p'_{i\mathbf{T}}, \alpha_i, p_{i\mathbf{T}}) \Psi_d^{LF}$$

Expected reliability for nucleon momenta up to 700 MeV/c.

Backup - Nuclear Amplitude

LF perturbation rules applied to the propagating and instantaneous diagrams produces:

Propagating:

$$A^{\mu}_{prop} = -\bar{u}(p_f, s_f)\Gamma^{\mu}_{\gamma^* N}\,\frac{1}{p_i^+}\,\frac{(\not{p}_i + m_N)_{on}}{(p_d^- - p_r^- - p_{i,on}^-)}\,\bar{u}(p_r, s_r)\Gamma_{DNN}\chi^{s_d} \tag{1}$$

Instantaneous:

$$A^{\mu}_{inst} = -\bar{u}(p_f, s_f)\Gamma^{\mu}_{\gamma^* N}\frac{1}{p_i^+}\left(\frac{1}{2}\gamma^+\right)\bar{u}(p_r, s_r)\Gamma_{DNN}\chi^{s_d} \tag{2}$$

Factorization of the amplitude (PWIA):

$$A^{\mu} = A^{\mu}_{prop} + A^{\mu}_{inst} = \sum_{s_i} J^{\mu}_N\left(p_f s_f, p_i s_i\right)\,\frac{\psi^{s_i s_r s_d}_{LF}\left(\alpha, \mathbf{p_T}\right)}{\alpha}\sqrt{2\left(2\pi\right)^3}$$

Backup - Deuteron LF-WF

Dynamics for two interacting particles on the LF.

Schroedinger-like equation:

$$-i\frac{\partial}{\partial x^+}\Psi = H\Psi$$

LF 'time' ($\tau = x^+$) evolution.

In LF (kinematical boosts): $\Psi^d = \Psi^d_{cm(\mathbf{P}_{cm})}\Psi^d_{(\mathbf{k})}$

and,

$$\Psi^d(\mathbf{k}) = R^\dagger_{M1}(\mathbf{k})R^\dagger_{M2}(\mathbf{k})\frac{\psi^d(\mathbf{k})\left(\mathbf{k}_\mathbf{T}^2 + m^2\right)^{1/4}}{2(\alpha(1-\alpha))^{3/4}}$$

R_M (Melosh rotation) relates the LF $\mathbf{J}_{LF}$ and IF $\mathbf{J}_{cm}$ spin in the rest frame,

$$J_{LF} = R^\dagger_{M1}R^\dagger_{M2}\mathbf{J}_{cm}R_{M1}R_{M2} = \mathbf{L}(\mathbf{k}) + \mathbf{s}_1 + \mathbf{s}_2$$

with $\mathbf{J}_{LF}$, J^3_{LF} invariant, i.e. allows a partial wave decomposition:

$$\psi_d(\mathbf{k})^{s_d} = \bar{u}(\mathbf{p}_1)\left[\frac{\gamma^{s_d}}{\epsilon^{1/2}}\left(u(k) - \frac{1}{\sqrt{2}}w(k)\right) + \frac{(p_1 - p_2)^{s_d}}{\sqrt{2\epsilon}k^2}\left((\epsilon - m)u(k) + (2\epsilon + m)\right)w(k)\right]u(\mathbf{p}_2)$$

Known $u(k)$, $w(k)$ for $k < 500$ (MeV/c)