The Delta Functions in PDFs and the QCD Vacuum Structure

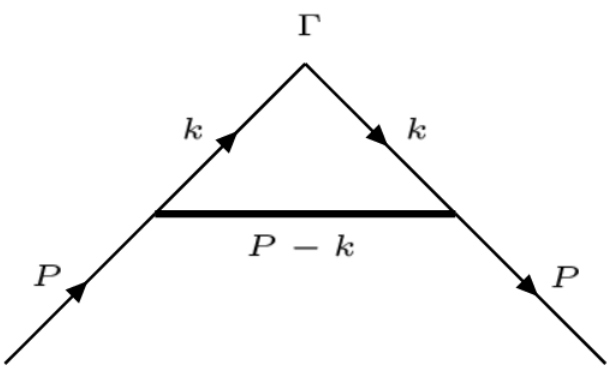
Fatma Aslan & Matthias Burkardt

Based on:
Matthias Burkardt, Light Front Quantization, 1995
Aslan, Burkardt, Singularities in Twist-3 Quark Distributions, 2018.

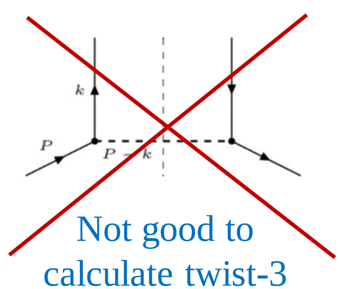
Model calculations of PDFs

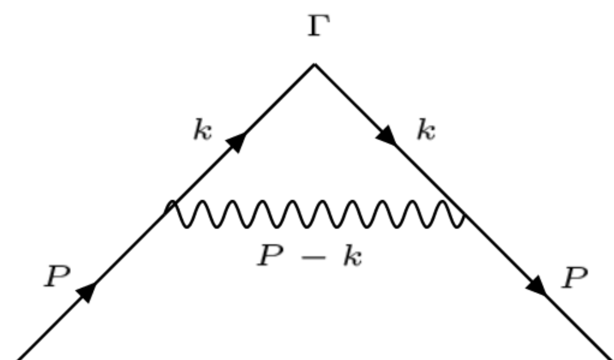
Twist-2 pdf	Measurement	Operator
f_1	Spin average	γ^+
g_1	Helicity difference	$\gamma^+\gamma_5$
h_1	Helicity flip	$i\sigma^{\perp+}\gamma_5$

Twist-3 pdf	Measurement	Operator
e	Spin average	**1**
h_L	Helicity difference	$i\sigma^{+-}\gamma_5$
g_T	Helicity flip	$\gamma^{\perp}\gamma_5$



Scalar diquark model





Quark target model

Singularities in twist-3 quark distributions

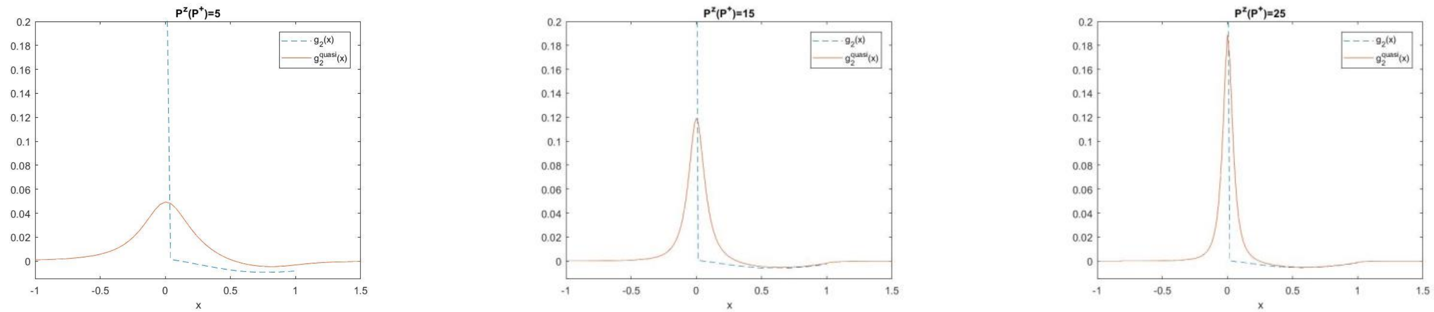
Twist-2 PDF	*SDM*	*QTM*	*Twist-3 PDF*	*SDM*	*QTM*
f_1	✗	✗	e	✓	✓
g_1	✗	✗	h_L	✓	✓
h_1	✗	✗	g_T (g_2)	✓	✗

✓: There is a $\delta(x)$ & ✗ : There is no $\delta(x)$

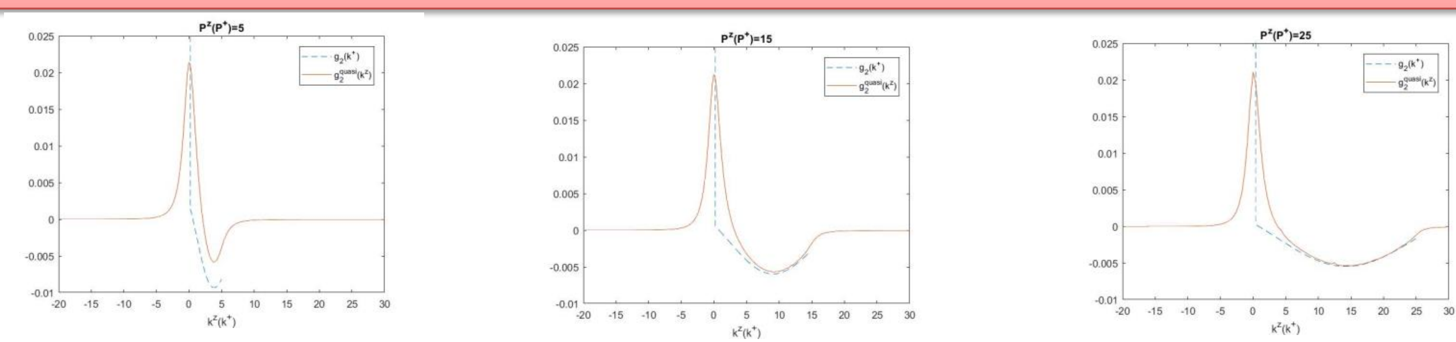
Burkardt, Koike, Violation of sum rules for Twist-3 parton distributions in QCD, 2001.

Aslan, Burkardt, Singularities in Twist-3 Quark Distributions, 2018.

At twist-3 there is something that does not exist in twist-2: There are delta functions.



We identify these delta functions with momentum components in the nucleon state that do not scale as the nucleon is boosted to the infinite momentum.



Singularities in twist-3 quark distributions

➢ The delta functions contribute both to the mass and the qgq correlation term.

$$h_L(x) = h_L^{WW}(x) + h_L^m(x) + h_L^3(x)$$

Burkardt & Koike, Violation of Sum Rules for Twist 3 Parton Distributions in QCD, 2001

➢ They do not go away with regularization.

$\delta(x)$ remains	$\delta(x)$ is recovered
Transverse momentum cut off	Pauli-Villars regularization
Dimensional regularization	
Adding form factors	
Adding axial diquark contribution	

Aslan, Burkardt, Lorentz invariance of twist-3 quark distributions, in prep.

➢ Sum rules are violated if they are not taken into account.

$$\int_{-1}^{1} dx g_1(x) = \int_{-1}^{1} dx g_T(x)$$

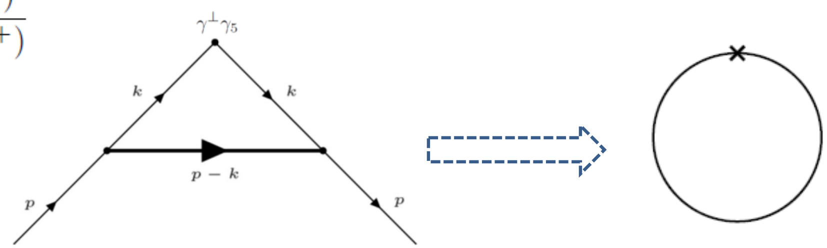
Burkardt & Koike, Violation of Sum Rules for Twist 3 Parton Distributions in QCD, 2001

The origin of δ(x): The zero modes

$$g_T(x) = ig^2 \int \frac{d^2k^\perp dk^-}{(2\pi)^4} \frac{(x + \frac{m}{M})(2k^- P^+ + mM)}{(k^2 - m^2 + i\epsilon)^2[(P-k)^2 - \lambda^2 + i\epsilon]}$$

$$k^- = \frac{M^2}{2p^+} - \frac{[(p-k)^2 - \lambda^2]}{2(p^+ - k^+)} - \frac{(k_\perp^2 + \lambda^2)}{2(p^+ - k^+)}$$

$$\int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2}$$



$$\text{for } k^+ \neq 0, \quad \int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2} = \int \frac{dk^-}{\left[2k^+\left(k^- - \frac{(k_\perp^2 + m^2)}{2k^+} + \frac{i\epsilon}{2k^+}\right)\right]^2} = 0$$

