Exclusive diffractive processes including saturation effects at next-to-leading order

Samuel Wallon

Sorbonne Université

and

Laboratoire de Physique Théorique

CNRS / Université Paris Sud

Orsay

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based on works with:

R. Boussarie, A. V. Grabovsky, D. Yu. Ivanov, L. Szymanowski
References


- On the one loop $\gamma^{(*)} \rightarrow q\bar{q}$ impact factor and the exclusive diffractive cross sections for the production of two or three jets, R. Boussarie, A. V. Grabovsky, L. Szymanowski, S. W., JHEP 1611 (2016) 149 [arXiv:1606.00419 [hep-ph]]


Example: DIS

The various regimes governing the perturbative content of the proton

- “usual” regime: $x_B$ moderate ($x_B \gtrsim 0.01$):
  Evolution in $Q$ governed by the QCD renormalization group
  (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equation)
  \[
  \sum_n (\alpha_s \ln Q^2)^n + \alpha_s \sum_n (\alpha_s \ln Q^2)^n + \cdots
  \]
  LLQ
  NLLQ

- Perturbative Regge limit: $s_{\gamma^*p} \to \infty$ i.e. $x_B \sim Q^2/s_{\gamma^*p} \to 0$
  in the perturbative regime (hard scale $Q^2$)
  (Balitski Fadin Kuraev Lipatov equation)
  \[
  \sum_n (\alpha_s \ln s)^n + \alpha_s \sum_n (\alpha_s \ln s)^n + \cdots
  \]
  LLs
  NLLs
One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative Regge limit $s \gg -t$.

Based on theoretical grounds, one should identify and test suitable observables in order to test this peculiar dynamics:

\[
\begin{align*}
    h_1(M_1^2) & \quad \rightarrow \quad h'_1(M'_1^2) \\
    s & \quad \rightarrow \quad \text{vacuum quantum number} \\
    h_2(M_2^2) & \quad \rightarrow \quad h'_2(M'_2^2)
\end{align*}
\]

hard scales: $M_1^2, M_2^2 \gg \Lambda_{QCD}^2$ or $M'_1^2, M'_2^2 \gg \Lambda_{QCD}^2$ or $t \gg \Lambda_{QCD}^2$

where the $t-$channel exchanged state is the so-called hard Pomeron.

Inclusive processes: the above picture applies at the level of cross-sections (optical theorem $\Rightarrow t = 0$).

Diffractive processes: gap in rapidity between two clusters in the detector. The above picture applies at the level of amplitudes.
How to test QCD in the perturbative Regge limit?

What kind of observable?

- perturbation theory should be applicable: selecting external or internal probes with transverse sizes \( \ll \frac{1}{\Lambda_{QCD}} \) (hard \( \gamma^* \), heavy meson (\( J/\Psi \), \( \Upsilon \)), energetic forward jets) or by choosing large \( t \) in order to provide the hard scale.

- governed by the "soft" perturbative dynamics of QCD and not by its collinear dynamics

\[ p \to 0 \]

\[ m = 0 \]

\[ \theta \to 0 \]

\[ m = 0 \to 0 \]

\[ \implies \text{select semi-hard processes with } s \gg p_{T1}^2 \gg \Lambda_{QCD}^2 \text{ where } p_{T1}^2 \text{ are typical transverse scale, all of the same order.} \]
### Kinematics

**Lightcone Sudakov vectors**

\[ n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1 \]

**Lightcone coordinates:**

\[ x = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x}) \]

\[ x^+ = x^- = (x \cdot n_2) \quad x^- = x^+ = (x \cdot n_1) \]
Rapidity separation

\[ ~p^+ n_1 \]

\[ k^+ > e^\eta p^+ \]

\[ k^+ < e^\eta p^+ \]

\[ ~p^- n_2 \]

