Improved opacity expansion for in-medium parton splitting

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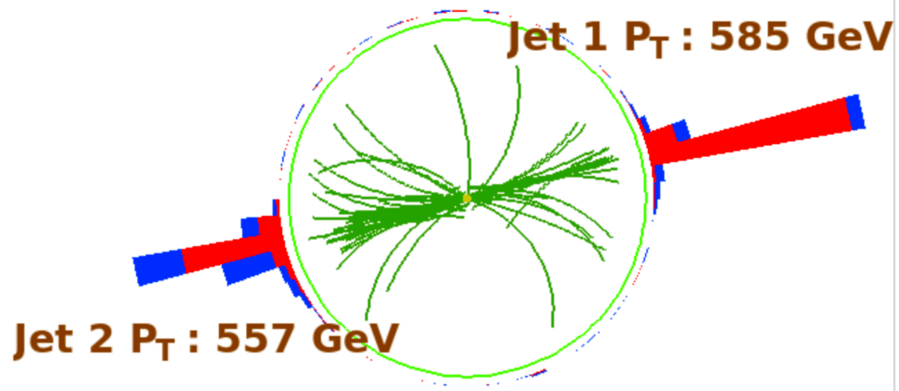
Outline

- Jet quenching in heavy ion collisions
- QCD analog of the LPM effect and its analytic limits: single hard and multiple soft scattering approximations
- Improved opacity expansion
- Medium-induced gluon spectrum
- Outlook

Jet quenching

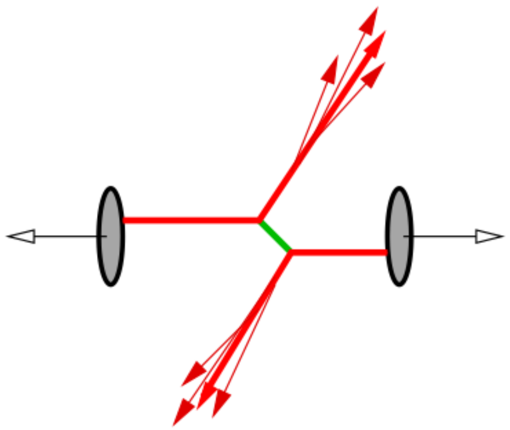
Jet quenching in heavy ion collisions

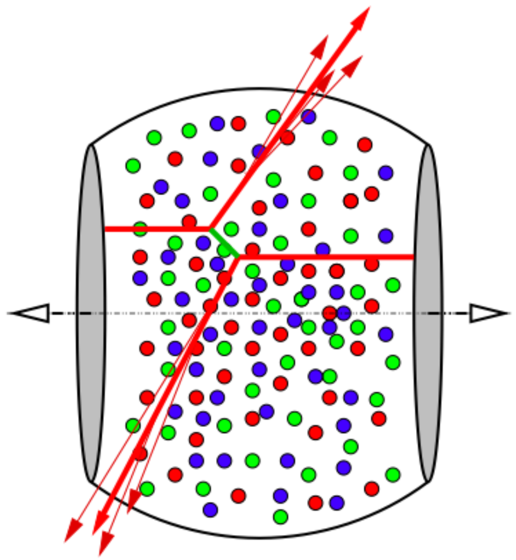
- Jets are collimated spray of particles observed in high energy collisions (e+e-, electron-proton, proton-proton, ion-ion)

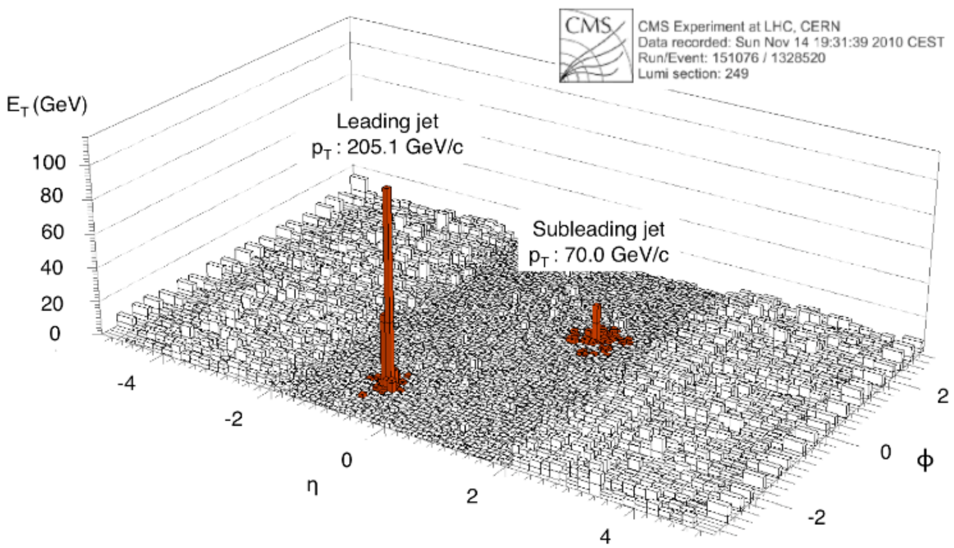


(CMS collaboration)

- High energy jets traversing a deconfined matter lose energy via radiative processes → Jet quenching



