Overview of GPDs

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QCD on the Light Cone: from Hadrons to Heavy Ions, Palaiseau, September 16th, 2019

Outline

- Introduction
- Experimental campaign
- Recent progress
- Summary

Nucleon is not a point-like particle, it is made out of partons:

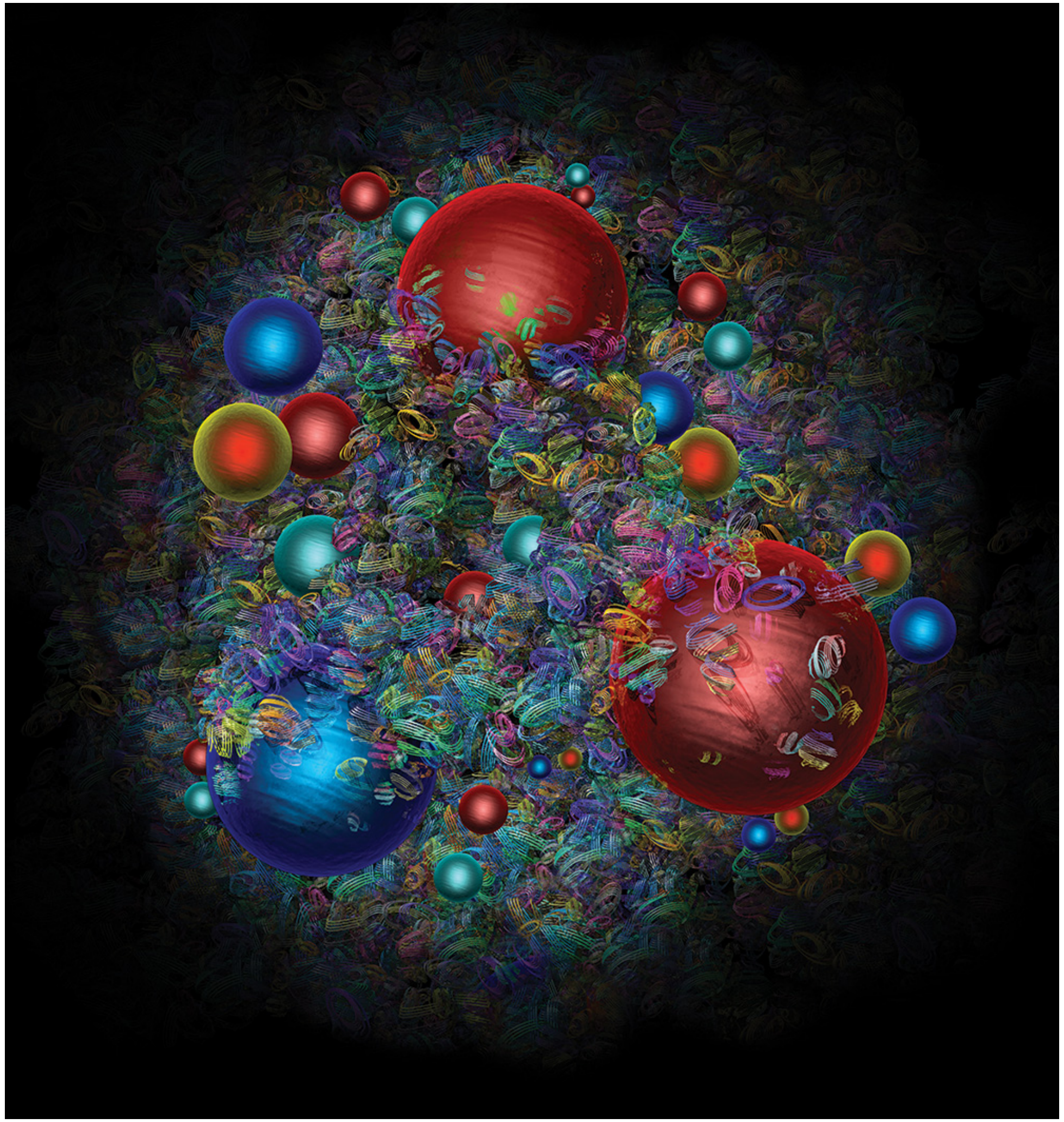
- valence quarks
- sea quarks
- gluons

How can we recover basic properties of nucleon from those of its constituents?

- charge
- spin
- mass

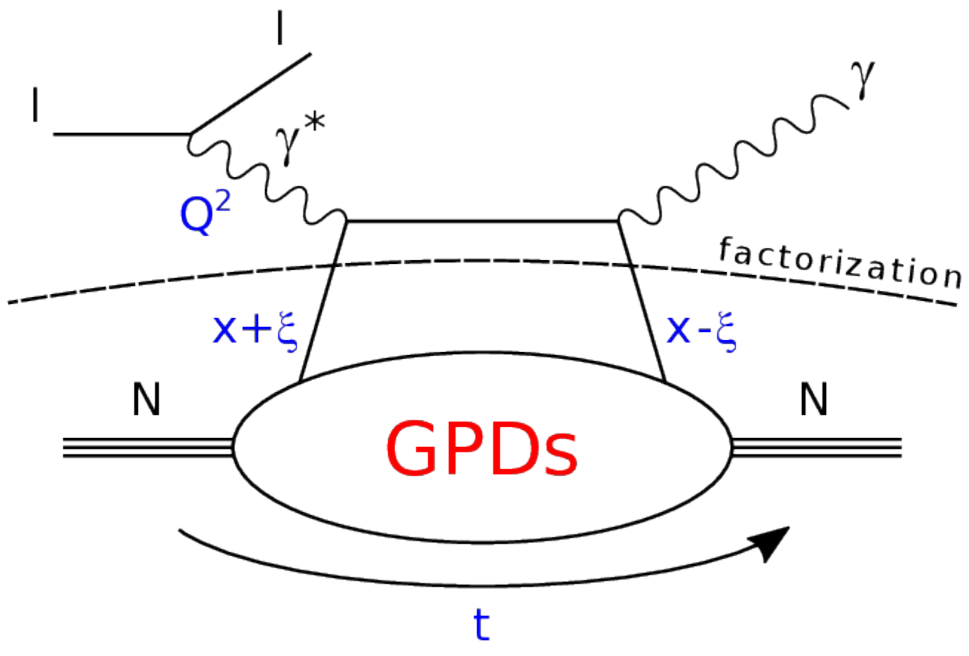
How partons are distributed inside nucleon?

- momentum (longitudinal and transverse)
- position
- polarisation
- mechanical properties



from CERN Courier / D. Dominguez

Deeply Virtual Compton Scattering (DVCS)



factorization for |t|/Q² « 1

Chiral-even GPDs:
(helicity of parton conserved)

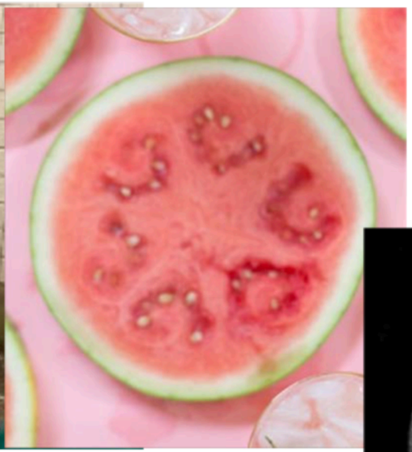
$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

Study of internal structure of a watermelon:

Exclusive measurements

A-A (RHIC)

1) Violent collision of melons



2) Cutting the watermelon with a knife

Violent DIS e-A (EIC)

