PDFs from lattice

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Work within ETMC: Extended Twisted Mass Collaboration

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K. Jansen	Desy – Zeuthen
H. Panagopoulos	University of Cyprus
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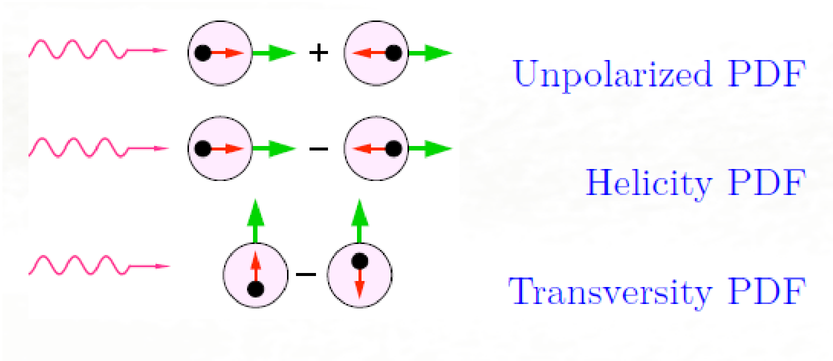
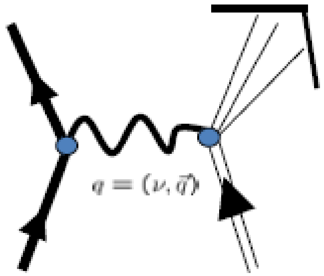
Introduction

Complete set of twist-2 parton distribution functions



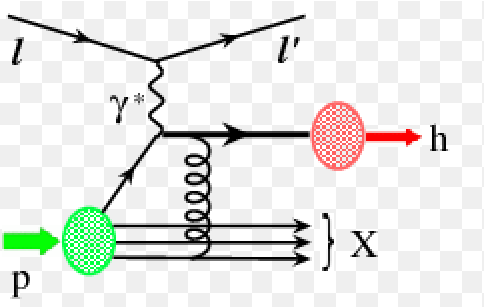
Cross sections are measured:

Totally inclusive



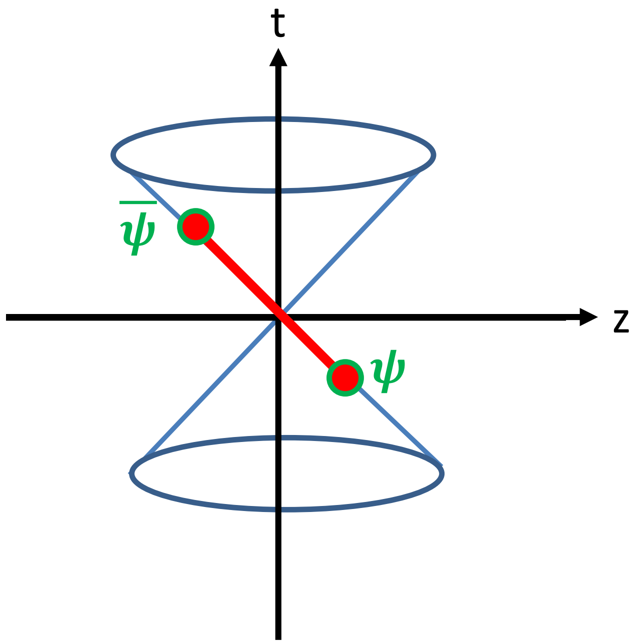
Have access to the chiral-even distributions $f_1(x)$ (unpolarized) and $g_1(x)$ (helicity)

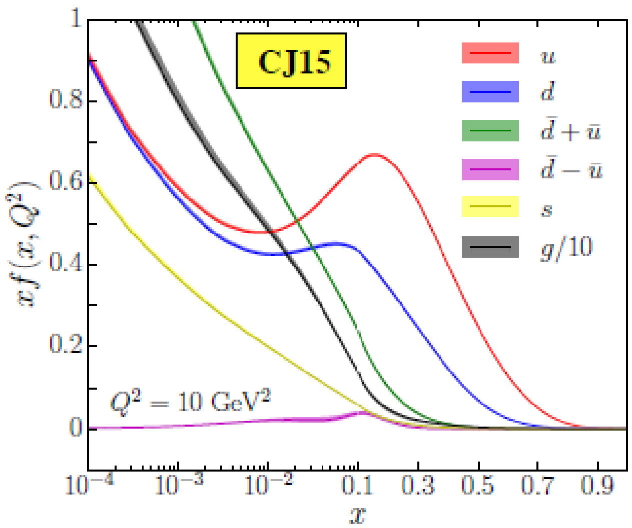
Semi-inclusive



Have access to the chiral-odd distribution $h_1(x)$ (transversity). Naturally more difficult to obtain data on transversity

Light-cone PDFs





Quark distribution is given
by a light-cone correlation

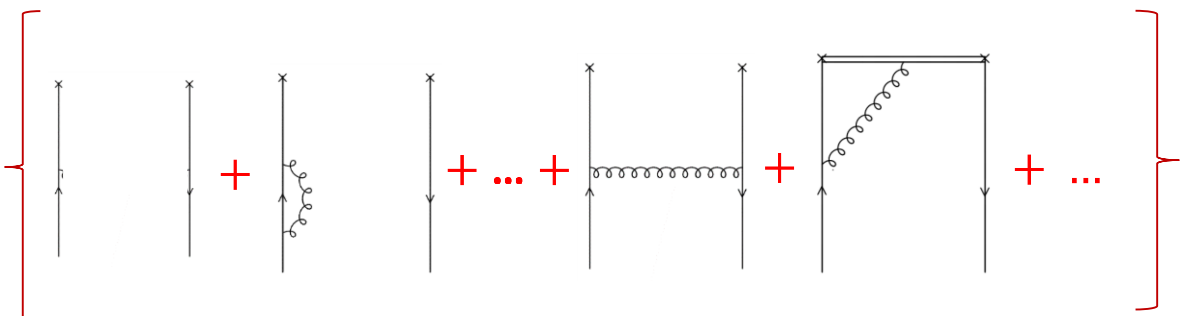
$$q(x) = \frac{1}{4\pi} \int d(n \cdot \xi) e^{-i\xi P^+ n\cdot\xi} \langle P|\bar{\psi}(n \cdot \xi)\gamma^+ W\,(n \cdot \xi, 0)\psi(0)|P\rangle, \qquad n \cdot \xi = z^-$$

Dirac Structure

Wilson line

Our focus: isovector quark distributions, $q(x) \equiv u(x) - d(x)$

Perturbative correction to isovector quark distributions :

$q(x,\Lambda)=$  $\otimes q_{bare}(y) + \mathcal{O}(\alpha_s^2)$

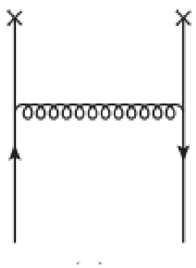
$\delta(1-x/y)$ $\quad \Pi(\Lambda)\delta(1-x/y)$ $\quad \Gamma(x/y,\Lambda)$

Self-energy $\quad$ Vertex

Regulator of IR and UV divergences

$$q(x,\Lambda) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi}\Pi(\Lambda)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \Gamma\left(\frac{x}{y},\Lambda\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

Simplest diagram



$$= -ig^2 C_F \int \frac{dk^+ dk^- d^2k_\perp}{(2\pi)^4} \frac{\bar{u}(p)\gamma^\mu k\cdot\gamma\gamma^+ k\cdot\gamma\gamma_\mu u(p)}{(k^2)^2(p-k)^2}\,\delta\left(y - \frac{k^+}{p^+}\right)$$

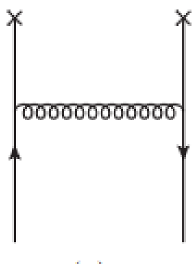
$$p = (\xi P^+, 0,0,0);\;\; \xi = \frac{p^+}{P^+}$$

$$k^2 + i\epsilon = 2yp^+\left(k^- - \frac{k_\perp^2}{2yp^+} + i\epsilon\right)$$

$$(p-k)^2 + i\epsilon = -2p^+(1-y)\left(k^- + \frac{k_\perp^2}{2p^+(1-y)} - i\epsilon\right)$$

For $0 < y < 1$, one pole in the upper half and other in the lower half of the complex plane

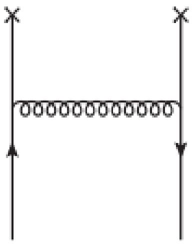
For $y > 1$ or $y < 0$, the poles are either on the lower half or on the upper half of the complex plane



$$= 2\alpha_s C_F(1-y)\int \frac{d^2k_\perp}{(2\pi)^2}\frac{\bar{u}(p)\gamma^+ u(p)}{k_\perp^2} = \frac{\alpha_s}{2\pi}4p^+(1-y)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} + \ln\left(\frac{\mu^2}{\mu_F^2}\right)\right)$$

DR used for IR and UV divergences

Infinite momentum frame (IMF)

