LIGHT CONE 2019

LC2019 - QCD ON THE LIGHT CONE: FROM HADRONS TO HEAVY IONS

Glueballs as gravitons in holographic approaches

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and

Vicente Vento

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DIPARTIMENTO DI FISICA E GEOLOGIA

Università degli Studi di Perugia, Italia

INFN

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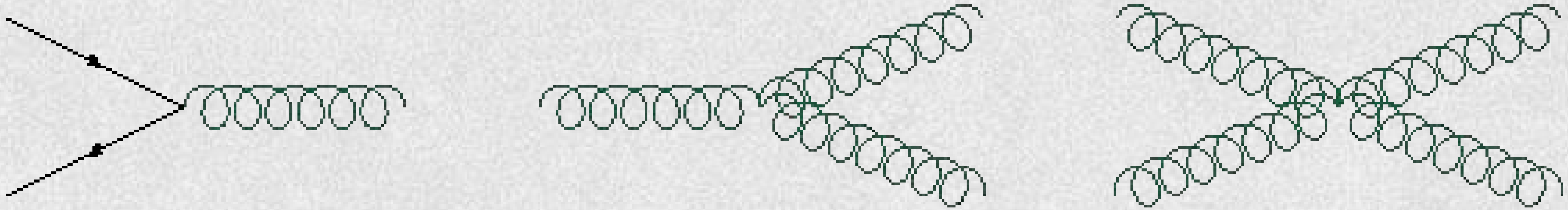
1 Open questions in Glueball Physics

The QCD, the gauge theory describing strong interactions

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum \bar{\Psi}\left(i\gamma \cdot D - m\right)\Psi$$

gluon field strength tensor: $G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abs} A^b_\mu A^c_\nu$



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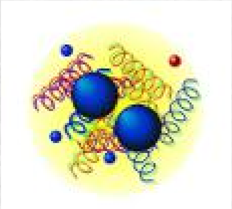
Mesons

$\mathbf{3} \otimes \bar{\mathbf{3}}$



Baryons

$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$

Exotic states

$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$

HYBRIDS

$\mathbf{8} \otimes \mathbf{8}$

$\mathbf{8} \otimes \cdots\cdots \otimes \mathbf{8}$

GLUEBALLS

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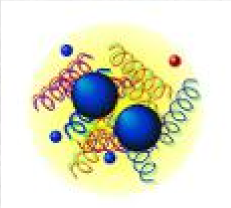

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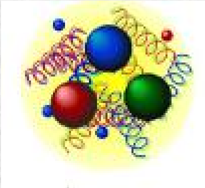

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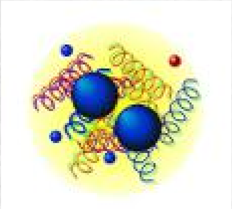

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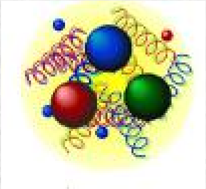

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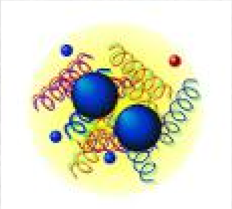

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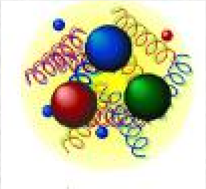

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GLUEBALLS

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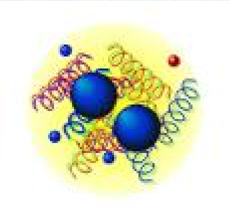

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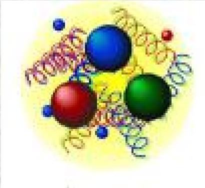

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GLUEBALLS

Glueball spectroscopy is a unique laboratory to test non perturbative QCD and CONFINEMENT

However :

1) several mesons have similar mass and quantum number ⟹ MIXING
2) Their measurements represent a very hard task
3) Theoretical calculations of decay are very difficult! Models could help!

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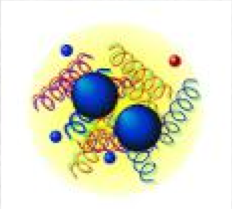

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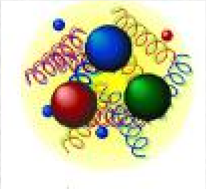

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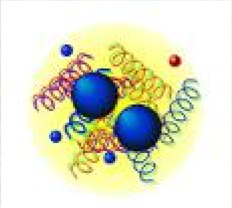

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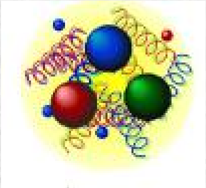

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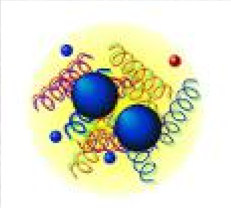

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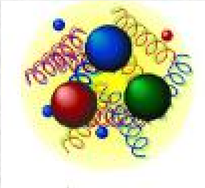

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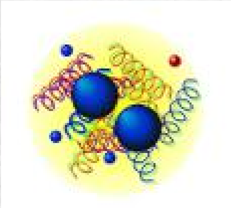

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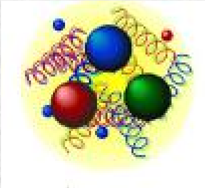

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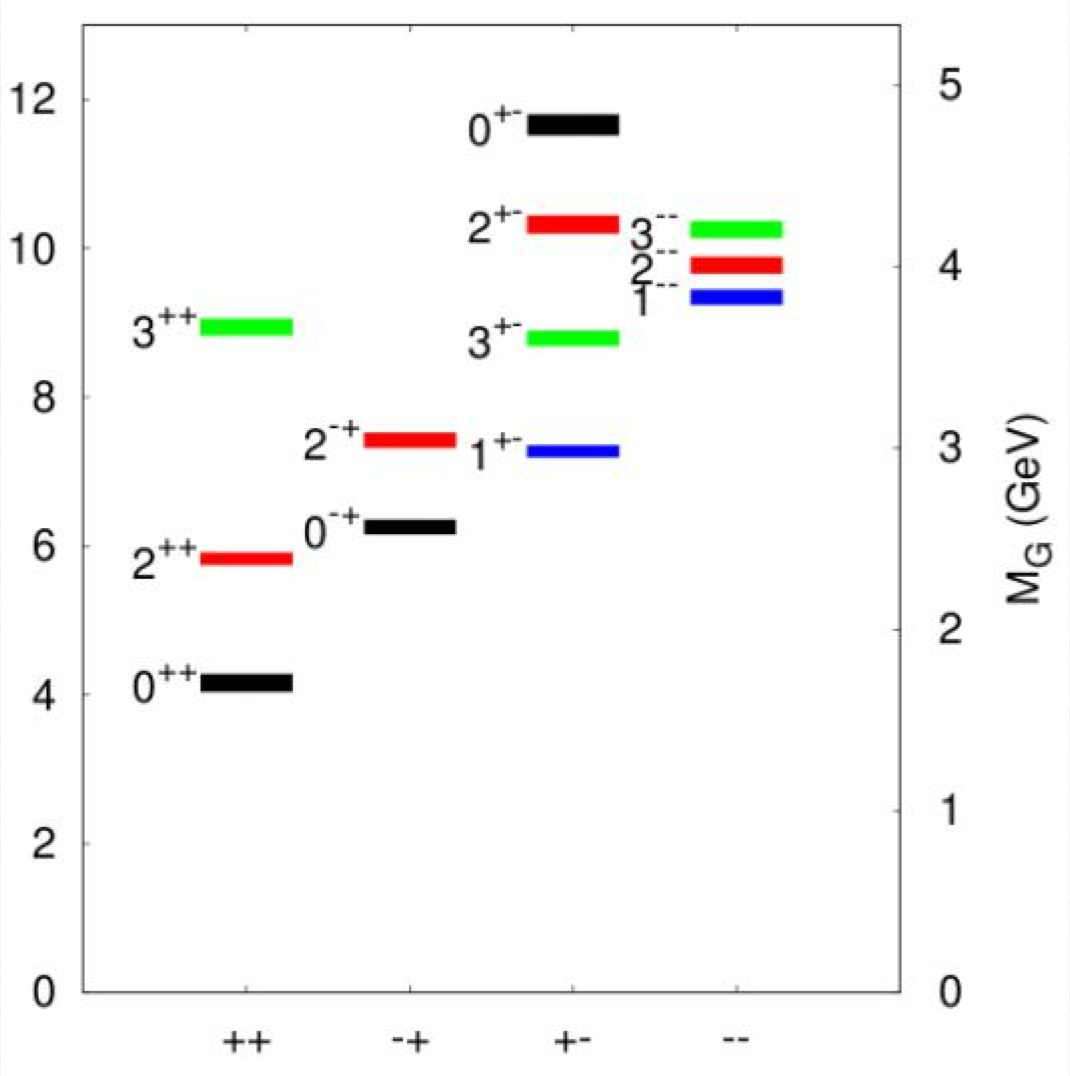

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QCD! BUT also in this framework we have problems:

MP: **C.J. Morningstar et al, PRD 60, 034509 (1999)** YC: **Y. Chen et al, PRD 73, 014516 (2006)** LTW: **B. Lucini et al, JHEP 06, 012 (2004)**

	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277

The mass of the lightest state is very hard to estimate

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Could model help in this scenario?
We used AdS/QCD models!

1 Open questions in Glueball Physics

One of the main difficulties in the observation of glueballs is related to their mixing with mesons!

For example:

J^{PC}	0^{++}	2^{++}	0^{++}
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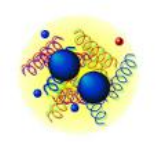
Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	1992 ± 16	2101 ± 7	2189 ± 13



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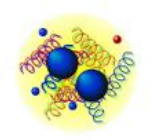

Mixing?

We use AdS/QCD models to study the MIXING problems and "predict" the kinematic conditions where pure glueball states could be observed.

2 Introduction to AdS/QCD

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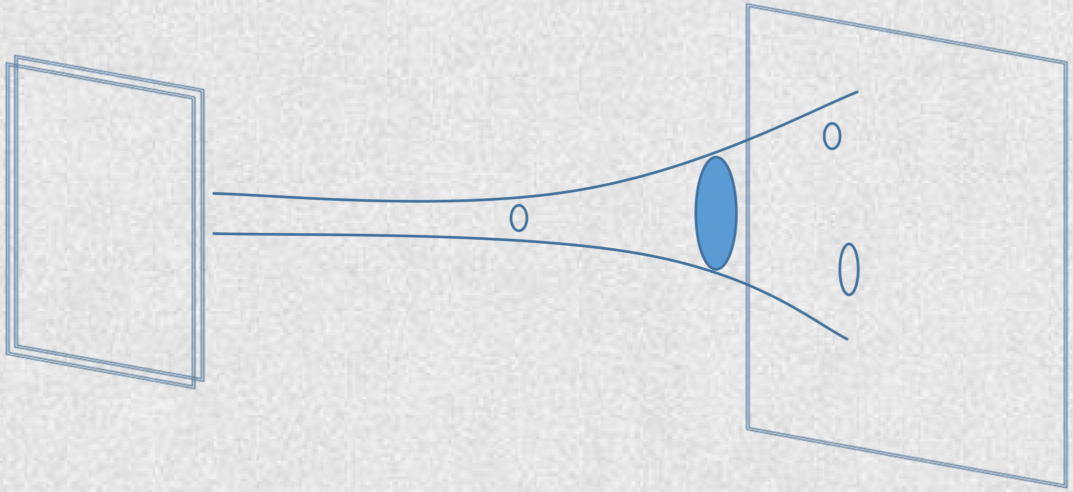
From Maldacena conjecture: AdS/CFT