Let us split the gluonic field between "fast" and "slow" gluons

\[ \mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a} (|k^+| > e^\eta p^+, k^-, \vec{k}) \quad \text{quantum part} \]

\[ + b_\eta^{\mu a} (|k^+| < e^\eta p^+, k^-, \vec{k}) \quad \text{classical part} \]

\[ e^\eta \ll 1 \]
Large longitudinal boost: $\Lambda \propto \sqrt{s}$

\[ b^\mu(x) \rightarrow b^-(x) \, n_2^\mu \simeq \delta(x^+) \, B(\vec{x}) \, n_2^\mu \]

Shockwave approximation

Multiple interactions with the target can be resummed into path-ordered Wilson lines attached to each parton crossing lightcone time 0:

\[ \tilde{U}^\eta(p) = \int d^{D-2} \vec{z} \, e^{-i(p \cdot \vec{z})} U^\eta_{\vec{z}}, \quad U^\eta_{\vec{z}} = U^\eta_{\vec{z}_i} = P e^{ig \int b^-_{\vec{z}_i} (z_i^+, \vec{z}_i) \, dz_i^+} \]
Factorized picture in the projectile frame

Factorized amplitude

\[ \mathcal{A}^\eta = \int d^{D-2} \vec{z}_1 d^{D-2} \vec{z}_2 \, \Phi^\eta(\vec{z}_1, \vec{z}_2) \langle P' \left[ \text{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c \right] | P \rangle \]

Dipole operator \( U^\eta_{ij} = \frac{1}{N_c} \text{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1 \)

Written similarly for any number of Wilson lines in any color representation
Evolution for the dipole operator

**B-JIMWLK hierarchy of equations**

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

\[
\frac{\partial U_{12}^\eta}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\mathbf{z}_3 \frac{\bar{z}_{12}^2}{\bar{z}_{13}^2 \bar{z}_{23}^2} \left[ U_{13}^\eta + U_{32}^\eta - U_{12}^\eta + U_{13}^\eta U_{32}^\eta \right]
\]

\[
\frac{\partial U_{13}^\eta U_{32}^\eta}{\partial \eta} = \ldots
\]

Mean field approximation (large \( N_C \))

\[ \Rightarrow \text{BK equation} \ [\text{Balitsky, 1995} ] \ [\text{Kovchegov, 1999}] \]

\[
\frac{\partial \langle U_{12}^\eta \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\mathbf{z}_3 \frac{\bar{z}_{12}^2}{\bar{z}_{13}^2 \bar{z}_{23}^2} \left[ \langle U_{13}^\eta \rangle + \langle U_{32}^\eta \rangle - \langle U_{12}^\eta \rangle + \langle U_{13}^\eta \rangle \langle U_{32}^\eta \rangle \right]
\]

Non-linear term: saturation
Practical use of the formalism

- **Compute** the upper impact factor using the effective Feynman rules
- **Build** non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- **Solve** the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $Y_0$.
- Evaluate the solution at a typical projectile rapidity $Y$, or at the rapidity of the slowest gluon
- **Convolute** the solution and the impact factor

$$ A = \int d\vec{z}_1 \ldots d\vec{z}_n \Phi(\vec{z}_1, \ldots, \vec{z}_n) \times \langle P' | U_{\vec{z}_1} \ldots U_{\vec{z}_n} | P \rangle $$

Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude
Exclusive dijet diffractive production

Framework

- **Regge-Gribov limit**: $s \gg Q^2 \gg \Lambda_{QCD}$

- **Otherwise** completely general kinematics

- **Shockwave (CGC)** Wilson line approach

- **Transverse dimensional regularization** $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$
Exclusive dijet diffractive production

LO diagram

\[
\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{e}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_\bar{q}, z_0)]_{jk} \theta(-z_0^+) \\
= \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_0(\vec{p}_1, \vec{p}_2) \\
\times C_F \langle P' | \hat{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle \\
\hat{U}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2 e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} \left[ \frac{1}{N_c} \text{Tr}(U_{\vec{z}_1} U_{\vec{z}_2}^\dagger) - 1 \right]
\]
Exclusive dijet diffractive production
NLO open $q\bar{q}$ production