3) MRI of a watermelon

Non-Violent e-A (EIC)

from A. Deshpande EIC@U. of Chicago 14/01/19

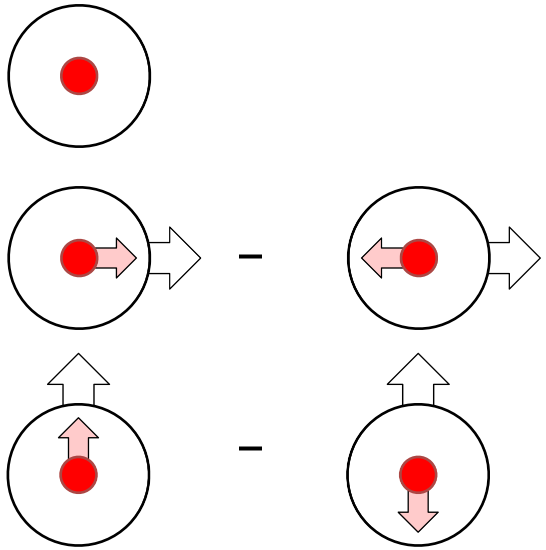
Introduction

- Reduction to PDFs:

$$H^q(x,0,0) \equiv q(x)$$

$$\widetilde{H}^q(x,0,0) \equiv \Delta q(x)$$

$$H_T^q(x,0,0) \equiv h_1(x)$$



no corresponding relations exist for other GPDs

- Reduction to Elastic Form Factors (EFFs):

$$\int_{-1}^{1} \mathrm{dx}\, H^q(x,\xi,t) \equiv F_1^q(t) \qquad \int_{-1}^{1} \mathrm{dx}\, E^q(x,\xi,t) \equiv F_2^q(t)$$

$$\int_{-1}^{1} \mathrm{dx}\, \widetilde{H}^q(x,\xi,t) \equiv g_A^q(t) \qquad \int_{-1}^{1} \mathrm{dx}\, \widetilde{E}^q(x,\xi,t) \equiv g_P^q(t)$$

Polynomiality - non-trivial consequence of Lorentz invariance:

$$\int_{-1}^{1} \mathrm{d}x \; x^n H^q(x,\xi,t) = h_0^{q,n}(t) + \xi^2 h_2^{q,n}(t) + \ldots + \mathrm{mod}(n,2)\xi^{n+1} h_{n+1}^{q,n}(t)$$

$$\int_{-1}^{1} \mathrm{d}x \; x^n \widetilde{H}^q(x,\xi,t) = \tilde{h}_0^{q,n}(t) + \xi^2 \tilde{h}_2^{q,n}(t) + \ldots + \mathrm{mod}(n+1,2)\xi^{n} \tilde{h}_{n}^{q,n}(t)$$

Positivity bounds - positivity of norm in Hilbert space, e.g.:

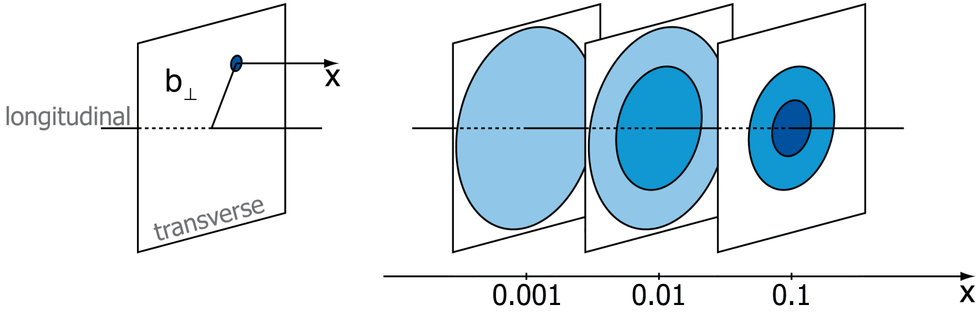
$$(1-\xi^2)\left(H^q - \frac{\xi^2}{1-\xi^2}E^q\right)^2 + \frac{t_0 - t}{4m^2}\left(E^q\right)^2 \leq q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)$$

strong constraint on GPD parameterizations!

Introduction

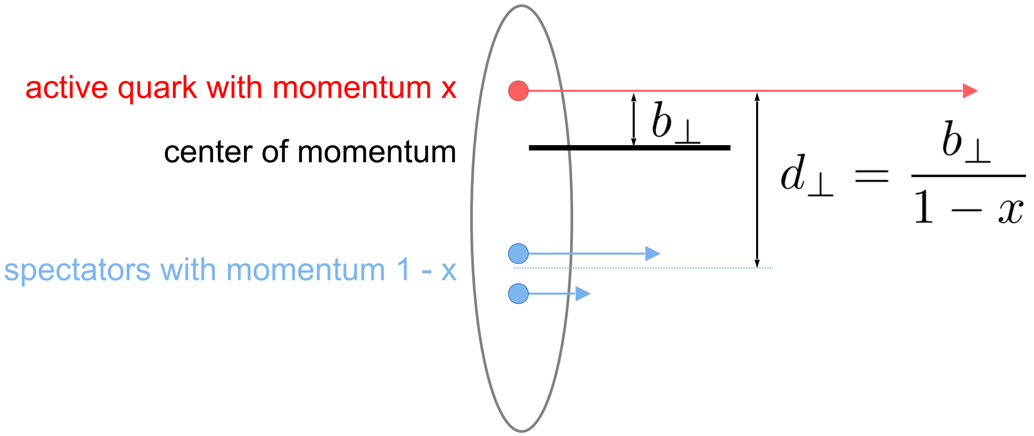
Nucleon tomography

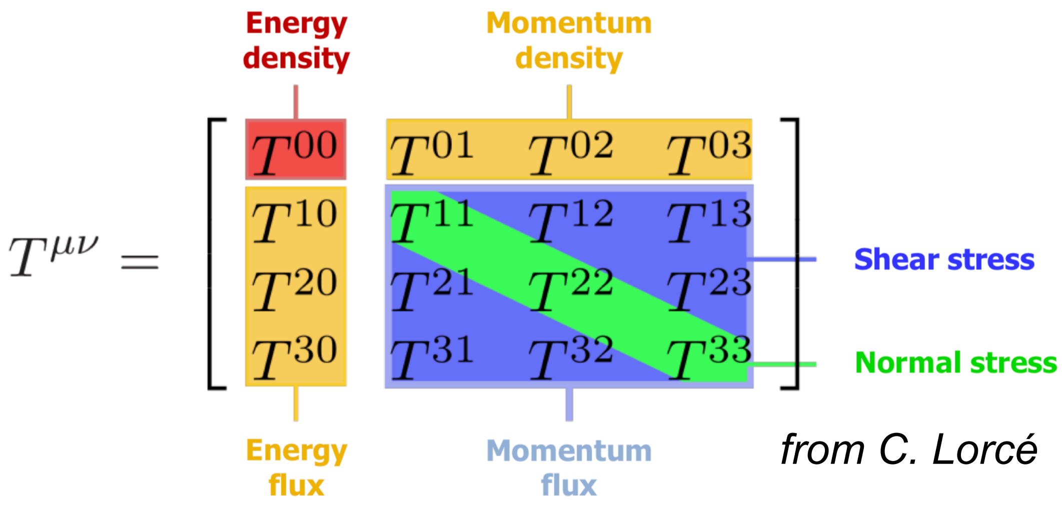
$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$



Study of long. polarization with GPD $\widetilde{H}$
Study of distortion in transv. polarized nucleon with GPD E