N=4 SU(N) SYM ⟷ String theory on $AdS_5 \times S_5$

Isometries between group symmetries



$$g_{YM}^2 N \underset{N\to\infty}{=} \frac{R^4}{l^4}$$

R= radius of the manifold

l = length

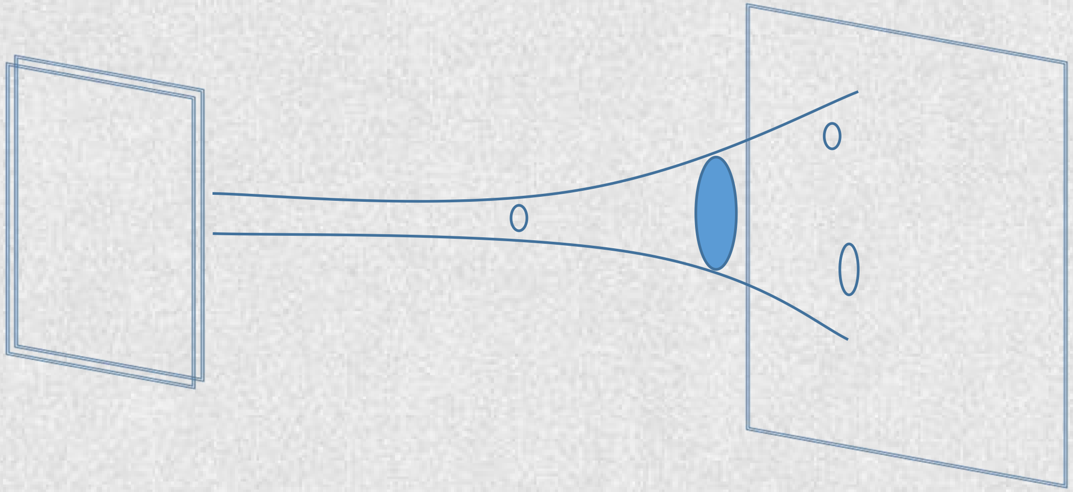
2 Introduction to AdS/QCD

From Maldacena conjecture: AdS/CFT

$\boxed{\mathcal{N}=4 \text{ SU(N) SYM}}$ $\xleftrightarrow[\text{Isometries between group symmetries}]{}$ String theory on $AdS_5 \times S_5$

This is not QCD!

- ❖ No supersymmetry
- ❖ Confinement
- ❖ Conformal symmetry broken
- ❖ N is finite



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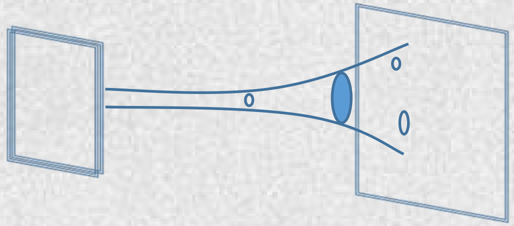

The dream is to understand QCD using its dual gravity theory!

Top-down Approach:

Find a gravitational theory dual to QCD

Advantages: duality is exact and well understanding of the theory

Disadvantages: a dual of QCD with fundamental flavors even at large *N* has not been found

Bottom-up Approach:

Starts from QCD and attempts to construct a five dimensional holographic dual

Advantages: some freedom in matching the model to features of QCD

Disadvantages: some discrepancies with data could be found

- No supersymmetry
- Confinement
- Conformal symmetry broken
- N is finite

Witten:

Supersymmetry could be neglected by compactifying one of the spatial direction and imposing antiperiodic boundary conditions.

Gauge fields at low energies

SUSY partners at the compactification scale

2 Introduction to AdS/QCD

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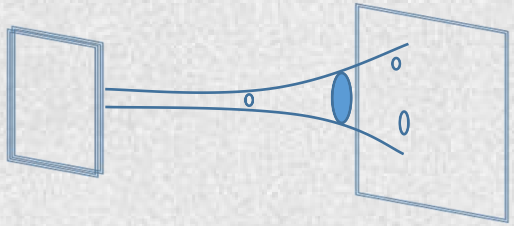

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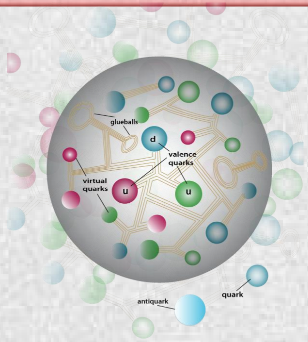


Hard-wall model

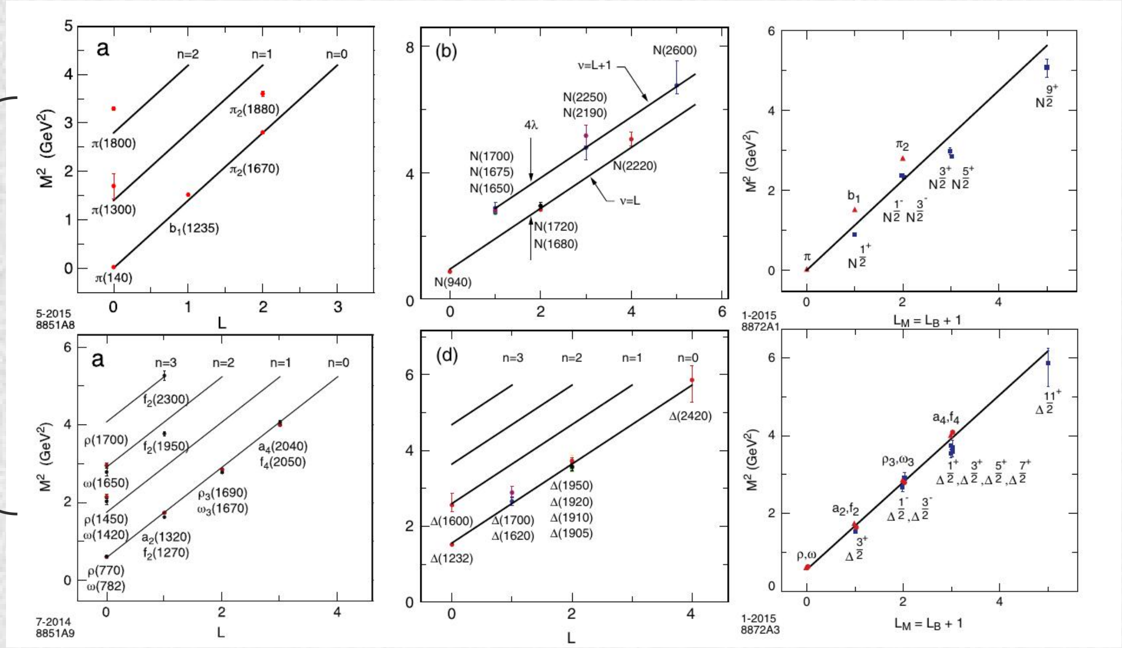
Compattification of the 5° dimension by hand. AdS geometry cut by two branes: UV and IR.

Soft-wall model

Soft cutoff of AdS space by introducing a dilaton field.

$$e^{-\varphi}$$

Introduction to AdS/QCD: applications



HADRON SPECTRUM:

S.J. Brodsky et al, Phys. Rep. 584 (2015)
H.G. Dosh et al PRD 91, 045040 (2015), 085016 (2015)

see Brodsky's talk on Thursday

2 Introduction to AdS/QCD: applications

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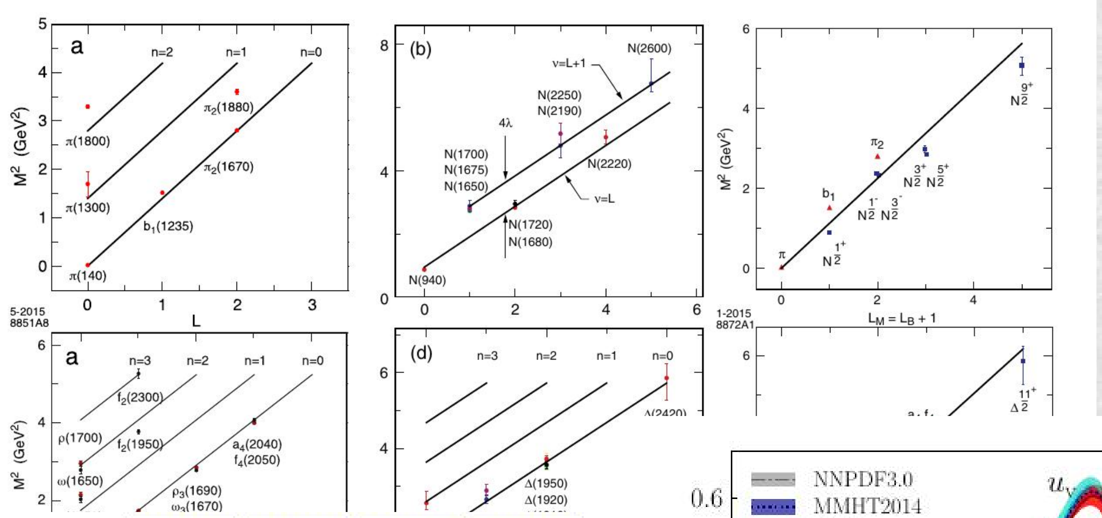

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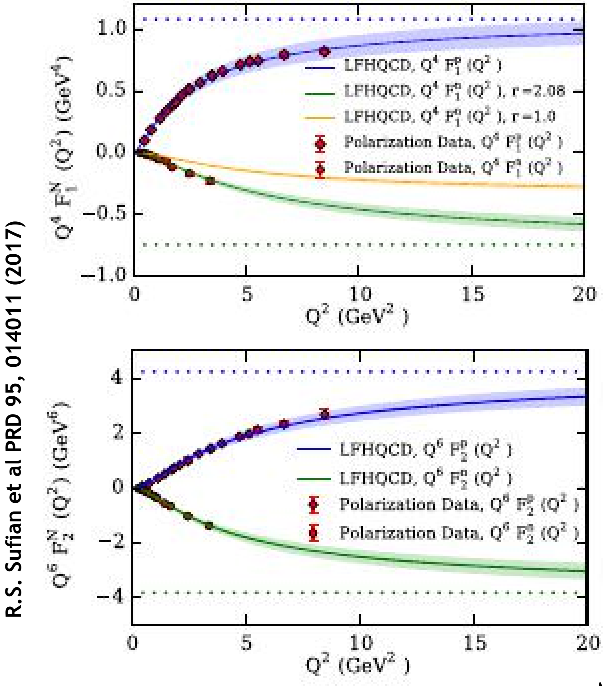
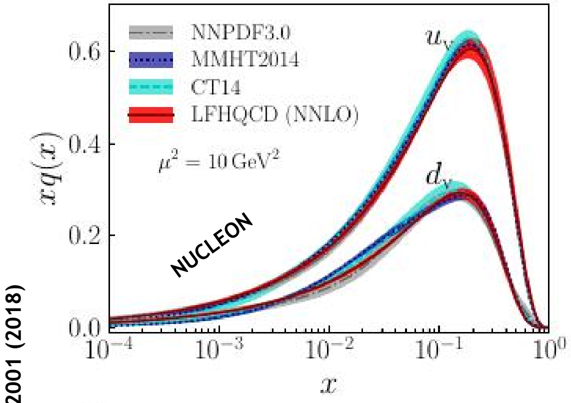
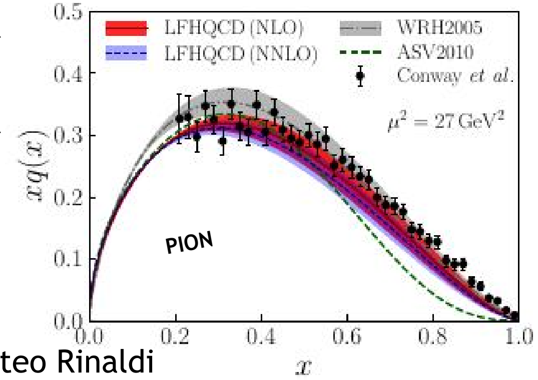
FORM FACTORS, PDFs & GPDs

R.S. Sufian et al PRD 95, 014011 (2017)