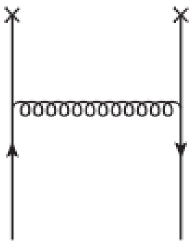


$$= -ig^2 C_F \int \frac{dk^0 dk^3 d^2k_\perp}{(2\pi)^4} \frac{\bar{u}(p)\gamma^\mu k\cdot\gamma\gamma^3 k\cdot\gamma\gamma_\mu u(p)}{(k^2)^2(p-k)^2}\delta\left(y - \frac{k^3}{p^3}\right)$$

$$k^2 + i\epsilon = \left(k^0 - \sqrt{k_\perp^2 + y^2(p^3)^2} + i\epsilon\right)\left(k^0 + \sqrt{k_\perp^2 + y^2(p^3)^2} - i\epsilon\right)$$

$$(p-k)^2 + i\epsilon = \left(k^0 - p^3 - \sqrt{k_\perp^2 + (1-y)^2(p^3)^2} + i\epsilon\right)\left(k^0 - p^3 + \sqrt{k_\perp^2 + (1-y)^2(p^3)^2} - i\epsilon\right)$$

Integrating in k^0 and taking the $p^3 \to \infty$ limit:



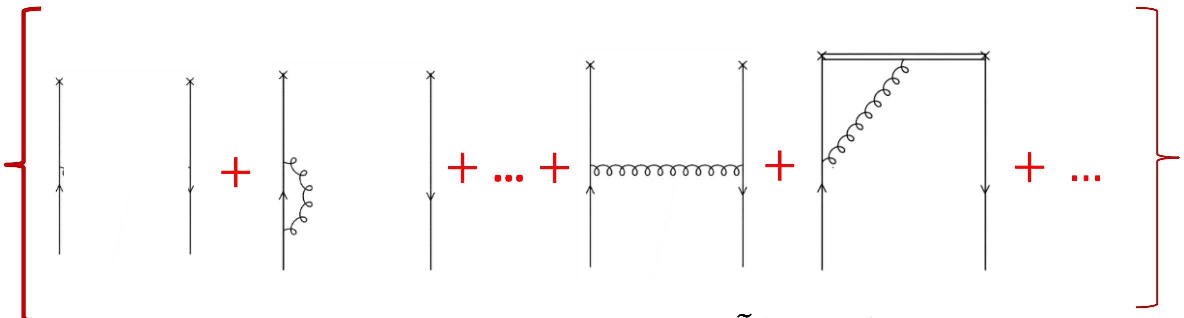
$$= 2\alpha_s C_F(1-y)\int \frac{d^2k_\perp}{(2\pi)^2}\frac{\bar{u}(p)\gamma^3 u(p)}{k_\perp^2} = \frac{\alpha_s}{2\pi}4p_3(1-y)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} + \ln\left(\frac{\mu^2}{\mu_F^2}\right)\right)$$

LC and IMF have the same IR and UV behaviour and are equivalente

Unfortunately, they can not be computed within LQCD

What if p_3 is kept finite?

$\tilde{q}(x,\Lambda)=$



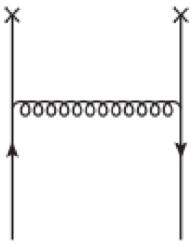
$\otimes q_{bare}(y) + \mathcal{O}(\alpha_s^2)$

$\delta(1-x/y)$ $\quad \widetilde{\Pi}(\Lambda)\delta(1-x/y)$ $\quad \tilde{\Gamma}(x/y,\Lambda)$

Regulator of IR and UV divergences

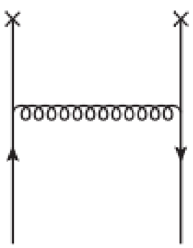
$$\tilde{q}(x,\Lambda) = q_{bare}(x)\left\{1+\frac{\alpha_s}{2\pi}\widetilde{\Pi}(\Lambda)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \tilde{\Gamma}\left(\frac{x}{y},\Lambda\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

Keeping p_3 finite



$$= -ig^2 C_F \int \frac{dk^0 dk^3 d^2k_\perp}{(2\pi)^4} \frac{\bar{u}(p)\gamma^\mu k\cdot\gamma\gamma^3 k\cdot\gamma\gamma_\mu u(p)}{(k^2)^2(p-k)^2}\,\delta\left(y-\frac{k^3}{p^3}\right)$$

Integrating in k^0 and keeping p_3 finite, we have an integral over k_T which is UV finite! But has an IR divergence. Using Dimensional Regularization:



$$= \frac{\alpha_s}{2\pi} 4p_3 \left((1-y)\left(-\frac{1}{\epsilon_{IR}} + \ln\left(\frac{p_3^2}{\mu_F^2}\right) + \ln(4y(1-y))\right) + 1\right), \qquad 0 < y < 1$$

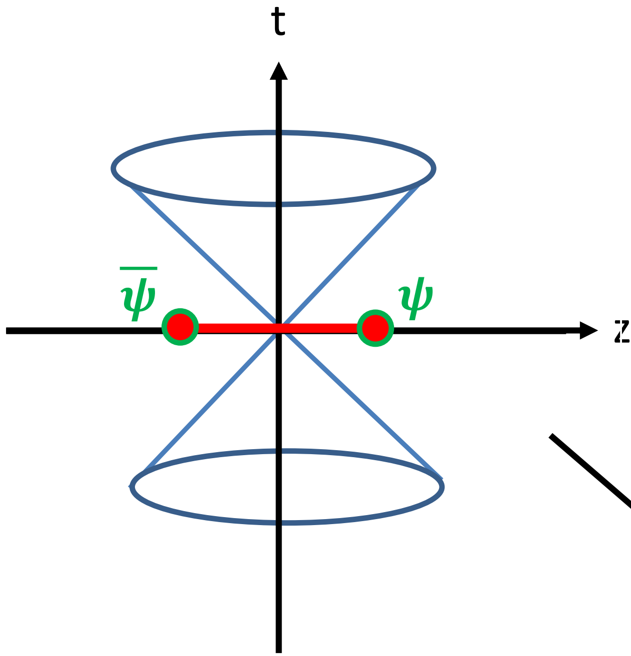
$$+ \frac{\alpha_s}{2\pi} 4p_3 \left((1-y)\ln\left(\frac{x}{x-1}\right) + 1\right), \qquad y > 1$$

$$+ \frac{\alpha_s}{2\pi} 4p_3 \left((1-y)\ln\left(\frac{x-1}{x}\right) - 1\right), \qquad y < 0$$

Support outside the physical region!

Same IR pole as in the LC and IMF cases

UV divergence appears only when integrating over all parton momentum fraction y

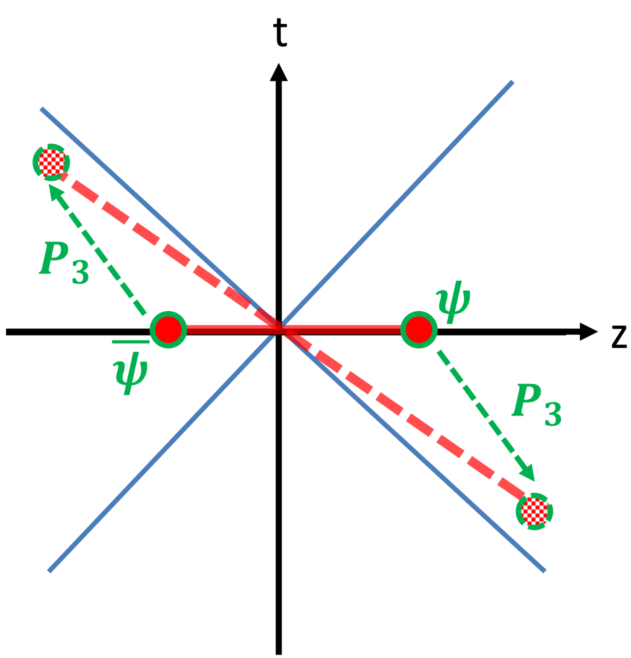


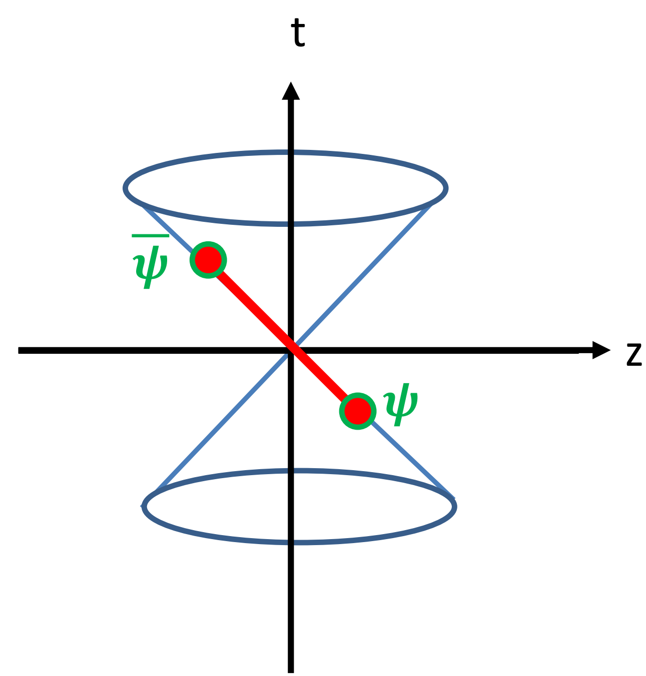
$$\tilde{q}(x) = \frac{2P_3}{4\pi} \int e^{-ixP_3 z} \left\langle P \middle| \bar{\psi}(z)\gamma^0 W\,(z,0)\psi(0) \middle| P \right\rangle$$