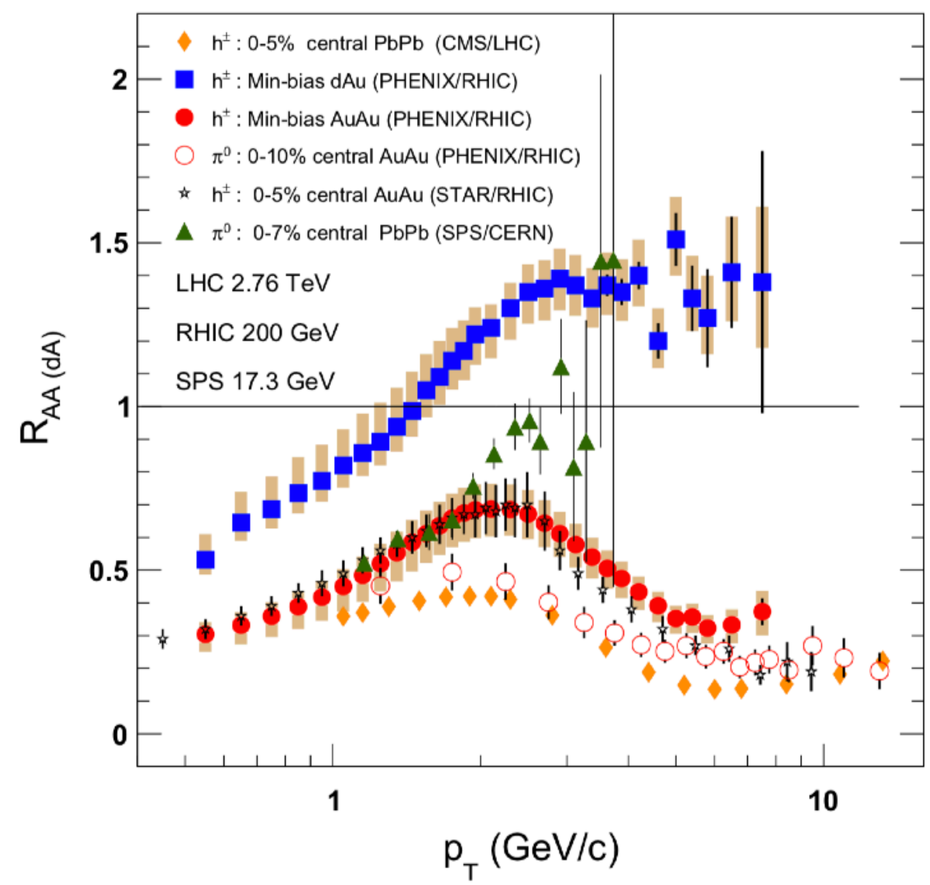




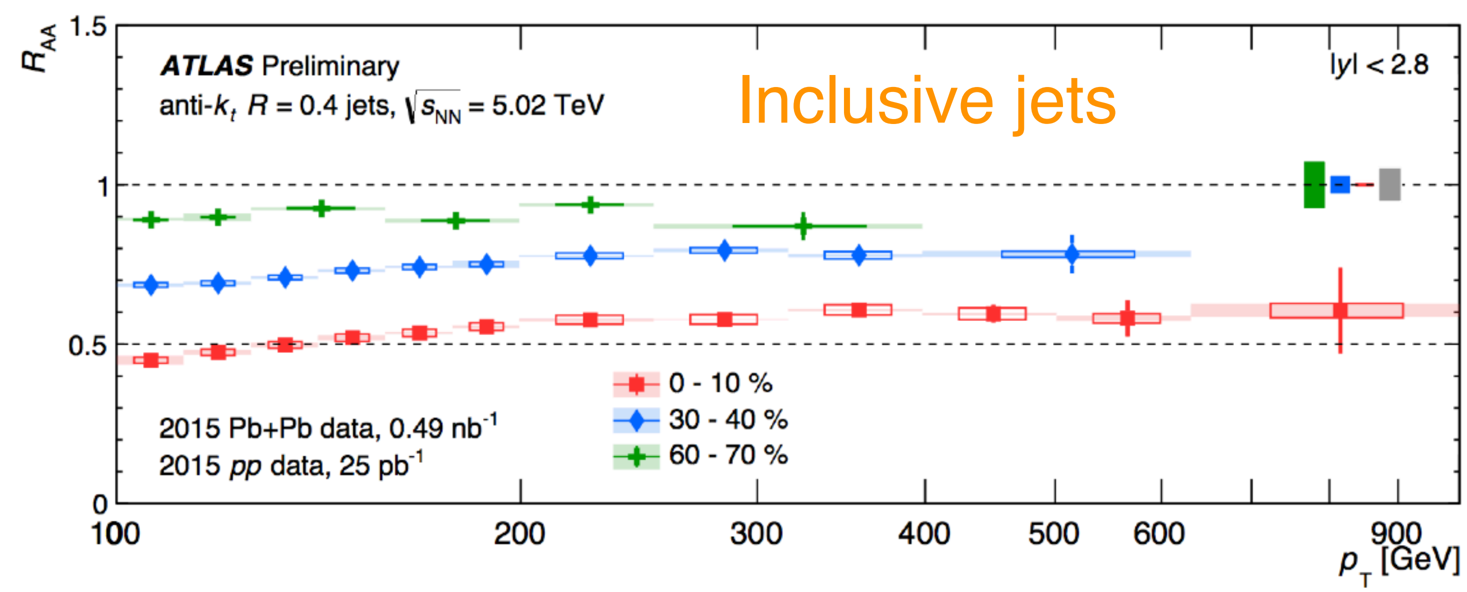
Jet quenching in heavy ion collisions

- Two decades after Bjorken prediction, jet quenching phenomenon was observed at RHIC in the suppression of high-pT hadrons and confirmed at LHC where a strong suppression of 1 TeV jets was observed
- Use jets as test particles to learn about the properties of of the Quark-Gluon-Plasma (QGP)

Inclusive hadrons



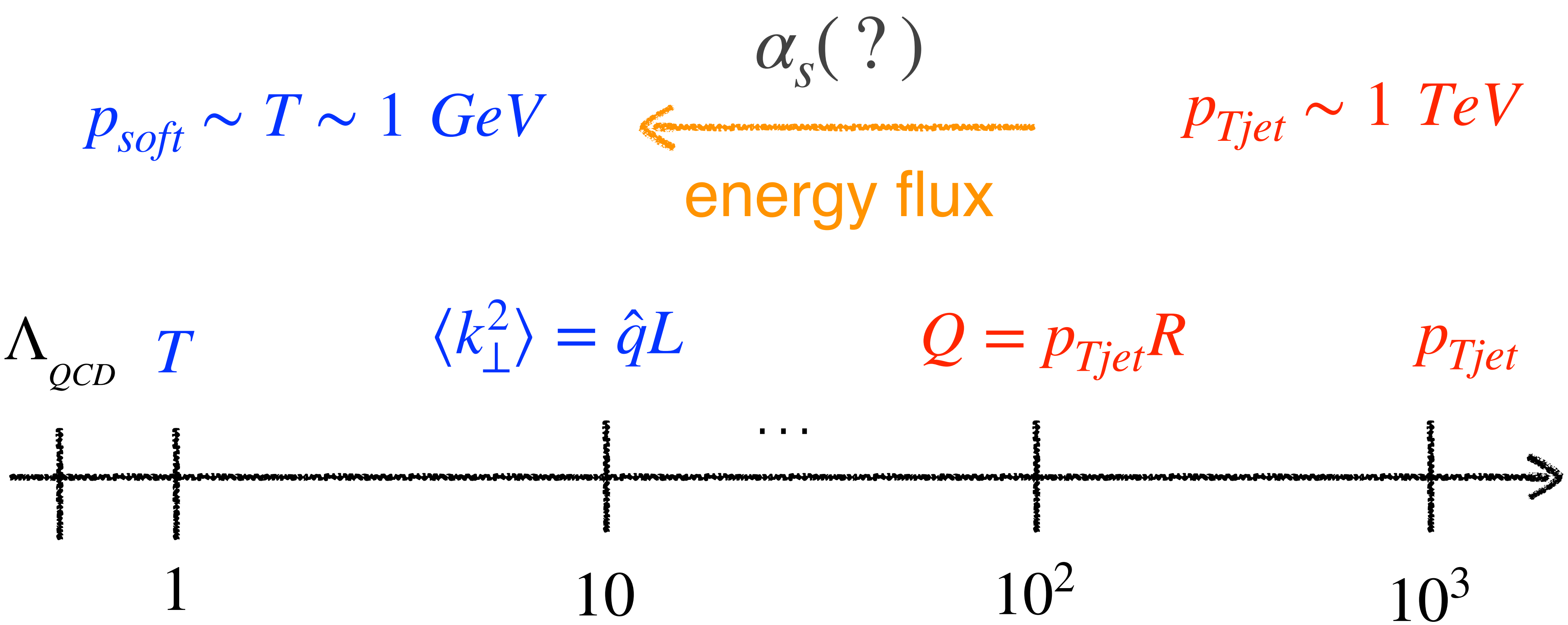
$$R_{AA} \equiv \frac{1}{N_{coll}} \frac{\mathrm{d}N_{AA}/\mathrm{d}p_T}{\mathrm{d}N_{pp}/\mathrm{d}p_T}$$



Jet quenching in heavy ion collisions

- Physics questions:

how does a jet as a multi-parton (and multi-scale) system interact with the QGP?



How is energy transported from energetic partons to low momenta and dissipated in the QGP?

Weak coupling picture of jet quenching

- **Elastic processes:** diffusion in transverse momentum space

$$\hat{q} \equiv \frac{\mathrm{d}\langle k_T^2 \rangle_{typ}}{\mathrm{d}t} \sim \alpha_s^2 C_R n \ln \frac{Q^2}{m_D^2} \sim \alpha_s^2 T^3$$

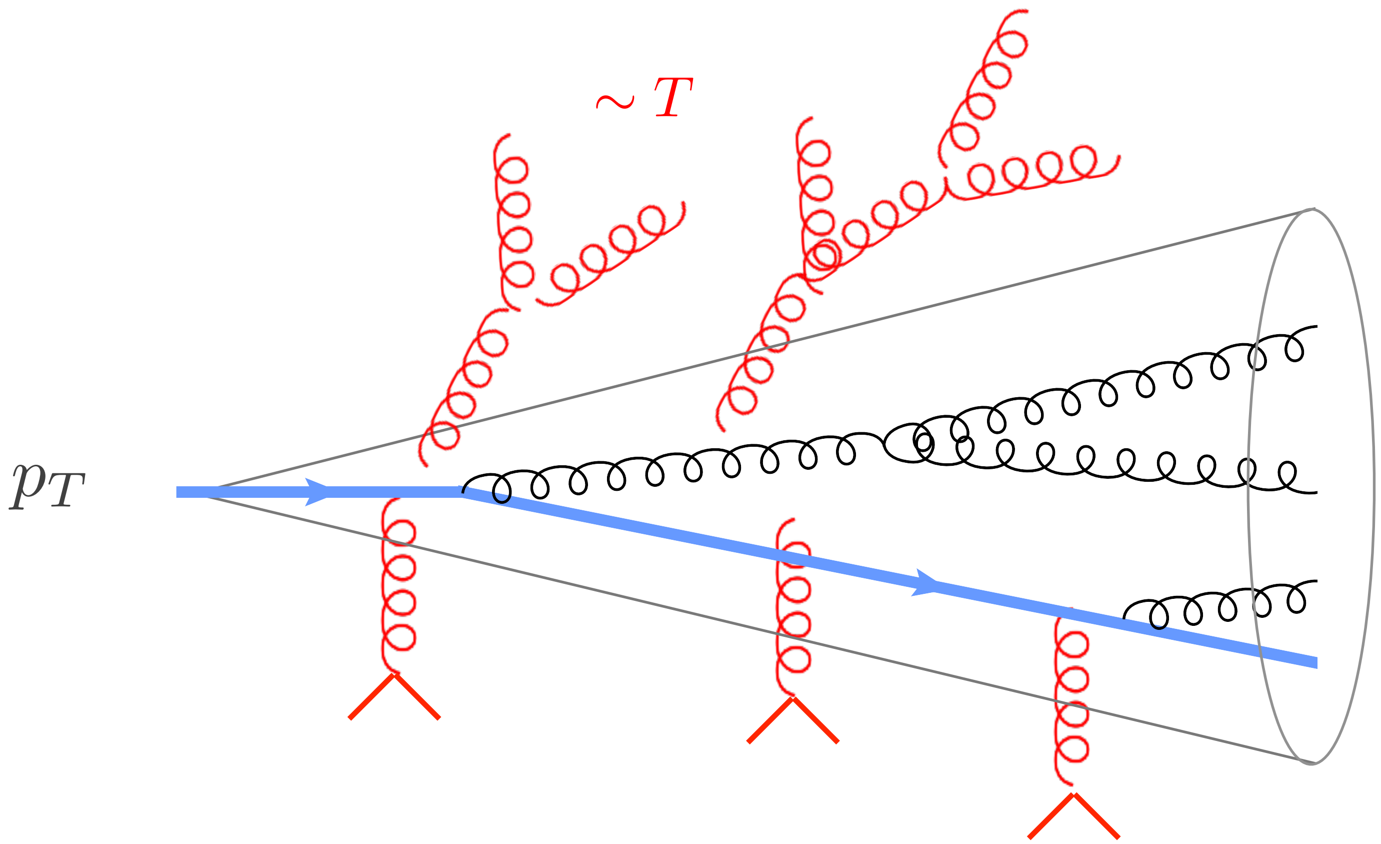
[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996) Arnold, Moore, Yaffe (2002)