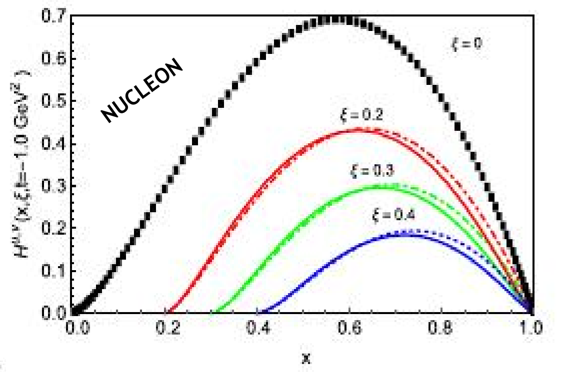


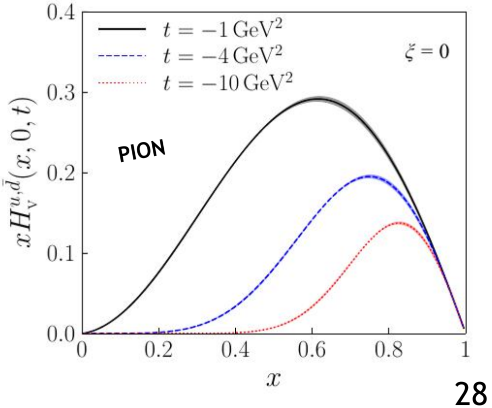
de Teramond et al, PRL 120,182001 (2018)





M. Rinaldi, PLB 771 (2017)

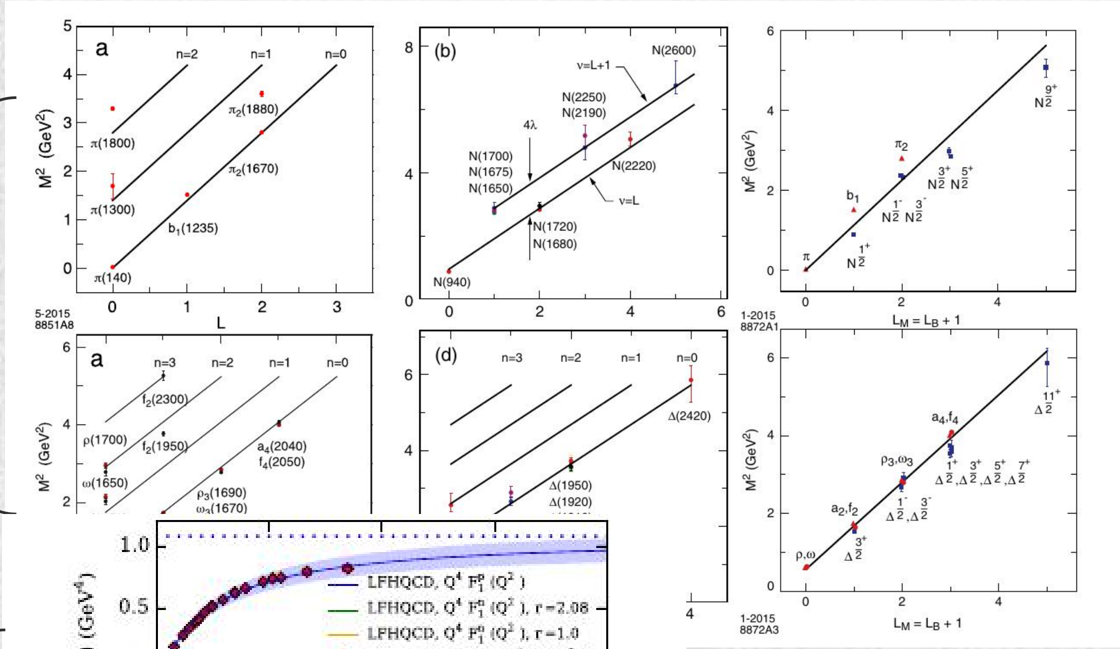




2 Introduction to AdS/QCD: applications

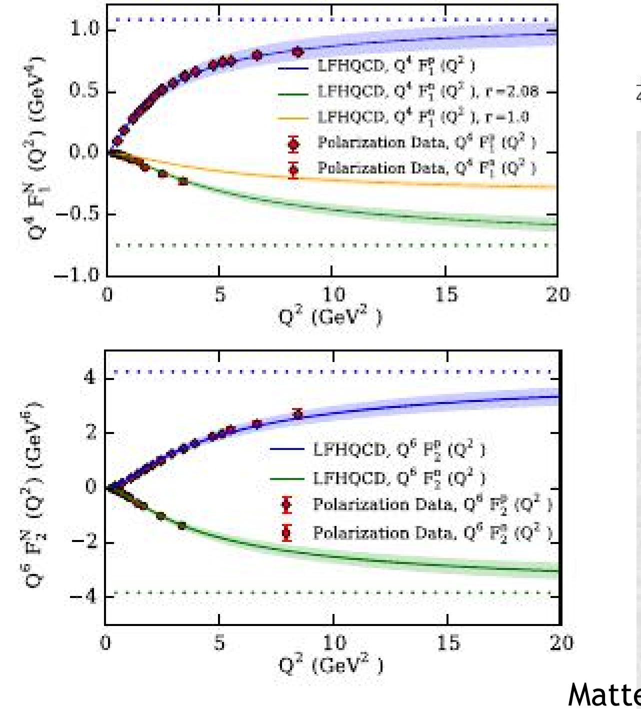
HADRON SPECTRUM:

S.J. Brodsky et al, Phys. Rep. 584 (2015)
H.G. Dosh et al PRD 91, 045040 (2015), 085016 (2015)



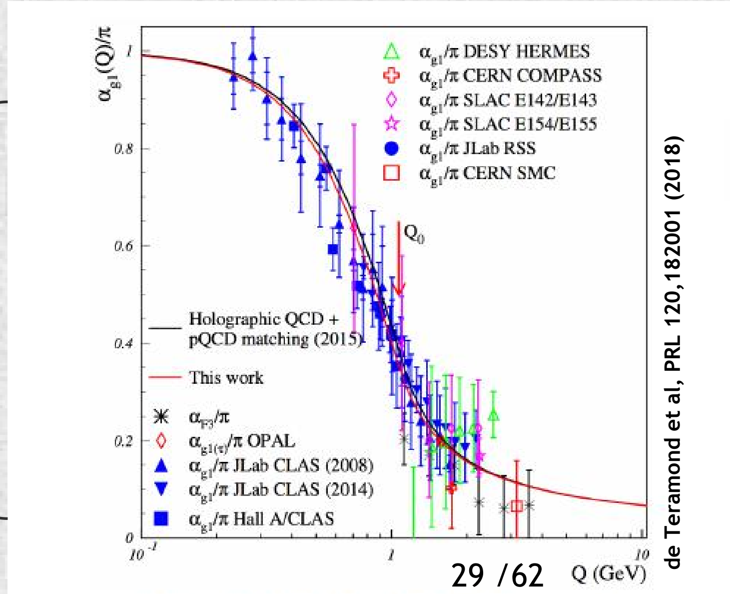
see Brodsky's talk on Thursday

FORM FACTORS, PDFs & GPDs



R.S. Sufian et al PRD 95, 014011 (2017)

MATCHING THE RUNNING COUPLING



de Teramond et al, PRL 120,182001 (2018)

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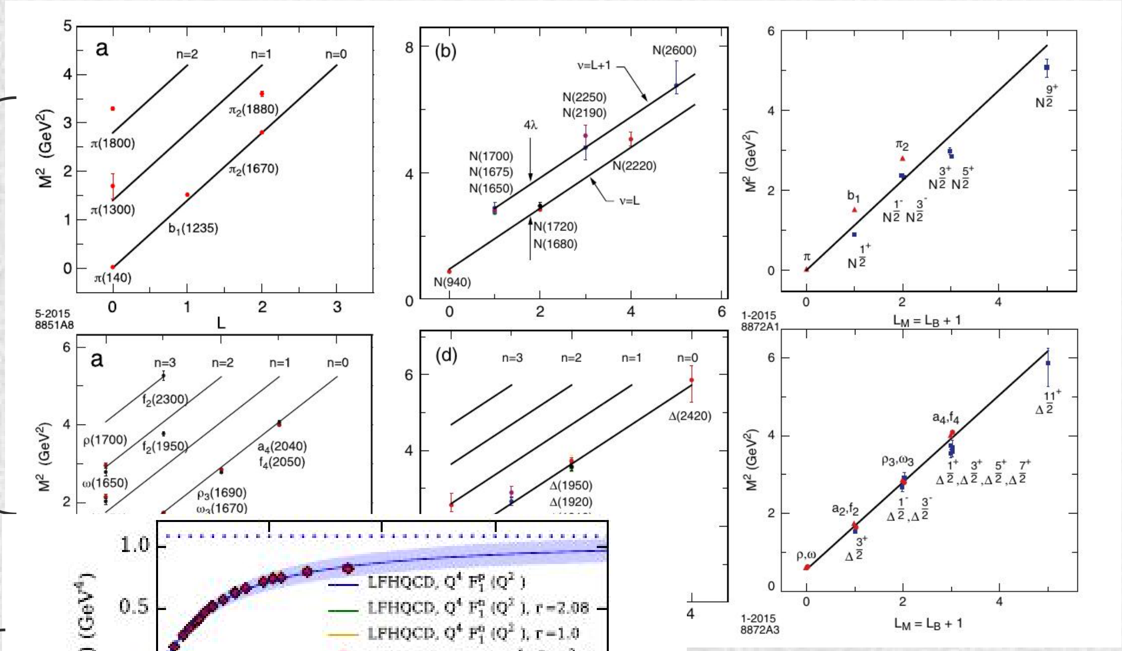

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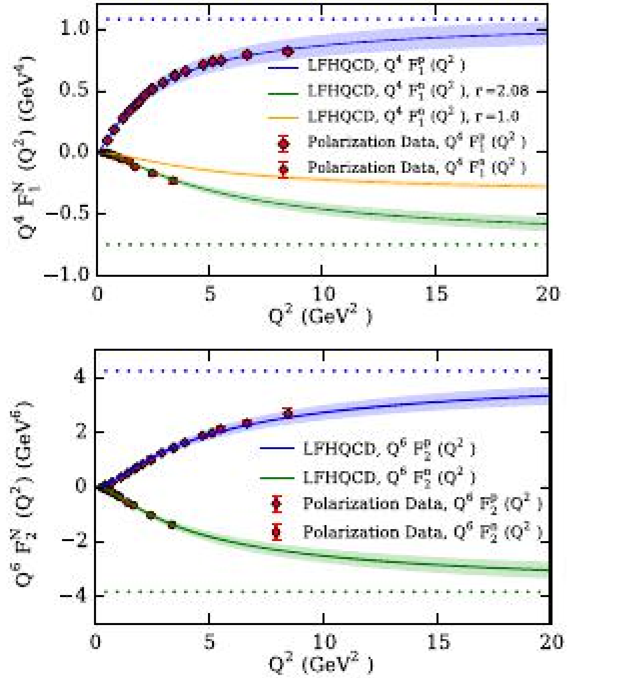