Purely spatial correlation

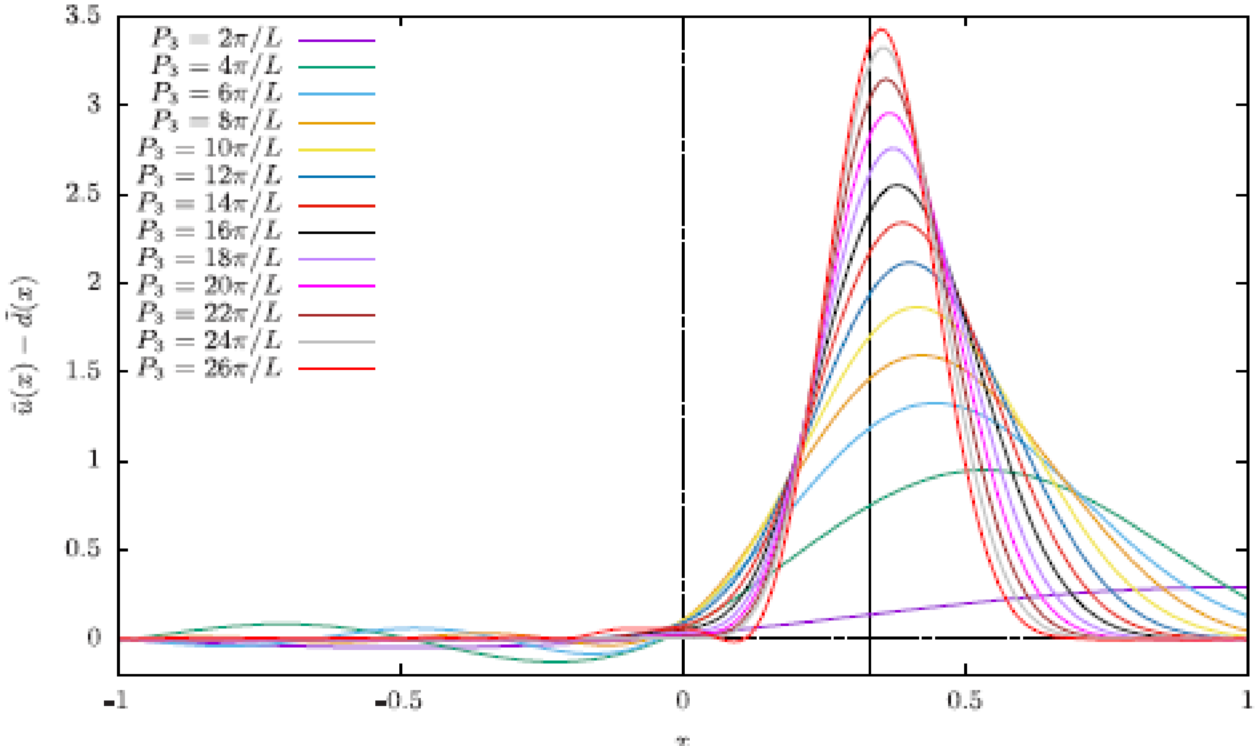
X. Ji, PRL 110 (2013) 262002.

We want to go from a purely spatial correlation to a light cone correlation





Free quark distributions for a $48^3 \times 96$ lattice



Each quark should carry 1/3 of the Nucleon momentum

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens and C. Wiese, ``Updated Lattice Results for Parton Distributions,'', PReD 96 (2017) no.1, 014513.

It tends to 1/3, as it should, as the nucleon momentum grows

We can do better than increasing P_3 indefinitely

As they have the same IR behaviour, PDFs and quasi-PDFs are related to each other:

$$q(x,\Lambda) = q_{bare}(x)\left\{1+\frac{\alpha_s}{2\pi}\Pi(\Lambda)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \Gamma\left(\frac{x}{y},\Lambda\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

1-loop correction in the LC or IMF

$$\tilde{q}(x,\Lambda) = q_{bare}(x)\left\{1+\frac{\alpha_s}{2\pi}\tilde{\Pi}(\Lambda)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \tilde{\Gamma}\left(\frac{x}{y},\Lambda\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

1-loop correction with finite P_3

Matching:

$$q^{\overline{MS}}(x,\mu) = \int_{-\infty}^{+\infty}\frac{dy}{|y|}C^{\overline{MS}}\left(\frac{x}{y},\frac{\mu}{p_3},\frac{\mu}{\mu_F}\right)\tilde{q}^{\overline{MS}}(y,P_3,\mu)$$

With:

$$C^{\overline{MS}}(\xi) = \delta(1-\xi) - \frac{\alpha_s}{2\pi}C_F\left(\left(\tilde{\Pi}^{\overline{MS}} - \Pi^{\overline{MS}}\right)\delta(1-\xi) + \tilde{\Gamma}(\xi) - \Gamma^{\overline{MS}}(\xi)\right)$$

Also

$$\int_{-\infty}^{+\infty} dx\,\tilde{q}^{\overline{MS}}(x,P_3,\mu) = h(0)$$

, for any P_3, μ

$$h(0) = \langle P|\bar{\psi}(0)\gamma^0 W(z,0)\psi(0)|P\rangle$$

Matching Kernel

$$C_{\gamma^0}^{\overline{\rm MS}}\left(\xi,\frac{\bar\mu}{p_3},\frac{\bar\mu}{\mu_F}\right)=\delta\left(1-\xi\right)$$

$$+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right)+1+\frac{3}{2\xi}\right)_{+(1)}-\frac{3}{2\xi}, & \xi>1\\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\left(\frac{p_3^2}{\bar\mu^2}\right)+\ln\left(4\xi(1-\xi)\right)\right]-\frac{\xi(1+\xi)}{1-\xi}\right)_{+(1)}, & 0<\xi<1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\left(\frac{-\xi}{1-\xi}\right)-1+\frac{3}{2(1-\xi)}\right)_{+(1)}-\frac{3}{2(1-\xi)}, & \xi<0\end{cases}$$

$$+\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\left(\frac{\mu_F^2}{4\bar\mu^2}\right)+\frac{5}{2}\right).$$

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato and F. Steffens, PRD 99 114504 (2019)

T.Izubuchi, X.Ji, L.Jin, I.W.Stewart and Y.Zhao, PRD 98 056004 (2018)

But, $\int_{-\infty}^{+\infty} dx\, \tilde{q}^{\overline{MS}}(x,\mu,P_3) \neq \int_{-1}^{+1} dx\, q^{\overline{MS}}(x,\mu) \longrightarrow$ quark number not preserved!

Origin: remaining $\ln(\xi)$ divergence

THIS NEEDS TO BE FIXED

Introduce a new scheme such that

$$\int_{-\infty}^{+\infty} dx\, \tilde{q}^{R}(x,\mu,P_3) = \int_{-1}^{+1} dx\, q^{R}(x,\mu)$$

Define a new scheme, where the remaining divergences are subtracted outside the physical region only, resulting in a minimal modification of $\overline{MS}$: Modified $\overline{MS}$ ($M\overline{MS}$):

$$\tilde{Z}^{\mathrm{M\overline{MS}}}_{\Gamma_{\gamma^0}}(\xi) = 1 - \frac{\alpha_s}{2\pi} C_F \frac{3}{2}\left(-\frac{1}{\xi}\theta(\xi-1) - \frac{1}{1-\xi}\theta(-\xi)\right) - \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\left(\frac{1}{4}\right) + \frac{5}{2}\right)$$

Momentum space

$$\begin{aligned} Z^{\mathrm{M\overline{MS}}}_{\Gamma_{\gamma^0}}(z\bar{\mu}) &= 1 - \frac{\alpha_s}{2\pi} C_F \left(\frac{3}{2}\ln\left(\frac{1}{4}\right) + \frac{5}{2}\right) \\ &+ \frac{3}{2}\frac{\alpha_s}{2\pi} C_F \left(i\pi\frac{|z\bar{\mu}|}{2z\bar{\mu}} - \mathrm{Ci}(z\bar{\mu}) + \ln(z\bar{\mu}) - \ln(|z\bar{\mu}|) - i\mathrm{Si}(z\bar{\mu})\right) \\ &- \frac{3}{2}\frac{\alpha_s}{2\pi} C_F e^{iz\bar{\mu}} \left(\frac{2\mathrm{Ei}(-iz\bar{\mu}) - \ln(-iz\bar{\mu}) + \ln(iz\bar{\mu}) + i\pi\mathrm{sgn}(z\bar{\mu})}{2}\right) \end{aligned}$$