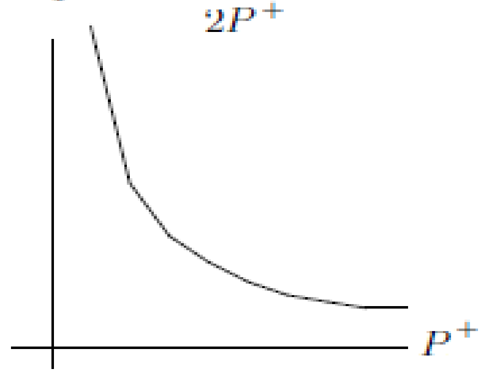
$$\text{for all } k^+ \quad \int dk^+ dk^- \frac{1}{(k^2 - m^2 + i\epsilon)^2} = \int d^2k_L \frac{1}{(k_L^2 - k_\perp^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2 + m^2}$$

$$\int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2 + m^2}\delta(k^+).$$

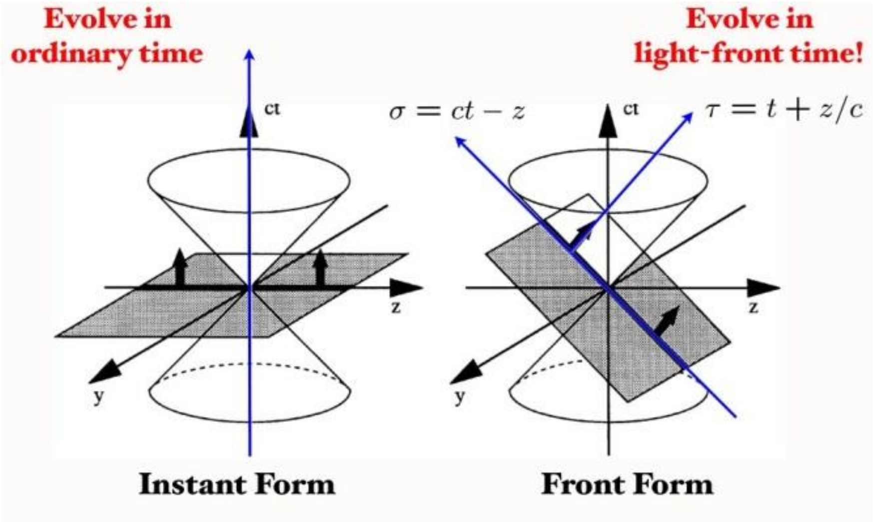
$$k^+ = 0$$

ZERO MODES

$$P^- = \frac{m^2 + \vec{P}_\perp^2}{2P^+}$$

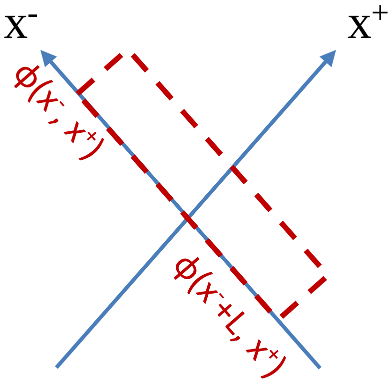


Issues with the LF approach



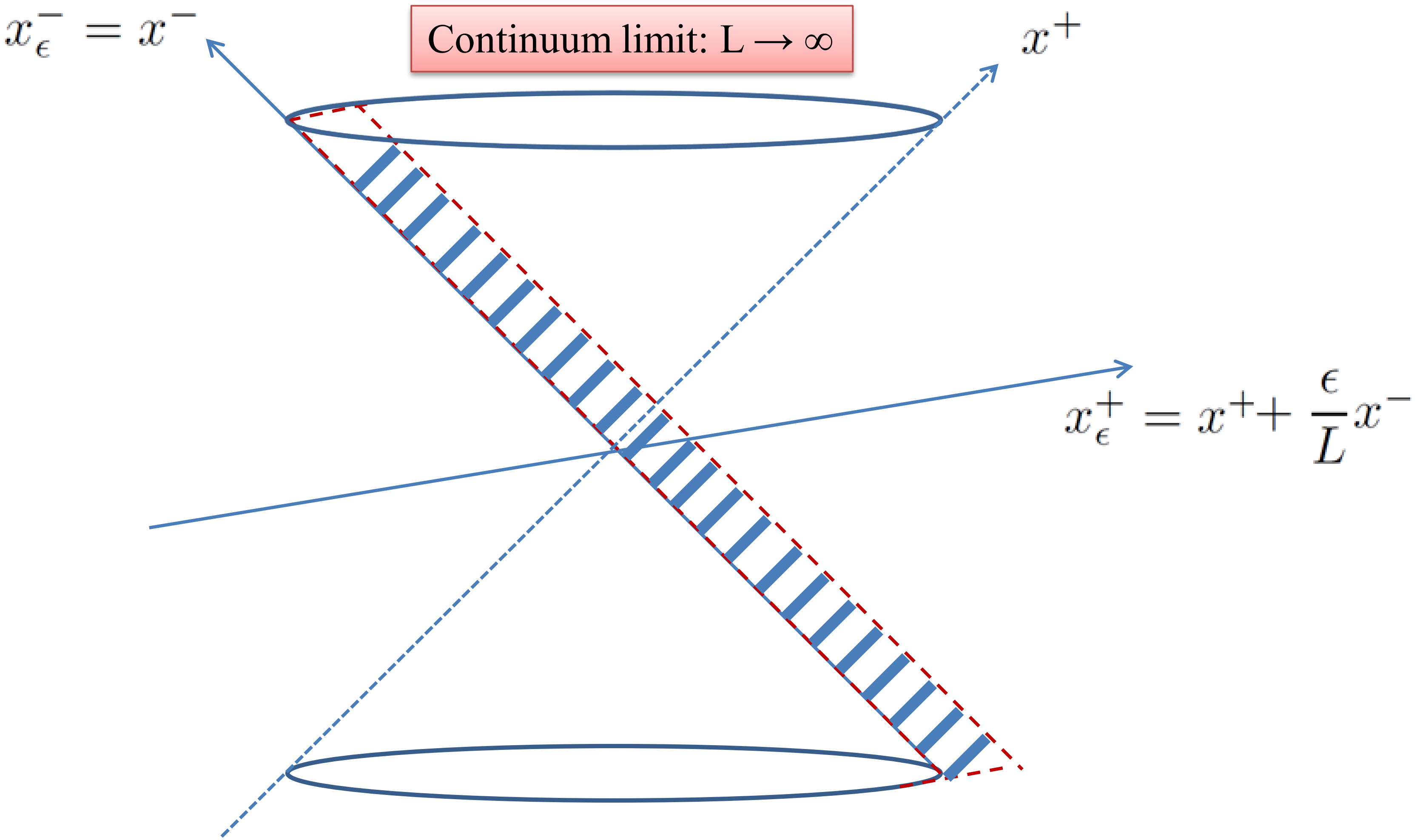
LF time!

- ❑ Do any conflicts arise with causality on the LF **?**
- ❑ How to keep the IR singularities under control with a "box" **?**



ε-COORDINATES

Provide a controlled and well-defined approach to the LF



Lenz, Thies, Yazaki and Levit
Hamiltonian formulation of two-dimensional gauge theories on the light cone (1991).