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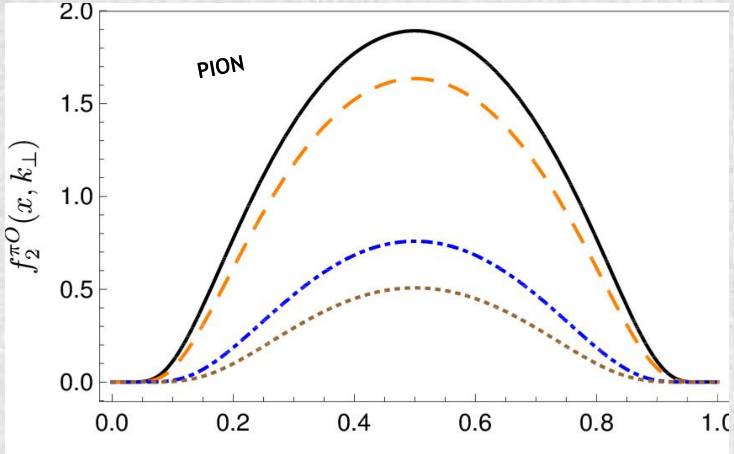
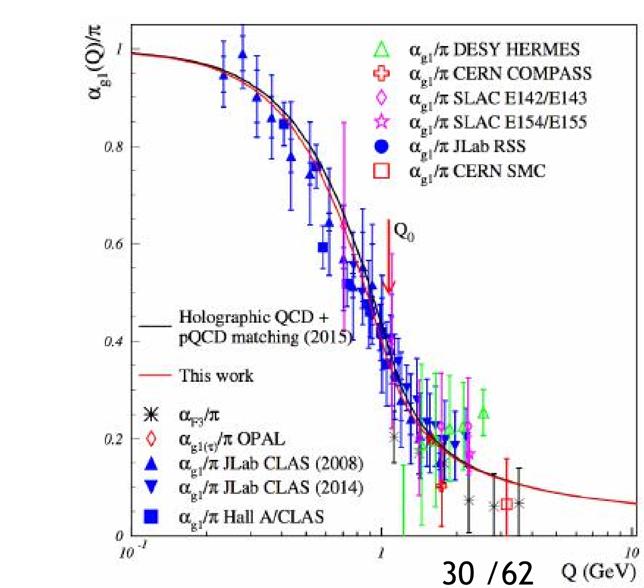
APPLICATIONS TO DOUBLE PARTON SCATTERING



M. Rinaldi et al, EPJC 78, 781 (2018)

MATCHING THE RUNNING COUPLING



de Teramond et al, PRL 120,182001 (2018)

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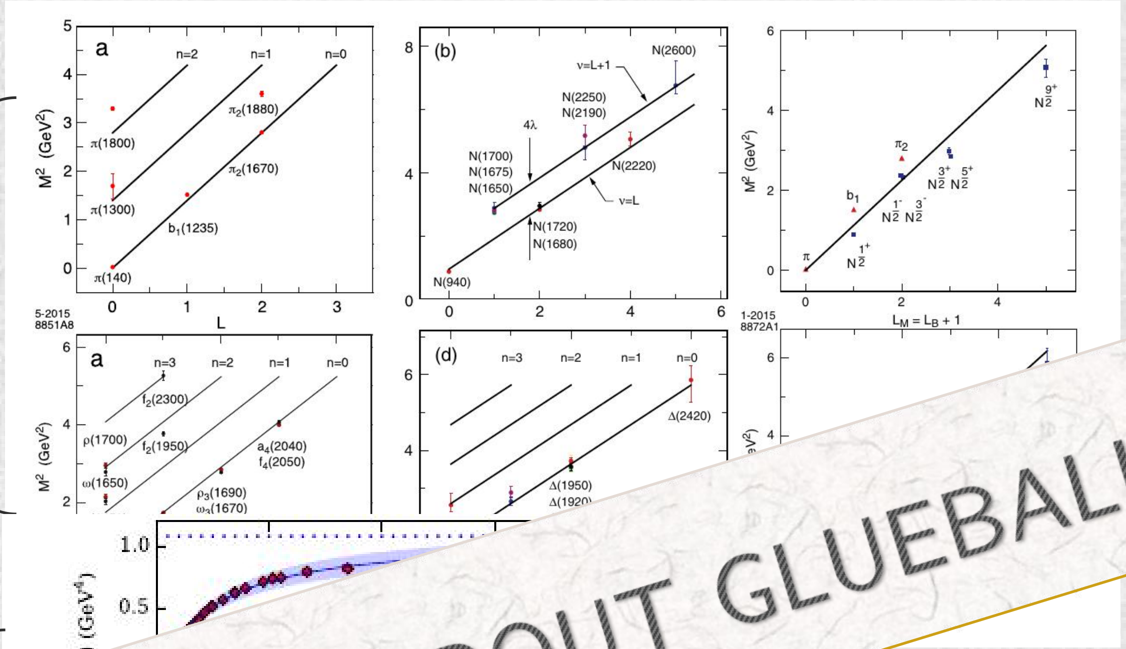

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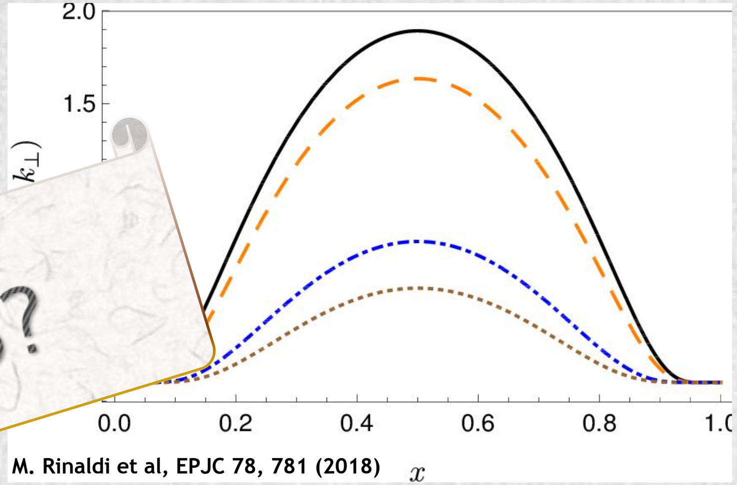

M. Rinaldi et al, EPJC 78, 781 (2018)

WHAT ABOUT GLUEBALLS?

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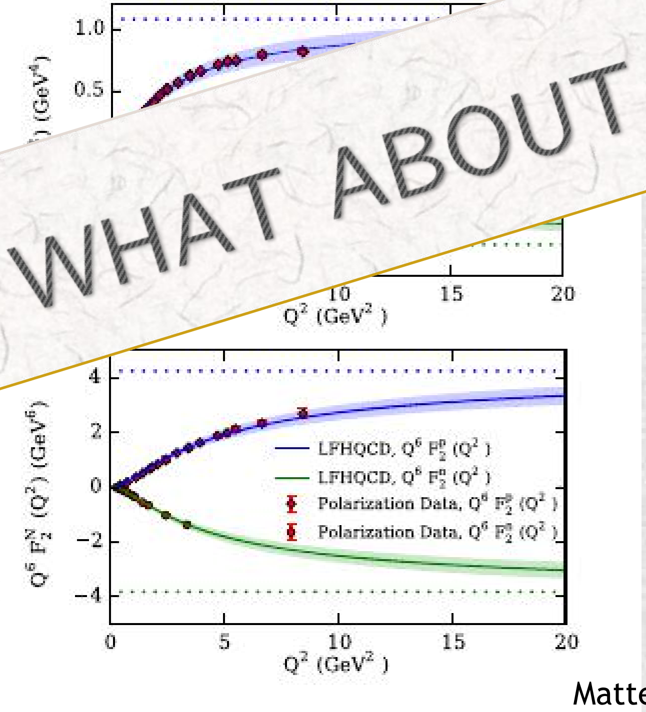

R.S. Sufian et al PRD 9

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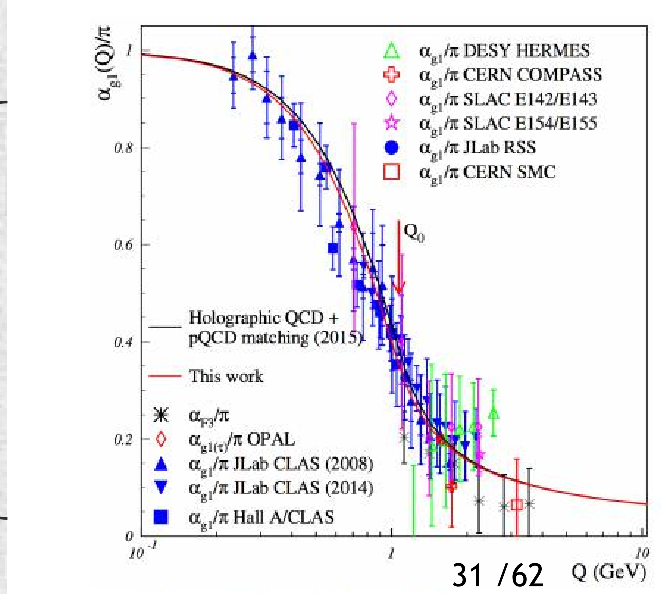

de Teramond et al, PRL 120,182001 (2018)

3 Glueballs in AdS/QCD

3 Glueballs in AdS/QCD: Hard-Wall model

In this case we have the following $AdS_5 \times S_5$ metric : $$ds^2 = g_{MN}dx^M dx^N + R^2 d\Omega_5 = \frac{R^2}{z^2}\left(dz^2 + \underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{\text{Minkowski space}}\right) + R^2 d\Omega_5$$

Holographic 5° dimension

Radius of the AdS space

In the hard-wall (HW) model confinement is implemented by imposing the following IR cutoff:

$$0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$$

WHAT ABOUT GLUEBALLS?

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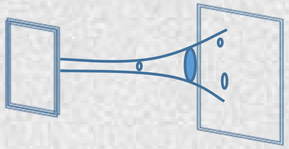

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Equation of motion of the scalar glueball can be obtained:

$$I = \int d^5x \, \sqrt{g}\left[g^{MN}\partial_M \mathcal{G}\partial_N \mathcal{G} + M_5^2 \mathcal{G}^2\right] \quad \begin{cases} \Delta = \text{conformal dimension} \\ \Delta = 2 + \sqrt{4 + M_5^2 R^2} \end{cases}$$

Mass in AdS space

1) scalar glueball state 0^{++} is represented by: $\mathcal{O}_{\Delta=4} = Tr\left(F^{\mu\nu}F_{\mu\nu}\right)$

2) For example for even spin J: $\mathcal{O}_{\Delta=4+j} = FD_{\{\mu 1 \dots} D_{\mu j\}}F$

The equation of motion for the scalar is:

$$\frac{d^2\phi(z)}{dz^2} - \frac{3}{z}\frac{d\phi(z)}{dz} + M^2\phi(z) = 0$$

where:

$\mathcal{G}(x,z) \sim \phi(z)e^{-iP_\mu x^\mu}, \quad P^2 = -M^2$

H. Boschi-Filho et al, JHEP 05, 009 (2003)
H. Boschi-Filho et al, PRD 73, 047901 (2006)
P. Colangelo et al, PLB 652, 73 (2007)

GRAVITON SPECTRUM:

Equation of motion for metric perturbation h_{MN} obtained from the linearized Einstein's equation :

R.C. Brower et al, Nucl. Phys. B 587, 249 (2000)

$$-\frac{1}{2}h_{ab;c}^{;c} - \frac{1}{2}h_{c;ab}^{c} + \frac{1}{2}h_{ac;b}^{;c} + \frac{1}{2}h_{bc;a}^{;c} + 4h_{ab} = 0$$

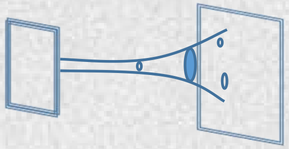
By choosing the gauge:

$$\begin{cases} h_{tt} = \left(z^{-2} - z^2\right)\phi(z)e^{-Mx_3} & \text{Scalar component} \\ h_{ij} = q_{ij}T(z)e^{-Mx_3} & \text{Tensor component} \end{cases}$$

"Tensor" wave-function

Same equation of motion of the scalar field for the scalar component.

3 Glueballs in AdS/QCD: Hard-Wall model



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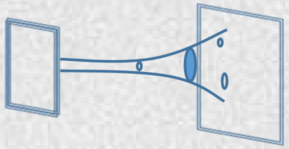
"Tensor" wave-function

Same equation of motion for the scalar field for the scalar component of the graviton.