Position space

In the limit of $z \to 0$, it agrees with the Ratio scheme of Izubuchi et al. arXiv:1801.03917

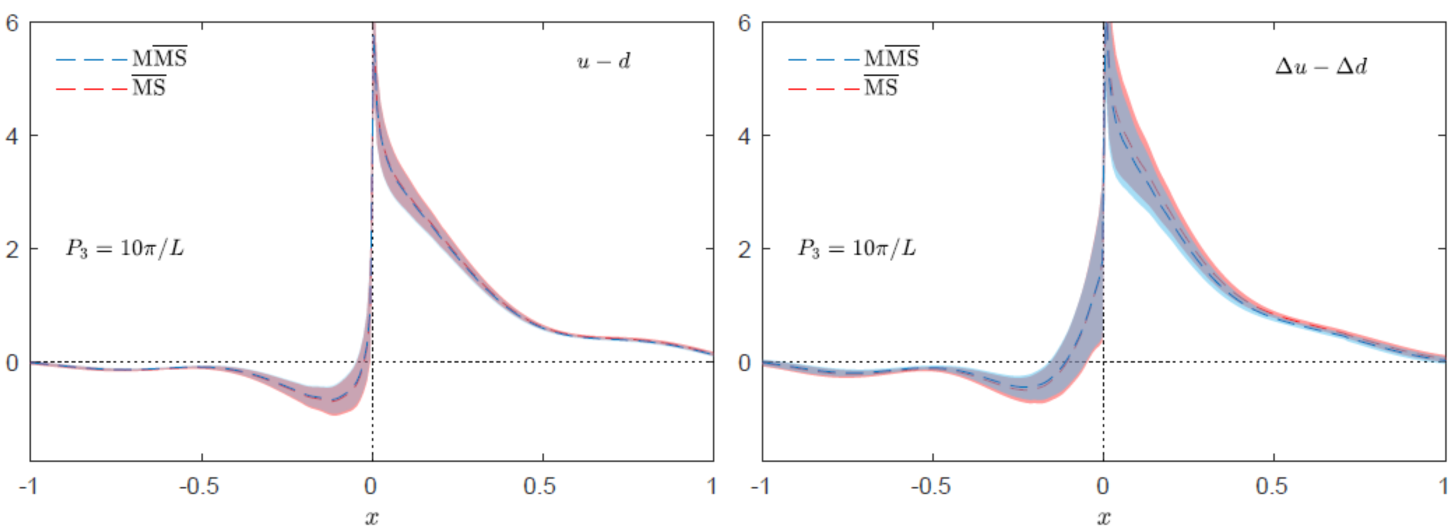
$$Z^{\mathrm{M\overline{MS}}}_{\Gamma_{\gamma^0}}(z \to 0) = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\left(\frac{\bar{\mu}^2 z^2 e^{2\gamma_E}}{4}\right) + \frac{5}{2}\right) = Z^{\mathrm{ratio}}_{\Gamma_{\gamma^0}}(z\bar{\mu})$$

It subtracts the $\ln(z^2 \to 0)$ divergence present in the $\overline{MS}$ scheme

$$q^{\overline{MS}}(x,\mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|}\, C^{M\overline{MS}}\left(\frac{x}{y},\frac{\mu}{p_3}\right) \tilde{q}^{M\overline{MS}}(y,P_3,\mu)\ ,\quad \int_{-\infty}^{+\infty} dx\, q^{\overline{MS}}(x,\mu,P_3) = \int_{-\infty}^{+\infty} dx\, \tilde{q}^{M\overline{MS}}(x,\mu)$$

Quark number preserved!

$$C^{\mathrm{M\overline{MS}}}_{\gamma^0,\gamma^3,\gamma^3\gamma^5}\left(\xi,\frac{\bar{\mu}}{p_3}\right) = \delta\left(1-\xi\right) + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right)+1+\frac{3}{2\xi}\right)_{+(1)}, & \xi>1, \\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\left(\frac{p_3^2}{\bar{\mu}^2}\right)+\ln\left(4\xi(1-\xi)\right)\right]-\frac{\xi(1+\xi)}{1-\xi}+2\iota(1-\xi)\right)_{+(1)}, & 0<\xi<1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\left(\frac{-\xi}{1-\xi}\right)-1+\frac{3}{2(1-\xi)}\right)_{+(1)}, & \xi<0, \end{cases}$$



Antiquarks Quarks Antiquarks Quarks

$\overline{MS}$: Previous calculation (C.Alexandrou et al., PRL 121, (2018), 112001), where $Z^{M\overline{MS}}$ had not been applied to lattice data

$M\overline{MS}$: Extra subtraction consistently applied

Nonperturbative Renormalization of Lattice Data

$$h^{R,u-d} = Z_h h^{u-d} = (Re[Z_h] + i\ Im[Z_h])\left(Re\left[h^{u-d}\right] + i\ Im\left[h^{u-d}\right]\right)$$

Z_h renormalizes both the usual log divergence
and the estra linear divergence associated with the Wilson line

Nonperturbative renormalization using the RI'-MOM to remove both divergences

C. Alexandrou et al., NPB 923 (2017) 394 (Frontier Article)
J-W. Chen et al., PRD 97 014505 (2018)
C. Alexandrou et al., 1807.00232

Convert the ME from RI'-MOM to $\overline{MS}$ using 1-loop perturbation theory

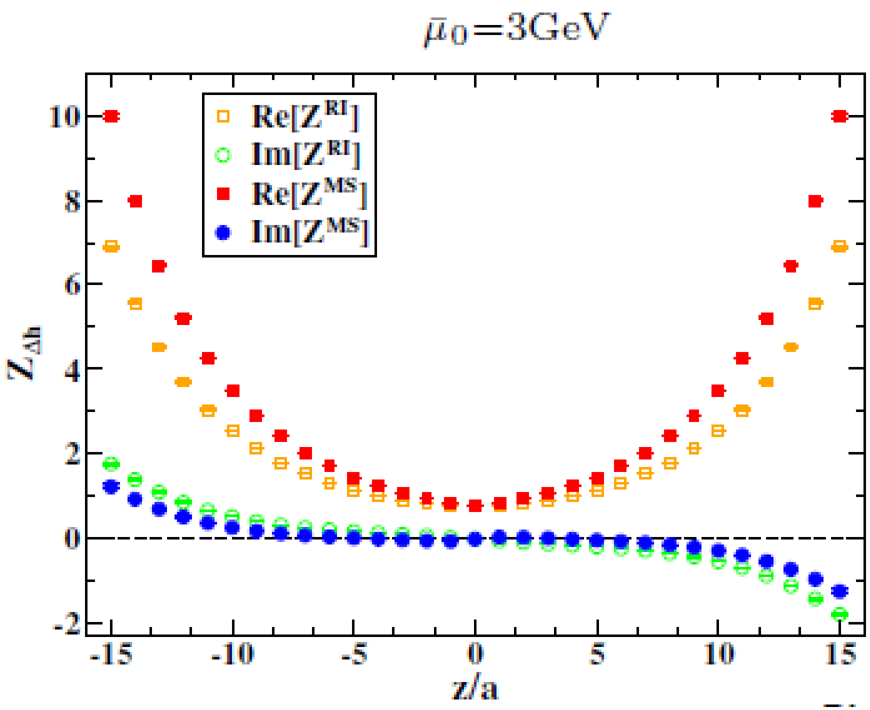
M. Constantinou, H. Panapaulos, PRD (2017)054506

We present results for the $\overline{MS}$ scheme

Example of the renormalization factor

RI'-MOM scheme at the scale $\bar{\mu}_0 = 3$ GeV

Perturbative conversion to $\overline{MS}$ scheme at the scale 2 GeV



$$Z_q^{-1}\, Z_{\mathcal{O}} \frac{1}{12} Tr[\nu(p,z)(\nu^{Born}(p,z))^{-1}]|_{p^2=\bar{\mu}_0^2} = 1$$

$$Z_q = \frac{1}{12} Tr[\left(S(p)\right)^{-1} S^{Born}(p)]|_{p^2=\bar{\mu}_0^2}$$

The vertex function ν contains the same divergences as the nucleon matrix elements

The factor $Z_{\mathcal{O}}$ subtracts both the linear and log divergences.

