Double parton distributions of the pion in the NJL model

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Motivation for multi-parton distributions

- **Old story (Fermilab), renewed interest (e.g., ATLAS measurement for pp → W+2 jets 2013)** [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979, Gaunt, Stirling 2010, Diehl, Ostermeier, Schäfer 2012, ...], reviews: Bartalani et al. 2011, Snigirev 2011, Rinaldi, Ceccopieri 2018] (see Matteo Rinaldi’s talk this afternoon)

- **Model exploration** [MIT bag: Chang, Manohar, Waalewijn 2013], valon [WB+ERA 2013], constituent quarks: Rinaldi, Scopetta, Vento 2013, Rinaldi, Scopetta, Traini, Vento 2018]


Double parton scattering

[example from Łuszczak, Maciuła, Szczurek 2011]

DPS can be comparable to SPS at the LHC

Assumption: $D_{gg}(x_1, x_2, b) = g(x_1)g(x_2)F(b)$

– no correlations, transverse-longitudinal factorization
Intuitive probabilistic definition:

**Multi-parton distribution = probability distribution that struck partons have LC momentum fractions \( x_i \)**

Field-theoretic definition of (spin-averaged) sPDF and dPDF [Diehl, Ostermeier, Schaeffer 2012] of a hadron with momentum \( p \):

\[
D_j(x) = \int \frac{dz^-}{2\pi} e^{ixz^-p^+} \langle p \mid O_j(0, z) \mid p \rangle \bigg|_{z^+=0, z=0}
\]

\[
F_{j_1j_2}(x_1, x_2, y) = 2p^+ \int dy^- \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} e^{i(x_1z_1^- + x_2z_2^-)p^+} \times \langle p \mid O_{j_1}(y, z_1) O_{j_2}(0, z_2) \mid p \rangle \bigg|_{z_1^+=z_2^+=y^+=0, z_1=z_2=0}
\]

\[
O_q(y, z) = \frac{1}{2} \bar{q}(y - \frac{z}{2}) \gamma^+ q(y + \frac{z}{2}), \ldots
\]

\( y \) plays the role of the transverse distance between the two quarks

\( v^\pm = (v^0 \pm v^3)/\sqrt{2} \)
dPDF in momentum space

Fourier transform in $y$

$$F_{j_1 j_2}(x_1, x_2, y) \rightarrow \tilde{F}_{j_1 j_2}(x_1, x_2, q)$$

Special case of $q = 0$:

$$D_{j_1 j_2}(x_1, x_2) = \tilde{F}_{j_1 j_2}(x_1, x_2, q = 0)$$
Gaunt-Stirling sum rules


Fock-space decomposition on LC + conservation laws →

\[
|P\rangle = \sum_N \int d[x, k]_N \Phi(\{x_i, k_i\}) |\{x_i, k_i\}\rangle_N \\

\begin{align*}
&\quad d[x, k]_N = \prod_{i=1}^{N} \left[ \frac{dx_i d^2 k_i}{\sqrt{2(2\pi)^3 x_i}} \right] \delta \left( 1 - \sum_{i=1}^{N} x_i \right) \delta(2) \left( 1 - \sum_{i=1}^{N} k_i \right)
\end{align*}
\]
Fock-space decomposition on LC + conservation laws →

\[ \sum_i \int_0^{1-x_2} dx_1 \ x_1 D_{ij}(x_1, x_2) = (1-x_2)D_j(x_2) \quad \text{(momentum)} \]

\[ \int_0^{1-x_2} dx_1 \ D_{i\text{val}j}(x_1, x_2) = (N_{i\text{val}} - \delta_{ij} + \delta_{\bar{i}j})D_j(x_2) \quad \text{(quark number)} \]

\( A_{i\text{val}} \equiv A_i - A_{\bar{i}} \)

\[ N_{i\text{val}} = \int_0^1 dx \ D_{i\text{val}}(x) \]

- Preserved by DGLAP evolution
- Non-trivial to satisfy with the (guessed) function
- Checked in light-front perturbation theory and in lowest-order covariant calculations in [Diehl, Plößl, Schäfer 2019]

Important and fundamental constraints!
Simple example (valon model)