3 Glueballs in AdS/QCD: Hard-Wall model



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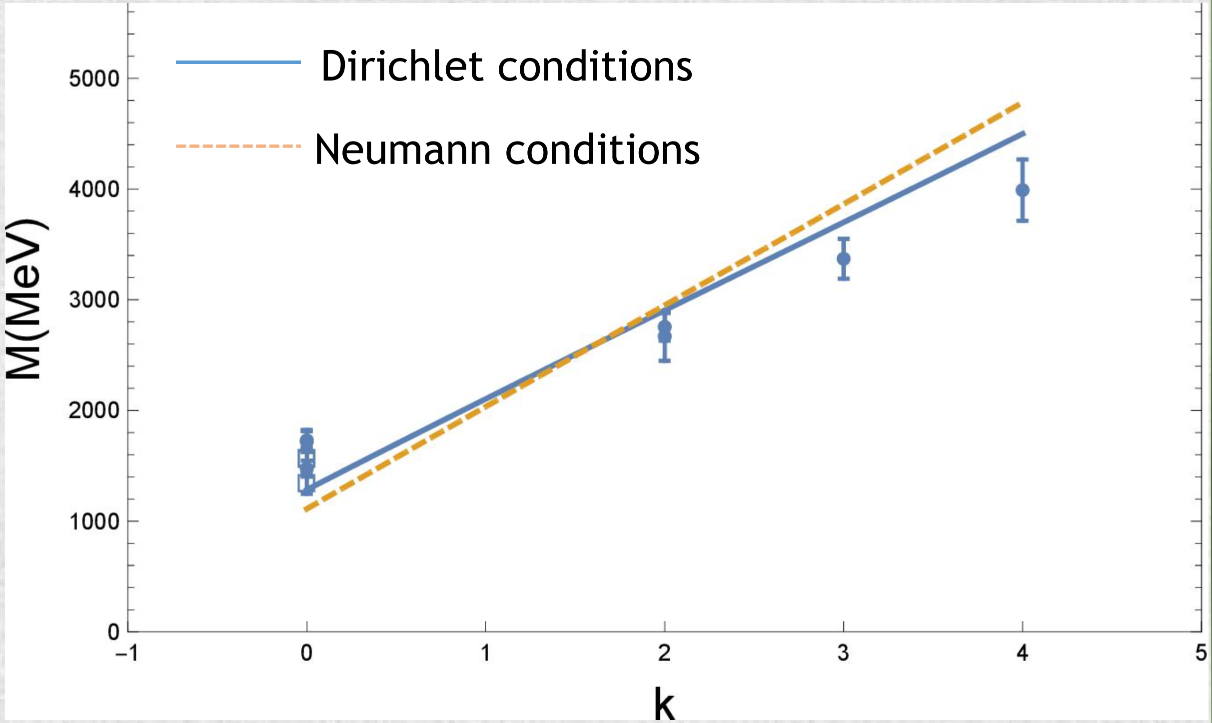
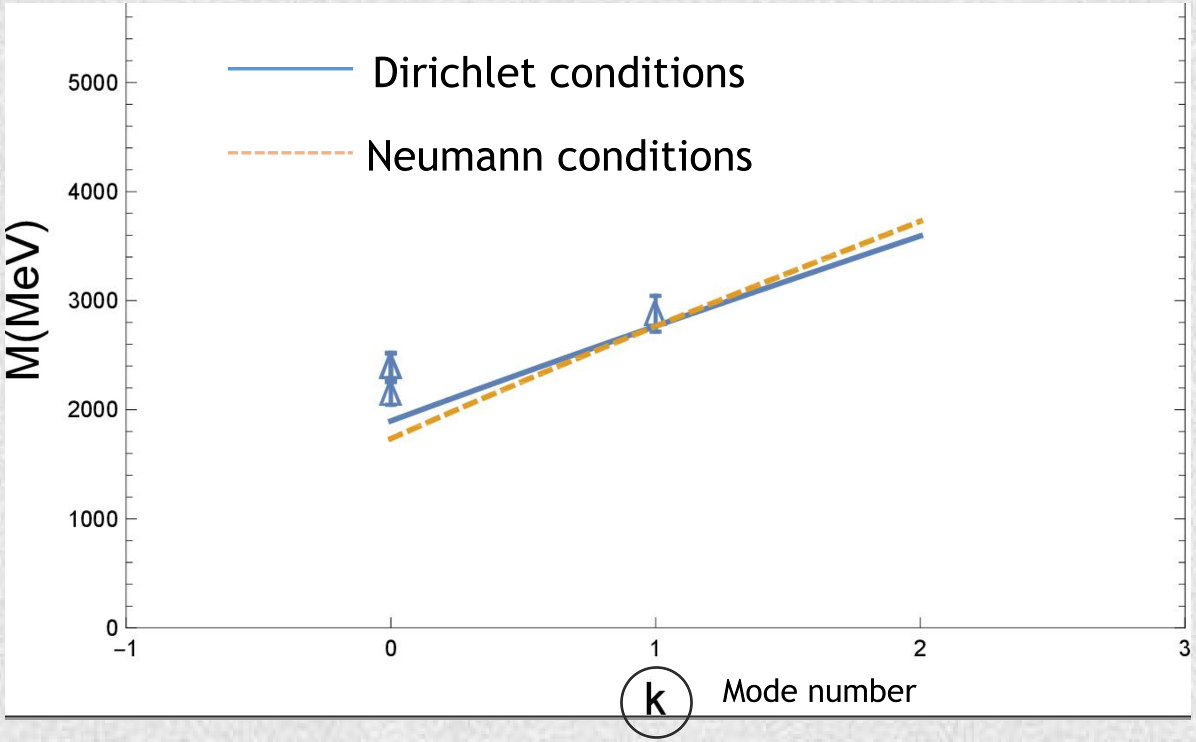
0++ GLUEBALL SPECTRUM

M.Rinaldi and V. Vento EPJA 54 (2018)

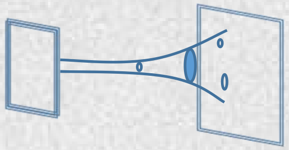


2++ GLUEBALL SPECTRUM

M.Rinaldi and V. Vento EPJA 54 (2018)



3 Glueballs in AdS/QCD: Hard-Wall model



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SCALR FIELD EQUATION:

GRAVITON SPECTRUM:

Within this model the spectrum of the scalar field is the same of that of the scalar component of the graviton!

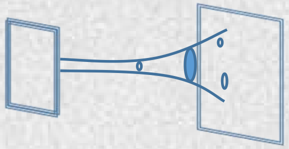
What about the tensor component?

k	1	2	3	4	5	...
D scalar	5.136	8.417	11.620	14.796	17.960	...
N scalar	3.832	7.016	10.173	13.324	16.471	...

k	1	2	3	4	5	...
D tensor	7.588	11.065	14.373	17.616	20.827	...
N tensor	5.981	9.537	12.854	16.096	19.304	...

Almost degeneracy!
The skip in the mode number is **equivalent to a mass contribution in the tensor sector!**

3 Glueballs in AdS/QCD: Hard-Wall model



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In the hard-wall (HW) model confinement is implemented by imposing the following IR cutoff: $0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$

$0^{++} \otimes 2^{++}$ GLUEBALL SPECTRA

M.Rinaldi and V. Vento EPJA 54 (2018)

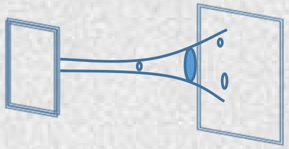
MP: C.J. Morningstar et al, PRD 60, 034509 (1999) YC: Y. Chen et al, PRD 73, 014516 (2006) LTW: B. Lucini et al, JHEP 06, 012 (2004)

LATTICE DATA:

	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277

These two states are almost degenerate

3 Glueballs in AdS/QCD: Hard-Wall model



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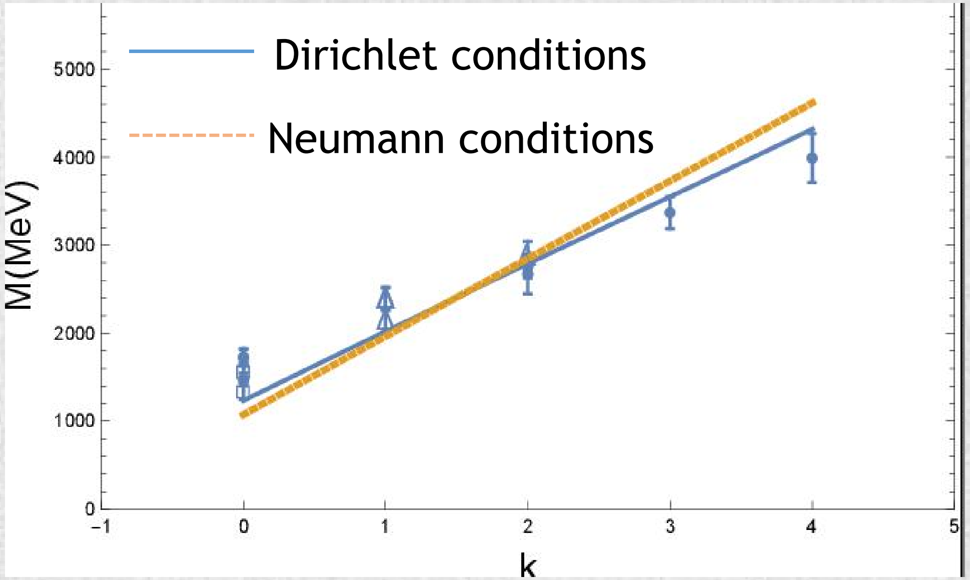
In the hard-wall (HW) model confinement is implemented by imposing the following IR cutoff: $0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$

$0^{++} \otimes 2^{++}$ GLUEBALL SPECTRA

M.Rinaldi and V. Vento EPJA 54 (2018)



Good agreement!

However the HW model does not reproduce the meson spectrum.

$$M_n^2 \sim n^2$$

In order to have a unified view we need another model, i.e.: the Soft-wall model!

3 Glueballs in AdS/QCD: The Soft-Wall

karch et al, PRD 74, 015005 (2006)

In the original model we have: $g_{MN} dx^M dx^N = \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$

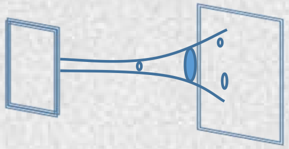
but a soft **cutoff** to space-time is obtained by adding a **dilaton** field in the action:

$$\mathcal{I} = \int d^5x \sqrt{-g}\, e^{-\varphi(x)} \mathcal{L}$$

Successful in describing the Regge behavior of the spectrum: $M^2_{n,J} \sim n + j, \qquad j \geq 0$

WHAT ABOUT GLUEBALLS?

