Paul Griffin:

Zero mode, zero mode
on the cone of light.
Zero mode, zero mode
shinest you so bright.

Much Ado About Nothing

an introduction to the LF vacuum

Matthias Burkardt

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September 20, 2019

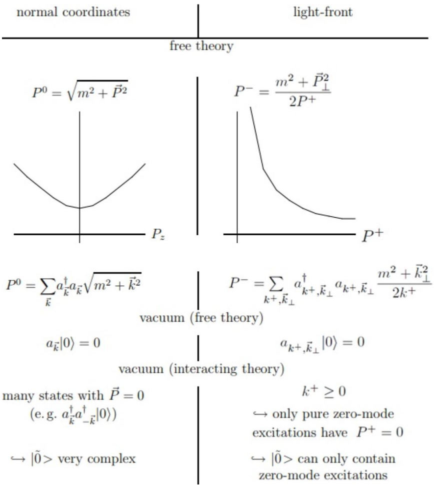
- P^+ conservation & P^+ purely kinematical

$\hookrightarrow$ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

$\hookrightarrow$ also nondegenerate state of lowest P^-

$\hookrightarrow$ exact ground state of theory



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issues with this result

- Higgs mechanism
- QCD vacuum:
  - lattice: $\langle 0|\bar{q}q|0\rangle \neq 0$
  - Gell-Mann, Oakes, Renner $f_\pi^2 m_\pi^2 = (m_u + m_d)\langle 0|\bar{q}q|0\rangle \neq 0$

possible resolutions

- $\langle 0|\bar{q}q|0\rangle \neq 0$ fake news!

↪ GOR made it up!

- LF formalism is fake!

↪ Dirac made it up!

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- maybe there is a 3^{rd} option

SSB in $1+1$ dimensions?

- no spontaneous symmetry breaking (SSB) in 1+1 (S.Coleman)
- however not valid for $N_C \longrightarrow \infty$ as Hartree-Fock approx. becomes exact

$\hookrightarrow$ SSB possible

't Hooft model

- $QCD_{1+1}(N_C \longrightarrow \infty)$
- LF quantization & gauge

$$M_n^2 \phi_n(x) = \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \phi_n(x) + \frac{g^2 C_F}{\pi} \int_0^1 dy \frac{\phi_n(x) - \phi_n(y)}{(x-y)^2}$$

- M^2 meson mass; x $(1-x)$ momentum fraction carried by q $(\bar{q})$
- trivial vacuum, lowest Fock sector for meson exact as $N_C \to \infty$
- infinite 'tower' of solutions
- lowest meson state $M_\pi^2 \propto m_q$

$\hookrightarrow$ hint that $\langle 0|\bar{q}q|0\rangle \neq 0$

- meson spectrum confirmed by Li, Wilets, Birse in ET/BS (1986)

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Zhitnitzky PLB 165B (1985) 405, Sov.JNP 43, 999; 44, 139 (1984)

- GMOR: $\lim_{m_q \to 0} \langle 0|\bar{q}q|0\rangle = -\frac{N_C}{\sqrt{12}} \sqrt{\frac{g^2 C_F}{\pi}}$
- confirmed by ET calculation: M. Li, PRD34 (1986) 3888
- nonperturbative analytic expression valid for all m_q: MB&N.Uraltsev, PRD 63 (2001) 014004

free lunch?

- Solving LF wave functions from diagonalizing LF Hamiltonian based on trivial vacuum yields same rusults (incl. condensate numbers - using GMOR) as complicated ET calculation!!!!!

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- Does that mean the vacuum is trivial or that it is not trivial?!?

explicit LF calculation (MB, F.Lenz,M.Thies)

- vacuum condensate $\langle 0|\bar{q}(0)q(0)|0\rangle$ ill-defined
- employ point-splitting in LF time x^+, i.e.
  $\langle 0|\bar{q}(0)q(0)|0\rangle \longrightarrow \langle 0|\bar{q}(0)Wq(\varepsilon)|0\rangle$ with $\varepsilon^2 \neq 0 \quad \Rightarrow \quad \varepsilon^+ \neq 0$
- W Wilson line gauge link
- same as heavy-light correlator: for straight Wilson line, W represents a 'static' heavy quark

$\hookrightarrow$ relate $\langle 0|\bar{q}(\varepsilon)Wq(0)|0\rangle$ to properties of heavy-light mesons (calculated using LF quantization: masses, decay constants)

- reproduced $\langle 0|\bar{q}(0)q(0)|0\rangle$ from GMOR (Zhitnitsky)
- take $\varepsilon^\pm \to 0$ (subtract free-field divergence)

implications for LF vacuum

- condensates (properly regularized) <u>nonzero</u>
- don't affect hadron structure/dynamics in QCD_{1+1}

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- no! ($\longrightarrow$ Fatma Aslan)

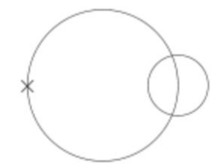
$\langle 0|\phi^2|0\rangle$

- LF: no particles popping out of vacuum (→SJB)