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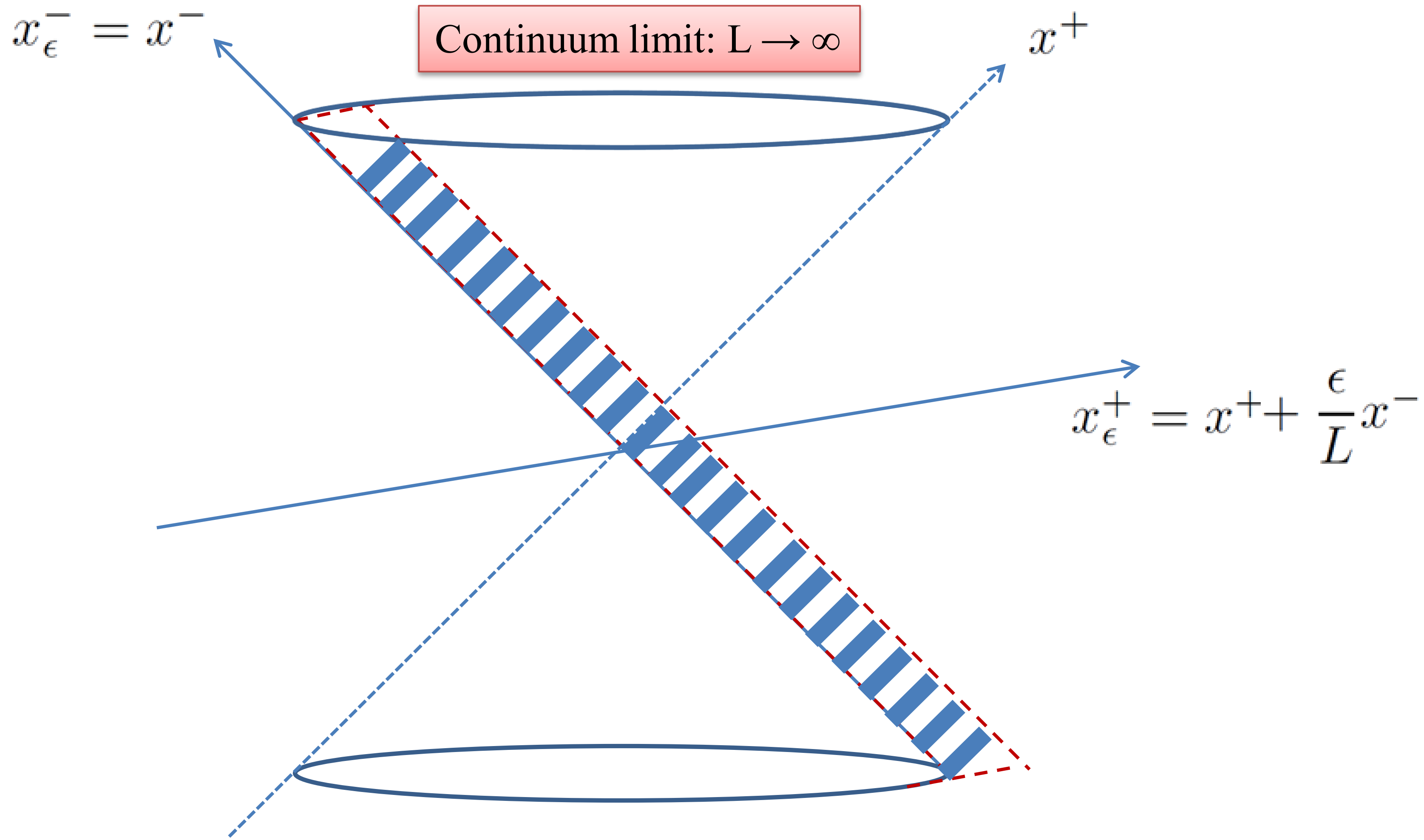

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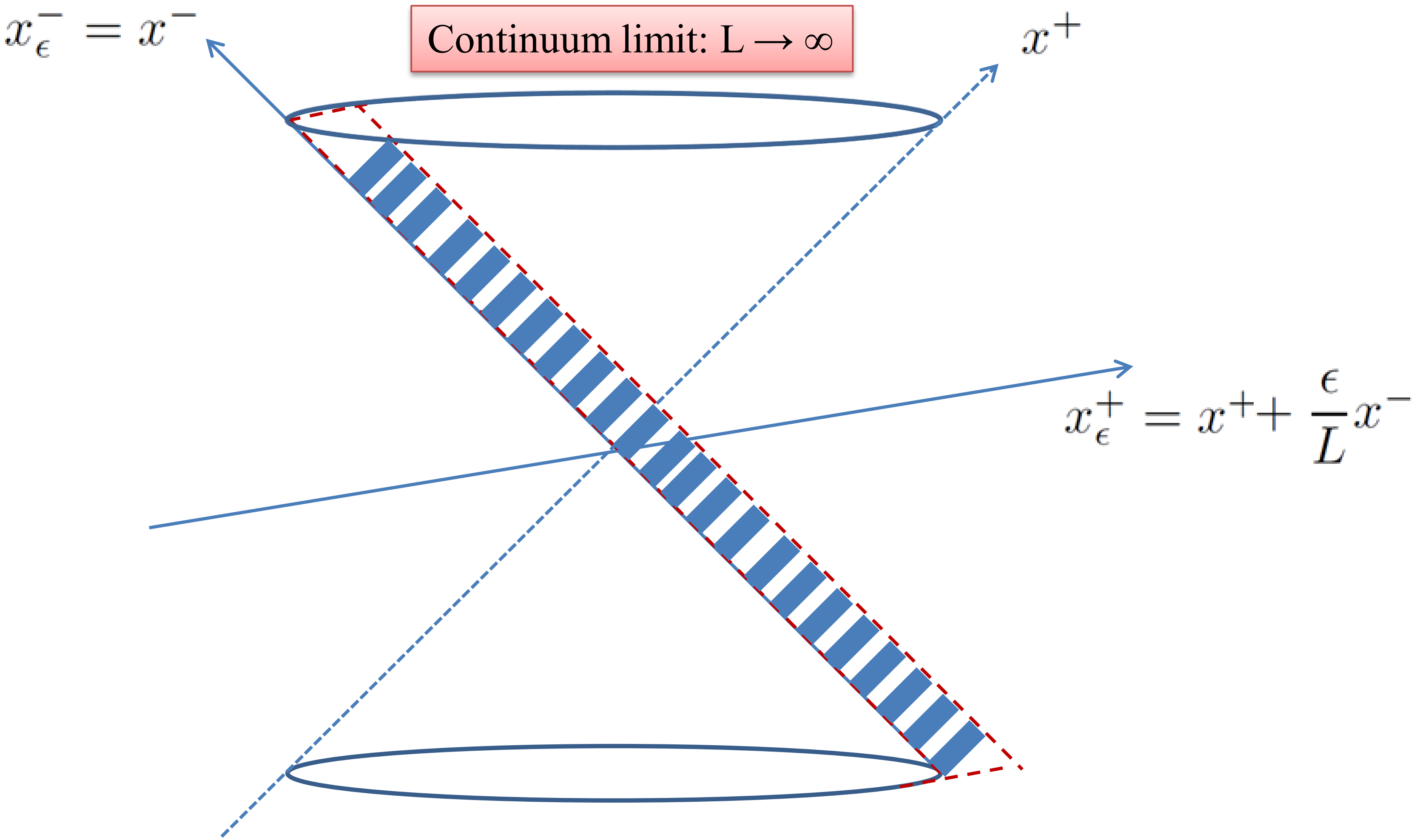

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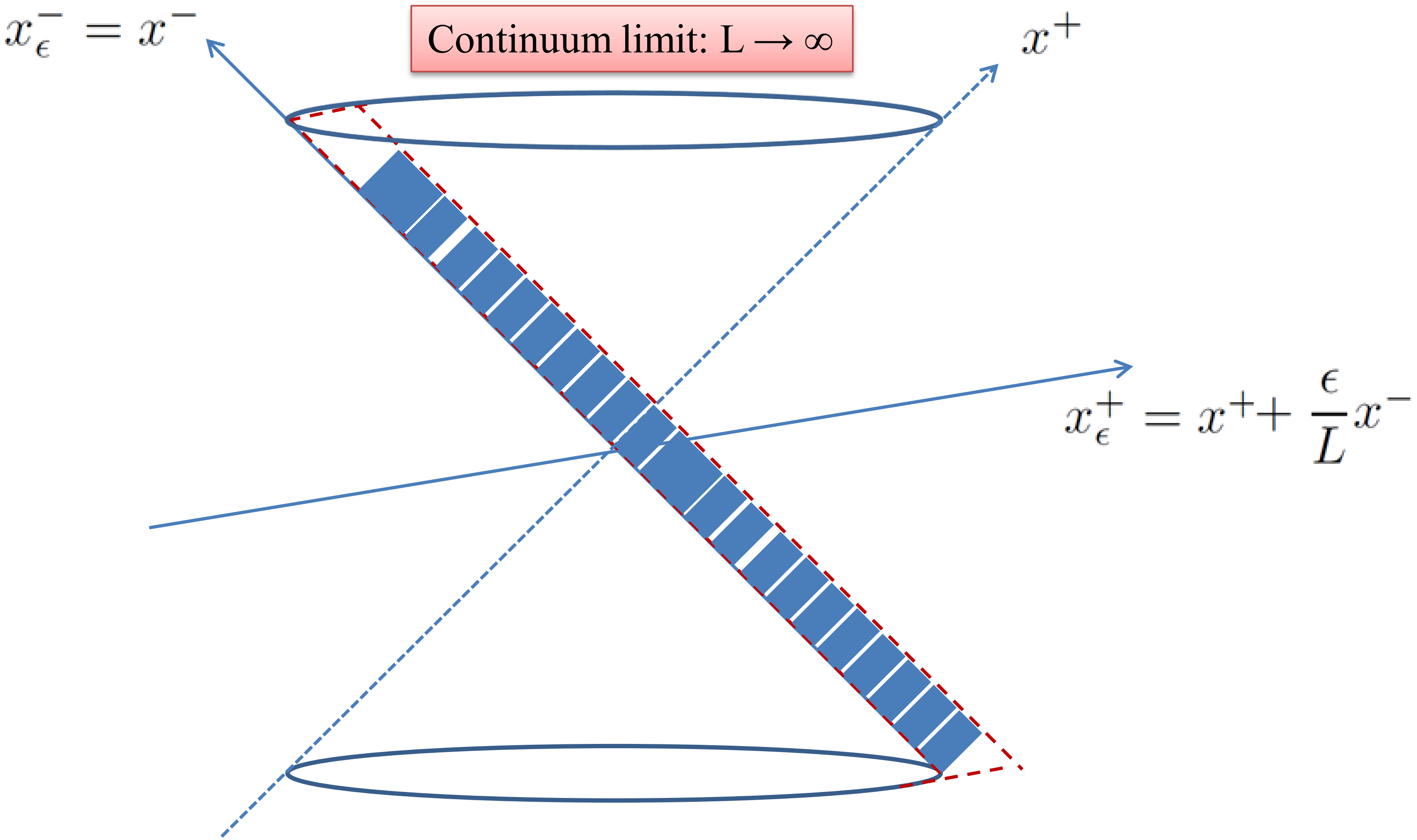