Impact parameter $\mathbf{b}_\perp$ defined w.r.t. center of momentum, such as $\sum x\,\mathbf{b}_\perp = 0$





from C. Lorcé

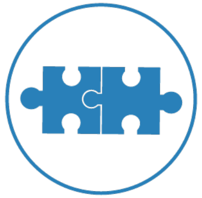
Energy momentum tensor in terms of form factors:

$$\langle p', s' | \widehat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Bigg[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C(t) + M \eta^{\mu\nu} \bar{C}(t) + \frac{P^\mu i \sigma^{\nu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i \sigma^{\mu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \Bigg] u(p, s)$$

Total angular momentum

$$A^q(0) + B^q(0) = \int_{-1}^{1} x\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right] = 2J^q$$

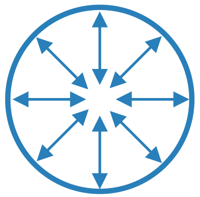
1/2

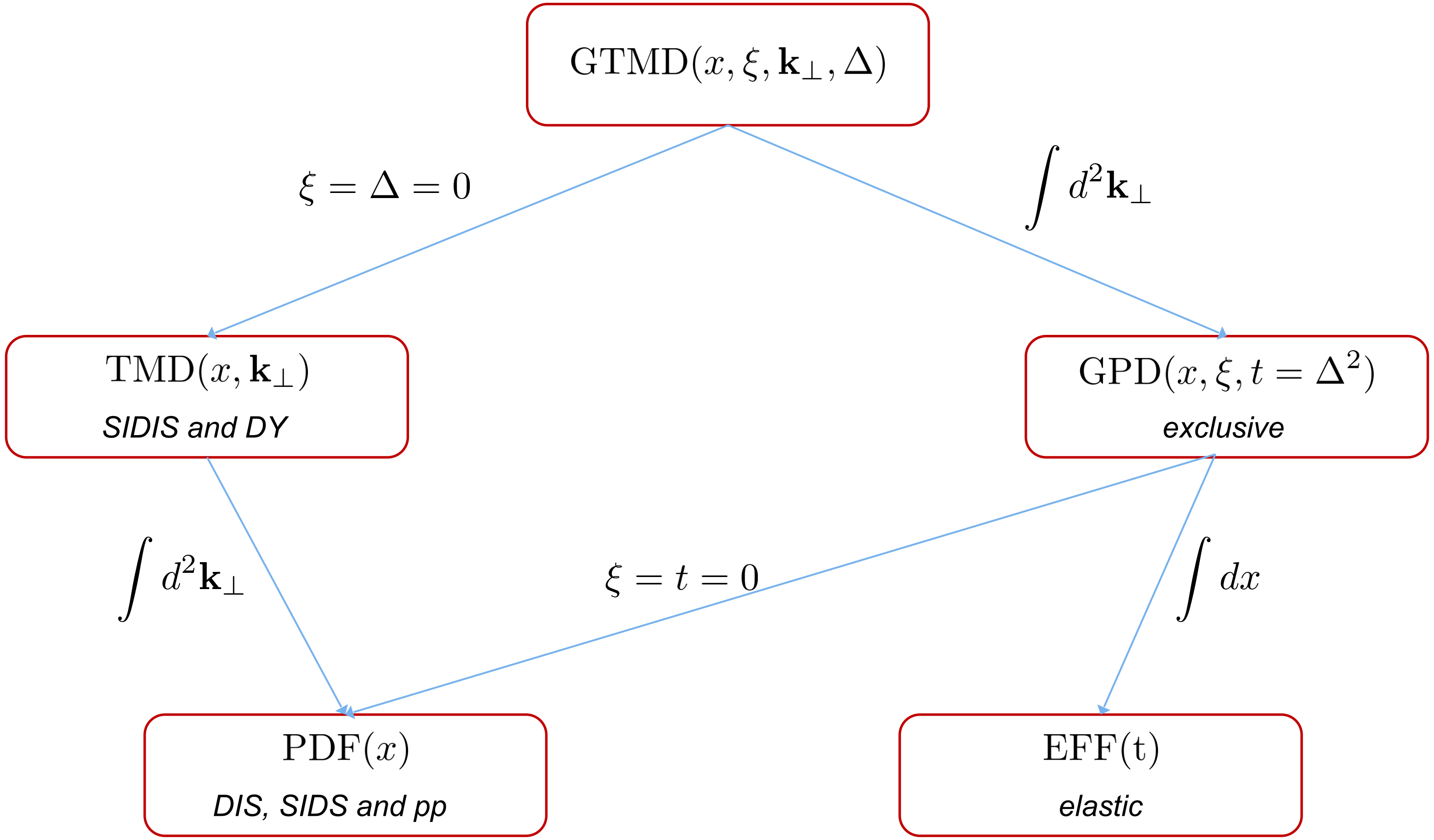


Ji's sum rule

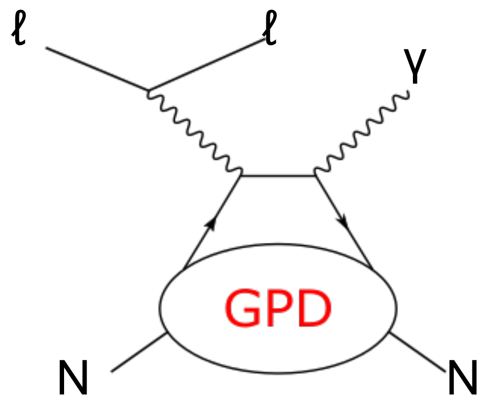
"Mechanical" forces acting on quarks, e.g. pressure in nucleon center

$$p(0) = \frac{1}{6\pi^2 M}\int_{-\infty}^{0} dt \sqrt{-t}\, t C(t)$$

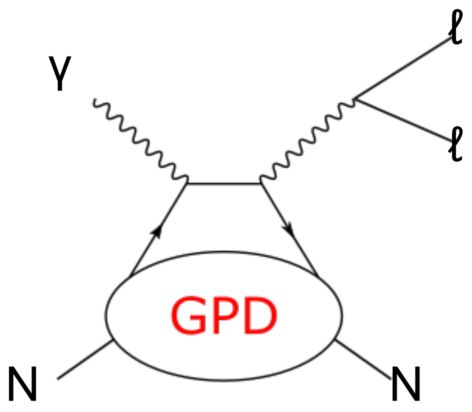



$\mathrm{GTMD}(x, \xi, \mathbf{k}_\perp, \Delta)$
$\xi = \Delta = 0$
$\int d^2\mathbf{k}_\perp$
$\mathrm{TMD}(x, \mathbf{k}_\perp)$
SIDIS and DY
$\mathrm{GPD}(x, \xi, t = \Delta^2)$
exclusive
$\int d^2\mathbf{k}_\perp$
$\xi = t = 0$
$\int dx$
$\mathrm{PDF}(x)$
DIS, SIDS and pp
$\mathrm{EFF}(\mathrm{t})$
elastic

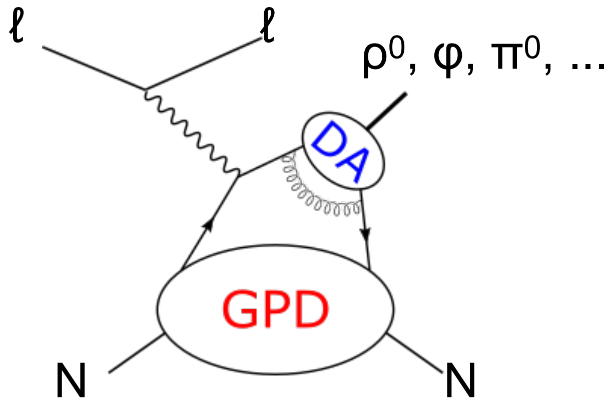
GPDs accessible in various production channels and observables
→ experimental filters



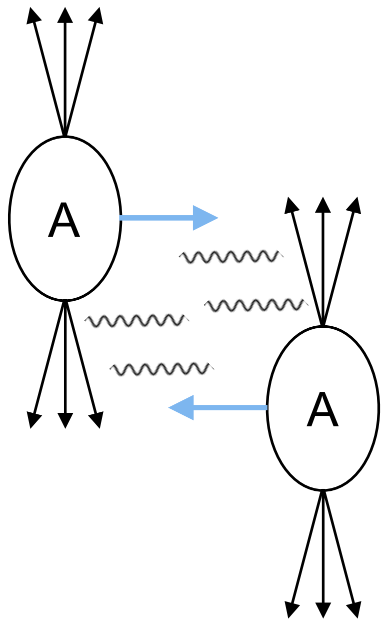
DVCS
Deeply Virtual Compton Scattering



TCS
Timelike Compton Scattering



HEMP
Hard Exclusive Meson Production



UPC
Ultra Peripheral Collisions

more production channels sensitive to GPDs exist!