- Multiple scattering trigger abundant soft gluon radiation (requires resummation)

- Large angle turbulent cascade
(constant flow of energy from p_T to T

→ minijet thermalization

[Blaizot, Iancu, MT (2013), Iancu, Wu (2015)]



Typical energy loss

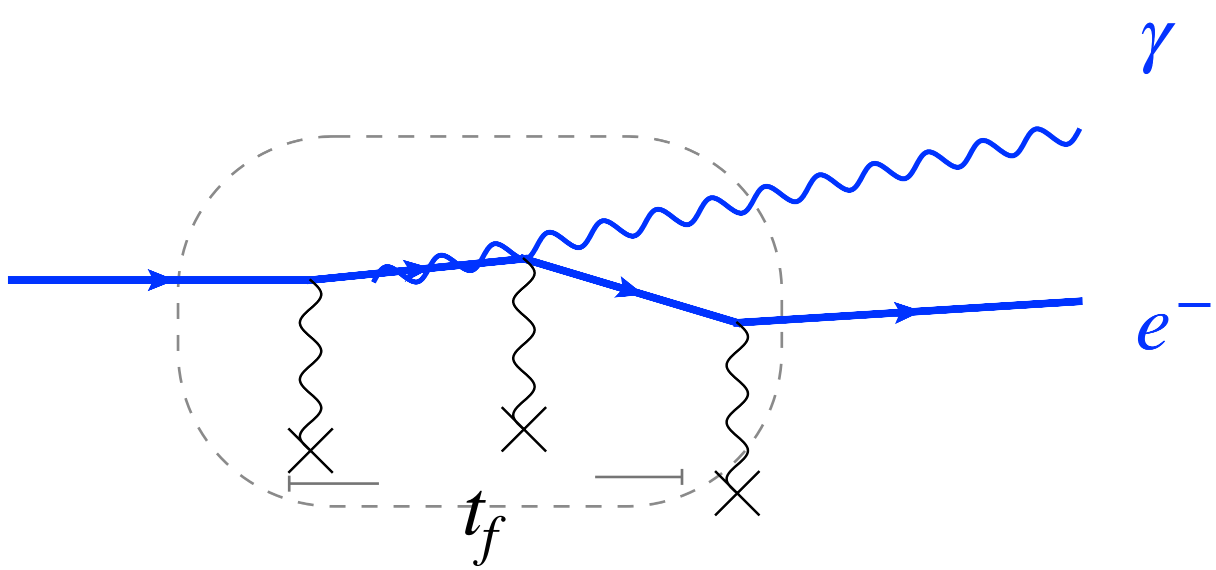
$$\langle E \rangle_{typ} \sim \bar{\alpha}^2 \, \hat{q} \, L^2$$

L: length of the medium

The LPM effect on the back of the envelop

The LPM effect on the back of the envelop

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects **(Landau-Pomeranchuk-Migdal 1953)**

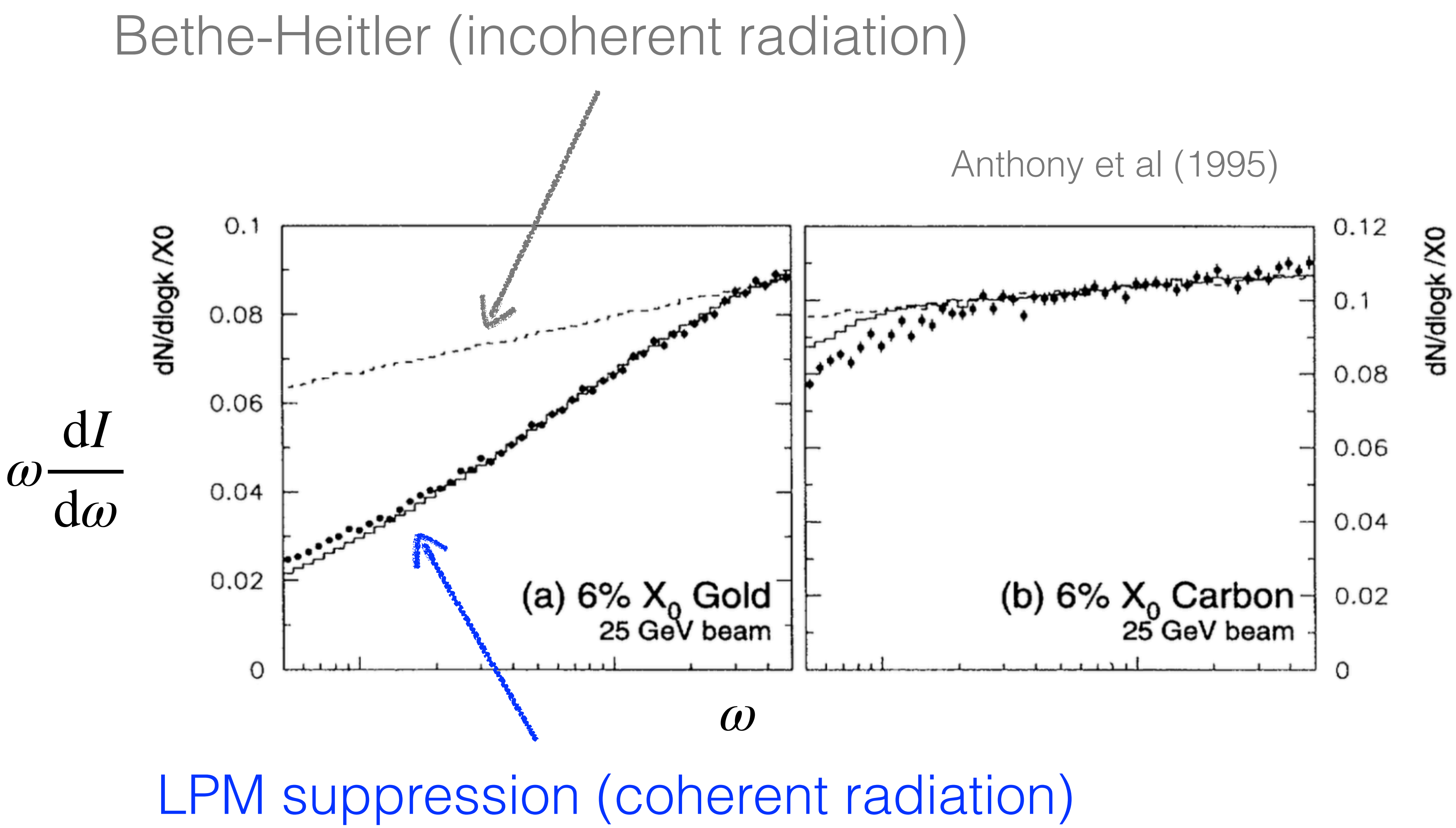


- During the quantum mechanical formation time (coherence length) N_{coh} scattering centers act coherently surpassing the radiation spectrum

$$\omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_e N_{eff} \sim \alpha_e \frac{N_{scatt}}{N_{coh}} \sim \alpha_e \frac{L}{t_f(\omega)}$$