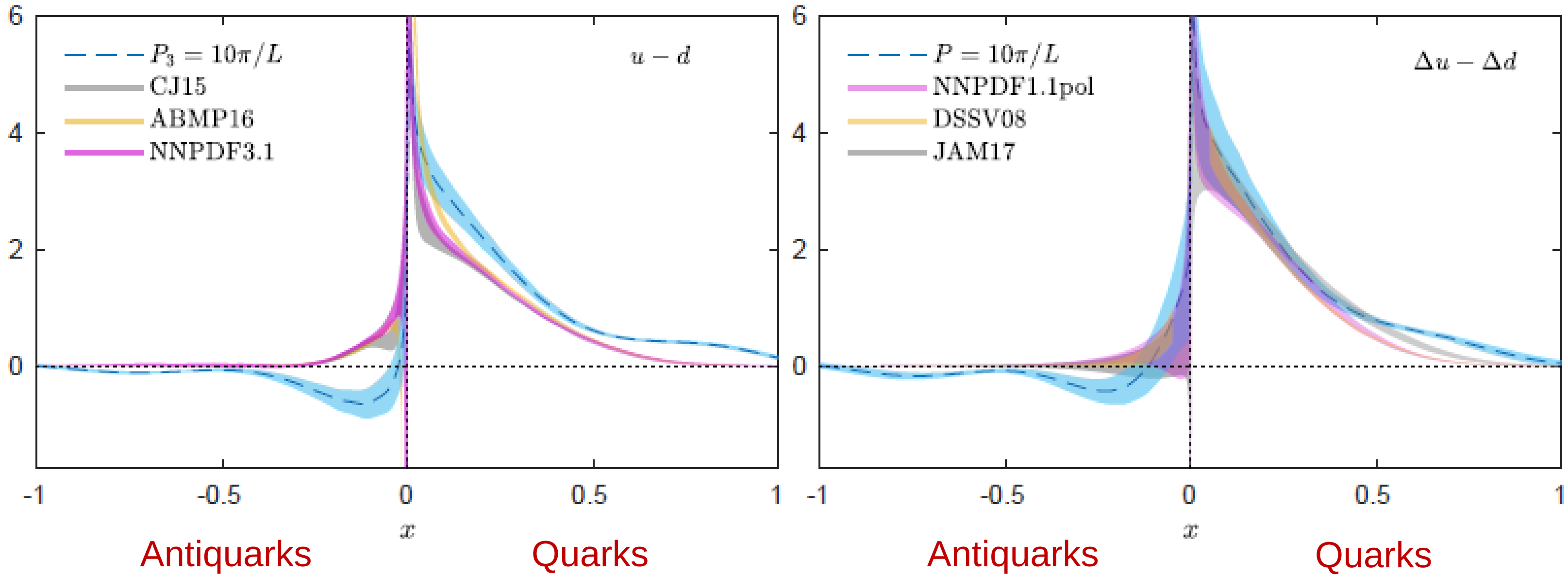
The linear divergence associated with the Wilson line makes $Z_{\mathcal{O}}$ to grow very fast for large z;

That makes the renormalized ME to have amplified errors at large z;

Isovector unpolarized and helicity distributions at the physical pion mass

Renormalized ME ⟶ FT ⟶ Matching⟶ Resulting PDF

Unpolarized $P_3 = 1.38\ GeV$ **Helicity**



C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato and F. Steffens, arXiv:1902.00587 [hep-lat].

$\bar{d}(x) - \bar{u}(x) < 0$ induced by the finite number of points in the Fourier transform?

$m_\pi \cong 130$ MeV

$48^3 \times 96$ lattice

$a \cong 0.093$ fm

See Savvas talk

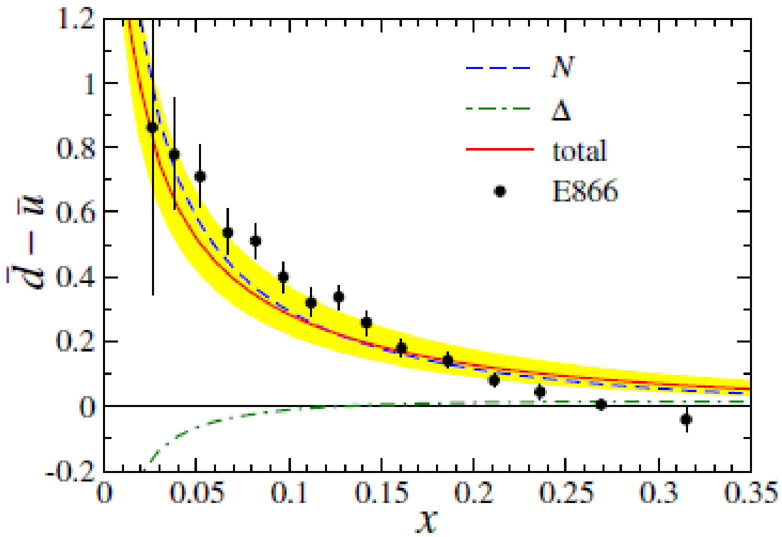
Nucleon sea

Gluons are flavour blind

But, from NMC data

$$\int_0^1 \frac{dx}{x}\left(F_2^p(x) - F_2^n(x)\right) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx(\bar{u}(x) - \bar{d}(x)) = 0.235 \pm 0.026$$

And from E886 data



$\bar{d}(x) \neq \bar{u}(x)$ has a nonperturbative origin

What is the physics behind it? Chiral Loops?

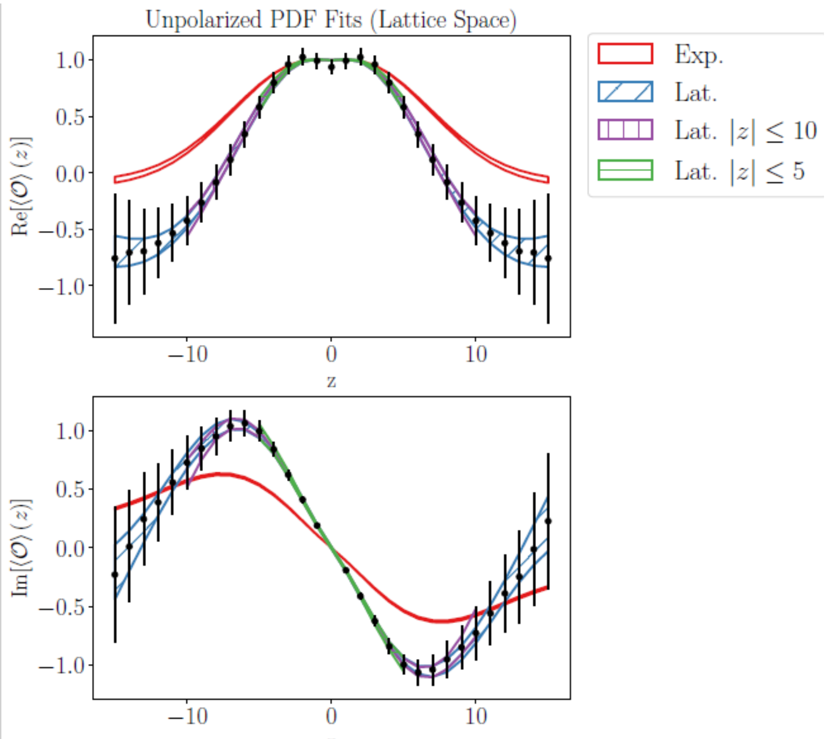
P. C. Barry, N. Sato, W. Melnitchouk and C. R. Ji, PRL **121** (2018) no.15, 152001

Fourier transform problem can be avoided by going in the opposite direction:

$$\langle \mathcal{O} \rangle (z) = -\int_{-\infty}^{\infty} dy\, e^{-iyP_3 z} \int_{-1}^{1} \frac{dx}{|x|}\, C\left(\frac{y}{x}, \frac{\mu}{xP_3}\right)\; f(x, \mu)$$

Lattice QCD matrix elements

LC PDFs fitted to reproduce the lattice QCD ME



Lattice QCD data to be treated with same status of experimental data

Use lattice data as experimental points in a global fitting analysis

Work in progress in collaboration with J. Bringewatt, M. Constatinnou, W. Melnitchouk, J.-W. Qui, N. Sato, FS