Experimental campaign

GPDs studied in various laboratories
→ need to cover a broad kinematic range

experiments

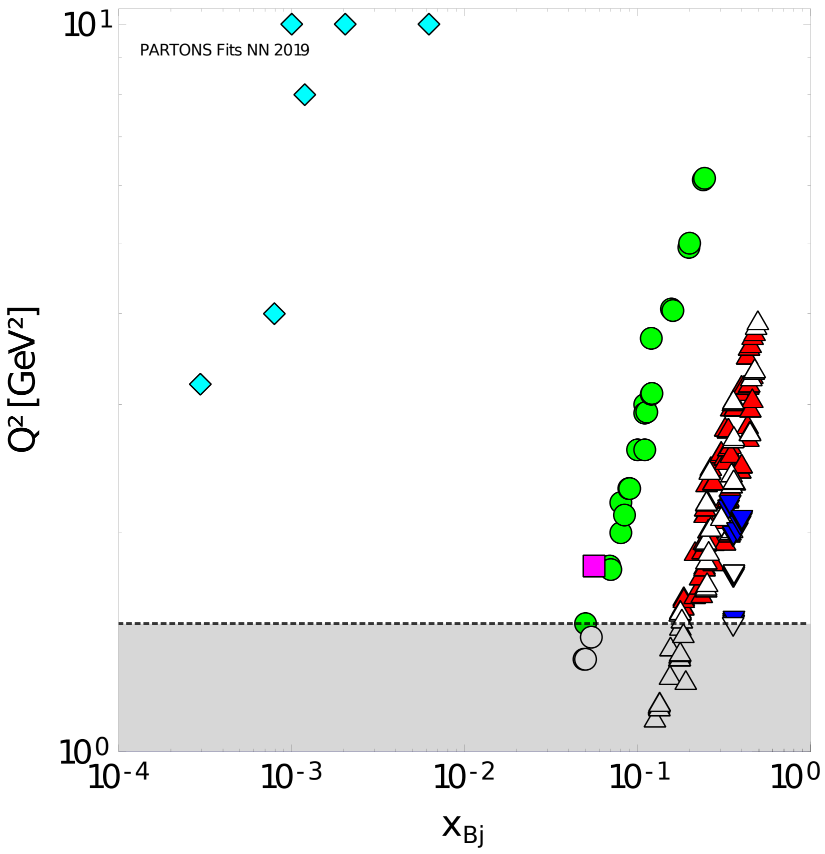
closed **active** **planned**

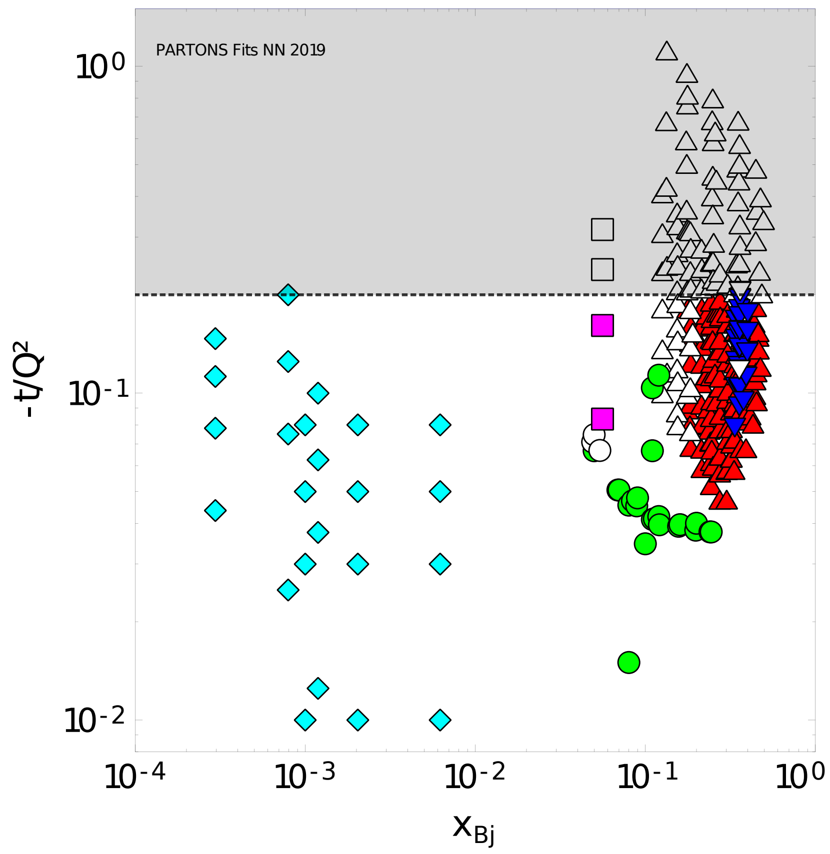
Kinematic cuts
used in our recent analyses:

$$Q^2 > 1.5\ \mathrm{GeV}^2$$

$$-t/Q^2 < 0.2$$

- ▼ HALL A
- ▲ CLAS
- ● HERMES
- ■ COMPASS
- ◆ H1 and ZEUS



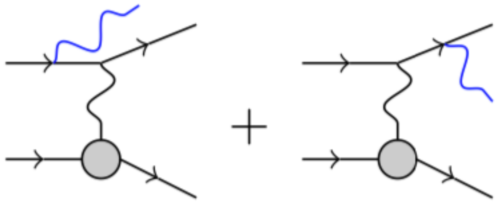


Compton Form Factors

Cross-section for single photon production $(l + N \rightarrow l + N + \gamma)$:

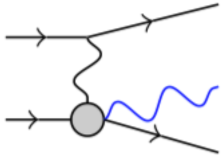
$$\sigma \propto |\mathscr{A}|^2 = |\mathscr{A}_{BH} + \mathscr{A}_{DVCS}|^2 = |\mathscr{A}_{BH}|^2 + |\mathscr{A}_{DVCS}|^2 + \mathscr{I}$$

Bethe-Heitler process



calculable within QED

DVCS



parametrised by CFFs

CFF - the most basic GPD-sensitive observables
- analogy with connection between structure functions and PDFs

Compton Form Factors

- imaginary part

$$Im\mathcal{G}(\xi,t) \quad = \pi G^{(+)}(\xi,\xi,t) = \pi \sum_q e_q^2 G^{q(+)}(\xi,\xi,t)$$

$$G^{q(+)}(x,\xi,t) = G^q(x,\xi,t) \mp G^q(-x,\xi,t)$$

$$G^{q(+)}(\xi,\xi,t) = G^{q_{\mathrm{val}}}(\xi,\xi,t) + 2G^{q_{\mathrm{sea}}}(\xi,\xi,t)$$