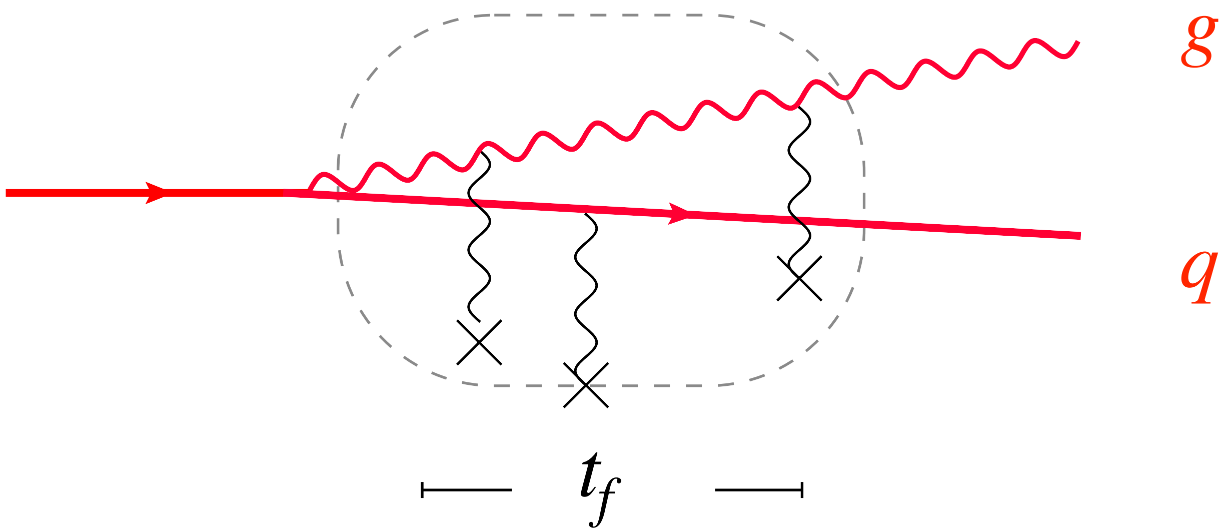
The LPM effect on the back of the envelop

- The LPM effect was observed at SLAC in 1995



The LPM effect on the back of the envelop

- Analog effect in QCD except the gluon interacts with the plasma and suffers "brownian kicks"



formation time

$$t_f(\omega) = \frac{\omega}{k_\perp^2}$$

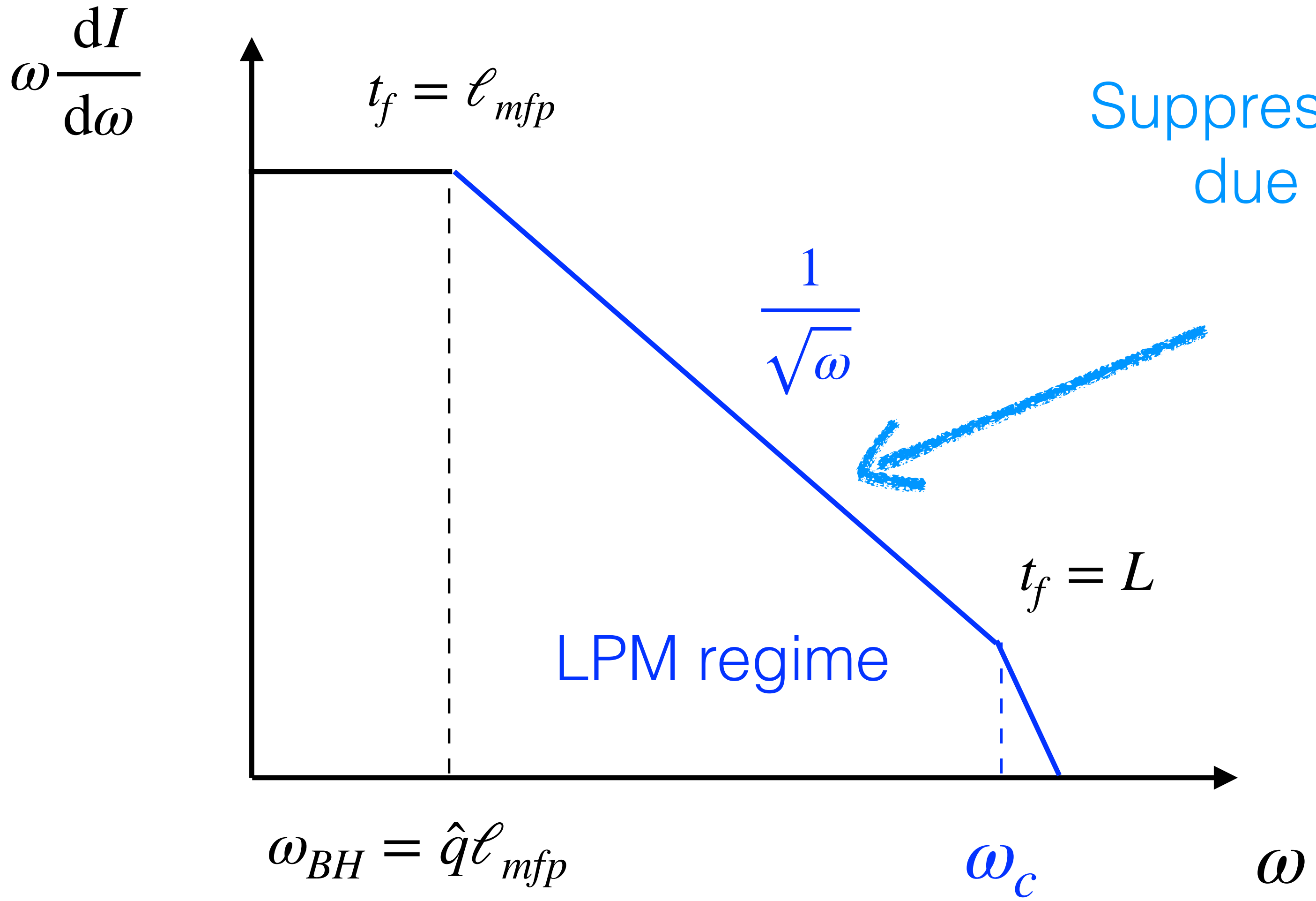
$$k_\perp^2 \sim \hat{q}\, t_f$$

- In QCD the spectrum is suppressed in the UV

$$t_f(\omega) = \sqrt{\frac{\omega}{\hat{q}}} \quad \text{and} \quad \omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_s \sqrt{\frac{\omega}{\hat{q}}}\, L \propto \frac{1}{\sqrt{\omega}}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

The LPM effect on the back of the envelop



Suppression of high frequencies due to coherence effects

- Maximum radiation frequency: $\omega_c = \hat{q}L^2$
- Minimum radiation angle (no mass singularity): $\theta_c = \frac{1}{\sqrt{\hat{q}L^3}}$
- Medium-induced gluon radiation spectrum is the building block of jet evolution in a QCD medium

Medium-Induced radiative gluon spectrum and its two limits

Elements of the formalism

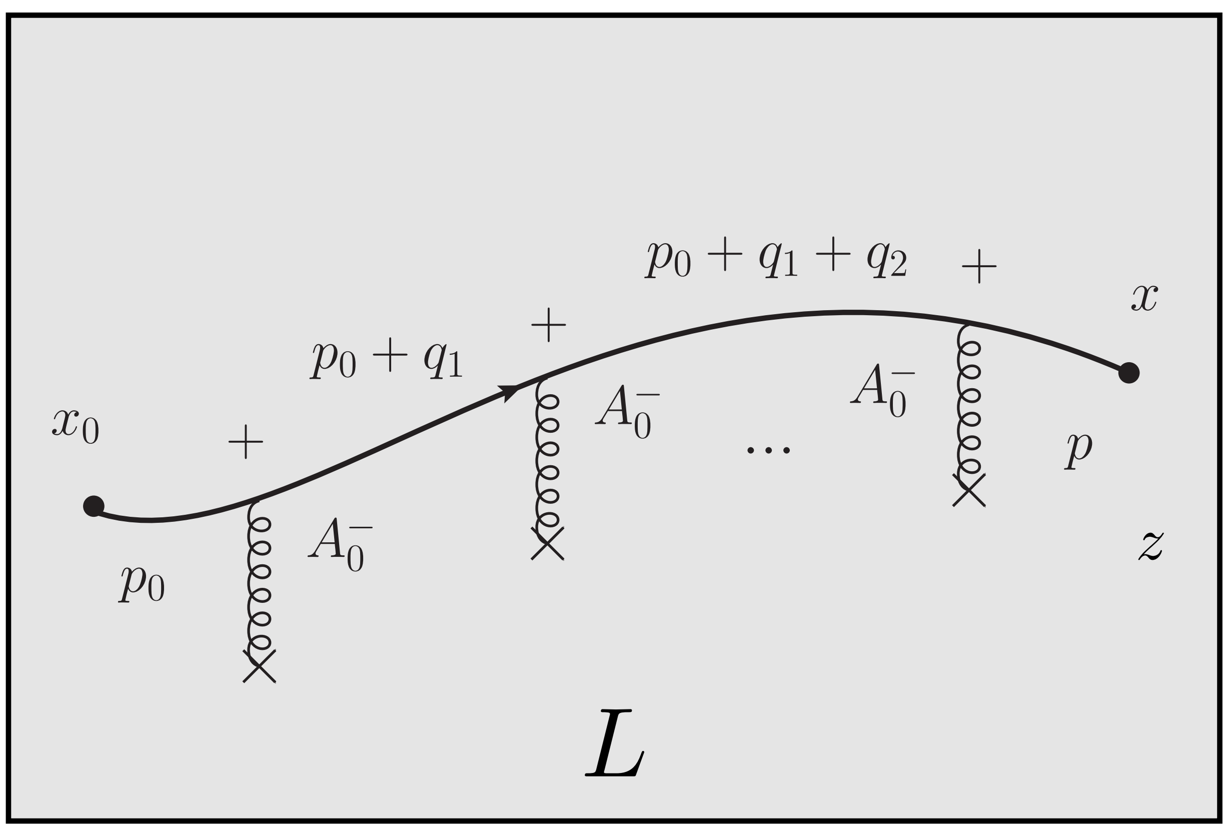
- Working assumption: neglect power corrections of the small momentum transfer $q^+ \ll p^+$

$$\text{eikonal vertex} \sim \delta(q^+)\, p^\mu \;\Leftrightarrow\; \mathcal{A}^-(x^+, x_\perp)$$