Diagrams contributing to the NLO correction
Exclusive dijet diffractive production

First kind of virtual corrections

\[ \mathcal{A}^{(1)}_{NLO} \propto \delta(p_1^+ + p_\bar{q} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_\bar{q} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_{V1}(\vec{p}_1, \vec{p}_2) \]

\[ \times C_F \langle P'| \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle \]
**Exclusive dijet diffractive production**

Second kind of virtual corrections

\[
\mathcal{A}^{(2)}_{NLO} \propto \delta(p_q^+ + p\bar{q} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3) \\
\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle] \quad \text{dipole contribution} \\
+ \Phi'_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{W}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle] \quad \text{double dipole contribution}
\]
Exclusive dijet diffractive production
LO open $q\bar{q}g$ production

\[ A_R^{(2)} \propto \delta(p_q^+ + p_\bar{q} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_\bar{q} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3) \]
\[ \times [\Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle + \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{V}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle] \]

\[ A_R^{(1)} \propto \delta(p_q^+ + p_\bar{q} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_\bar{q} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \]
\[ \times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle \]
Exclusive dijet diffractive production

Various types of divergences

Divergences

- Rapidity divergence $p_g^+ \rightarrow 0$
  \[ \Phi_{V2} \Phi_0^* + \Phi_0 \Phi_{V2}^* \]

- UV divergence $p_g^2 \rightarrow +\infty$
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^* \]

- Soft divergence $p_g \rightarrow 0$
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^* \]

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$
  \[ \Phi_{R1} \Phi_{R1}^* \]

- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \rightarrow 0$
  \[ \Phi_{R1} \Phi_{R1}^* \]
Exclusive dijet diffractive production

Rapidity divergence

Double dipole virtual correction $\Phi_{V_2}$

B-JIMWLK evolution of the LO term: $\Phi_0 \otimes K_{BK}$
**Introduction**

The shockwave approach

**Exclusive dijet diffractive production**

Light meson production

Phenomenology

**Conclusion**

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**Exclusive dijet diffractive production**

Rapidity divergence

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### B-JIMWLK equation for the dipole operator

\[
\frac{\partial \tilde{U}_{12}^\alpha}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left( \tilde{U}_{13}^\alpha \tilde{U}_{32}^\alpha + \tilde{U}_{13}^\alpha + \tilde{U}_{32}^\alpha - \tilde{U}_{12}^\alpha \right) 
\]

\[
\times \left[ \frac{\left( \vec{k}_1 - \vec{p}_1 \right) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi}{2} \frac{\Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left( \frac{\delta(\vec{k}_2 - \vec{p}_2)}{[(\vec{k}_1 - \vec{p}_1)^2]^{1-d/2}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{[(\vec{k}_2 - \vec{p}_2)^2]^{1-d/2}} \right) \right]
\]

**η rapidity divide**, which separates the upper and the lower impact factors

\[
\Phi_0 \tilde{U}_{12}^\alpha \rightarrow \Phi_0 \tilde{U}_{12}^\eta + 2 \log \left( \frac{e^\eta}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{W}_{123}
\]

Provides a counterterm to the \(\log(\alpha)\) divergence in the virtual double dipole impact factor:

\[
\Phi_0 \tilde{U}_{12}^\alpha + \Phi V^2 \tilde{W}_{123}^\alpha \text{ is finite and independent of } \alpha
\]
Exclusive dijet diffractive production

Various type of divergences

- Rapidity divergence

- UV divergence $\vec{p}_g^2 \to +\infty$

- Soft divergence $p_g \to 0$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

- Soft and collinear divergence $p_g = \frac{p_g}{p_q}p_q$ or $\frac{p_g}{p_{\bar{q}}}p_{\bar{q}}$, $p_g^+ \to 0$
Some null diagrams just contribute to turning UV divergences into IR divergences.

\[ \Phi = 0 \propto \left( \frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right) \]
Exclusive dijet diffractive production