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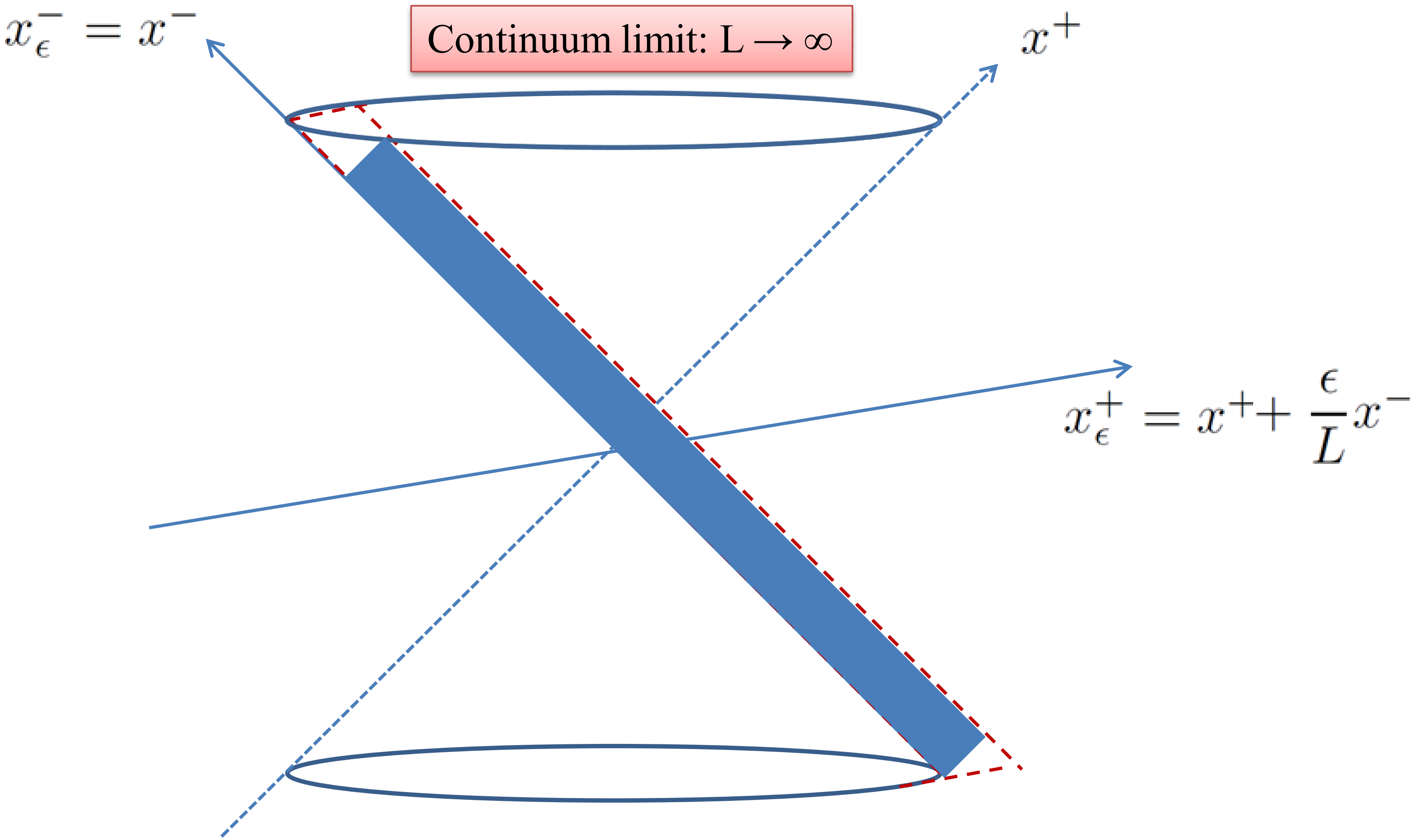

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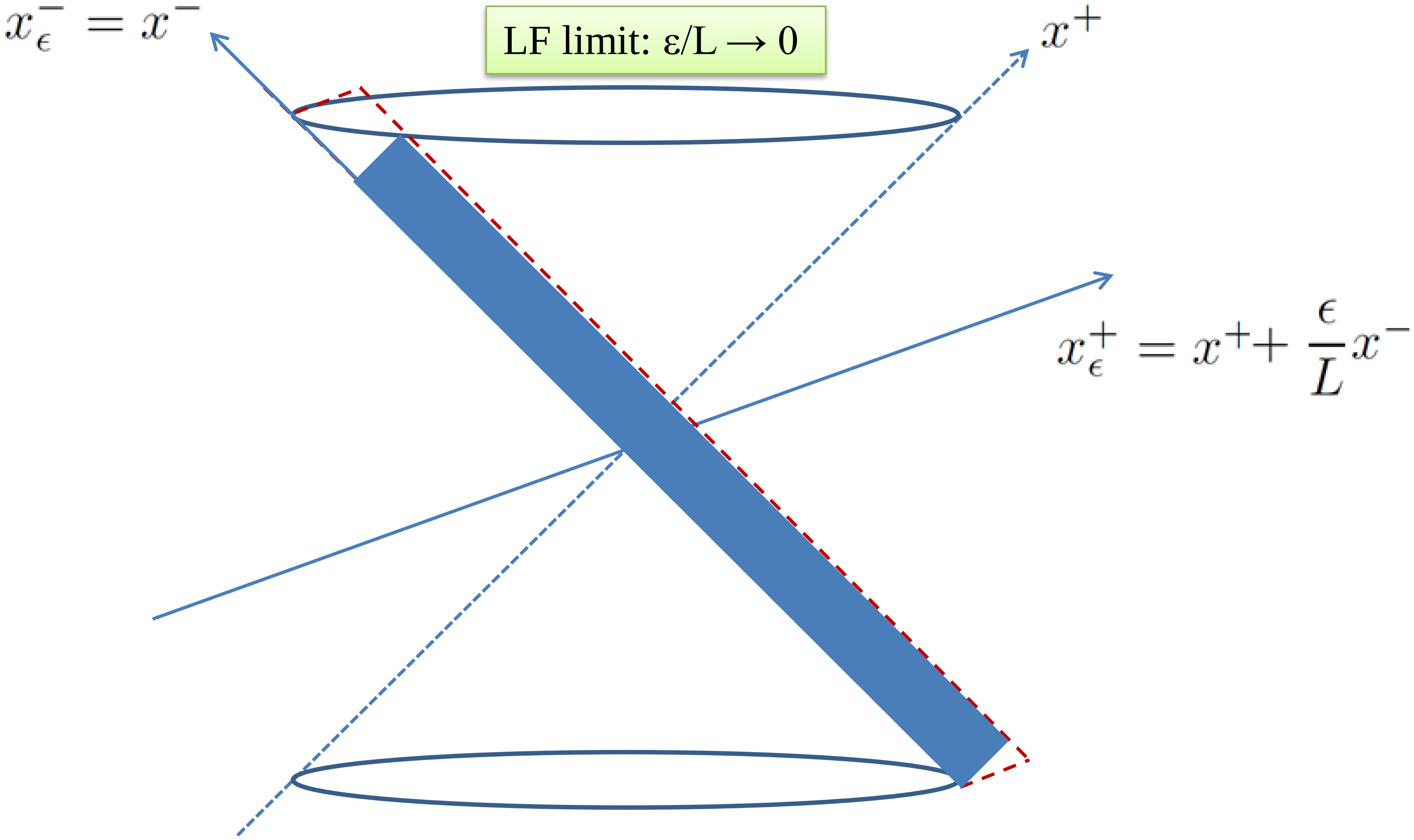