- Large medium: allow the gluon to explore the transverse plane between two scatterings. The Parton acquires an order one phase:

$$p^- L^+ \sim (p_\perp^2/p^+)\, L^+ \sim 1$$

$(p^+, 0^-, 0_\perp)$



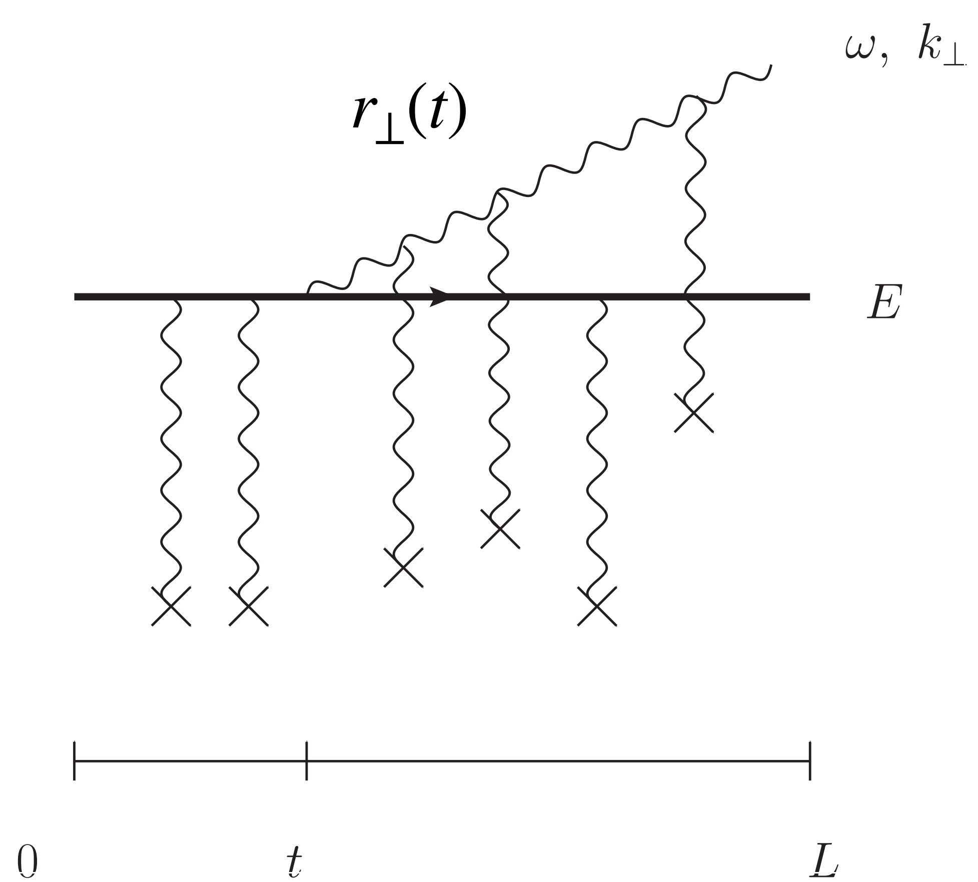
$$\left(p^+, \frac{q_\perp^2}{2p^+}, q_\perp\right)$$

Wilson line

$$\sim U[\mathcal{A}^-(x(t), t)]$$

Elements of the formalism

- The gluon spectrum is related to the 2-point correlation function



$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \langle \mathrm{tr}\, U_A(r(t))\, U^\dagger(0) \rangle_A$$

- Path ordered Wilson line

$$U_A(r(x^+)) \equiv \mathcal{P}_+ \exp\left[ig \int \mathrm{d}x^+ A^-(x^+, \boldsymbol{r}(x^+))\right]$$

- High energy limit → 2-D non-relativistic quantum mechanics

Independent multiple scattering approximation

- At weak coupling $\alpha_s \ll 1$ (kinetic description):

$$1/m_D \ll \ell_{mfp} \ll L$$

- Medium average: assume Gaussian random variable

$$\langle \mathcal{A}_a^-(q_\perp,t)\,\mathcal{A}_b^-({}'q_\perp,t')\rangle \equiv \delta^{ab}\,\delta(t-t')\,\delta(q_\perp - q'_\perp)\,\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

- Static scattering centers

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp} \equiv \frac{g^4 n}{(q_\perp^2+\mu^2)^2}$$

Gyulassy-Wang (1992)
Gyulassy-Levai-Vitev (2000)

- Thermal medium (HTL)

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp} \equiv \frac{g^2 m_D^2 T}{q_\perp^2(q_\perp^2+\mu^2)}$$

Aurenche-Gelis-Zakaret (2000)

- Large momentum transfer is given by the 2 to 2 QCD matrix element: $d\sigma_{el}/d^2q_\perp \sim q_\perp^{-4}$ for $q_\perp \gg \mu$

→ Average transverse momentum enhanced by a large Coulomb logarithm $k_\perp^2 \sim L\int^{Q^2} \mathrm{d}q_\perp q_\perp^2 \frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp} \propto nL\,\ln\frac{Q^2}{\mu^2}$

Medium-induced gluon radiation spectrum

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_R}{\omega^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1 \\ \times \boldsymbol{\partial}_x \cdot \boldsymbol{\partial}_y \left[\mathcal{K}(\boldsymbol{x}, t_2 | \boldsymbol{y}, t_1) - \mathcal{K}_0(\boldsymbol{x}, t_2 | \boldsymbol{y}, t_1) \right]_{\boldsymbol{x}=\boldsymbol{y}=\boldsymbol{0}}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

- The Green's function K obeys a Schrödinger equation

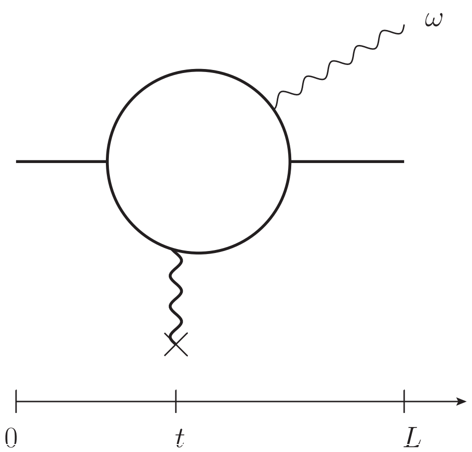
$$\left[i\frac{\partial}{\partial t} + \frac{\boldsymbol{\partial}^2}{2\omega} + i\sigma(\boldsymbol{x}) \right] \mathcal{K}(\boldsymbol{x}, t | \boldsymbol{y}, t_1) = i\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - t_1)$$

- The imaginary potential is rated to the dipole amplitude

$$\sigma(\boldsymbol{x}, t) = N_c \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2\boldsymbol{q}} \left(1 - \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right) \quad \sim x_\perp^2 \ln \frac{1}{\mu^2 x_\perp^2}$$

Medium-induced gluon radiation spectrum

- Two analytic approximations in the literature (implemented in various MC event generators)



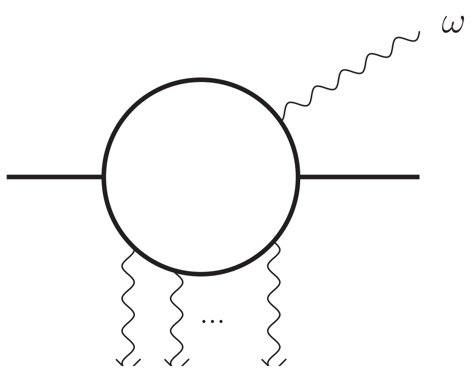
1. Dilute medium: single-hard scattering approximation (Opacity expansion, Higher-Twist)

Gyulassy-Levai-Vitev (2000) Guo, Wang (2000)