$|\Lambda\rangle = |uds\rangle$ (to avoid the complications of indistinguishable partons)

$$D_{uds}(x_1, x_2, x_3) = f(x_1, x_2, x_3)\delta(1 - x_1 - x_2 - x_3)$$

$$D_{ud}(x_1, x_2) = \int dx_3 D_{uds}(x_1, x_2, x_3) = f(x_1, x_2, 1 - x_1 - x_2)$$

$$D_{us}(x_1, x_3) = \ldots$$

$$D_u(x_1) = \int_0^{1-x_1} dx_2 D_{ud}(x_1, x_2) = \int_0^{1-x_1} dx_3 D_{us}(x_1, x_3)$$

$$\int_0^{1-x_1} dx_2 x_2 D_{ud}(x_1, x_2) + \int_0^{1-x_1} dx_3 x_3 D_{ud}(x_1, x_3)$$

$$= \int dx_2 dx_3 (x_2 + x_3) D_{uds}(x_1, x_2, x_3) = \int dx_2 dx_3 (1 - x_1) D_{uds}(x_1, x_2, x_3)$$

$$= (1 - x_1) D_u(x_1)$$
Attempts of bottom-up construction

- Gaunt, Stirling (2011)

\[ D_{ij}(x_1, x_2) = D_i(x_1) D_j(x_2) \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}} \]

(do not satisfy the GS sum rules)

- Lewandowska, Golec-Biernat 2014

\[ D_{ij}(x_1, x_2) = \frac{1}{1 - x_2} D_i \left( \frac{x_1}{1 - x_2} \right) D_j(x_2) \]

... (no parton exchange symmetry, negative \( D_{qq} \) at large \( x \))

- Can never be unique: marginal projections do not determine the two-particle distribution

Problems!
Construct the multiparticle distribution (model, data?) and go down with marginal projections

[cf. a similar in spirit “top-down” study by M. Rinaldi et al. 2018 with the Brodsky - de Teramond AdS/CFT soft wall pion wave function]
Chiral quark models

- $\chi_{SB}$ breaking $\rightarrow$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, $W$, $Z$)
- Large-$N_c$ $\rightarrow$ one-quark loop
- Regularization

Pion - Goldstone boson of $\chi_{SB}$, fully relativistic $q\bar{q}$ bound state of the Bethe-Salpeter equation

Quantities evaluated at the quark model scale (where constituent quarks are the only degrees of freedom)
Chiral quark models

- \( \chi_{SB} \) breaking \( \rightarrow \) massive quarks
- Point-like interactions
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Pion – Goldstone boson of \( \chi_{SB} \), fully relativistic \( q\bar{q} \) bound state of the Bethe-Salpeter equation

Need for evolution
Gluon dressing, renorm-group improved
sPDF in NJL

[Davidson, Arriola, 1995]

\[ q_{\text{val}}(x; Q_0) = 1 \times \theta[x(1 - x)] \]

(proper treatment of symmetries with regularization)

Quarks are the only degrees of freedom, hence saturate the sPDF sum rules:

\[ \int_0^1 dx \, q_{\text{val}}(x; Q_0) = 1 \text{ (valence), } 2 \int_0^1 dx \, x q_{\text{val}}(x; Q_0) = 1 \text{ (momentum)} \]
Scale and evolution

QM provide non-perturbative result at a low scale $Q_0$

$$F(x, Q_0)|_{\text{model}} = F(x, Q_0)|_{\text{QCD}}, \quad Q_0 - \text{the matching scale}$$

Quarks carry 100\% of momentum at $Q_0$, adjusted such that when evolved to $Q = 2$ GeV, they carry the experimental value of 47\% (radiative generation of gluons and sea quarks)

LO DGLAP evolution

$Q_0 = 313^{+20}_{-10}$ MeV

NLO close to LO

$$\sim (1 - x)^p \frac{4C_F}{\beta_0} \log \frac{\alpha(Q_0)}{\alpha(Q)}$$
Pion valence quark PDF, NJL vs E615

points: Fermilab E615 Drell-Yan, $\pi^{\pm} W \rightarrow \mu^+ \mu^- X$

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q = 4$ GeV

Many predictions for related quantities of the pion: DA, GPD, TDA, TMD, quasi/pseudo DA/PDF...
dPDF of the pion in NJL model

\[ D_{ud}(x_1, x_2) = 1 \times \delta(1 - x_1 - x_2) \theta(x_1) \theta(x_2) \]

- Special case of the valon model
- GS sum rules satisfied
- ... at the quark-model scale → need for evolution
dDGLAP evolution in the Mellin space

[Kirschner 1979, Shelest, Snigirev, Zinovev 1982]: method of solving dDGLAP based on the Mellin moments, similarly to sPDF

\[ M_j^n = \int_0^1 dx \, x^n \, D_j(x), \quad M_{j_1 j_2}^{n_1 n_2} = \int_0^1 dx_1 \int_0^1 dx_2 \, \theta(1-x_1-x_2) \, x_1^{n_1} \, x_2^{n_2} \, D_{j_1 j_2}(x_1, x_2) \]

\[ \frac{d}{dt} M_{j_1 j_2}^{n_1 n_2} = \sum_i P_{i \to j_1}^{n_1} M_{i j_2}^{n_1 n_2} + \sum_i P_{i \to j_2}^{n_2} M_{j_1 i}^{n_1 n_2} + \sum_i \left( P_{i \to j_1 j_2}^{n_1 n_2} + \tilde{P}_{i \to j_1 j_2}^{n_1 n_2} \right) M_{i}^{n_1+n_2} \]

\[ t = \frac{1}{2\pi \beta} \log \left[ 1 + \alpha_s(\mu) \beta \log(\Lambda_{\text{QCD}}/\mu) \right] \text{ (single scale for simplicity), } \beta = \frac{11N_c - 2N_f}{12\pi} \]