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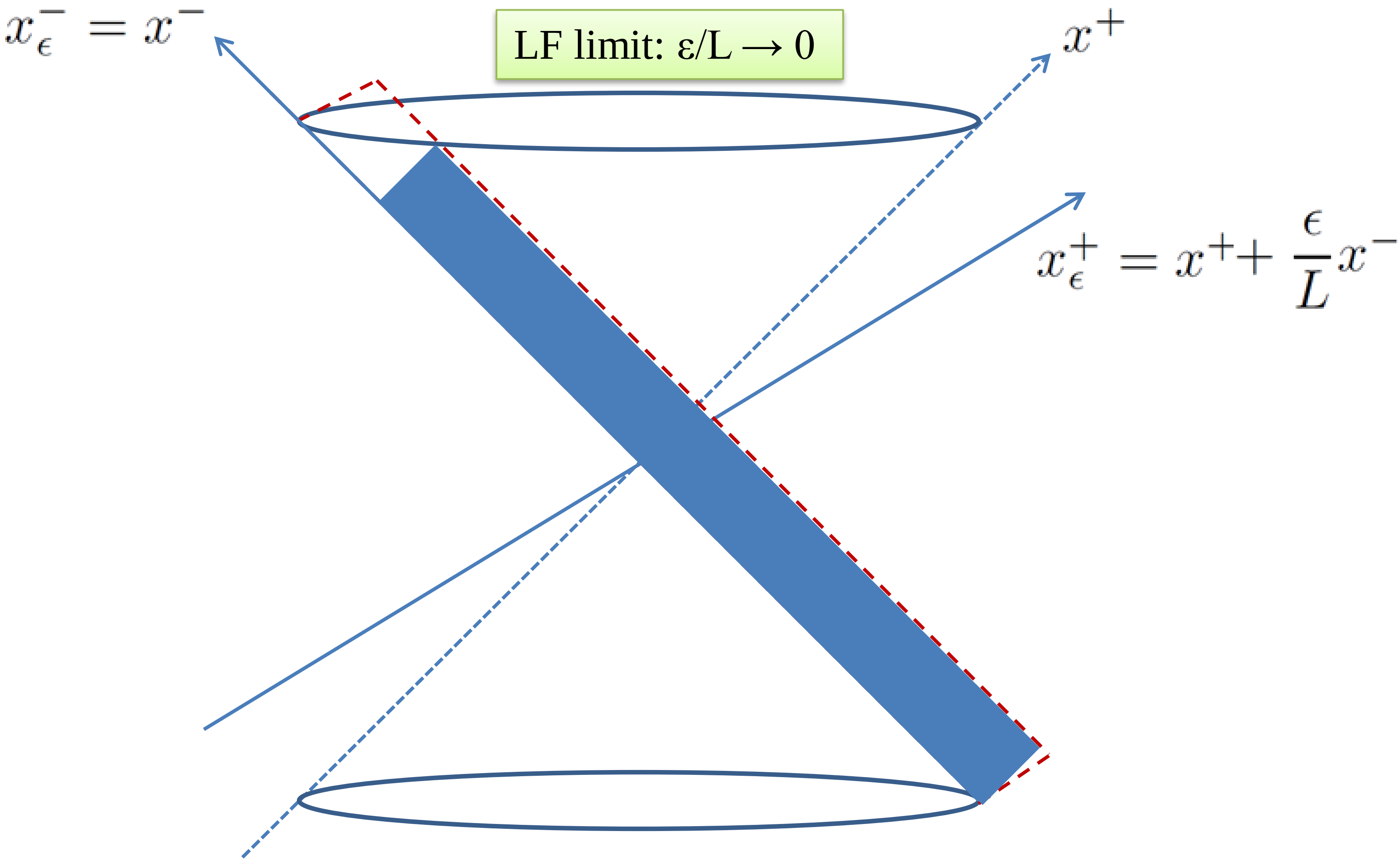

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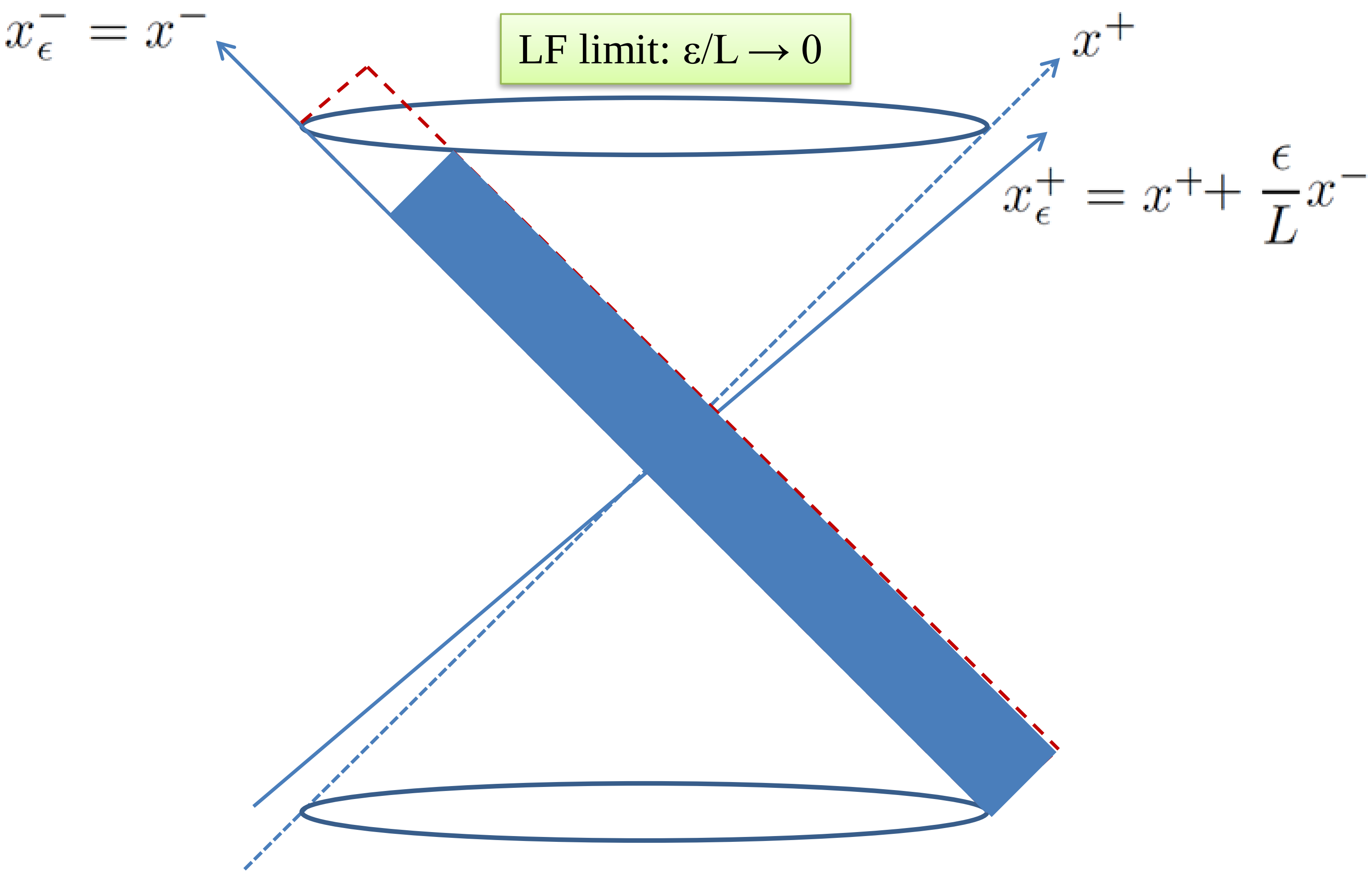

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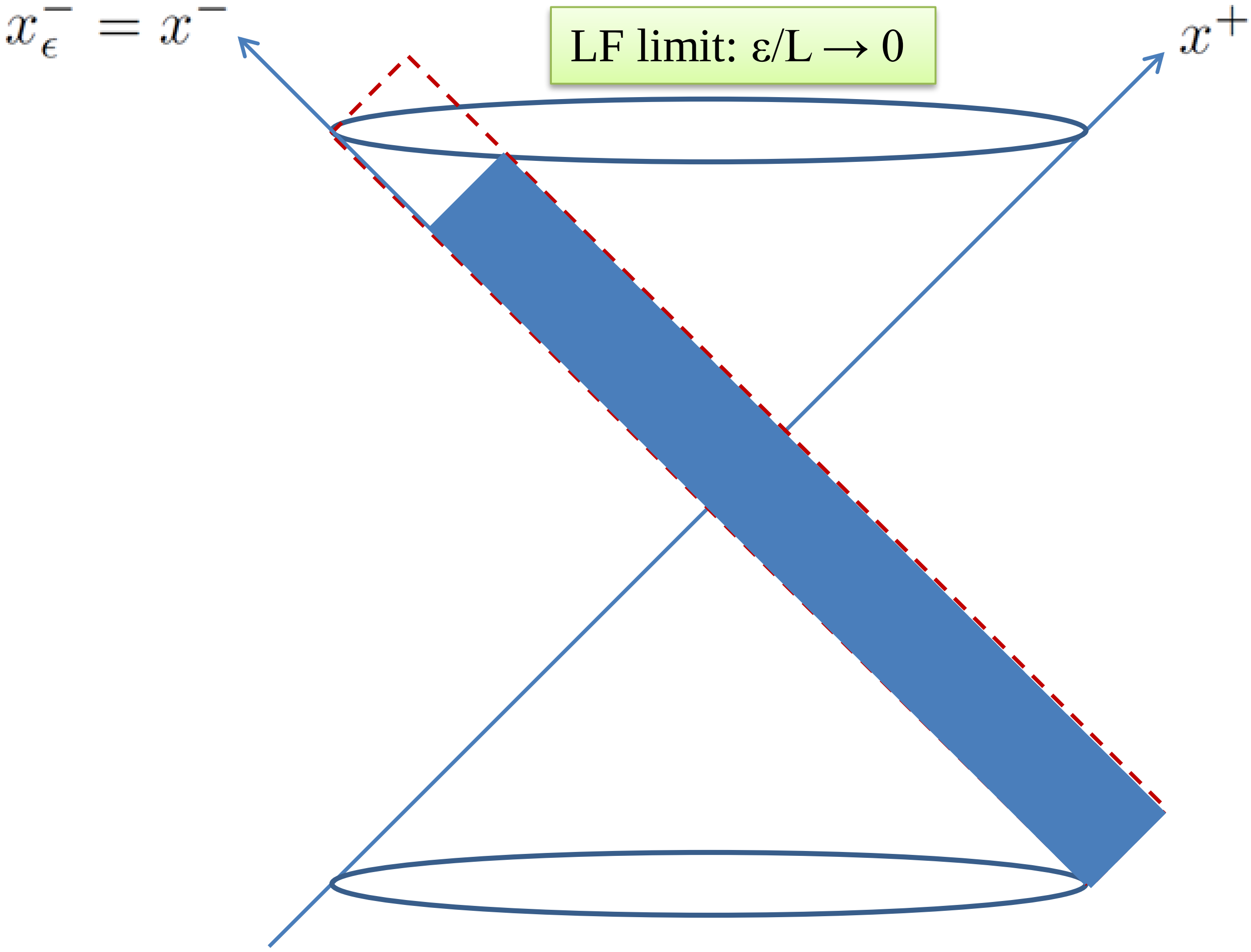