"−" for $G \in \{H, E\}$
"+" for $G \in \{\widetilde{H}, \widetilde{E}\}$

- real part

$$Re\mathcal{G}(\xi,t) = \mathrm{P.V.} \int_0^1 G^{(+)}(x,\xi,t) \left(\frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) \mathrm{d}x$$

$$Re\mathcal{G}(\xi,t) = \mathrm{P.V.} \int_0^1 G^{(+)}(x,x,t) \left(\frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) \mathrm{d}x + C_G(t)$$

$$C_H(t) = -C_E(t) \qquad C_{\widetilde{H}}(t) = C_{\widetilde{E}}(t) = 0$$

connected to EMT FF

Subtraction Constant

Relation between subtraction constant and D-term:

$$C_G^q(t) = 2\int_{-1}^{1} \frac{D^q(z,t)}{1-z}\mathrm{d}z \equiv 4D^q(t)$$

where

$$z = \frac{x}{\xi}$$

Decomposition into Gegenbauer polynomials:

$$D^q(z,t) = (1-z^2)\sum_{i=0}^{\infty} d_i^q(t) C_{2i+1}^{3/2}(z)$$

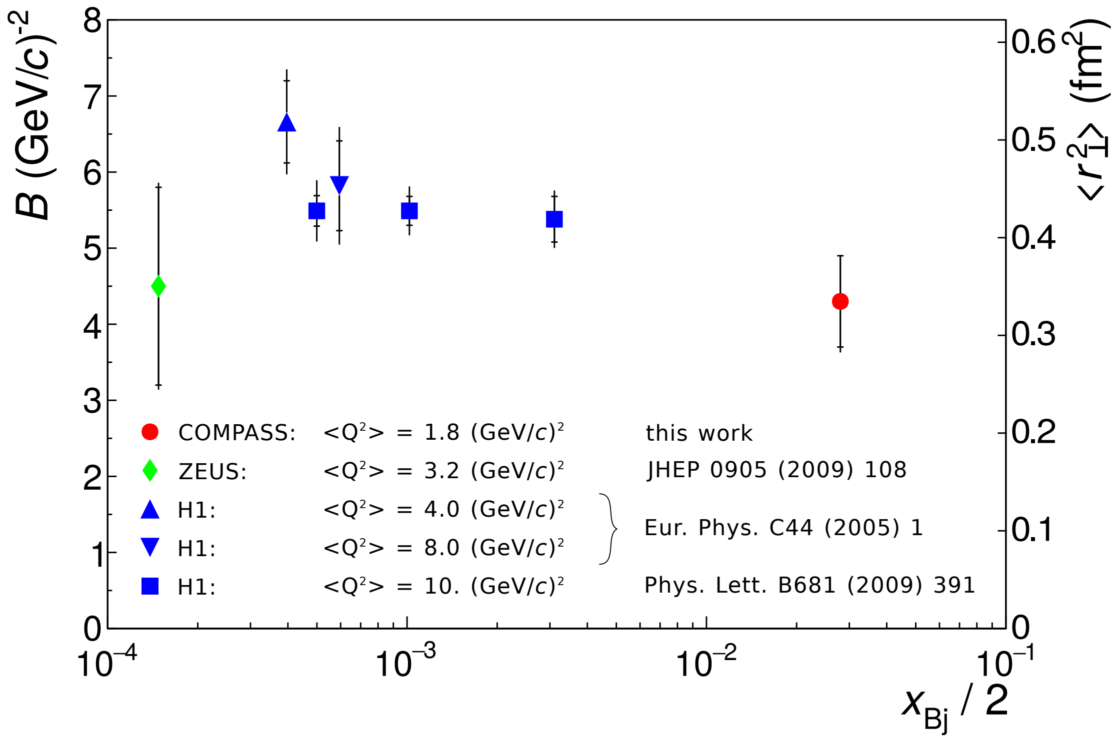
Connection to EMT FF:

$$D^q(t) = \sum_{\substack{i=1 \\ \mathrm{odd}}}^{\infty} d_i^q(t) \qquad d_1^q(t) = 5C^q(t)$$

Nucleon tomography

- “Direct” measurement at low-x

COMPASS Collaboration, Phys. Lett. B793 (2019) 188



Slope of t-dependance:

$$\frac{d\sigma^{\gamma^* p \to \gamma p}}{dt} \propto e^{-Bt}$$

related to transverse extension of quarks:

$$\langle r_\perp^2(x_{Bj}) \rangle \approx 2\langle B(x_{Bj}) \rangle$$

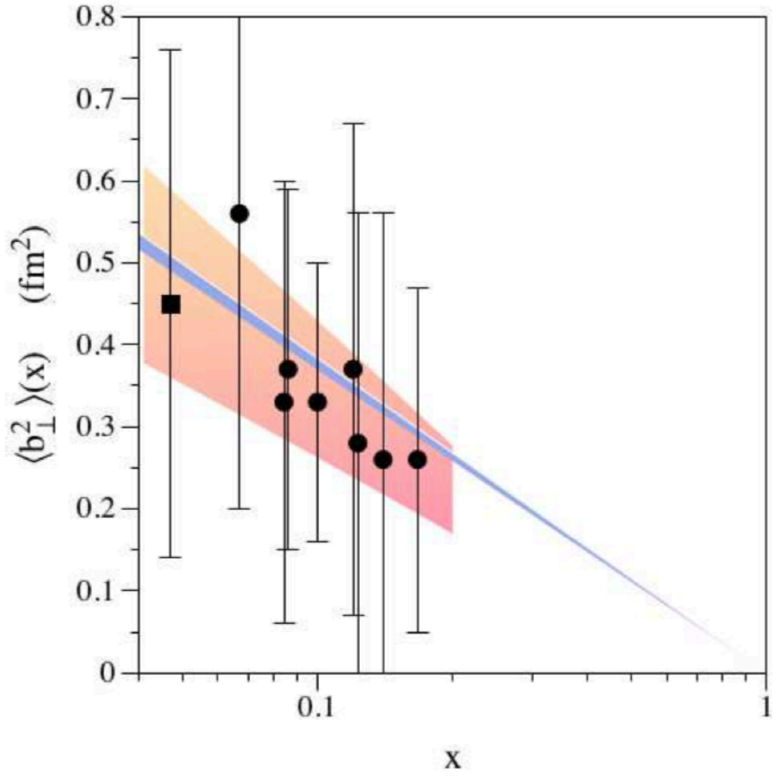
under following assumptions:

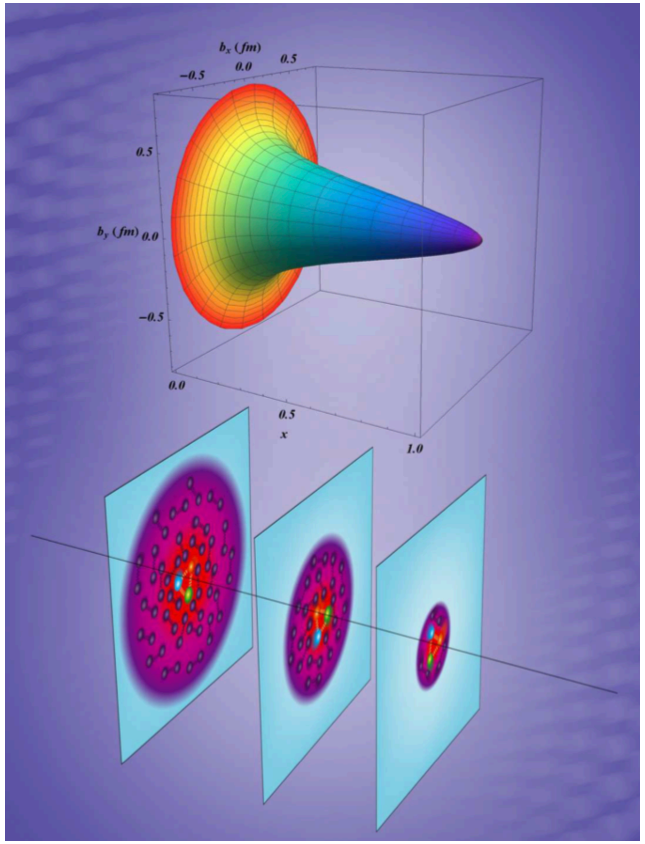
- single-exponential dependence
- dominance of CFF Im$\mathcal{H}$
- negligible “skewness effect” H(x, x, t) ~ H(x, 0, t)

which are applicable at low-x

- Local fits and de-skewness procedure at high-x

Dupre, Guidal, Vanderhaeghen, Phys. Rev. D95 (2017) no. 1, 011501
Dupré, Guidal, Niccolai, Vanderhaeghen, Eur. Phys. J. A53 (2017) no. 8, 171





The procedure:

1. Fit CFFs separately in each (x_B, t, Q^2) bin of data

2. Fit extracted ImH values with

$$Im\mathscr{H}(\xi, t, Q^2) = A(\xi)\ \exp(B(\xi)t)$$

$$A(\xi) = a_A \frac{1-\xi}{\xi}$$

$$B(\xi) = a_B \ln(1/\xi)$$

$$\xi \approx \frac{x_B}{2 - x_B}$$

- Global fits with skewness dependance encoded in Ansatz

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C78 (2018) 11, 890

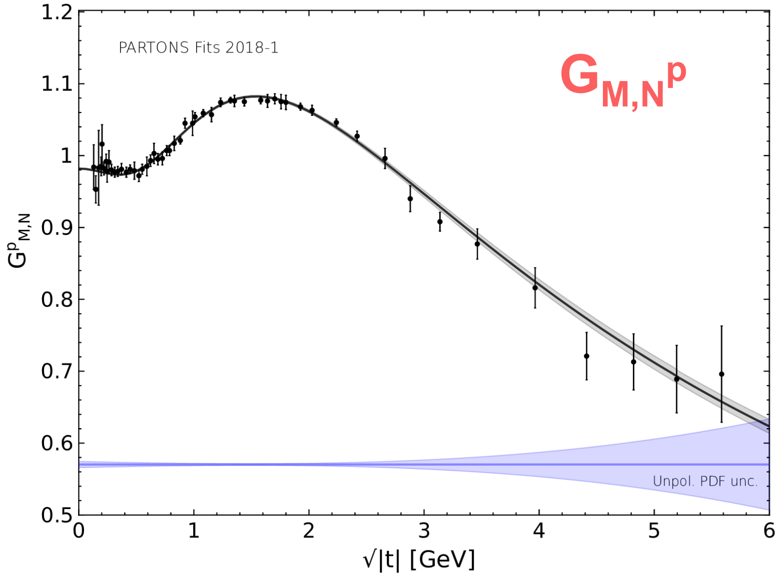
$$G^q(x,x,t) = G^q(x,0,t)\ g^q_G(x,x,t)$$

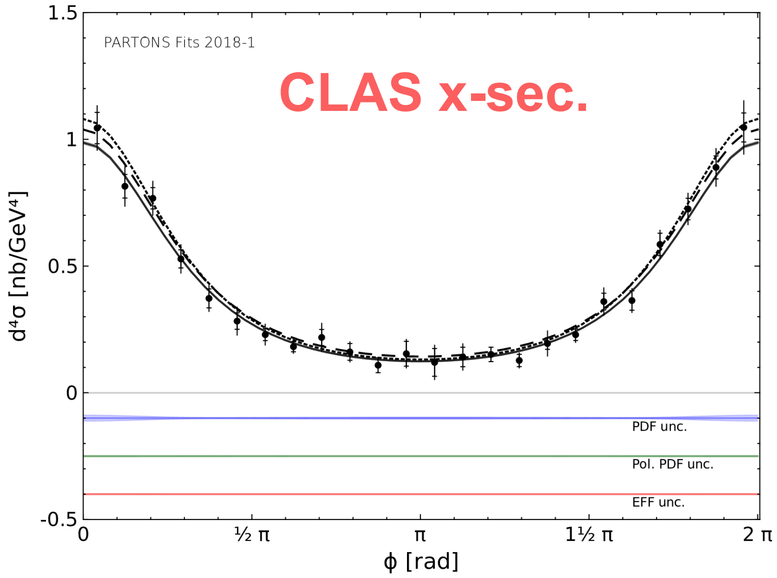
$$G^q(x,0,t) = \mathrm{pdf}^q_G(x)\ \exp(f^q_G(x)t)$$

$$g^q_G(x,x,t) = \frac{a^q_G}{(1-x^2)^2}\left(1+t(1-x)(b^q_G + c^q_G \log(1+x))\right)$$