(inhomogeneous term from coupling to sPDF’s)

For valence-valence distributions there are no partons \( i \) decaying into a pair of valence quarks \( (P_{i \to j_1 j_2} = 0) \) \( \rightarrow \) inhomogeneous term vanishes

\[
\begin{align*}
\text{dPDF :} & \quad \frac{d}{dt} M_{j_1 j_2}^{n_1 n_2}(t) = \left( P_{j_1 \to j_1}^{n_1} + P_{j_2 \to j_2}^{n_2} \right) M_{j_1 j_2}^{n_1 n_2}(t) \\
\text{sPDF :} & \quad \frac{d}{dt} M_j^n(t) = P_{j \to j}^{n} M_j^n(t)
\end{align*}
\]
Solution

[10 lines in Mathematica (!)]

\[ M_j^n(t) = e^{P_{j\rightarrow j}(t-t_0)} M_j^n(t_0) \]
\[ M_{j_1j_2}^{n_1n_2}(t) = e^{[P_{j_1\rightarrow j_1} + P_{j_2\rightarrow j_2}](t-t_0)} M_{j_1j_2}^{n_1n_2}(t_0) \]

inverse Mellin transform:

\[ D_j(x; t) = \int_C \frac{dn}{2\pi i} x^{-n-1} M_j^n(t) \]
\[ D_{j_1j_2}(x_1, x_2; t) = \int_C \frac{dn_1}{2\pi i} x_1^{-n_1-1} \int_{C'} \frac{dn_2}{2\pi i} x_2^{-n_2-1} M_{j_1j_2}^{n_1n_2}(t) \]

\( n \) and \( n' \) are complex variables and the contours \( C \) and \( C' \) lie right to singularities of \( M \)

- correlations \( M_{j_1j_2}^{n_1n_2}(t) \neq M_{j_1}^{n_1}(t) M_{j_2}^{n_2}(t) \) – no separability
- valence-valence: \( M_{j_1j_2}^{n_1n_2}(t)/[M_{j_1}^{n_1}(t) M_{j_2}^{n_2}(t)] \) independent of \( t \)
$x_1 x_2 D^\pi_{ud} (x_1, x_2)$
$D_{u\bar{d}}^{\pi^+}(x_1, x_2) - \text{log scale}$

\begin{align*}
\mu &= 0.8\text{GeV} \\
\mu &= 2\text{GeV} \\
\mu &= 10^3\text{GeV} \\
\mu &= 10^{12}\text{GeV}
\end{align*}
\[ \frac{D_{u \bar{d}}^{\pi^+}(x_1, x_2)}{D_u(x_1)D_{\bar{d}}(x_2)} \]

\( \mu = 0.8 \text{GeV} \)

\( \mu = 2 \text{GeV} \)

\( \mu = 10^3 \text{GeV} \)

\( \mu = 10^{12} \text{GeV} \)
Gluon correlation

[Golec-Biernat, Lewandowska, Serino, Snyder, Staśto 2015]

\[ x_2 = 0.01 \]

\[ D_{gg}^{Q^2} = \begin{cases} 1 \text{ GeV}^2 \\
10 \text{ GeV}^2 \end{cases} \]

\[ \text{our prod} \]

\[ \text{ratio} \]

\[ Q^2 = \begin{cases} 1 \text{ GeV}^2 \\
10 \text{ GeV}^2 \end{cases} \]
Gluon correlation

[Golec-Biernat, Lewandowska, Serino, Snyder, Staśto 2015]

$x_2 = 0.5$

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Valence moments in NJL

\[
\frac{\langle x_1^nx_2^m \rangle}{\langle x_1^n \rangle \langle x_2^m \rangle} = \frac{(1+n)!(1+m)!}{(1+n+m)!} \quad (\text{NJL, any scale})
\]

(independent of the evolution scale)

Double moments reduced compared to product of single moments

[lattice results coming shortly, Zimmermann et al.]
Top-down strategy of constructing multi-parton distributions → formal features guaranteed, in particular GS sum rules
Phenomenological sPDF’s as constraints
NJL → valon initial condition, \( \text{const} \times \delta(1 - x_1 - x_2) \), dDGLAP
Correlations decrease with increasing evolution scale and are probably not very important (±25%) in the range probed by experiments, justifying the product ansatz in that limit
Moments measure the \( x_1 - x_2 \) factorization breaking; can be verified in forthcoming lattice calculations