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Hamiltonian formulation of two-dimensional gauge theories on the light cone (1991).

Order of operations and Infinitely many zero modes

Example: Simple tadpole with a mass insertion in 1+1 dimensions.

$$\mathcal{I} = \int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2 - m^2 + i\epsilon)^2} = \frac{i}{4\pi m^2}$$

$$\mathcal{I}_\epsilon = \frac{1}{L} \sum \int \frac{dk^-}{2\pi} \frac{1}{\left(\frac{2\epsilon}{L} k^{-2} + 2k^+k^- - m^2 + i\epsilon\right)^2} = \frac{i}{4\sqrt{2\epsilon L}} \sum \left[\frac{(2\pi n)^2}{2\epsilon L} + m^2 \right]^{-3/2}$$

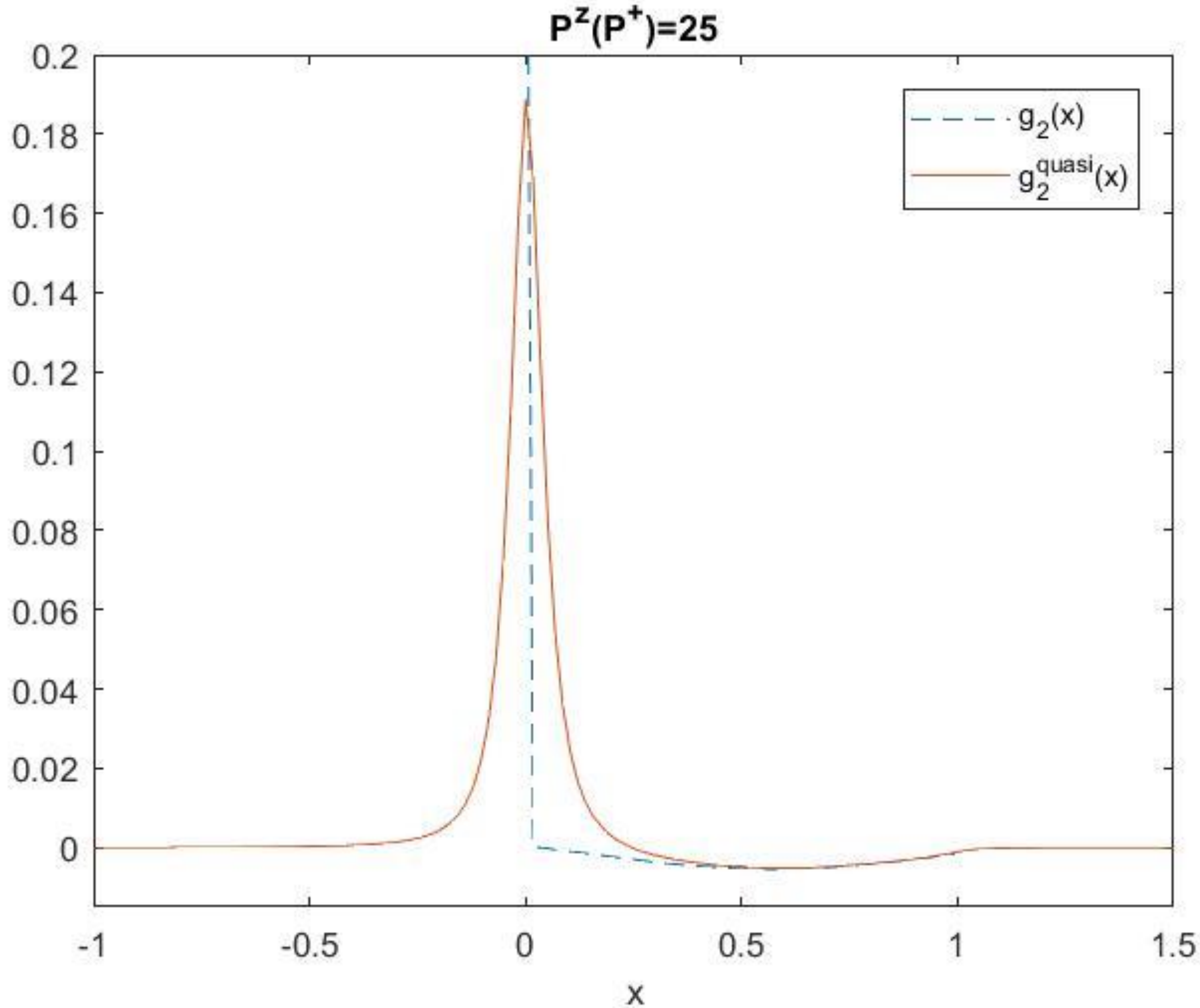
1st operation	2nd operation	Results in
LF limit ($\varepsilon \to 0$, L fixed)	Continuum Limit ($L \to \infty$, ε/L fixed)	Divergent contribution from the zero mode
Continuum Limit ($L \to \infty$, ε/L fixed)	LF limit ($\varepsilon \to 0$, L fixed)	Infinitely many zero modes contribute

Order of operations reveal that there is not only one zero mode but infinitely many.

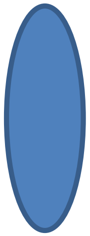
quasi-pdfs shows that the zero modes spread out as we go away from the light-cone.