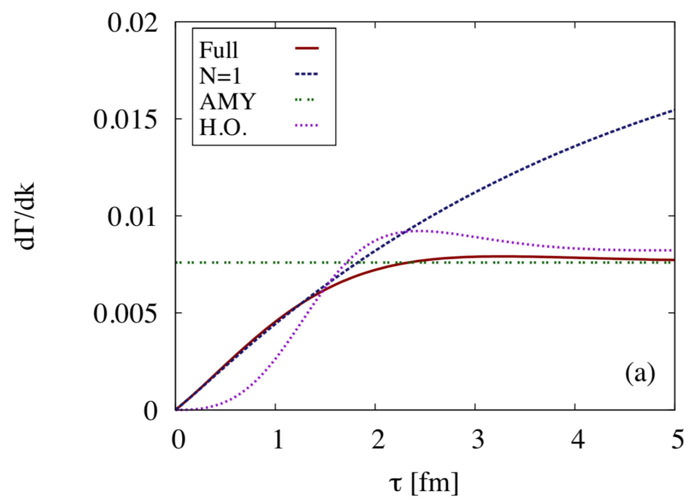
2. Dense medium: multiple-soft scattering. All order resummation by neglecting the Coulomb logarithm: Harmonic oscillator

$$\sigma(x_\perp) \sim x_\perp^2$$

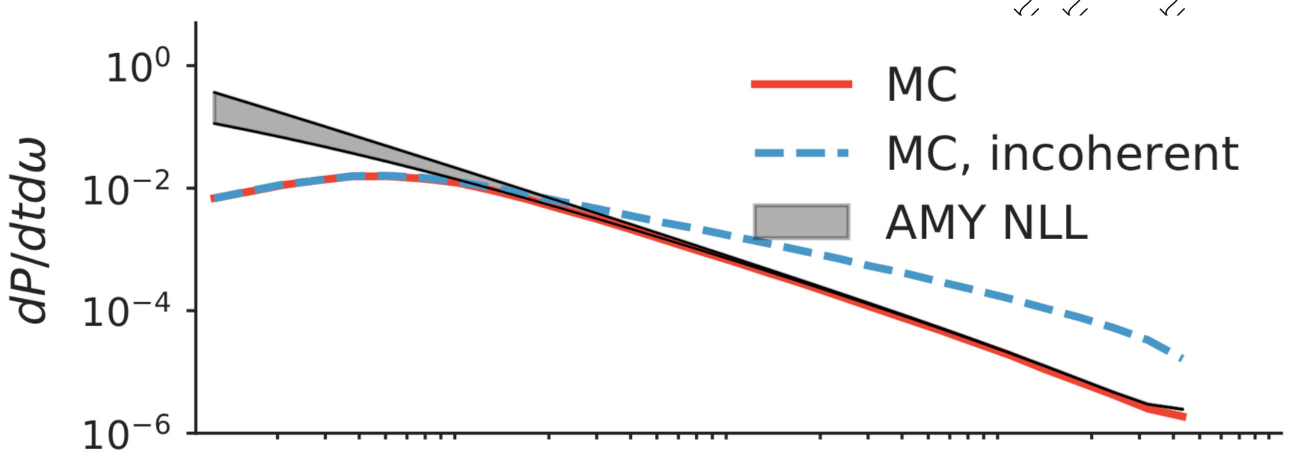
Baier, Dokshitzer, Mueller, Peigné, Schiff (1996) Zakharov (1997)



- Exact numerical solutions:



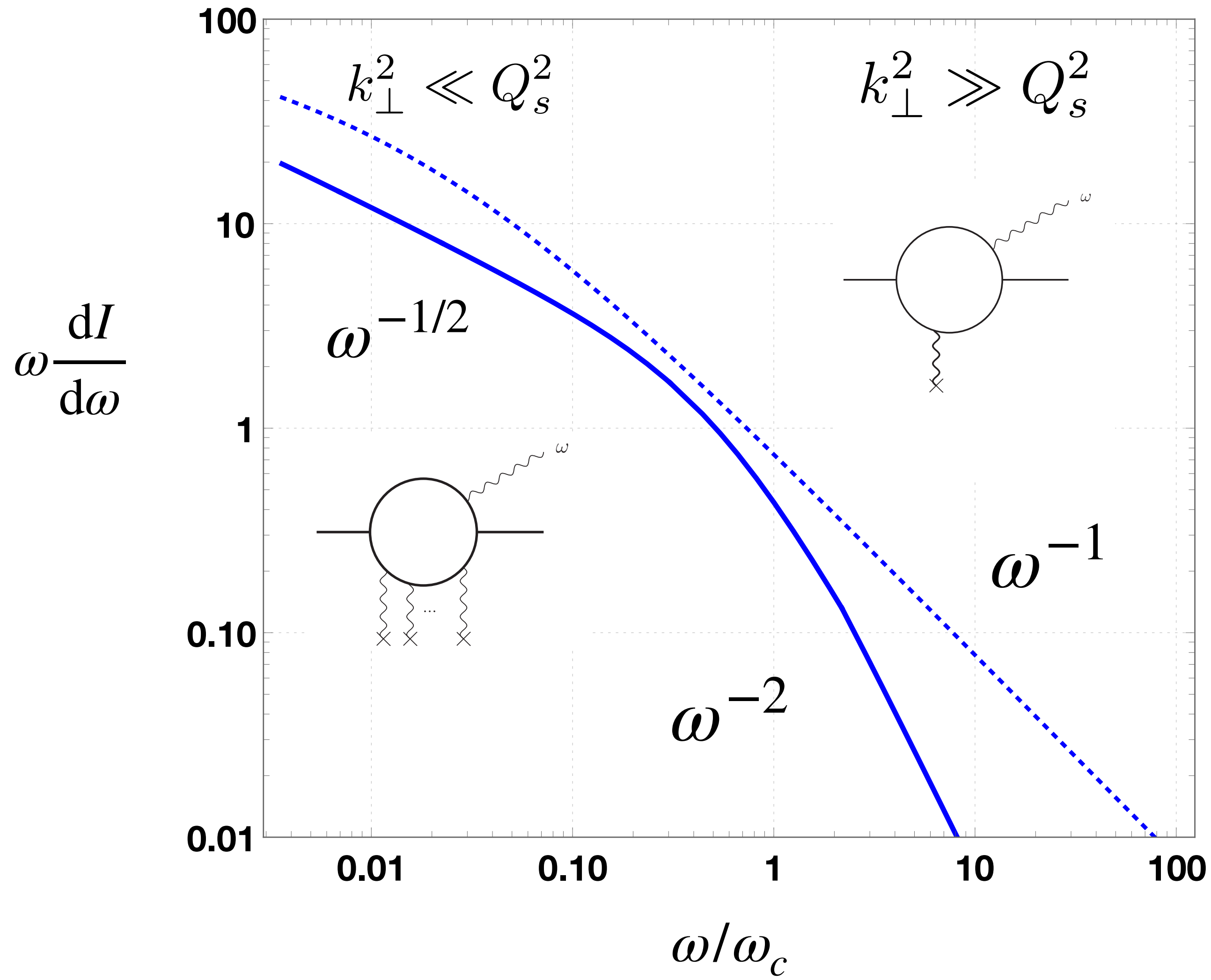
Caron-Huot and Gale (2010)



Ke , Xu, Bass (2018)

Medium-induced gluon radiation spectrum

- The Harmonic oscillator is a good approximation in the soft sector but fails in the UV due to the absence of the $k_\perp^{-4}$ tail



$$n = 0.1\, GeV^{-3}$$
$$\mu = 0.3\, GeV$$
$$L = 3\, fm$$
$$\omega_c = nL^2 \simeq 22.5 GeV$$

$$Q_s^2 \equiv \hat{q}L \sim 5 - 10\, GeV^2$$

Domains of validity of the HO and SH approximations:

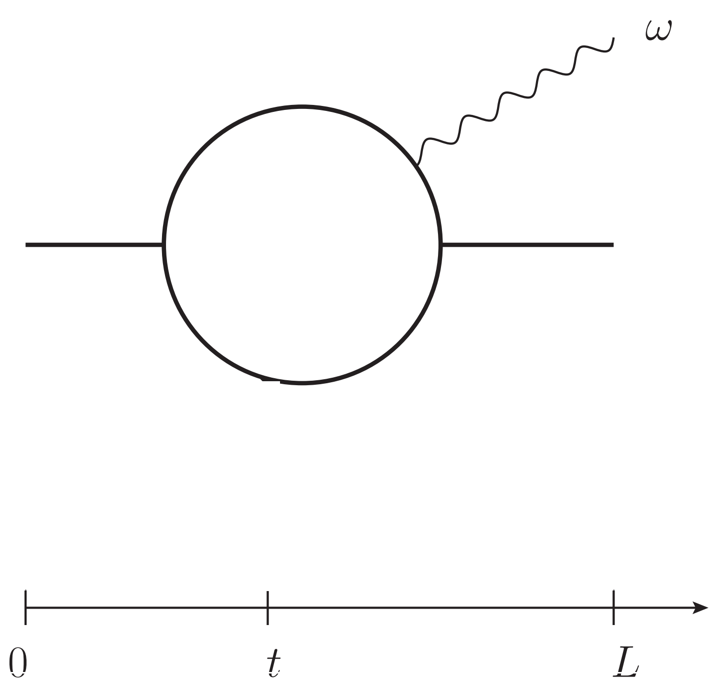
- Multiple-soft scattering $< \omega_c$
- Single hard scattering $> \omega_c$