Various type of divergences

- Rapidity divergence

- UV divergence

- Soft divergence $p_g \rightarrow 0$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

- Soft and collinear divergence $p_g = \frac{p_g^+}{p_{q}^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \rightarrow 0$
Cone jet algorithm at NLO (Ellis, Kunszt, Soper)

- Should partons \((|p_1|, \phi_1, y_1)\) and \((|p_2|, \phi_2, y_2)\) combined in a single jet? \(p_i\) = transverse energy deposit in the calorimeter cell \(i\) of parameter \(\Omega = (y_i, \phi_i)\) in \(y - \phi\) plane

- define transverse energy of the jet: \(p_J = |p_1| + |p_2|\)

- jet axis:

\[
\begin{align*}
\Omega_c \quad \left\{ \begin{array}{c}
y_J = \frac{|p_1| y_1 + |p_2| y_2}{p_J} \\
\phi_J = \frac{|p_1| \phi_1 + |p_2| \phi_2}{p_J}
\end{array} \right.
\]

\[
|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2 \quad (i = 1 \text{ and } i = 2)
\]

\(\Longrightarrow\) partons 1 and 2 are in the same cone \(\Omega_c\)

Applying this (in the small \(R^2\) limit) cancels our soft and collinear divergence
Exclusive dijet diffractive production

Various type of divergences

- Rapidity divergence
- UV divergence
- Soft divergence \( p_g \to 0 \)
  \[ \Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^* \]
- Collinear divergence \( p_g \propto p_q \) or \( p_{\bar{q}} \)
  \[ \Phi_{R1} \Phi_{R1}^* \]
- Soft and collinear divergence

  The remaining divergences cancel the standard way:
  virtual corrections and real corrections cancel each other

This is done after combining:

- the (LO + NLO) contribution to \( q\bar{q} \) production
- the part of the contribution of the \( q\bar{q}g \) production where the gluon is either soft or collinear to the quark or to the antiquark, so that they both form a single jet
Light meson production
Collinear factorization: basic principle

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator

\[ \int_x (H_2(x))_{ij}^{\alpha \beta} \left\langle \rho \left| \bar{\psi}_i^\alpha (x) \psi_j^\beta (0) \right| 0 \right\rangle \quad \int_{x_1, x_2} (H_3^{\mu}(x_1, x_2))_{ij, c}^{\alpha \beta} \left\langle \rho \left| \bar{\psi}_i^\alpha (x_1) A_\mu^c (x_2) \psi_j^\beta (0) \right| 0 \right\rangle \]

\(H\) and \(S\) are connected by:

- convolution
- summation over spinor and color indices

Once factorization in the \(t\) channel is done, now factorize in the \(s\) channel with collinear factorization: expand the impact factor in powers of the hard scale
Collinear factorization at twist 2

- Leading twist DA for a **longitudinally polarized** light vector meson
  \[
  \langle \rho \mid \bar{\psi}(z) \gamma^\mu \psi(0) \mid 0 \rangle \rightarrow p^\mu f_\rho \int_0^1 dx e^{ix(p \cdot z)} \phi_1(x)
  \]

- Leading twist DA for a **transversely polarized** light vector meson
  \[
  \langle \rho \mid \bar{\psi}(z) \sigma^{\mu\nu} \psi(0) \mid 0 \rangle \rightarrow i(p^\mu \varepsilon^\nu - p^\nu \varepsilon^\mu) f^T_\rho \int_0^1 dx e^{ix(p \cdot z)} \phi_\perp(x)
  \]

The twist 2 DA for a transverse meson is **chiral odd**, thus $\gamma^* A \rightarrow \rho T A$ starts at twist 3
Light meson production