$$f^q_G(x) = A^q_G \log(1/x) + B^q_G(1-x)^2 + C^q_G(1-x)x$$

Allows for a global fit of both elastic FF and DVCS data

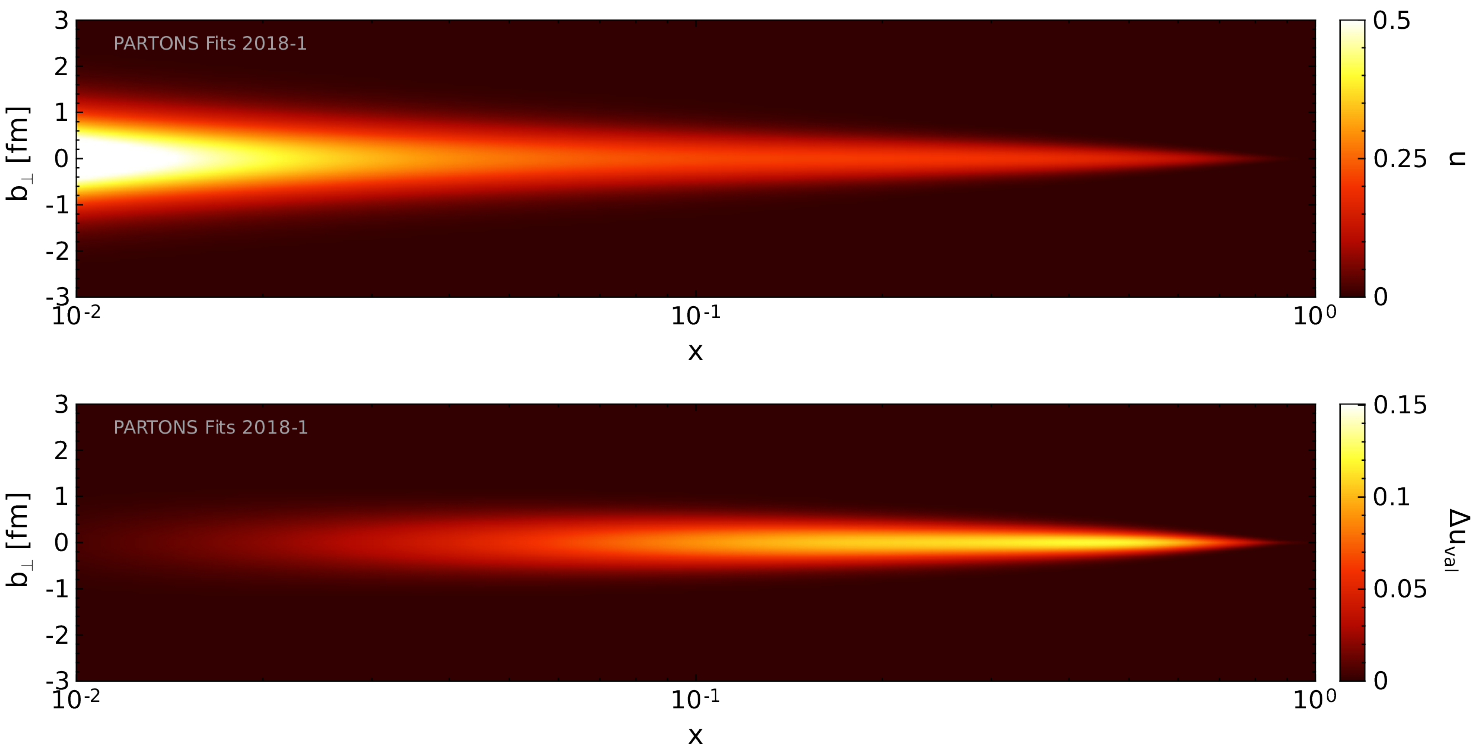


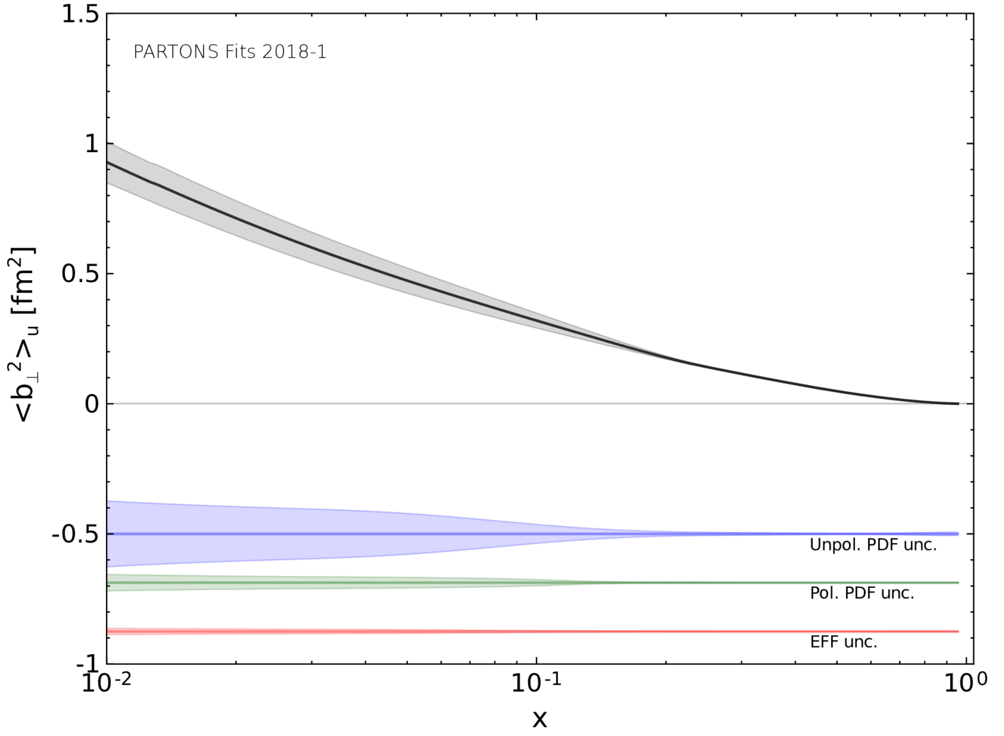


Nucleon tomography

- Global fits with skewness dependance encoded in Ansatz

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C78 (2018) 11, 890





$$Q^2 = 2 \text{ GeV}^2$$

Ji's sum rule - TOAM

No recent progress here :-(

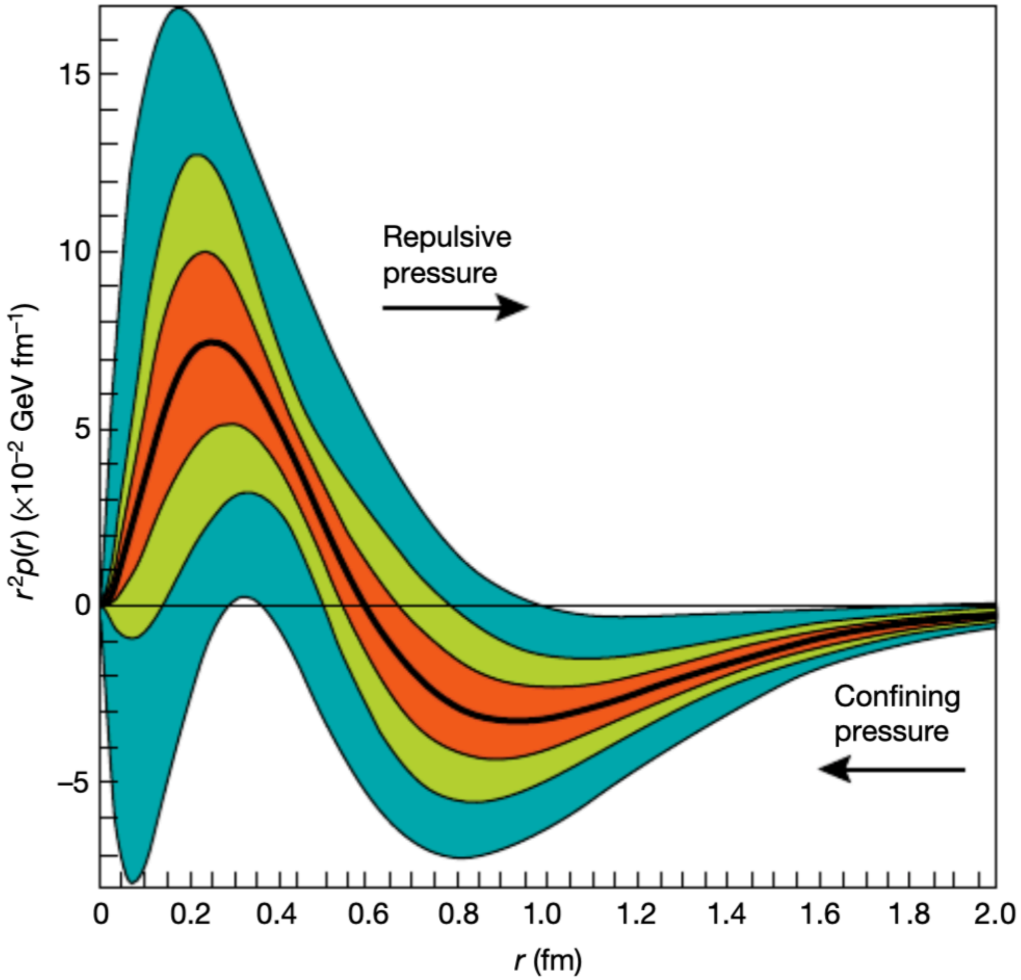
Need for more neutron data or data taken with transversely polarized targets

Need for modern GPD models to access H/E(x, const ξ, t=0) from exclusive data

Elastic data provide important constraints on GPDs E (however, only for valance contribution), see for instance Eur. Phys. J. C73 (2013) no. 4, 2397

Subtraction constant

V.D. Burkert, L. Elouadrhiri, F.X. Girod, Nature 557 (2018) no. 7705, 396



The procedure:

1. Extract subtraction constant using KM model and CLAS data

2. Fit extracted values with

$$d_1(t) = d_1(0)\left(1 - \frac{t}{M^2}\right)^{-\alpha}$$

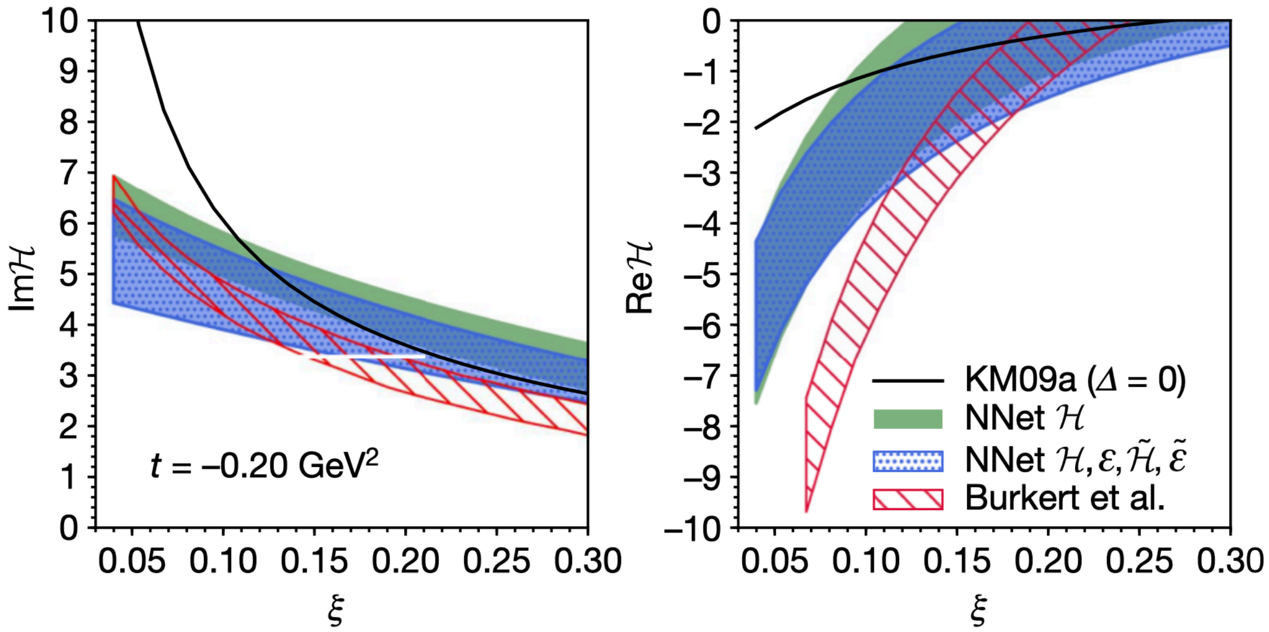
without JLab 6 GeV data

with JLab 6 GeV data

with JLab 12 GeV data (projected)

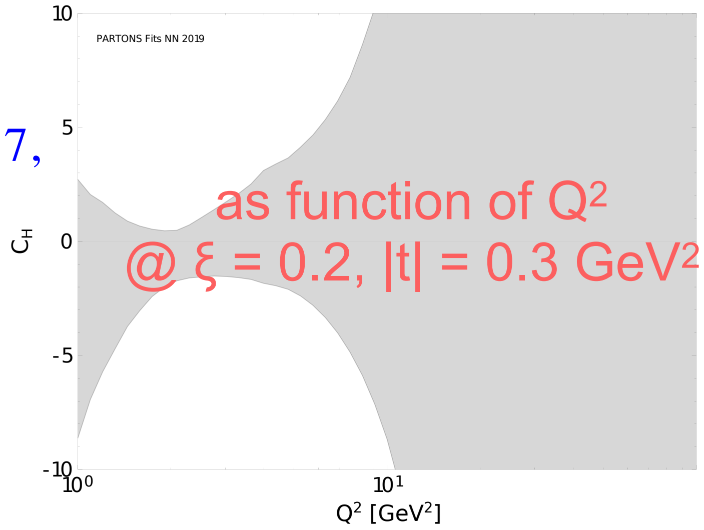
Subtraction constant

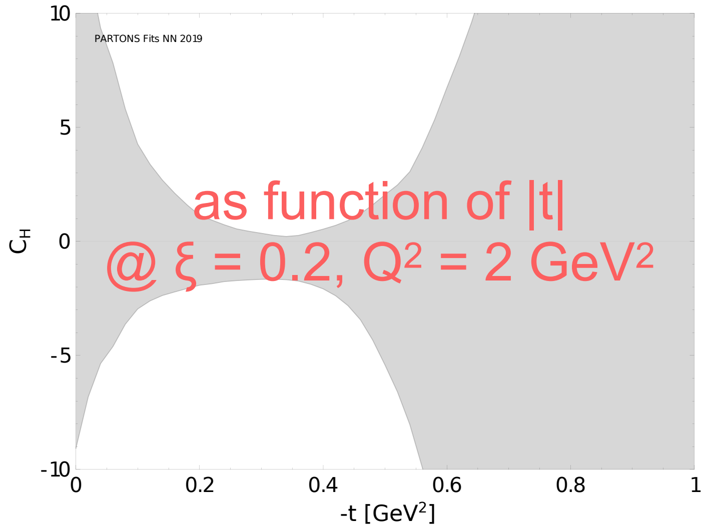
K. Kumerički, Nature 570 (2019) no.7759, E1



Result: C(t) = 0.78 ± 1.5
almost no t-dependence

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C79 (2019) no. 7, 614





Dedicated softwares to study GPDs:

GeParD

see e.g. K. Kumericki's talk at "Prospects for extraction of GPDs" workshop, Warsaw 2019

PARTONS

Eur. Phys. J. C78 (2018) 6, 478
http://partons.cea.fr

Tools

Dedicated softwares to study GPDs:

GeParD

- conformal space evolution of GPDs: up to NLO and partialy to NNLO (conformal scheme)
- unpol. DIS hard-scattering coefficient functions to NNLO
- unpol. DVCS hard-scattering coefficient functions to NNLO
- unpol. DVMP hard-scattering coefficient functions to NLO (not thoroughly tested)
- Compton Form Factors
- DVMP transition form factor
- small-x σ^{DVCS} and $\sigma^{DV\rho P}$
- complete (Belitsky, Muller et al.) DVCS formulas
- all measured DVCS observables

PARTONS

- GPD models: GK, VGG, Vinnikov (evolution), MPSSW13 (NLO study), MMS13 (DD study)
- DVCS CFFs: LO, NLO, NO Noritzsch, DR
- DVCS formulas: VGG, GV, BMJ
- all kinds of DVCS observables
- GPD evolution: Vinnikov code

next release with TCS coming soon!
work on DVMP ongoing!

Available generators:

- Pythia 6.4 (able to generate exclusive events, but is not very accurate)
- HEPGen (mainly used by COMPASS experiment)
- MILOU (comes from HERA times)
- …

- All in FORTRAN (makes maintaining, development and integration difficult)
- Clear need for a modern MC generator dedicated to GPD physics → EIC!
- Radiative corrections!

Summary

What was discussed today?

- Introduction to GPDs
- Recent progress in phenomenology
- Review of available software

What was't discussed today?

- Recent theory developments
- Experimental results
- GPD modeling
- Nuclear GPDs
- …

Generalised Parton Distributions

- novel way to describe partonic structure of nucleon
- allows to study (highlights):
  - → nucleon tomography
  - → total angular momentum of partons
  - → "mechanical" properties of parton distributions