Improved opacity expansion

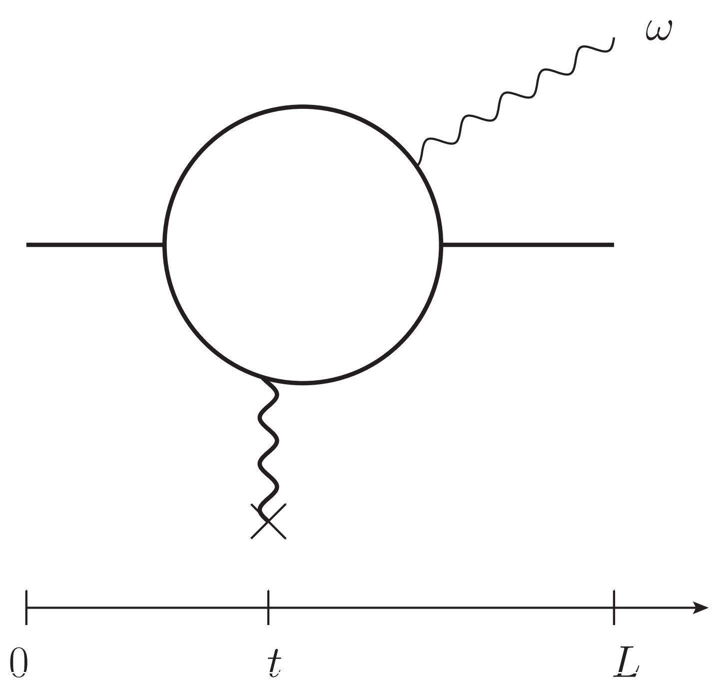
Improved opacity expansion

$$H = H_0 + H_I$$
$$= \frac{\boldsymbol{p}^2}{2p^+} + \sigma(\boldsymbol{x})$$

- Opacity expansion: expand in powers of $H_I \ll H_0$



LO (N=0): in-vacuum radiation



NLO (N=1):single scattering

Improved opacity expansion

- Extract the leading logarithm from the dipole cross-section

$$\sigma(\boldsymbol{x}) = N_c\, \boldsymbol{x}^2 \left(\ln \frac{Q^2}{\mu^2} + \ln \frac{1}{\boldsymbol{x}^2 Q^2} \right) \qquad Q^2 \equiv \langle x_\perp^2 \rangle_{\rm HO}$$

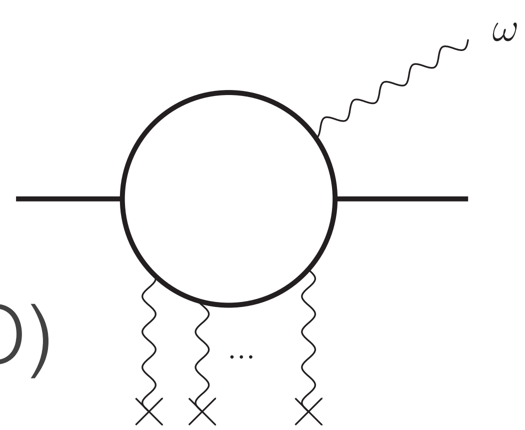
Application to momentum broadening: Molière (1948) Iancu, Itakura Triantafyllopoulos (2004)

- Perturbation around the harmonic oscillator

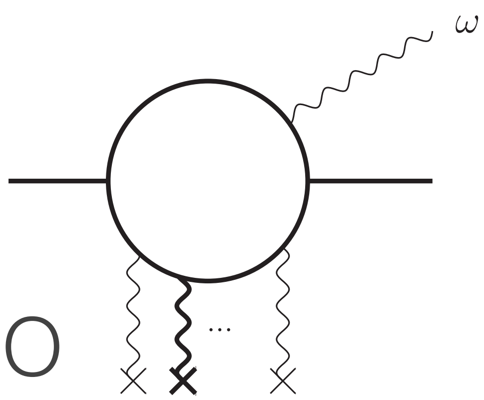
$$H \to H_{\rm HO} + H_I'$$

$$H_{\rm HO} \equiv H_0 + N_c\, \boldsymbol{x}^2 \ln \frac{Q^2}{\mu^2} \quad \gg \quad H_I' \equiv N_c\, \boldsymbol{x}^2 \ln \frac{1}{\boldsymbol{x} Q^2}$$

LO (HO)



NLO



- Expansion parameter $\left(\ln \frac{Q^2}{\mu^2} \right)^{-1}$

Improved opacity expansion

JHEP 1907 (2019) 057

- The radiative spectrum to NLO in the expansion around the Harmonic oscillator (LO)

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \simeq 2\,\bar{\alpha}\,\ln|\cos(\Omega L)| + \frac{1}{2}\,\bar{\alpha}\,\hat{q}_0\,\mathrm{Re}\int_0^L \mathrm{d}s\,\frac{1}{k^2(s)}\left[\ln\frac{k^2(s)}{Q^2} + \gamma\right]$$

where

$$Q^2 \simeq \sqrt{\omega\hat{q}(Q^2)} \equiv \sqrt{\omega\hat{q}_0\ln(Q^2/\mu^2)} \simeq \sqrt{\omega\hat{q}_0\ln(\sqrt{\omega\hat{q}_0}/\mu^2)}$$

$$k^2(s) = i\frac{\omega\Omega}{2}(\cot(\Omega s) - \tan(\Omega(L-s)))$$

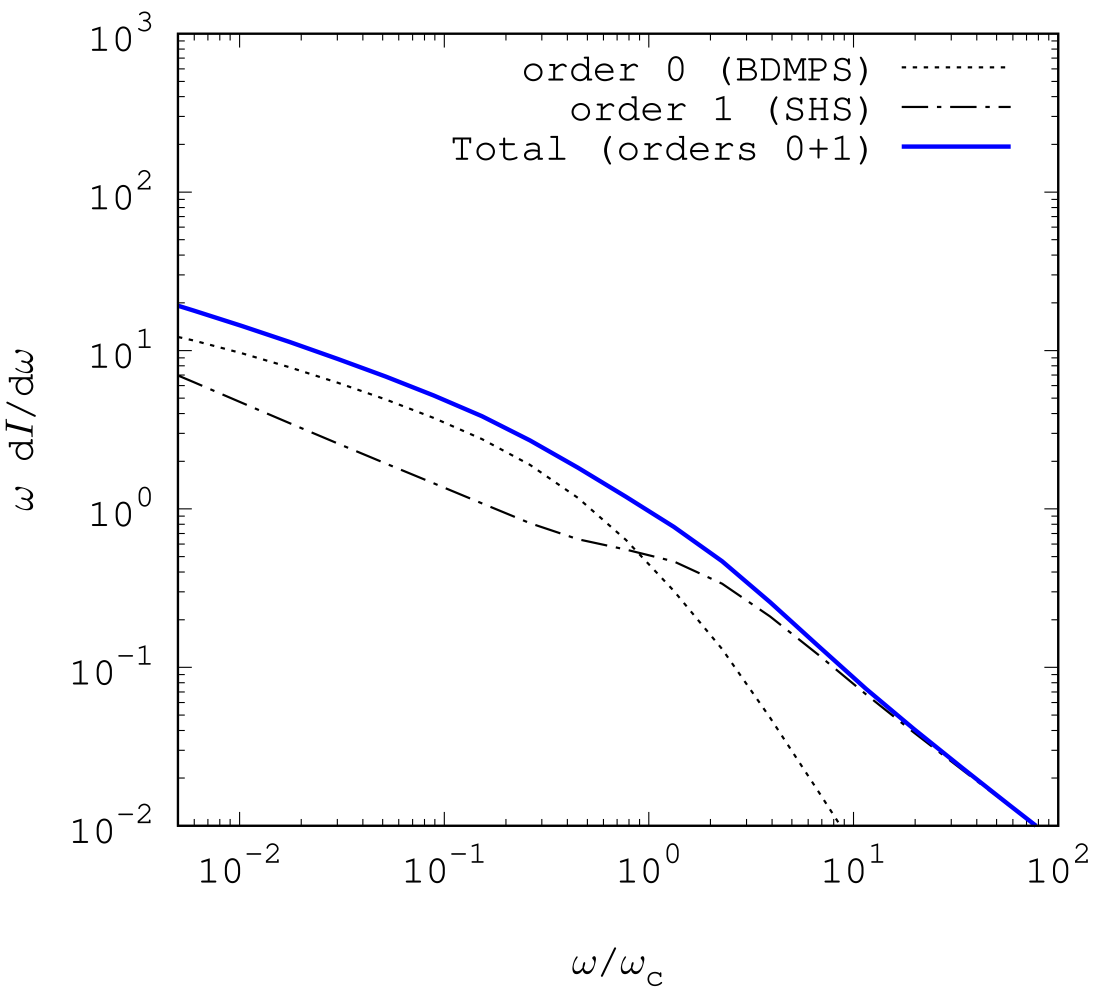
$$\Omega \equiv \frac{1-i}{2}\sqrt{\frac{\hat{q}}{\omega}}$$