Exclusive diffractive production of a light neutral vector meson

\[
\mathcal{A} = -\frac{e_V f_V \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel(x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d} \frac{d^d \vec{p}_3}{(2\pi)^d} \\
\times (2\pi)^{d+1} \delta(\vec{p}_V^+ - \vec{p}_\gamma^+) \delta(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3) \\
\times \left[ \left( \Phi_0^\beta(x, \vec{p}_1, \vec{p}_2) + C_F \Phi_{V1}^\beta(x, \vec{p}_1, \vec{p}_2) \right) \tilde{U}_{12}^\eta (2\pi)^d \delta(\vec{p}_3) \\
+ \Phi_{V2}^\beta(x, \vec{p}_1, \vec{p}_2, \vec{p}_3) \tilde{W}_{123}^\eta \right]
\]

Probes gluon GPDs at low $x$, as well as twist 2 DAs
Divergences

- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution

- UV, soft divergence, collinear divergence
  - Mostly cancel each other, but requires renormalization of the operator in the vacuum-to-meson matrix element $\to$ ERBL evolution equation for the DA

We thus built a finite NLO exclusive diffractive amplitude with saturation effects
Rapidity gap events at HERA

Experiments at HERA: about 10% of scattering events reveal a rapidity gap

DIS events

DDIS events
Rapidity gap events at HERA

Experiments at HERA: about 10% of events reveal a rapidity gap

ZEUS, 1993

H1, 1994
Theoretical approaches for DDIS using pQCD

- **Collinear factorization** approach
  - Relies on QCD factorization theorem, using a hard scale such as the virtuality $Q^2$ of the incoming photon
  - One needs to introduce a **diffractive distribution function** for partons within a **pomeron**

- **$k_T$ factorization** approach for two exchanged gluons
  - low-$x$ QCD approach: $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a **two-gluon color-singlet** state
Collinear factorization approach

Direct

Resolved
Phenomenology
Theoretical approaches for DDIS using pQCD

$k_T$-factorization approach: two gluon exchange

Bartels, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization
a ZEUS diffractive exclusive dijet measurements was performed
the azimuthal distribution of the two jets was obtained
Phenomenology
Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data

\[ \frac{d\sigma_{ep}}{d\beta d\phi} = \frac{1}{\pi} \frac{d\sigma}{d\beta} [1 + A \cos 2\phi] \quad \phi \in [0, \pi] \]

Bjorken variable normalized to the pomeron momentum:
\[ \beta = \frac{Q^2}{Q^2 + M^2_{dijet} - t} \sim \frac{Q^2}{Q^2 + M^2_{dijet}} \]

Collinear factorization approach: \( A > 0 \)
\( k_T \)-factorization approach: \( A < 0 \)
large \( M_{\text{dijet}}^2 \):

the dominant contribution comes from the \( q\bar{q} \) jet + \( g \) jet configuration (dominance of the exchange of a \( t \)–channel gluon with large \( s = M_{\text{dijet}}^2 \))
Exclusive $k_t$ jet algorithm for three partons

Distance between two particles:

$$d_{ij} = 2 \min(E_i^2, E_j^2) \frac{1 - \cos \theta_{ij}}{M^2} = \min \left( \frac{E_i}{E_j}, \frac{E_j}{E_i} \right) \frac{2p_i \cdot p_j}{M^2}$$

$E_{i,j}, \theta_{ij}$: particle’s energies and relative angle between them in c.m.f.

Two particles belong to one jet if $d_{ij} < y_{cut}$

$y_{cut}$ regularizes both soft and collinear singularities

**ZEUS**: $y_{cut} = 0.15 \Rightarrow$ we will rely on a small $y_{cut}$ approximation
Phenomenology
Towards a NLO CGC approach

- **ZEUS** cuts:
  - $5 \text{ GeV} < Q$
  - $5 \text{ GeV} < M_{2 \text{ jets}} < 25 \text{ GeV}$
  - $2 \text{ GeV} < p_{\perp \text{ min}}$

- at Born level, this removes aligned jets configurations (i.e. with a very small longitudinal momentum fraction $x$)

  $\Rightarrow$ suppression of the leading twist contribution which normally dominates in the Golec-Biernat Wüsthoff saturation model

- the typical hard scale in the impact factor is $\sim p_{\perp \text{ min}}^2 > Q_s^2$

- this justifies an expansion in powers of $Q_s$:
  - **ZEUS** experiment is dominated by the linear BFKL regime

- we restrict ourselves to the dominant contributions:
  - Born cross section
  - real correction with dipole $\times$ dipole and double dipole $\times$ double dipole configurations
\[ \frac{d\sigma_{\text{Lep}}}{d\beta} \text{ (pb)} \]

Cross-sections

\[ ep \to ep + 2jets \] cross-section in the case of a longitudinal photon.