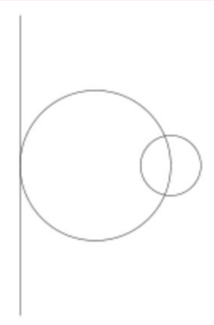
↪ beyond 1 loop, no contri to $\langle 0|\phi^2|0\rangle$ beyond one loop

- covariant calculation:contri to $\langle 0|\phi^2|0\rangle$ to all orders
- relevant since corresponding tadpoles contribute to self-energy - or not!

example for diagram that contributes to $\langle 0|\phi^2|0\rangle$, but cannot be generated by LF Hamiltonian



example for contri to self-energy, that cannot be generated by H_{LF}



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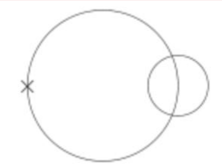


$\int dk^- \frac{\Pi(k^2)}{(k^2-m^2+i\varepsilon)^n}$

- issue arises for all integrals of above type!
- $\Pi(k^2)$ same pole structure as $\frac{1}{k^2-m^2+i\varepsilon}$

↪ $\delta(k^+)$ (talks by: F.Aslan & P.Mannheim)

- first studied by Yan, Ma

J.Collins, LC workshop 2018

- considered $\int d^2x \langle 0|\phi^2(0)\phi^2(x)|0\rangle e^{iqx}$
- for $q^+ = 0$ same pole structure as generalized tadpoles
- naively vanishes for $q^+ = 0$

$\hookrightarrow$ regulated by taking $q^+ \neq 0$

- support only for $0 < k^+ < q^+$ with k^+, $q^+ - k^+$ momentum of one of the particles created by $\phi^2|0\rangle$
- $\lim_{q^+\to 0}$ yields finite result
- in terms of k^+, rep. of $\delta(k^+)$

connection of singularities in twist-3 GPDs/PDFs

- pole structure similar to above vacuum corellator
- in GPDs $q^+ \neq 0$, 'regulates' $\delta(x)$ present in PDFs
- rep. of $\delta(x)$ as $q^+ \to 0$

bad news

- LF calc. misses whole class of diagrams:
  generalized tadpoles
- improper treatment of zero modes

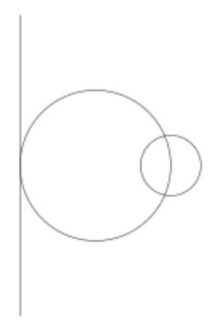
good news MB, PRD (1993)

- all of the missed diagrams only contribute constants

↪ can be taken care of by renormalization

↪ $m_{eff}^2 = m^2 + \lambda\langle 0|\phi^2|0\rangle$

example for contri to self-energy, that cannot be generated by H_{LF}



determining m_{eff}^2

- only match physical quantities during renorm.
- determine $\lambda\langle 0|\phi^2|0\rangle$ by point-splitting in LF time & inserting complete set of states (MB, S.Chabysheva, J.Hiller, PRD (2016))

good news MB, PRD47 (1993) 4628

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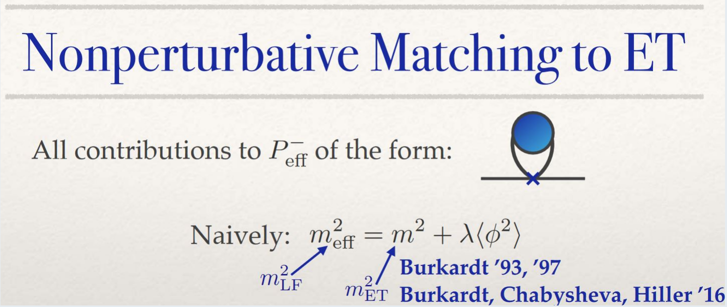
$\hookrightarrow$ can be taken care of by renormalization

$\hookrightarrow$ $m^2_{eff} = m^2 + \lambda\langle 0|\phi^2|0\rangle$

Matthew Walters - this workshop



free lunch! (almost)

- can 'ignore' zero-modes in ϕ^4 when interested in bound states
- some subtleties with σ-term sum rules (F.Aslan, this morning)

- zero-modes high-energy (k^-) degrees of freedom

$\hookrightarrow$ plausible that 'integrating out' zero modes leads to $P^- \longrightarrow P^-_{eff}$

- by construction, P^-_{eff} containe no zero-mode degrees of freedom

no tadpoles!?

- naively tadpole issue absent
- k^- from Dirac numerators can cancel one propagator:
  $k^- = p^- - \frac{(p_\perp - k_\perp)^2 + \lambda^2}{2(p^+ - k^+)} - \frac{(p-k)^2 - \lambda^2}{2(p^+ - k^+)}$
  - cancels one denominator
  - 'canonical term' (incl. instantaneous)

↪ self-energies contain pieces with same pole structure as generalized tadpoles

↪ condensates matter!

- renormalization can fix it...! (e.g. vertex mass $\neq$ kin. mass)

Summary

- naively LF vacuum trivial
- apparent contradiction with pheno & lattice
- regularization (point splitting in ε^+) yields nonzero condensates
- consistent with covariant in $QCD_{1+1}\&\phi^n$
- $P^- \longrightarrow P^-_{eff}$ embodies effect of zero modes non-zero modes
- done for ϕ^4 & Yukawa