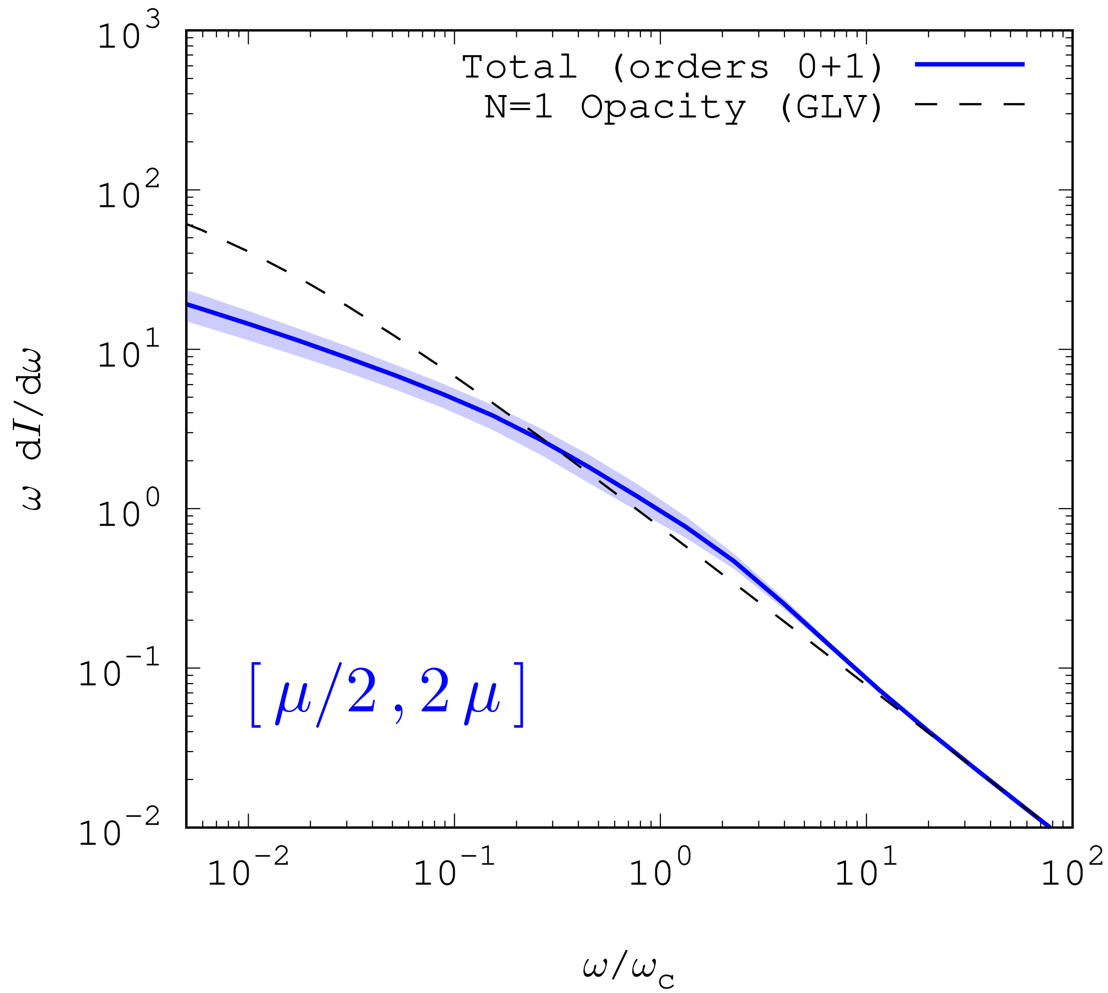
→ Encompasses the large frequency limit of N=1 opacity (GLV spectrum)

Numerical results

JHEP 1907 (2019) 057

Medium-induced gluon spectrum for $L = 3\,fm$ and $\hat{q} \simeq 1.4\,GeV^2/fm$ ($\omega_c = nL^2 = 22\,GeV$)





- First two orders. Order zero corresponds to the BDMPS spectrum.
- Improved opacity expansion compared to N=1 Opacity (GLV spectrum)

Summary and outlook

- We have introduced a new expansion scheme to perform analytic calculation of jet quenching observables beyond multiple soft scattering approximation by expanding around the harmonic oscillator (faster convergence)

- We have calculated the first two orders that encompass multiple-soft (IR) and single hard (UV) scattering regimes

- Under perturbative control for large media (hard scale much larger than the Debye mass)

- Outlook: generalize to transverse momentum dependent distribution. MC implementation. Phenomenology.

Back up

Numerical results II

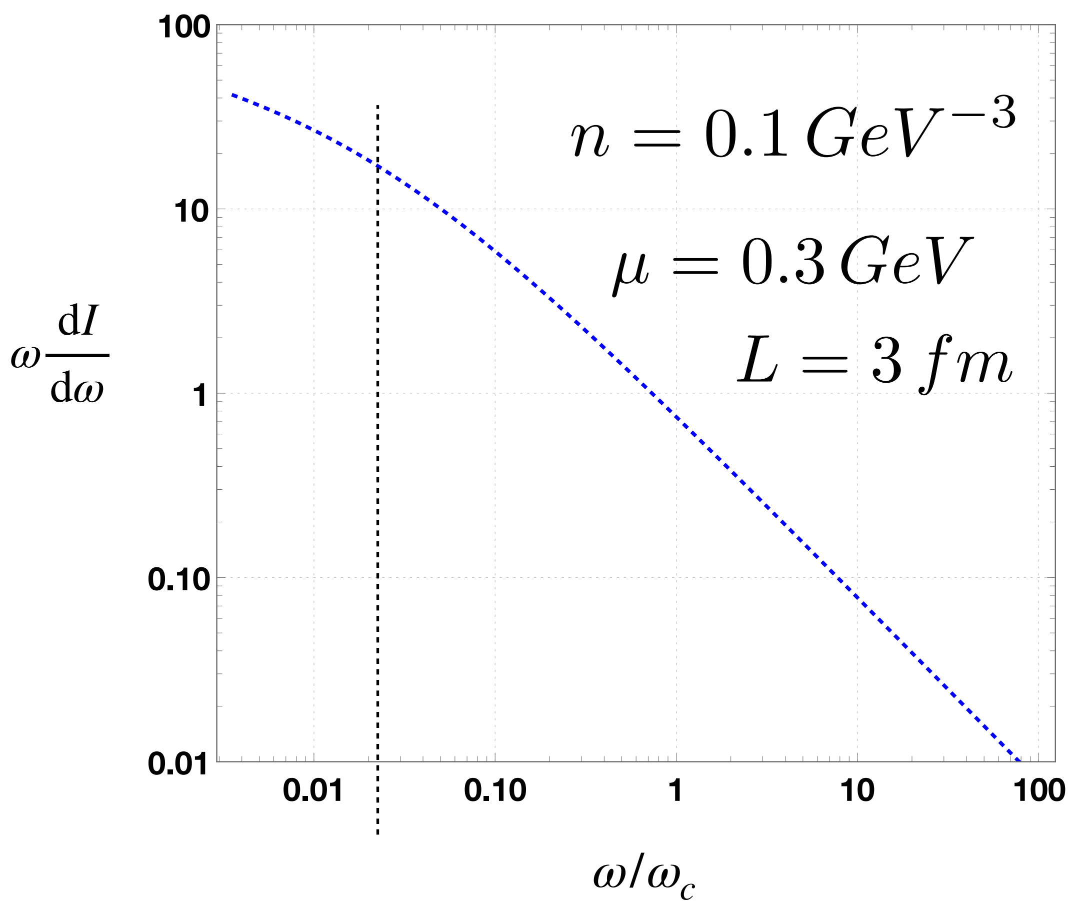
	N=1 (GLV)	full
$\Delta E(\omega < 100\, GeV)$	83 GeV	88 GeV
$N(\omega > 10^{-2}\omega_c)$	40	29

- The mean energy loss is dominated by single hard scattering
- Multiplicity is dominated by multiple soft scattering

N=1 Opacity (Gyulassy-Levai-Vitev (2000))

- Assuming a dilute medium and expand to leader order in

$$\omega \frac{dI_{\rm GLV}}{d\omega} \simeq 2\bar{\alpha} n\, L \begin{cases} \ln \frac{\bar{\omega}_c}{\omega} & \text{for} \quad \omega \ll \bar{\omega}_c \\ \frac{\pi}{4} \left(\frac{\bar{\omega}_c}{\omega} \right) & \text{for} \quad \omega \gg \bar{\omega}_c \end{cases}$$



$$\bar{\omega}_c = \frac{1}{2}\mu^2 L \simeq \ 0.7\, GeV$$

$$\omega_c = nL^2 \simeq \ 22.5 GeV$$