$P^z = P^+ = 25$

$g_2(x), \ g_2{}^{quasi}(x)$

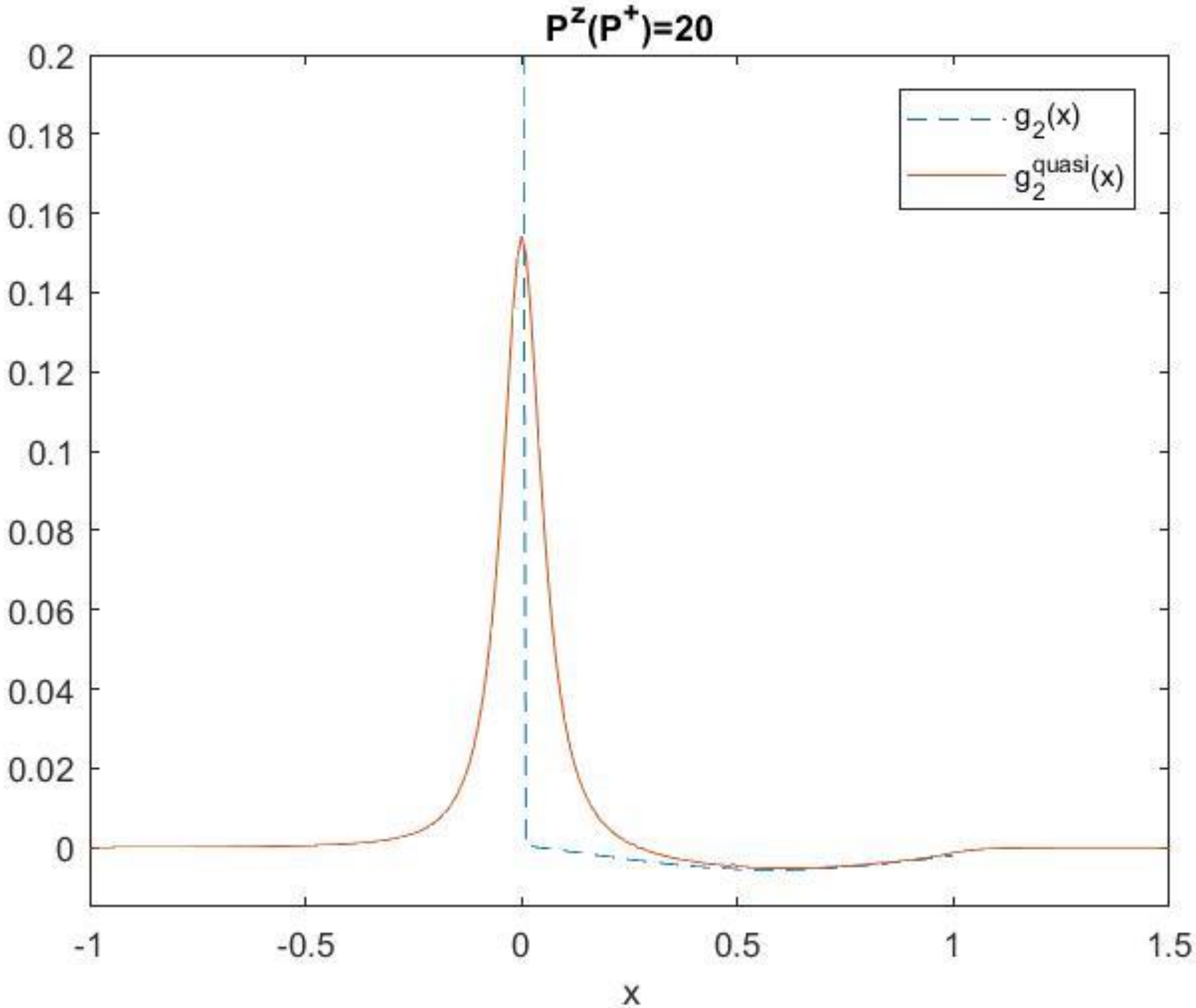


quasi-pdfs shows that the zero modes spread out as we go away from the light-cone.



$P^z = P^+ = 20$

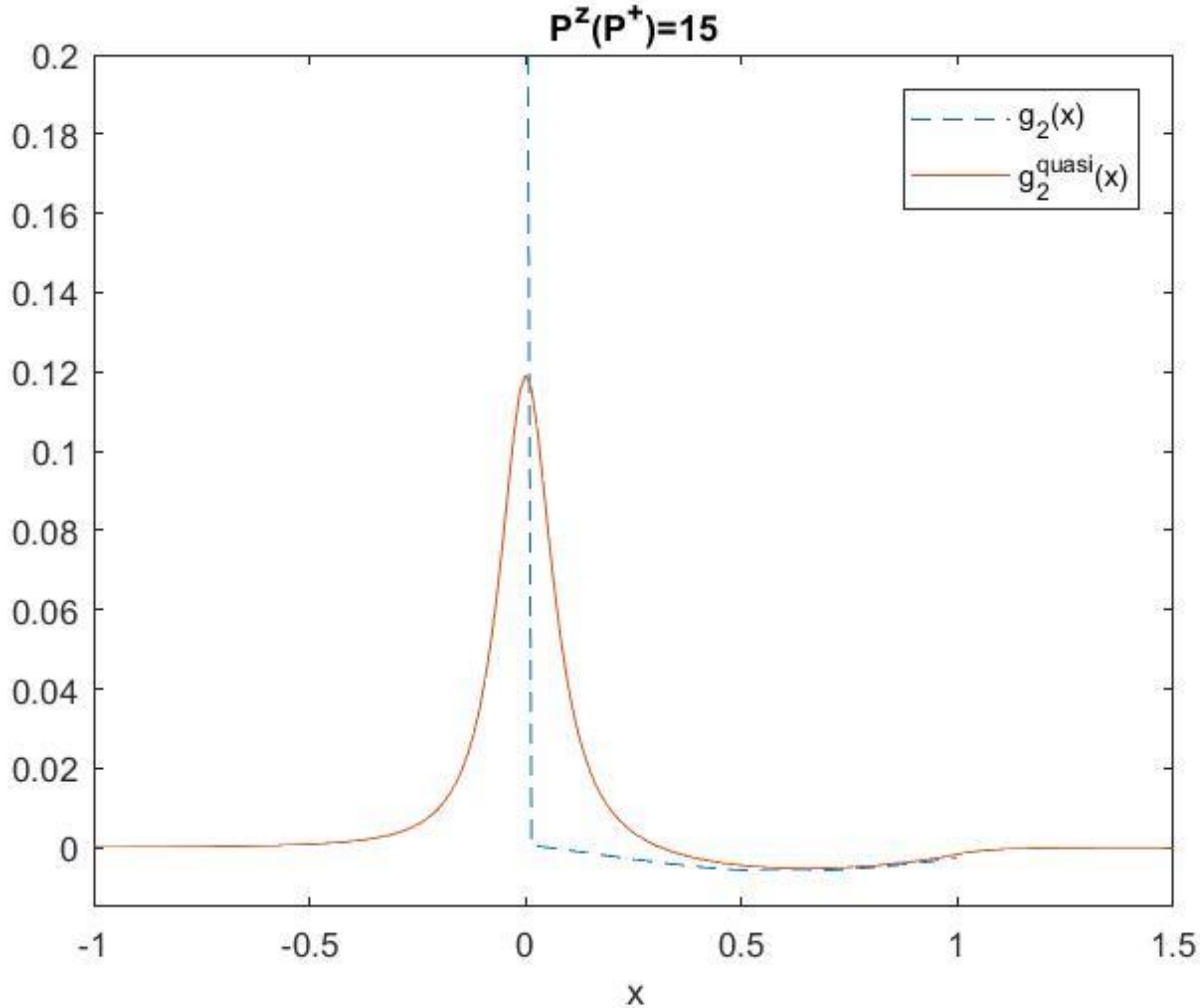
$g_2(\boldsymbol{x}), \; g_2{}^{quasi}(\boldsymbol{x})$



quasi-pdfs shows that the zero modes spread out as we go away from the light-cone.

$P^z = P^+ = 15$

$g_2(x), \ g_2{}^{quasi}(x)$



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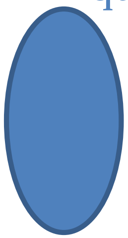
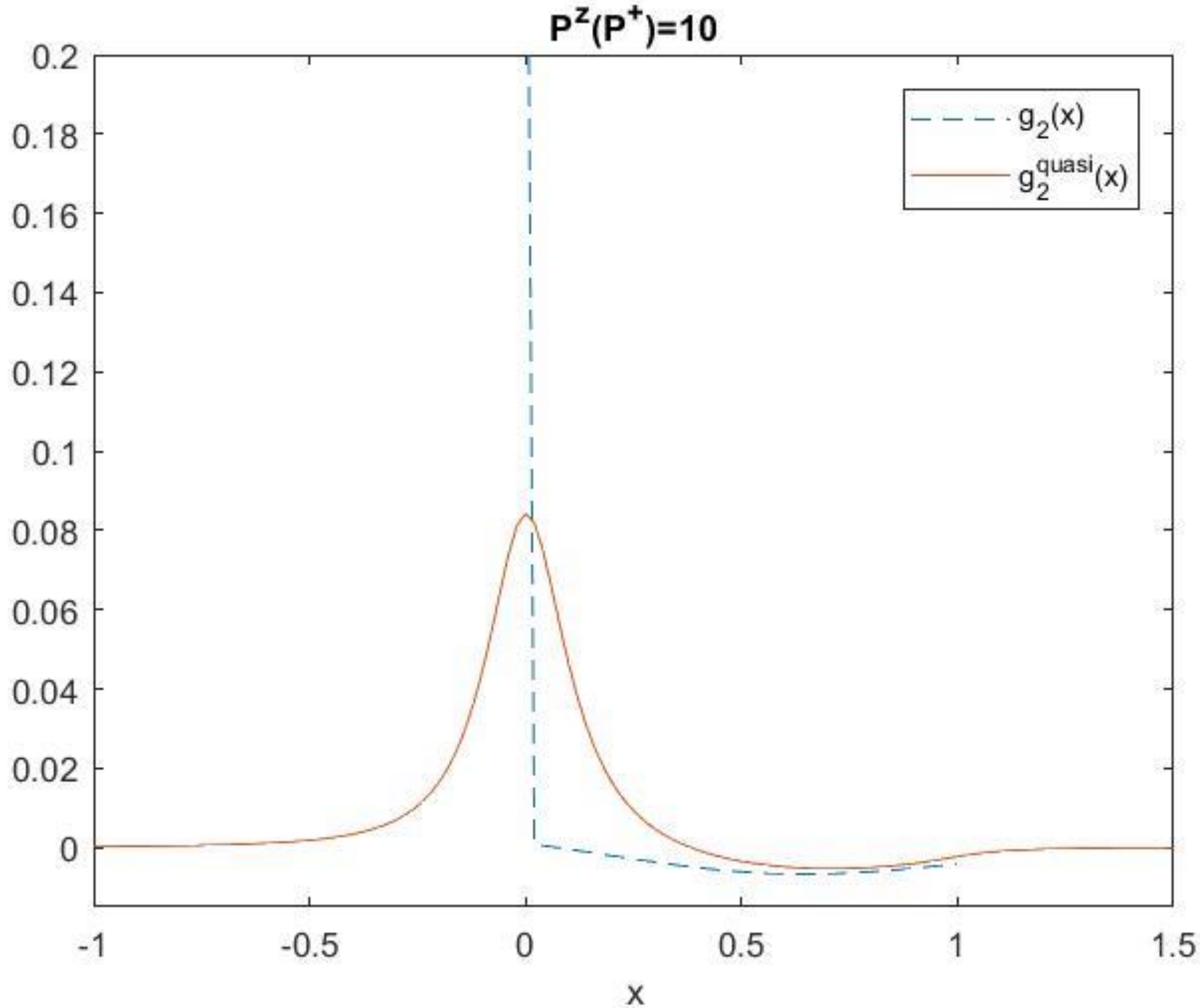
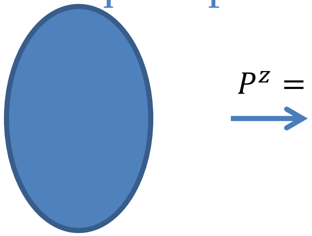