3 Glueballs in AdS/QCD: Soft-Wall model



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We consider the profile function: $\varphi(z) = \kappa z^2$

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Equation of motion of the scalar glueball can be obtained:

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Dilaton field

$\Delta =$ conformal dimension

$$\Delta = 2 + \sqrt{4 + M_5^2 R^2}$$

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The equation of motion for the scalar is:

$$-\Psi''(z) + \left[z^2 + \frac{15}{4z^2} + 2\right]\Psi(z) = M^2\Psi(z)$$

where:

$$\mathcal{G}(x,z) = e^{iP_\mu x^\mu}\left(\frac{z}{R}\right)^{3/2} e^{\kappa^2 z^2/2}\Psi(z), \qquad P^2 = -M^2$$

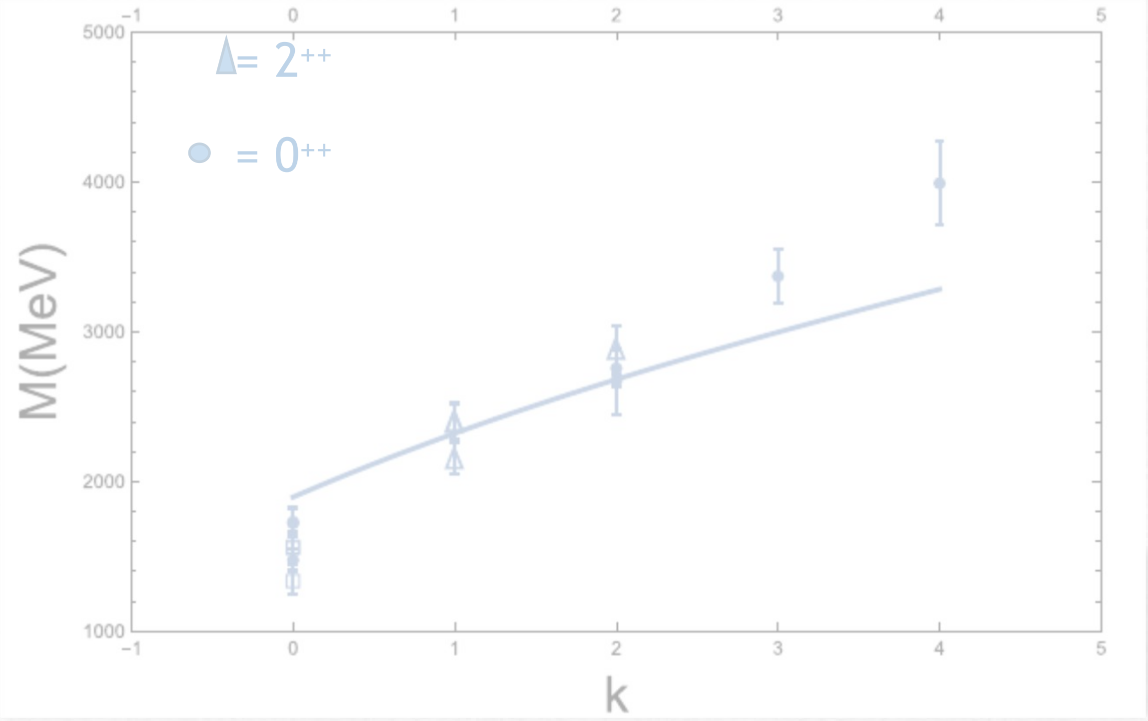
Eduardo Folco Capossoli et al, PLB 753 (2019) 419-423

SCALAR GLUEBALL SPECTRUM:

$$M_J^2 = 4k + 4 + 2\sqrt{4 + J(J+4)} = 4k + 8$$

$k = 0, 1 \ldots$ scalar

$k = 1, 2 \ldots$ tensor



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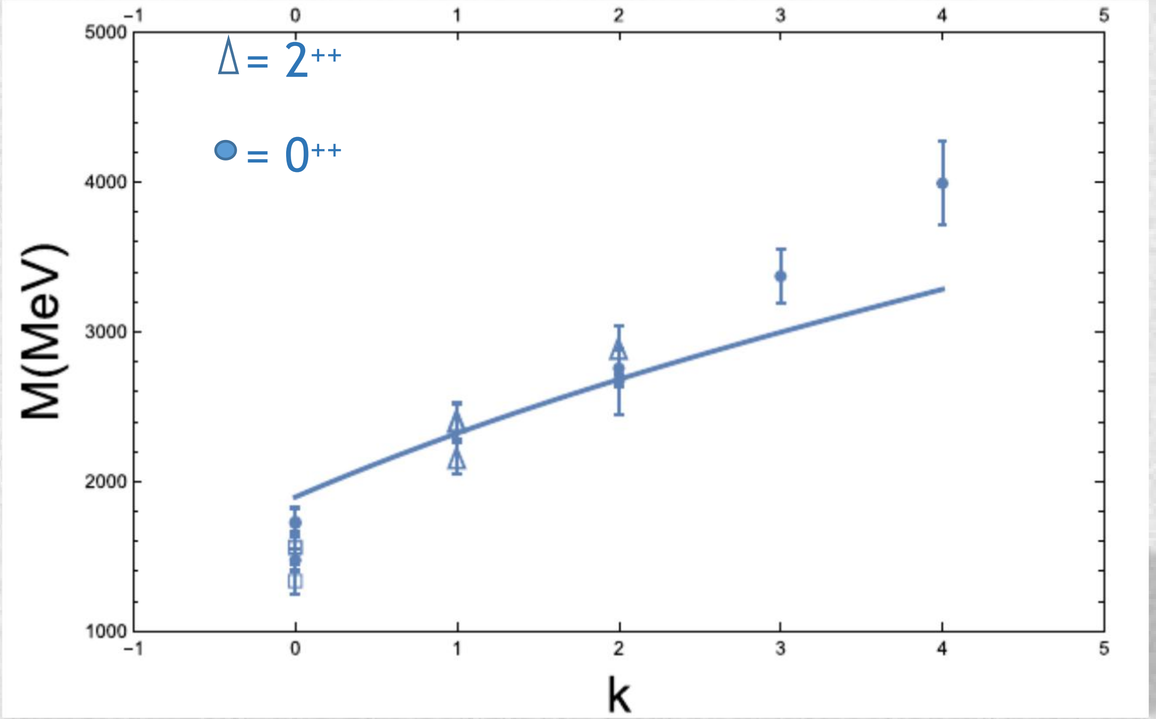

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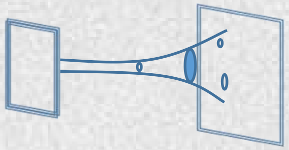

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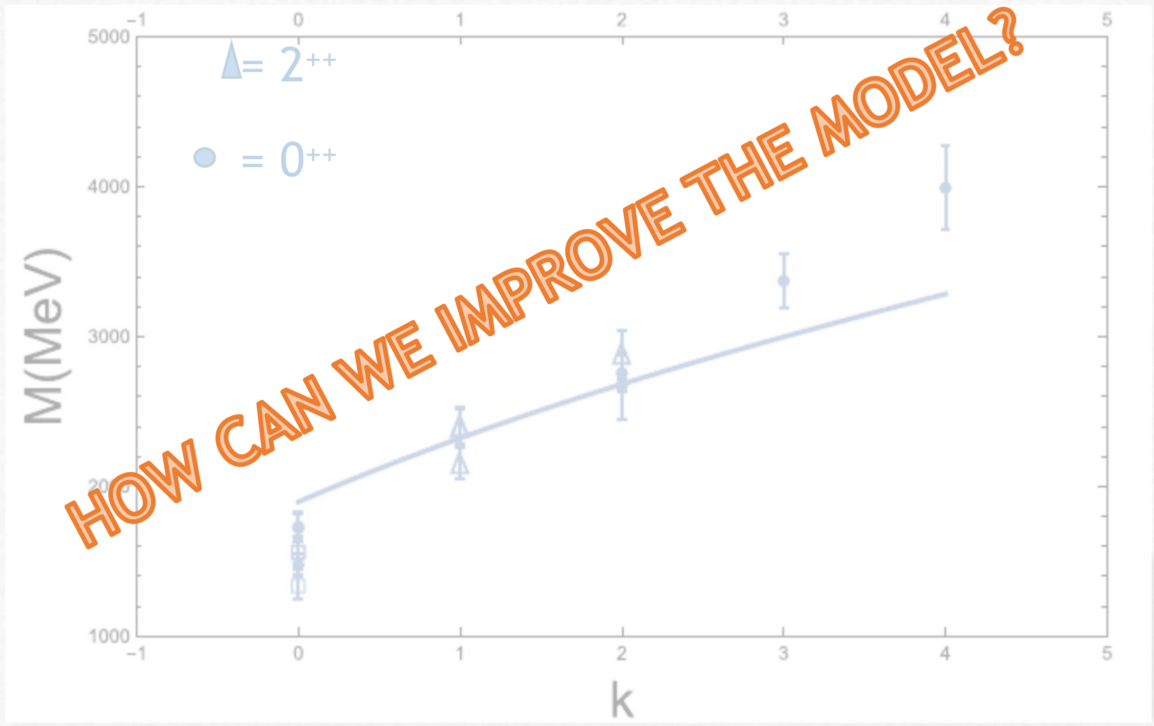

3 Glueballs in AdS/QCD: The Soft-Wall II

In M.Rinaldi and V. Vento EPJA 54 (2018) we propose to use a soft-wall graviton (GSW) model.
In this case a dilatonic cutoff is incorporated in the metric:

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

IR deformation — **QCD scale**

However, a dilatonic contribution in the action can still be kept:

$$\tilde{\mathcal{I}} = \int d^5x \sqrt{-\tilde{g}}\, e^{-\beta\varphi(x)} \mathcal{L}$$

In order to preserve the good description of the hadronic spectrum we require:

$$\int d^5x \sqrt{-\tilde{g}}\, e^{-\beta\varphi(x)} \mathcal{L} \sim \int d^5x \sqrt{-g}\, e^{-\varphi(x)} \mathcal{L}$$

kinetic term

$$\boxed{\frac{3\alpha}{2} + \beta = 1}$$

Modified Soft-Wall model in e.g.:
E. F. Capossoli et al, PLB 753, 419-423 (2006)
O. Andreev arXiv: 1902.10458
E. F. Capossoli et al, arXiv: 1903.06269
W. de Paula et al, PRD 79, 075019 (2009)

3 Glueballs in AdS/QCD: The Soft-Wall II

In M.Rinaldi and V. Vento EPJA 54 (2018) we propose to use a soft-wall graviton (GSW) model.
In this case a dilatonic cutoff is incorporated in the metric:

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2} \left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

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WHAT ABOUT GLUEBALLS?

3 Glueballs in AdS/QCD: The Soft-Wall II

In this case we have the following AdS_5 metric : $\tilde{g}_{MN} dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}_{\alpha\kappa^2}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$

In M.Rinaldi and V. Vento EPJA 54 (2018) we consider $\alpha\kappa^2$ as the only **one parameter!**

GRAVITON EoM and SPECTRUM

EoM for metric perturbation is obtained from the Einstein's equation: $-\frac{1}{2}\tilde{h}_{ab;c}^{;c} - \frac{1}{2}\tilde{h}^{c}_{c;ab} + \frac{1}{2}\tilde{h}_{ac;b}^{;c} + \frac{1}{2}\tilde{h}_{bc;a}^{;c} + 4\tilde{h}_{ab} = 0$

By choosing the gauge:

$$\begin{cases} \tilde{h}_{tt} = (z^{-2} - z^2)\phi(z)e^{-Mx_3} & \text{Scalar component} \\ \tilde{h}_{ij} = q_{ij}T(z)e^{-Mx_3} & \text{Tensor component} \end{cases}$$

"Tensor" wave-function

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$$\begin{cases} t = i\alpha z/\sqrt{2} \\ \Lambda^2 = \dfrac{M^2}{\alpha^2} \\ V_G(t) = \dfrac{e^{2t^2}}{t^2} - \dfrac{17}{4t^2} + 14 - 15t^2 \end{cases}$$

1) The scalar and tensor components have the same EoM

2) Bound states are found for $\alpha<0$

3) From the fitting procedure we found that: $a \le \kappa \le \beta$

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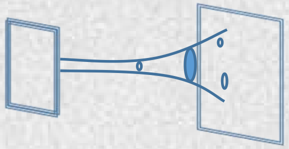

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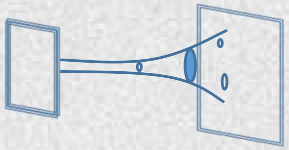

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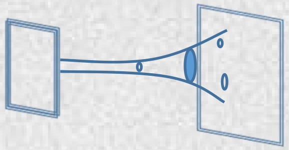

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Guy F. de Teramond et al, PRL 120, 182001 (2018)