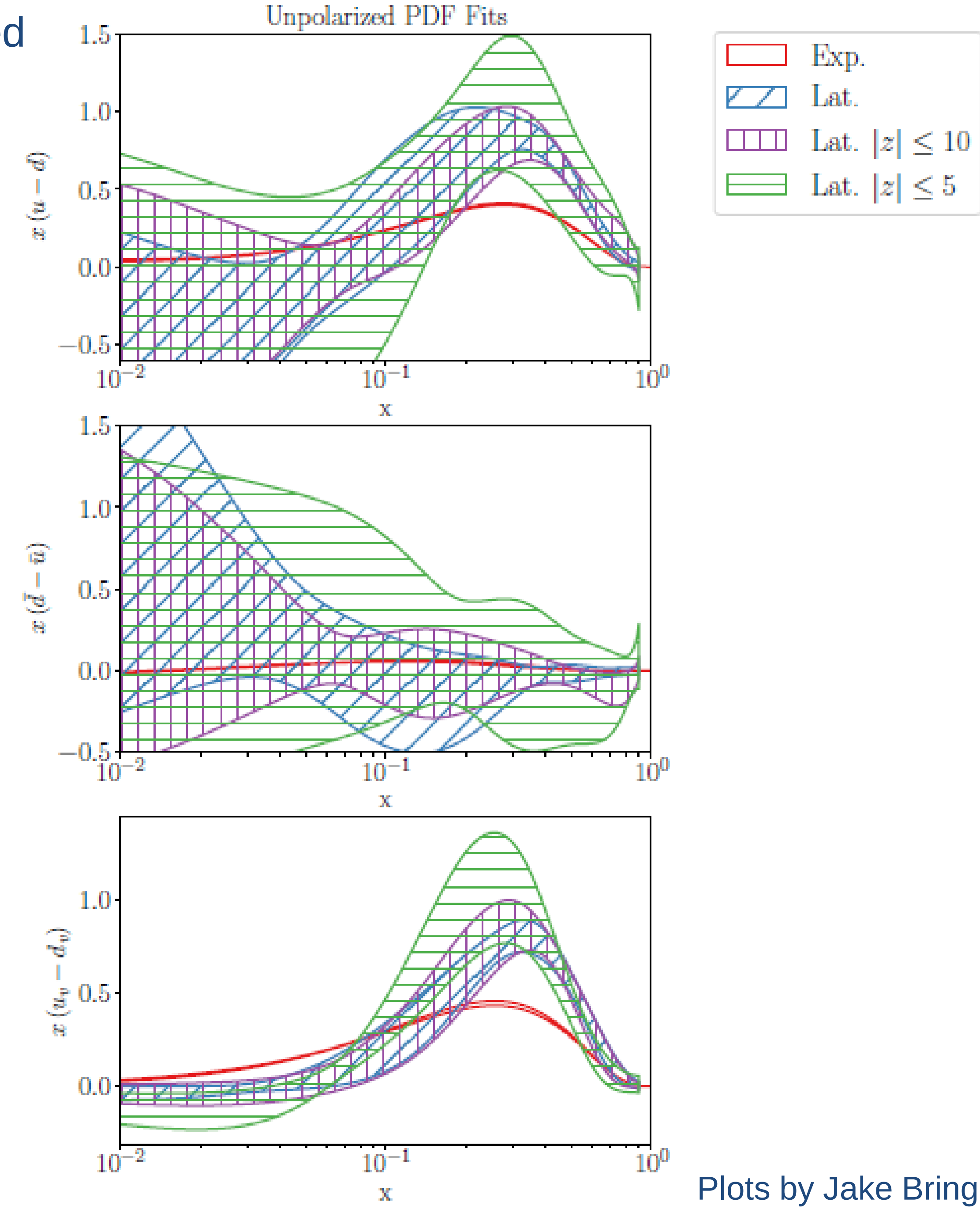
See also recent work of K. Cichy, L. Del Debbio, T. Giane, arXiv:1907.06037

100 Monte Carlo Samples used

$x(u(x) - d(x))$

$x(\bar{d}(x) - \bar{u}(x))$

$x(u_v(x) - d_v(x))$



Large z region is important to constrain the distributions

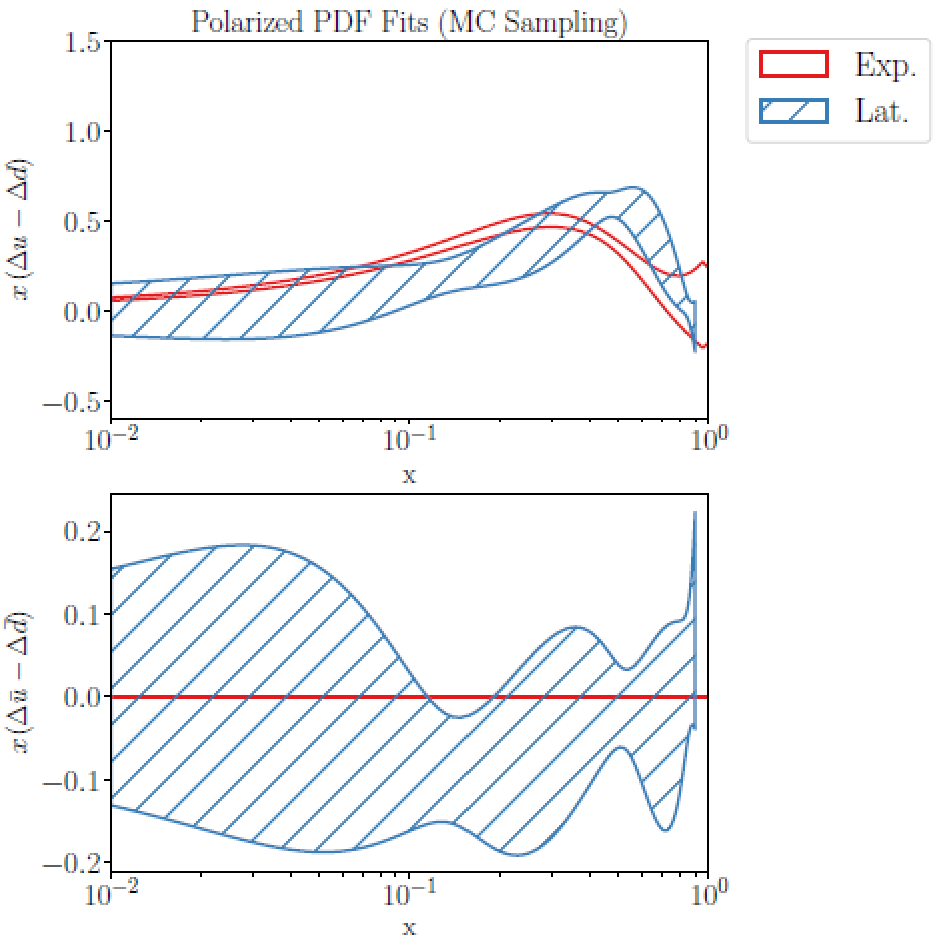
Plots by Jake Bringewatt

Helicity isovector

100 Monte Carlo Samples used

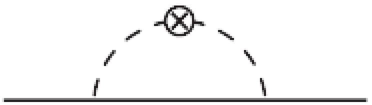
$x(\Delta u(x) - \Delta d(x))$

$x\left(\Delta\bar{d}(x) - \Delta\bar{u}(x)\right)$



Sea asymmetries and chiral loops

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\bar{\psi}_N\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\psi_N - \frac{1}{(2f_\pi)^2}\bar{\psi}_N\gamma^\mu\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\psi_N,$$



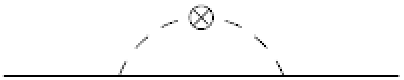
Usual Rainbow

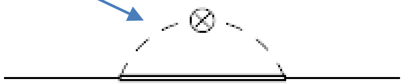
Weiberg-Tomozawa

Contributions from nucleon and delta intermediate states

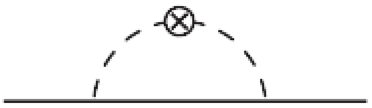
$$(\bar{d}-\bar{u})^p(x) = 2[(f_{N\to N\pi} - f_{N\to\Delta\pi})\otimes\overline{q_v}^{\pi}](x)$$

Antiquark distributions in the pion





Loop correction very much as before, with nucleons and pions replacing quarks and gluons:



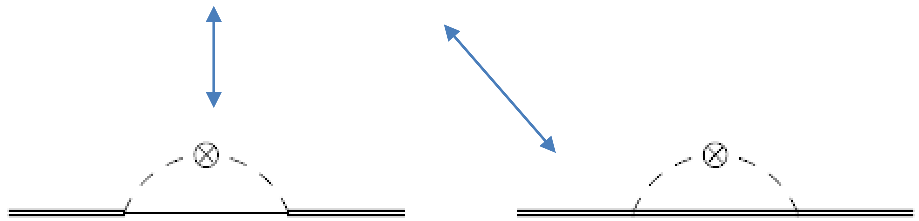
$$= 4M\left(\frac{g_A}{2f_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\bar{u}(p)(\gamma_5 k\cdot\gamma)\frac{i(p\cdot\gamma - k\cdot\gamma - M)}{D_N}(\gamma_5 k\cdot\gamma)u(p)\frac{i}{D_\pi}\frac{i}{D_\pi}\frac{2k^+}{p^+}\delta\left(y-\frac{k^+}{p^+}\right)$$

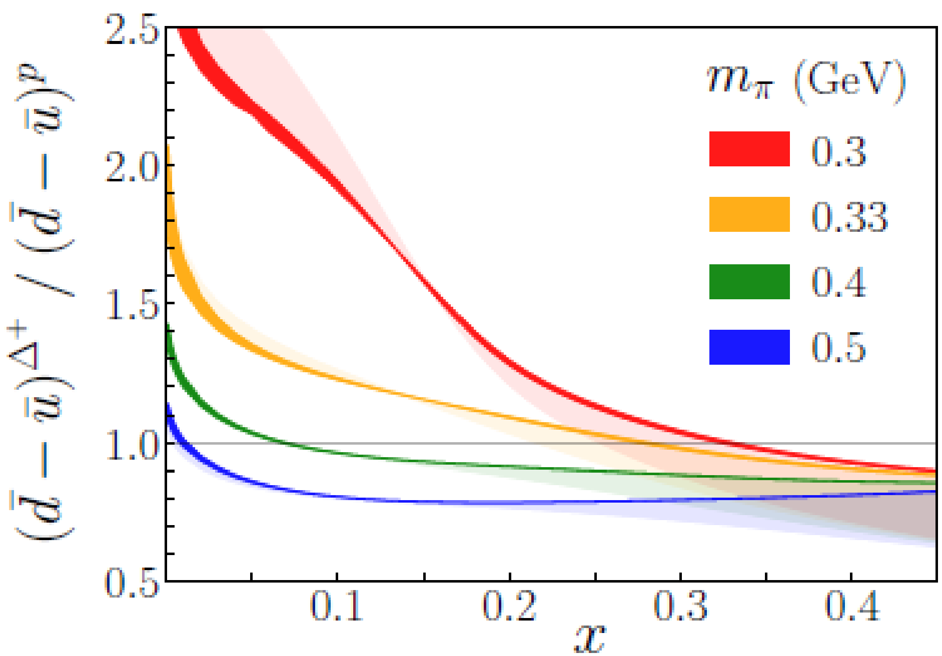
$$D_\pi = k^2 - m_\pi^2 + i\epsilon$$

$$D_N = (p-k)^2 - M^2 + i\epsilon$$

$\bar{d}(x) - \bar{u}(x)$ asymmetry in the Delta

$$(\bar{d} - \bar{u})^{\Delta^+}(x) = [(f_{\Delta\to N\pi} + 2f_{\Delta\to\Delta\pi})\otimes\overline{q_v}^{\pi}](x)$$