Born and gluon dipole contributions.
Phenomenology

Cross-sections

\[ \frac{d\sigma_{ep}}{d\beta} \text{ (pb)} \]

- \( d\sigma_{TT} \) (Born)
- \( d\sigma_{TT5} \) (double-dipole * double-dipole)
- \( d\sigma_{TT4} \) (double-dipole * dipole)
- \( d\sigma_{TT3} \) (dipole * dipole)
- Sum

\[ ep \rightarrow ep + 2jets \] cross-section in the case of a transverse photon.

Born and gluon dipole contributions.
Phenomenology

Results

Cross-sections

Born and total gluon dipole contributions to cross section versus ZEUS experimental data

- large $\beta$: good agreement with data
- small $\beta$: poor agreement with data, similar to the two gluon model of Bartels et al.
Phenomenology

Results

Azimuthal distribution

First 5 panels:
dependence of the cross-section on $\phi$
for each experimental $\beta$ bin

Good agreement at large $\beta$

Last panel:
$\beta$ dependence of the coefficient $A$.

$\Rightarrow$ The experimental result for $A$
at large $\beta$ is puzzling
Results

Summary

• using a small $y$ limit, and for large $\beta$, there is a good agreement with a Golec-Biernat Wüsthoff model (in the small $Q_s$ expansion) combined with our NLO impact factor

• within ZEUS kinematical cuts, the linear BFKL regime dominates

• our agreement is a good sign that perturbative Regge-like description are favored with respect to collinear type descriptions

• EIC should give a direct access to the saturated region

• a complete description of ZEUS data, in the whole $\beta$-range, requires to go beyond the small $y$ approximation: next highly non-trivial step!!
Conclusion

- We provided the **full computation** of the $\gamma^{(*)} \to Jet Jet$ and $\gamma^{*,T}_L \to \rho_L$ impact factors at **NLO accuracy**

- Our results are **perfectly finite**, even for photoproduction (at large $t$ for $\rho$)

- The computation can be adapted for **twist 3** $\gamma^{(*)} \to \rho_T$ NLO production in the Wandzura-Wilczek approximation, removing factorization breaking end-point singularities even at NLO for a process which would not factorize in a full collinear factorization scheme

- Exclusive diffractive processes are perfectly suited for **precision saturation physics** and **gluon tomography** with $b_\perp$ dependence at the EIC. Dijet production probes the **dipole Wigner** distribution, $\rho$ meson production probes **gluon GPDs** at small $x$.

- At **HERA**, due to the kinematical cuts, one does not enter the saturation regime through exclusive diffractive dijet production.

- The large $\beta$ region is well described, while the low $\beta$ requires to include every NLO contribution.
The ultimate picture

6D/5D

Wigner distributions for hadrons

\[ W(x, \vec{b}, k_T) \]

Experimentally inaccessible directly

3D

uPDFs (gluons)

\[ \int d^3\vec{b} \]

\[ \int d^2k_T \int d\vec{b}_T \]

Impact parameter distributions

\[ f(x, b_T) \]

Semi-inclusive processes

Transverse momentum dependent distributions

GPDs

\[ \xi = 0 \]

generalised parton distributions

exclusive processes

1D

PDFs

\[ f(x) \]

inclusive and semi-inclusive processes

parton distributions

FFs

\[ G_E, M(t) \]

elastic processes

form factors

GFFs

\[ \int dx x^{n-1} \]

generalized form factors

lattices