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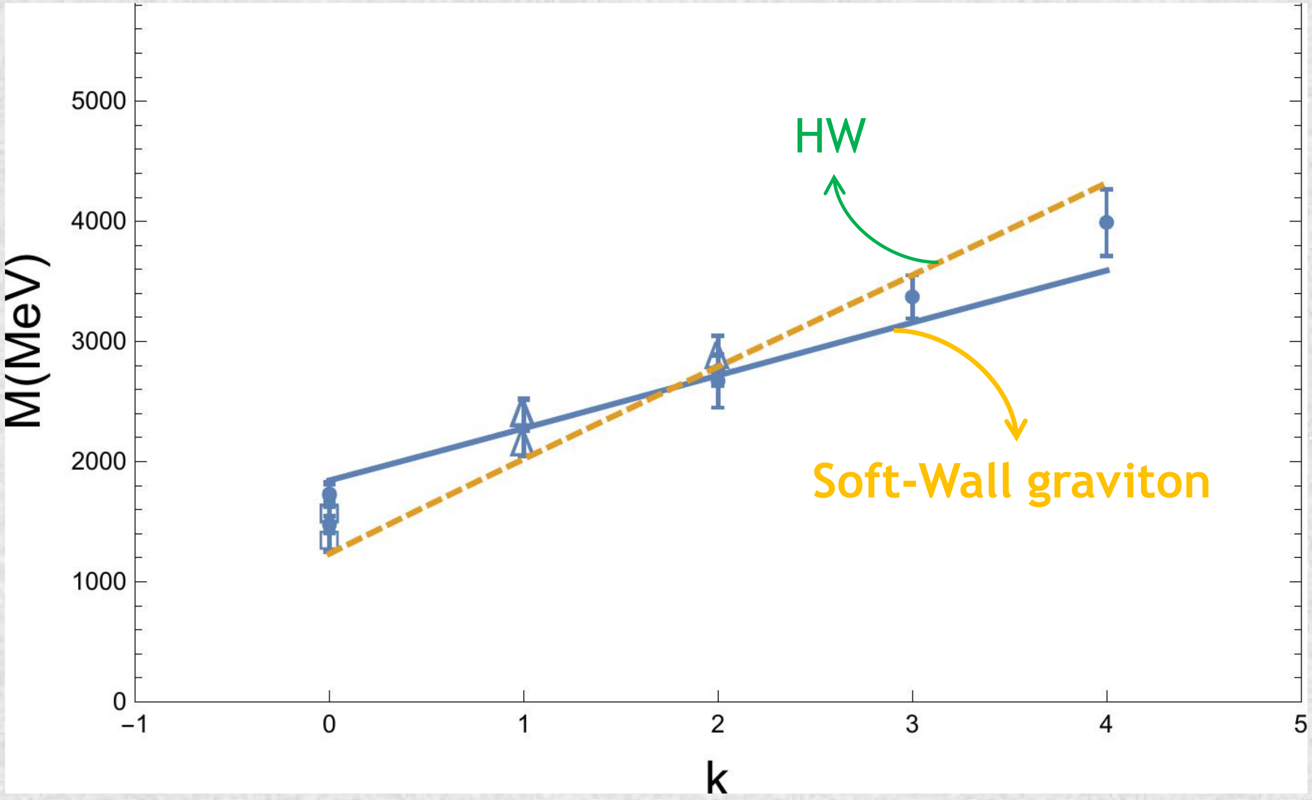

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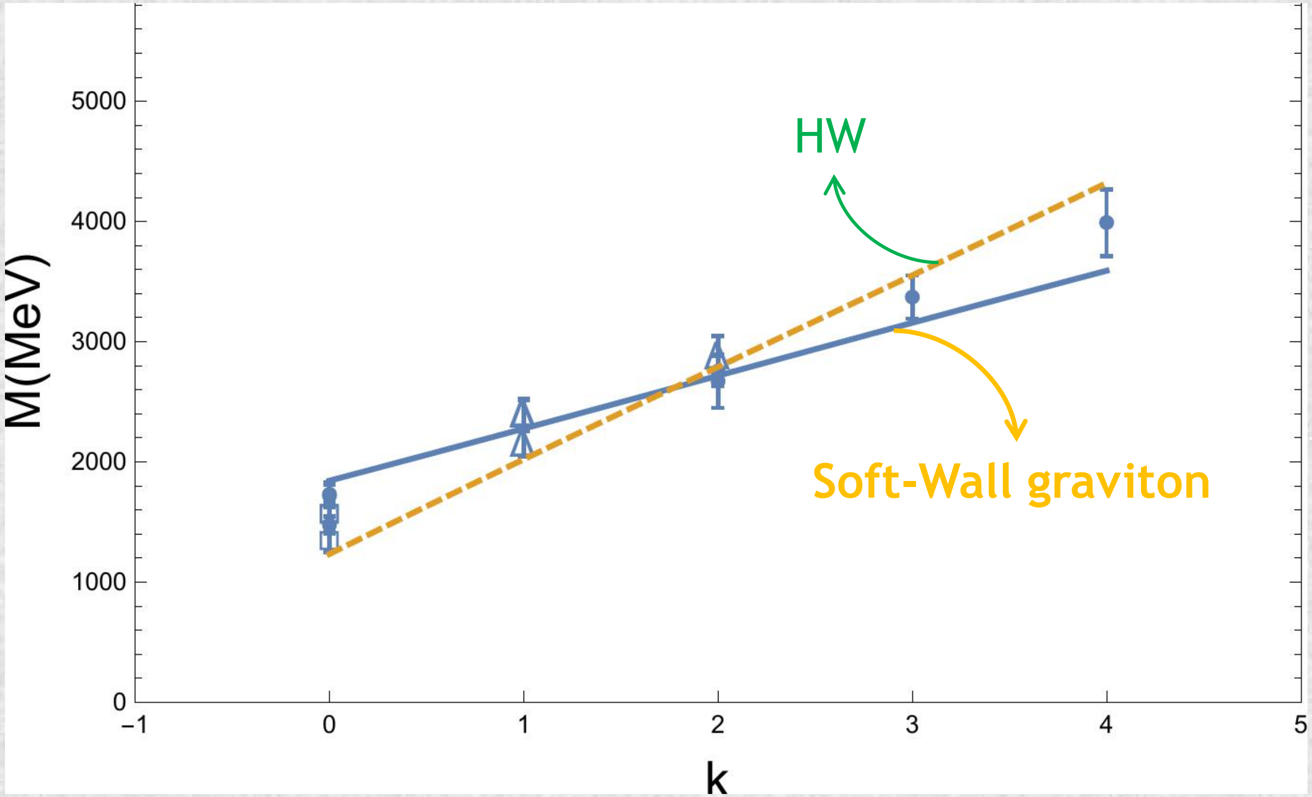

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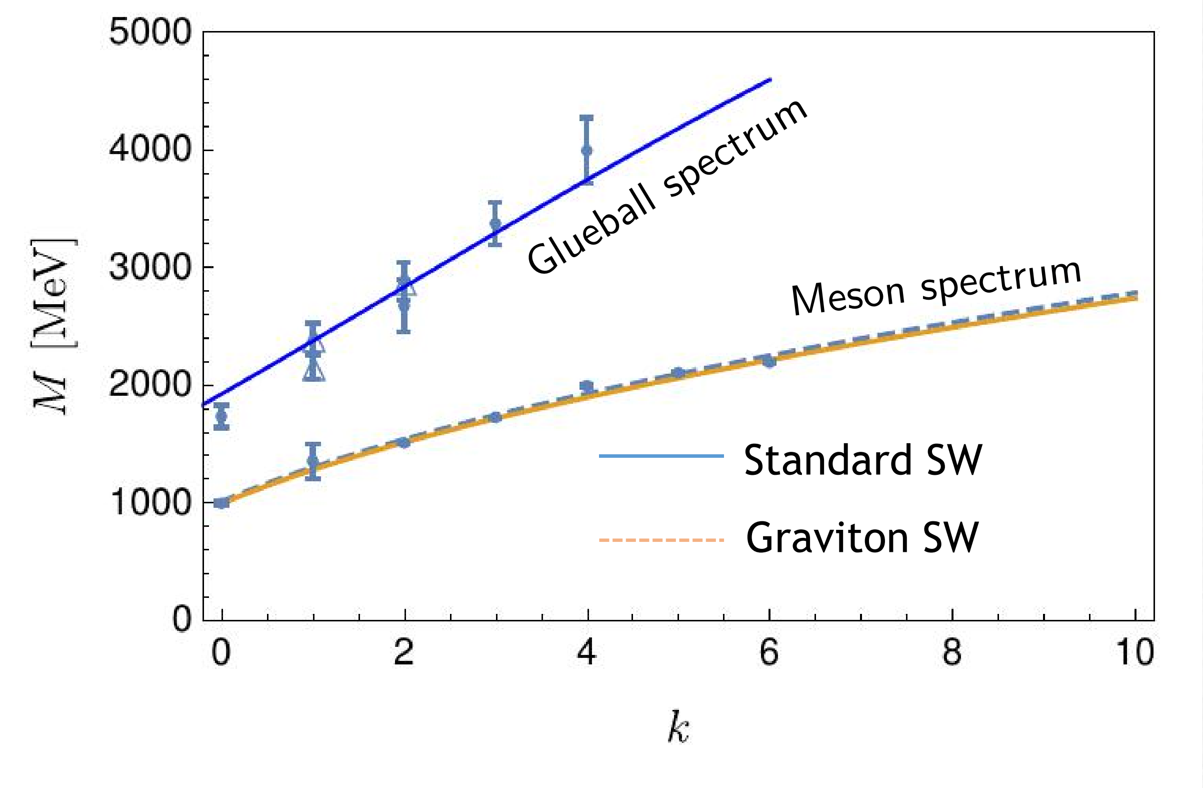

4

The mixing problem in AdS/QCD

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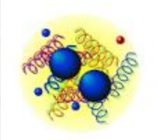
Glueball and meson states could mix!

J^{PC}	0^{++}	2^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222
YC	1719 ± 94	2390 ± 124	
LTW	1475 ± 72	2150 ± 104	2755 ± 124

Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	1992 ± 16	2101 ± 7	2189 ± 13



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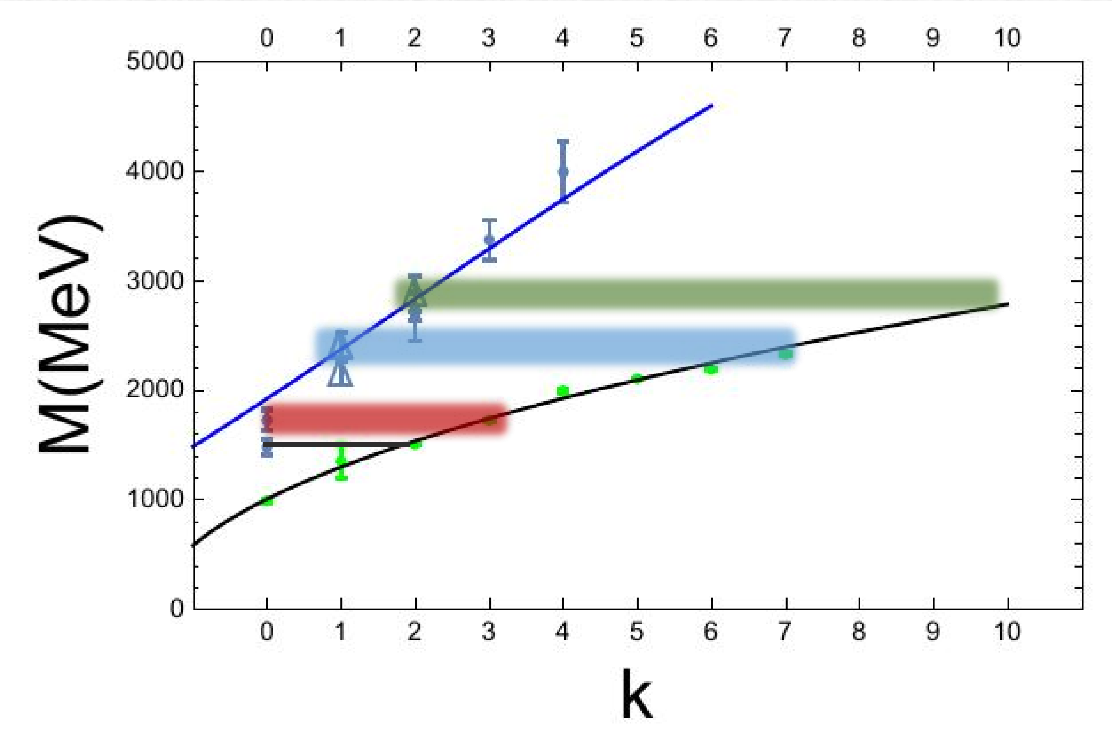
In terms of modes numbers:



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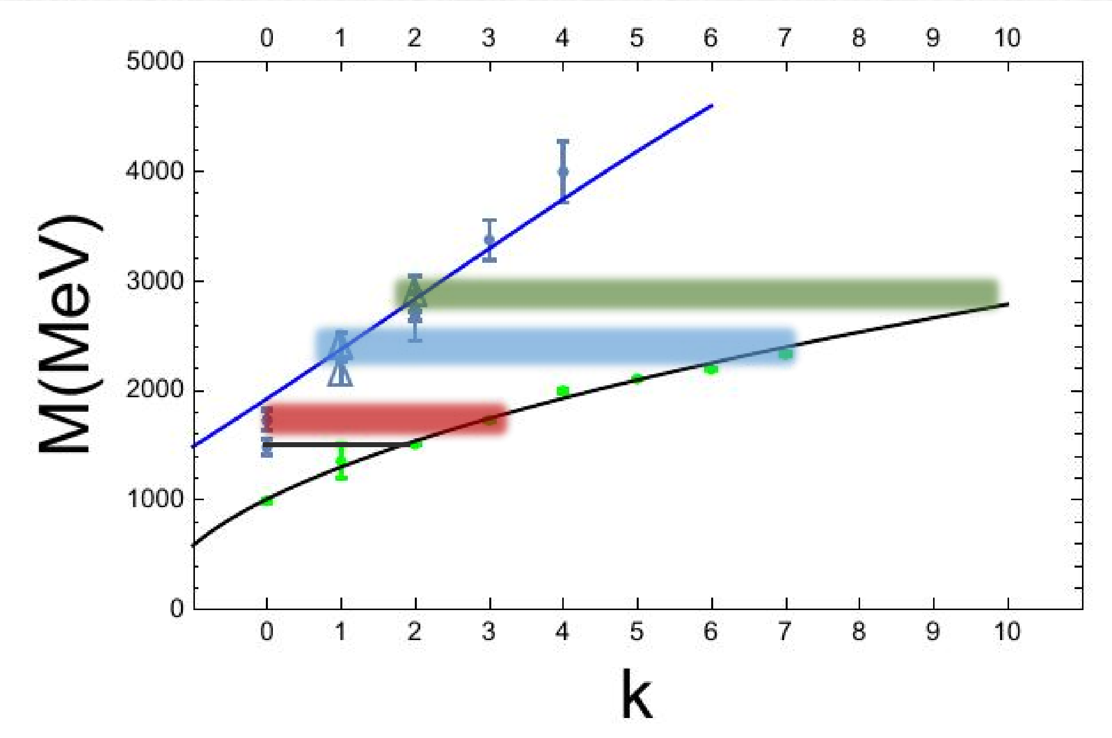

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Since the soft-wall model reproduces both the glueball and meson spectra, we can use it to study the mixing condition!

M.Rinaldi and V.Vento arXiv:1803.05738

4 The mixing problem in AdS/QCD

We consider the Light-Front formulation of the EoM in therms of the Hamiltonian. Within this framework the latter would be defined by the AdS/QCD model.

M.Rinaldi and V.Vento arXiv:1803.05738

$$H_{LC}|\Psi_k\rangle = M^2|\Psi_k\rangle$$

We consider its representation in a 2-D meson-glueball subspace: $\{ \, |\Psi^m\rangle, |\Psi^g\rangle \, \}$

$$[H] = \begin{pmatrix} m_m & \alpha \\ \alpha & m_g \end{pmatrix}$$

$$m_g = \langle \Psi^g | H | \Psi^g \rangle \qquad m_m = \langle \Psi^m | H | \Psi^m \rangle$$

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Mixing parameter!

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$$\alpha = \langle\Psi^m|H|\Psi^g\rangle \propto \langle\Psi^m|\Psi^g\rangle$$

OVERLAP

Mixing parameter!

4 The mixing problem in AdS/QCD

We define the probability for NO MIXING as: $P_{mg} \equiv 1 - |\langle \Psi^g | \Psi^m \rangle|^2$

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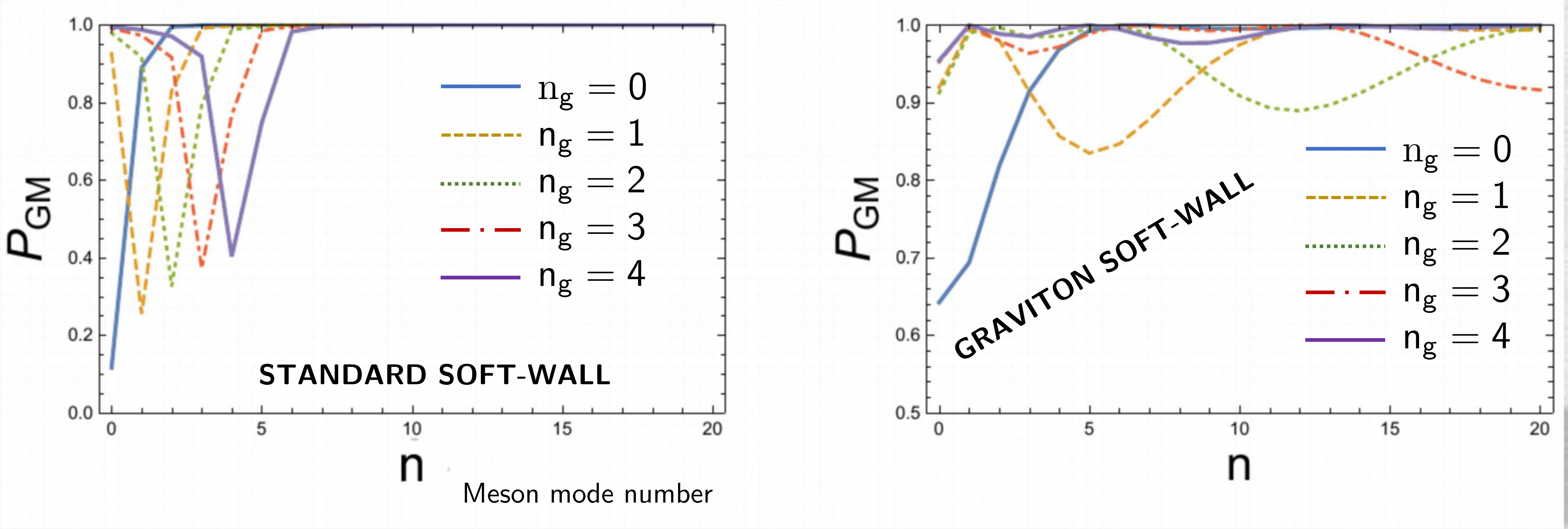

For heavy glueballs (e.g. n_g=2,3...) which would have similar mass of meson (e.g. n_m=10,13...) the probability of mixing is small!!

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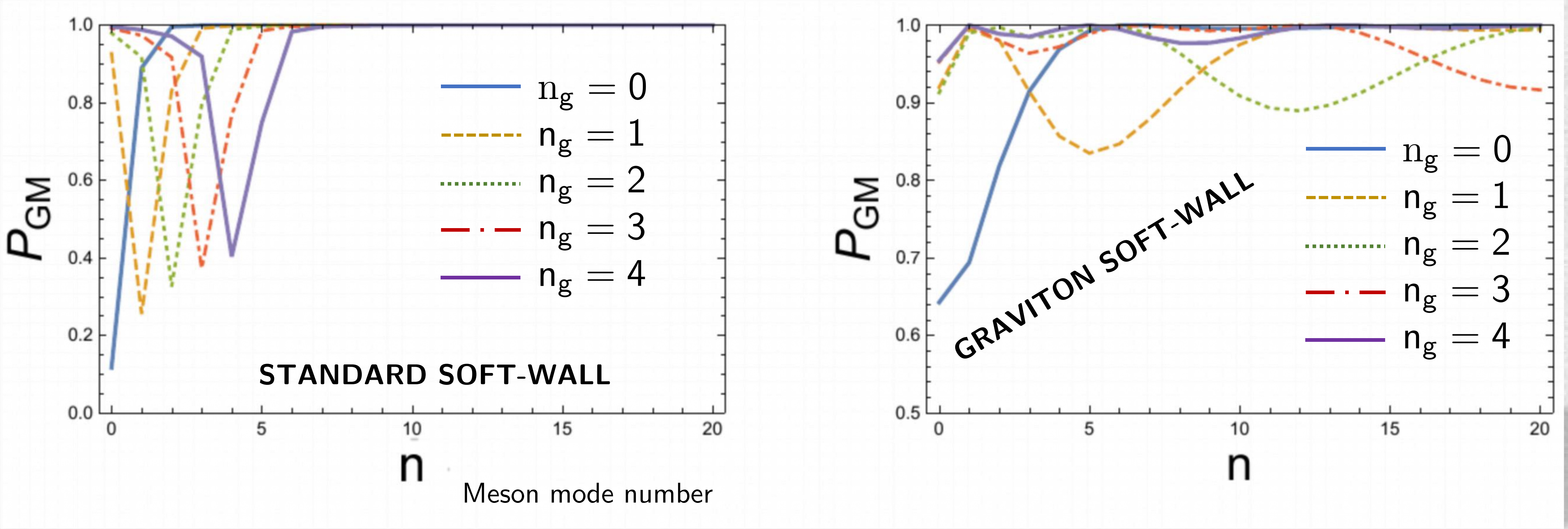
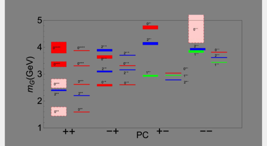
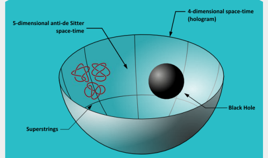


Within the Soft-Wall AdS/QCD models (standard and with graviton) pure glueballs in the scalar sector exist in the mass range above 2 GeV!

CONCLUSIONS

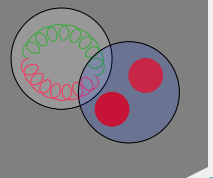
WE CONSIDER THE GLUEBALL SPECTRUM

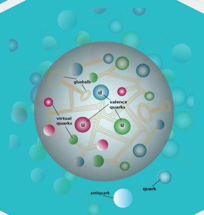




WE STUDIED THE GLUEBALL SPECTRUM WITHIN DIFFERENT AdS/QCD MODELS

WE FACED THE GLUEBALL-MESON MIXING PROBLEM





WE ADOPTED THE GRAVITON SOFT-WALL MODEL WHICH DESCRIBES BOTH THE MESON AND GLUEBALL SPECTRA

$$\langle \Psi^m | \Psi^g \rangle$$

WE FOUND THAT PURE SCALAR GLUEBALLS COULD BE FOUND ABOVE 2 GeV

THANKS

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SCALR FIELD EQUATION:

Equation of motion of the scalar glueball can be obtained:

$$\tilde{I} = \int d^5x \sqrt{g}\, e^{-\varphi(z)} \left[g^{MN} \partial_M \mathcal{S} \partial_N \mathcal{S} + e^{-\alpha\varphi(z)} M_5^2 \mathcal{S}^2 \right]$$

Dilaton field

Graviton contribution

1) scalar glueball state 0^{++} is represented by: $\mathcal{O}_{\Delta=4} = \mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})$
2) For example for even spin J: $\mathcal{O}_{\Delta=4+j} = FD_{\{\mu 1 \ldots} D_{\mu j\}} F$

The equation of motion for the scalar is (for small α):

$$-\psi''(z) + \left[\kappa^2 z^2 + \frac{15}{4z^2} + 2\kappa + M_5^2\left(\frac{R^2}{z^2}\right) - M_5^2 R^2 \alpha\kappa\right]\psi(z) = M^2\psi(z)$$

SCALAR GLUEBALL SPECTRUM:

$$M_n = \left[4n + 4 + 2\sqrt{M_5^2 R^2 + 4} - \alpha M_5^2 R^2\right] \begin{array}{ll} \rightarrow k = 0, 1\ldots. & \textbf{scalar} \\ \rightarrow k = 1, 2\ldots. & \textbf{tensor} \end{array}$$