Same calculation that reproduces $(\bar{d} - \bar{u})^p$

Enhancement from the opening of the decay channel when $m_\pi \sim M_\Delta - M$

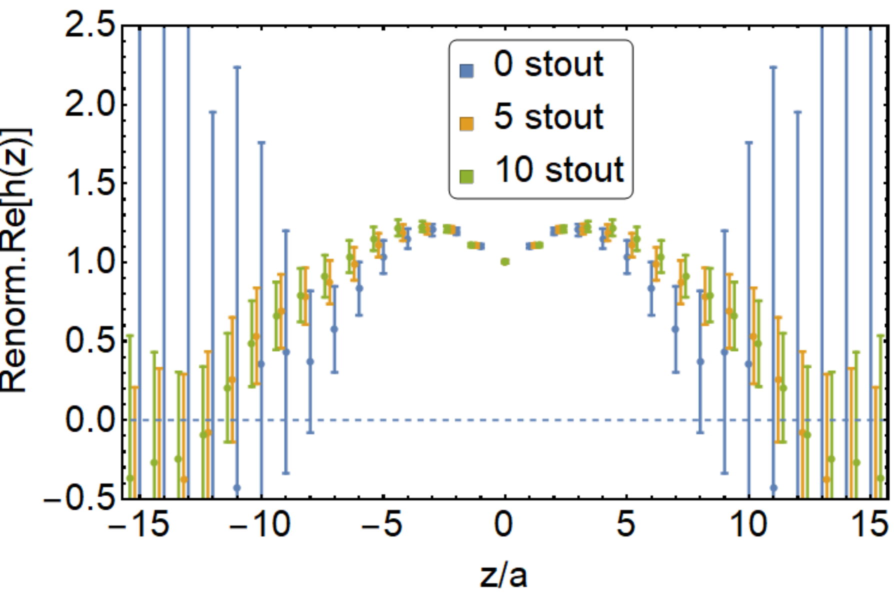
Can be tested in a Lattice computation!

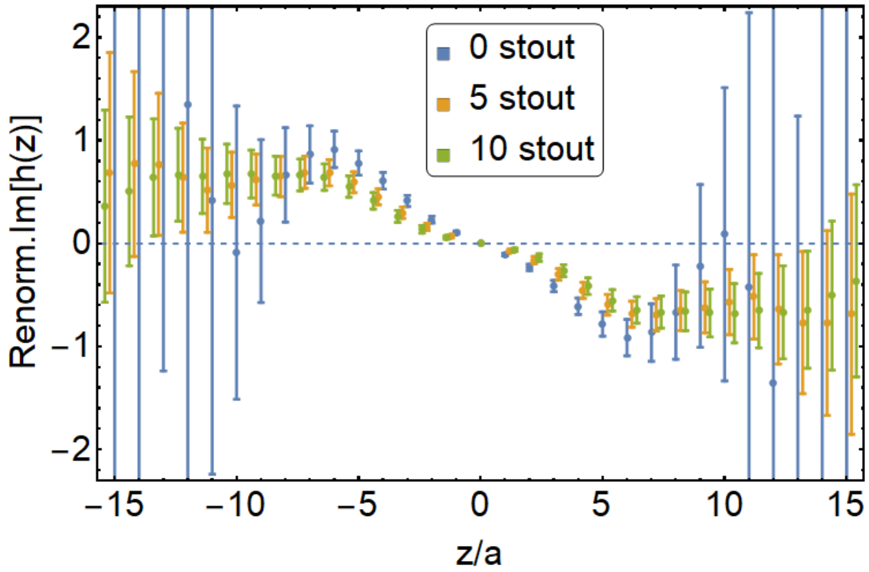
Darker bands: uncertainties on the pion PDFs

Ligher bands: dependence on the choice of regulator

J. J. Ethier, W. Melnitchouk, F. Steffens and A. W. Thomas,PRD 100 (2019), 014508, arXiv:1809.06885 [hep-ph].

Preliminary results for the Delta renormalized ME



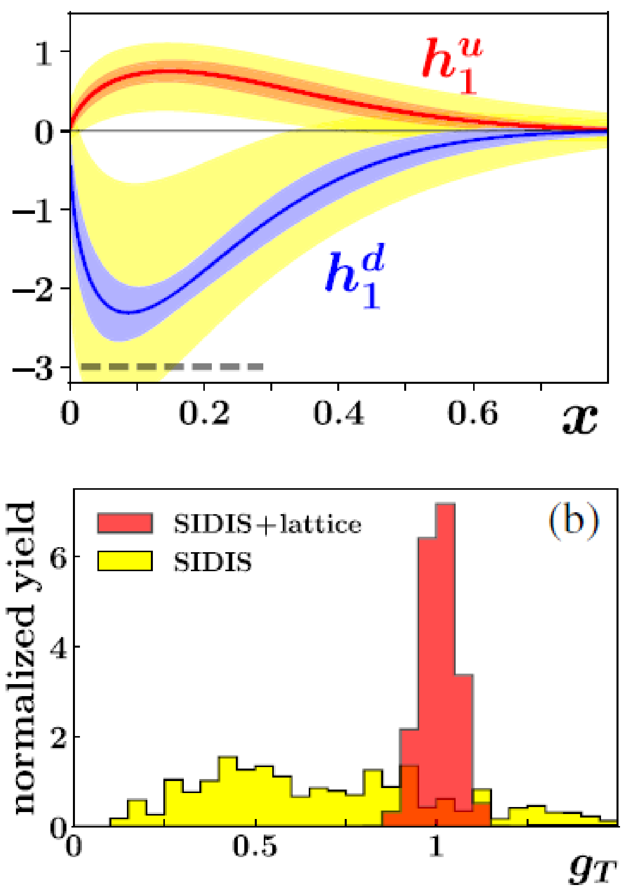


Renormalized Delta ME for $P_3 \cong 0.82$ GeV

$32^3 \times 64$ Lattice, $a = 0.094$ fm, $m_\pi = 250$ MeV

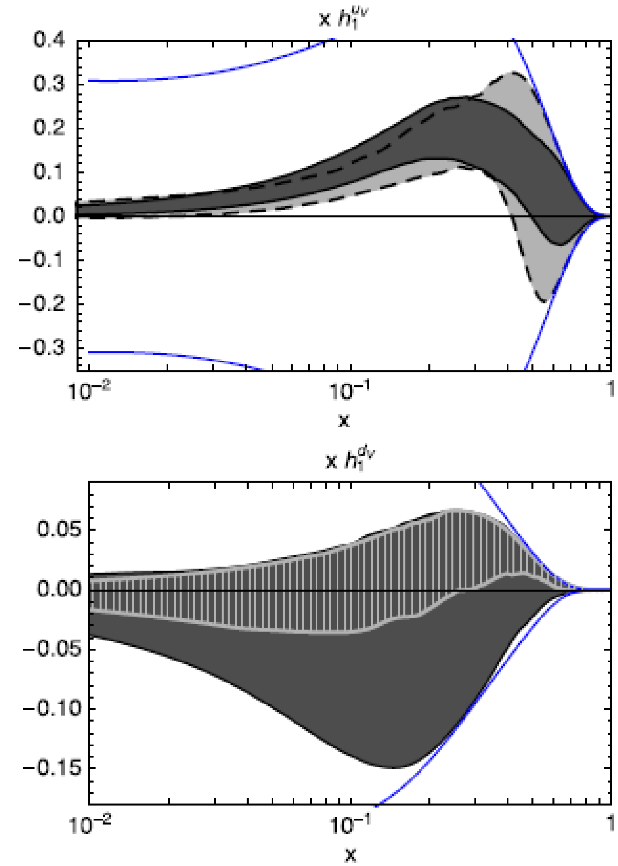
Yahui Chai et al., arXiv:1907.09827

Transversity: two recent extractions



$$g_T = \int_0^1 dx \left(h_1^u(x) - h_1^d(x) \right) = 1.0(1)$$

H.-W. Lin PRL 120, 152502 (2018)



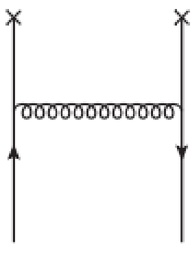
$$g_T = 0.53(25)$$

Radici and Bacchetta PRL 120, 192001 (2018)

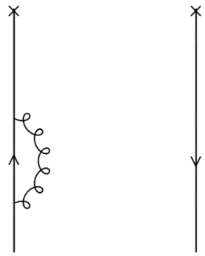
Can we have an *ab initio* calculation of $h_1^q(x)$?

Transversity case

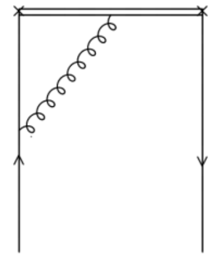
$$\tilde{h}_1(x) = \frac{1}{4\pi}\int e^{-ixP^3 z}\,\langle P|\bar{\psi}(z)\gamma^3\gamma^j W\,(z,0)\psi(0)|P\rangle$$



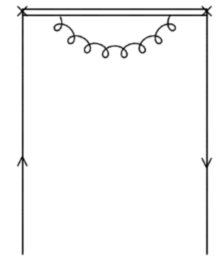
$= 0$



$\neq 0$



$\neq 0$



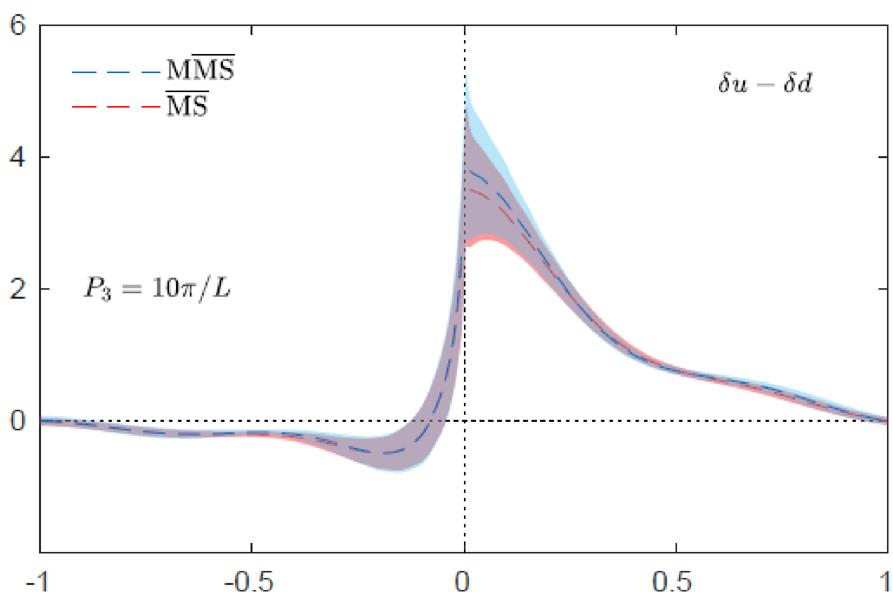
$\neq 0$

Easier than previous case

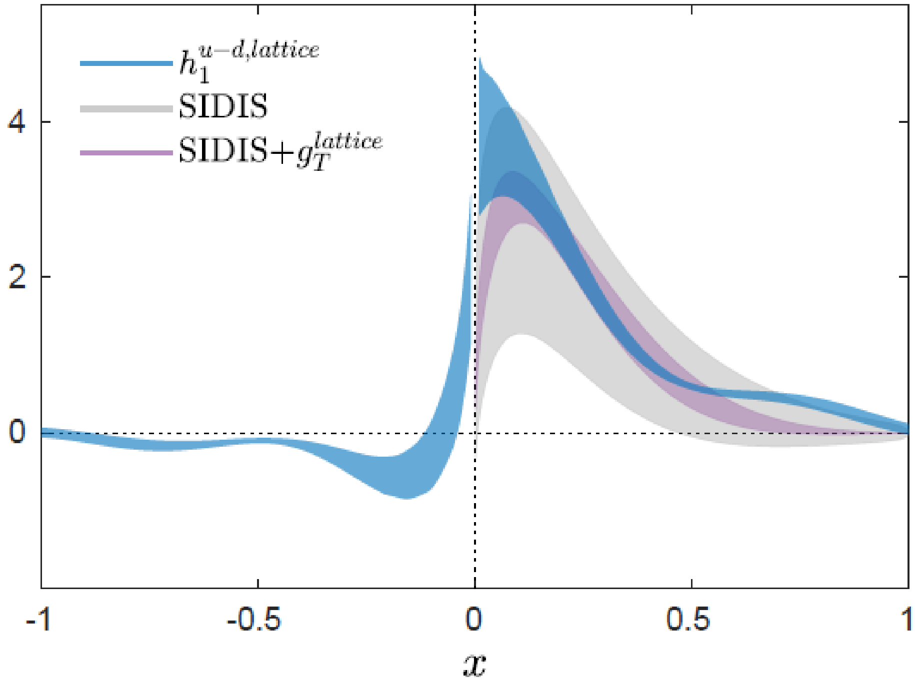
$$\Gamma^{\overline{MS}}_{\gamma^3\gamma^j}\left(\xi, \frac{\mu}{\mu_F}\right) = \frac{2\xi}{1-\xi}\left(-\frac{1}{\epsilon_{IR}} + \ln\frac{\mu^2}{\mu_F^2}\right)$$

$$\tilde{\Gamma}_{\gamma^3\gamma^j}(\xi, p_3/\mu_F) = \begin{cases} \frac{2\xi}{1-\xi}\ln\frac{\xi}{\xi-1}, & \xi > 1 \\ \frac{2\xi}{1-\xi}\left(-\frac{1}{\epsilon_{IR}} + \ln\frac{4\xi(1-\xi)(p_3)^2}{\mu_F^2}\right) - \frac{2\xi}{(1-\xi)}, & 0 < \xi < 1 \\ \frac{2\xi}{1-\xi}\ln\frac{\xi-1}{\xi}, & \xi < 0 \end{cases}$$

$$C^{\mathrm{M\overline{MS}}}_{\gamma^3\gamma^j}\left(\xi, \frac{\bar{\mu}}{p_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{2\xi}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right) + \frac{2}{\xi}\right]_{+(1)}, & \xi > 1, \\ \left[\frac{2\xi}{1-\xi}\left(\ln\left(\frac{p_3^2}{\bar{\mu}^2}\right) + \ln(4\xi(1-\xi))\right) - \frac{2\xi}{1-\xi}\right]_{+(1)}, & 0 < \xi < 1 \\ \left[-\frac{2\xi}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right) + \frac{2}{1-\xi}\right]_{+(1)}, & \xi < 0\,. \end{cases}$$



Comparing with phenomenology:



C. Alexandrou et al., PRD98, 091503 (2018)

$$g_T = \int_{-1}^{+1} dx h_1^{u-d} = 1.09(11)$$

This should be compared to: $g_T = 1.06(1)$ from dedicated lattice QCD calculation

C. Alexandrou et al., PRD95, 114514 (2017)

$g_T = 1.0(1)$ from Monte Carlo global analysis

H.-W. Lin PRL 120, 152502 (2018)

$g_T = 0.53(25)$ from global analysis of ep and pp data

Radici and Bacchetta PRL 120, 192001 (2018)

Lots of interest in recent years using not only the quasi-PDF approach

Pseudo-PDF approach – See Savvas talk

Lattice Cross-section – Y. -Q. Ma and J. Qiu PRD 98 (2018), PRL 120 (2018

Pion and nucleon PDFs using quasi-PDF, pseudo-PDF, and LCS – See:

- Savvas talk
- Colin Egner and David Richards talks at Lattice 2019

Gluon quasi-PDFs – Z. y. Fan, Y. B. Yang, A. Anthony, H. W. Lin PRL 121 (2018) 242001

Summary

LC and IMF PDFs are equivalent. However, lattice can not be used;

Calculations in a frame where the nucleon has finite but large momentum in the third direction are possible. Connection to the LC PDFs is made through a matching procedure;

A norm preserving matching has to be used;

$\bar{d}(x) - \bar{u}(x) < 0$ probably induced by Fourier transform;

Better path: go from momentum to position space. First results indicate that antiquarks are not strongly constrained by present data. Large z region is very important;

Hoe compatible is lattice data with other sets of data (E866 data, for example) in a global fitting analysis?

Non-trivial structure of the nucleon sea seems to be deeply connected to the breaking of chiral symmetry. Same physics that describes $\bar{d}(x) - \bar{u}(x)$ in the proton can can be applied to the Δ^+;

Strong enhancement of the asymmetry found close to the Δ^+ decay channel. Lattice QCD can test this effect;

Transversity distributions poorly constrained by current experimental data. Lattice QCD can help;

Data at one single point at the physical pion mass. Systematics have to be addressed;

Intense activity during last 4 years. Nucleon and pion quark distributions computations involving quasi-PDFs, Pseudo-PDFs and LCS.