$P^z = P^+ = 10$

$g_2(x), \; g_2{}^{quasi}(x)$

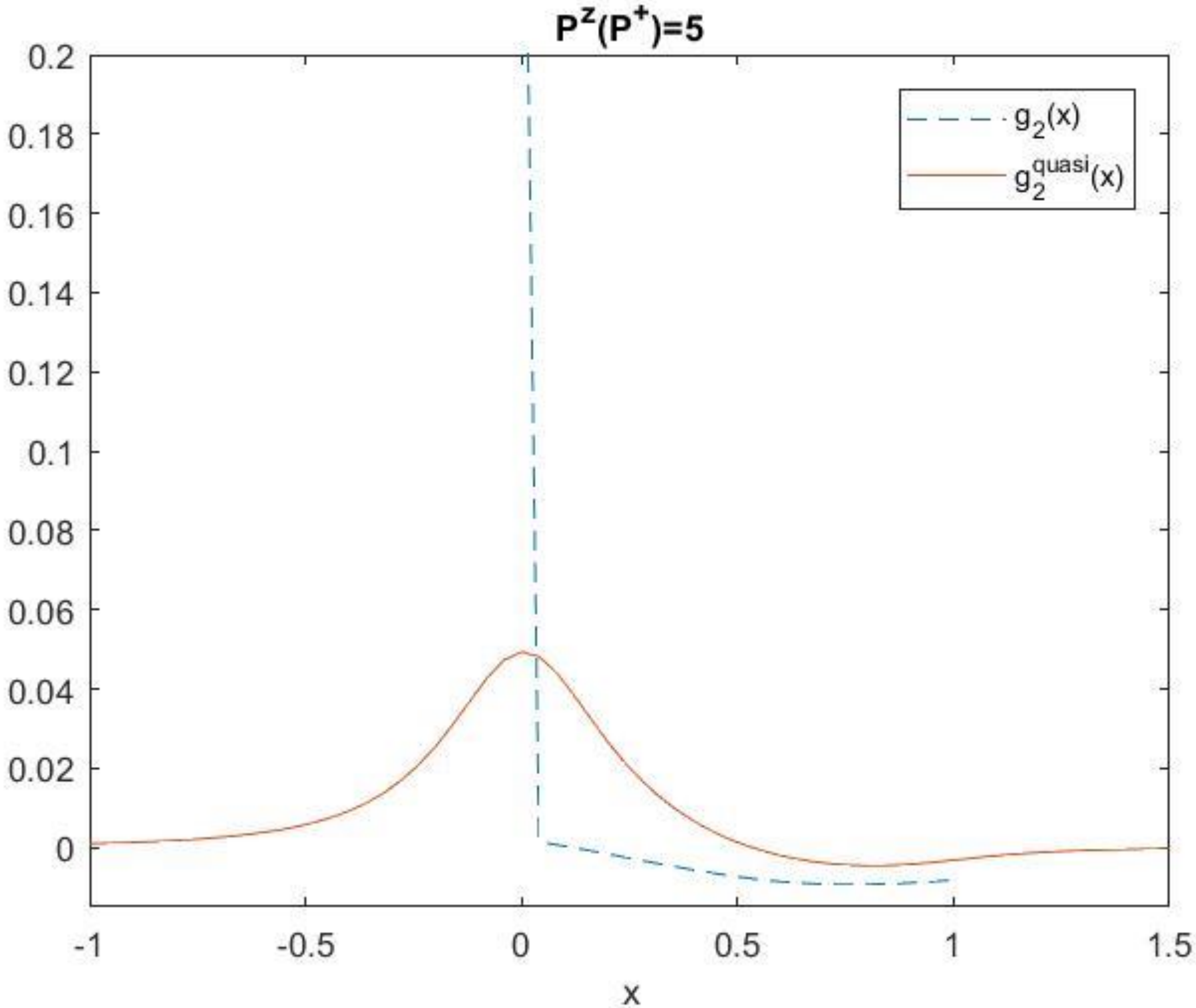


quasi-pdfs shows that the zero modes spread out as we go away from the light-cone.

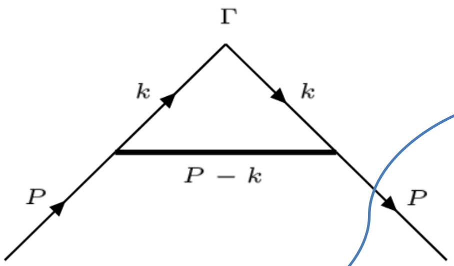


$P^z = P^+ = 5$

$g_2(\boldsymbol{x}),\ \ g_2{}^{quasi}(\boldsymbol{x})$



The discontinuities in GPDs



The parameterization

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle P', S' | \overline{q}(-\frac{z^-}{2})\gamma^j q(\frac{z^-}{2}) | P, S\rangle$$

$$= \frac{1}{2p^+}\overline{u}(P',S')\Big[\frac{\Delta_\perp^j}{2M}G_1 + \gamma^j(H+E+G_2) + \frac{\Delta_\perp^j}{p^+}\gamma^+ G_3 + \frac{i\epsilon_T^{jk}\Delta_\perp^k}{p^+}\gamma^+\gamma_5 G_4\Big]\overline{u}(P,S)$$

The model

$$-\frac{ig^2}{2}\int \frac{d^4k}{(2\pi)^4}\delta(k^+ - xp^+)\overline{u}(P',S')\gamma^\mu \frac{(\not k + \frac{\not\Delta}{2} + m)}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon]}\gamma^\perp \frac{(\not k - \frac{\not\Delta}{2} + m)}{[(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}\gamma^\nu \times \Big[g_{\mu\nu} - \frac{n_\nu(p_\mu - k_\mu)}{p^+ - k^+} - \frac{n_\mu(p_\nu - k_\nu)}{p^+ - k^+}\Big]\frac{1}{[(p-k)^2 - \lambda^2 + i\epsilon]}u(P,S)$$

The divergent part of G_2 is calculated as,

$$-ig^2\int \frac{d^2k_\perp dk^-}{(2\pi)^4}\frac{k^- 8(p^+)^2(1+x)}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon][(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon][(p-k)^2 - \lambda^2 + i\epsilon]}.$$

Using $(P-k)^2 - \lambda^2 = 2(P^+ - k^+)(P^- - k^-) - k_\perp^2 - \lambda^2$, k^- in the numerator can be replaced by the following expression

$$k^- = \frac{M^2}{2p^+} - \frac{[(p-k)^2 - \lambda^2]}{2(p^+ - k^+)} - \frac{(k_\perp^2 + \lambda^2)}{2(p^+ - k^+)}$$

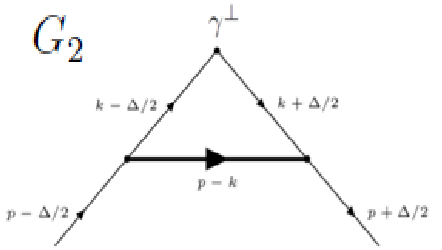
The second term cancels the propagator in the denominator leading to the following contribution which is nonzero only in the ERBL region, $-\xi < x < \xi$.

$$ig^2 4p^+\frac{(1+x)}{(1-x)}\int \frac{d^2k_\perp dk^-}{(2\pi)^4}\frac{1}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon][(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}.$$

What happens in different models ?

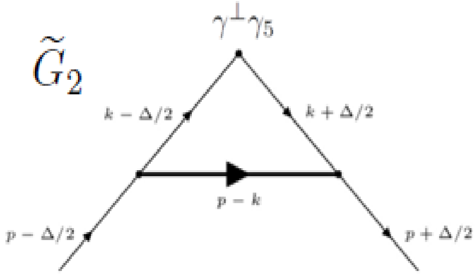
G_2 and $\widetilde{G}_2$ in Scalar Diquark Model

G_2

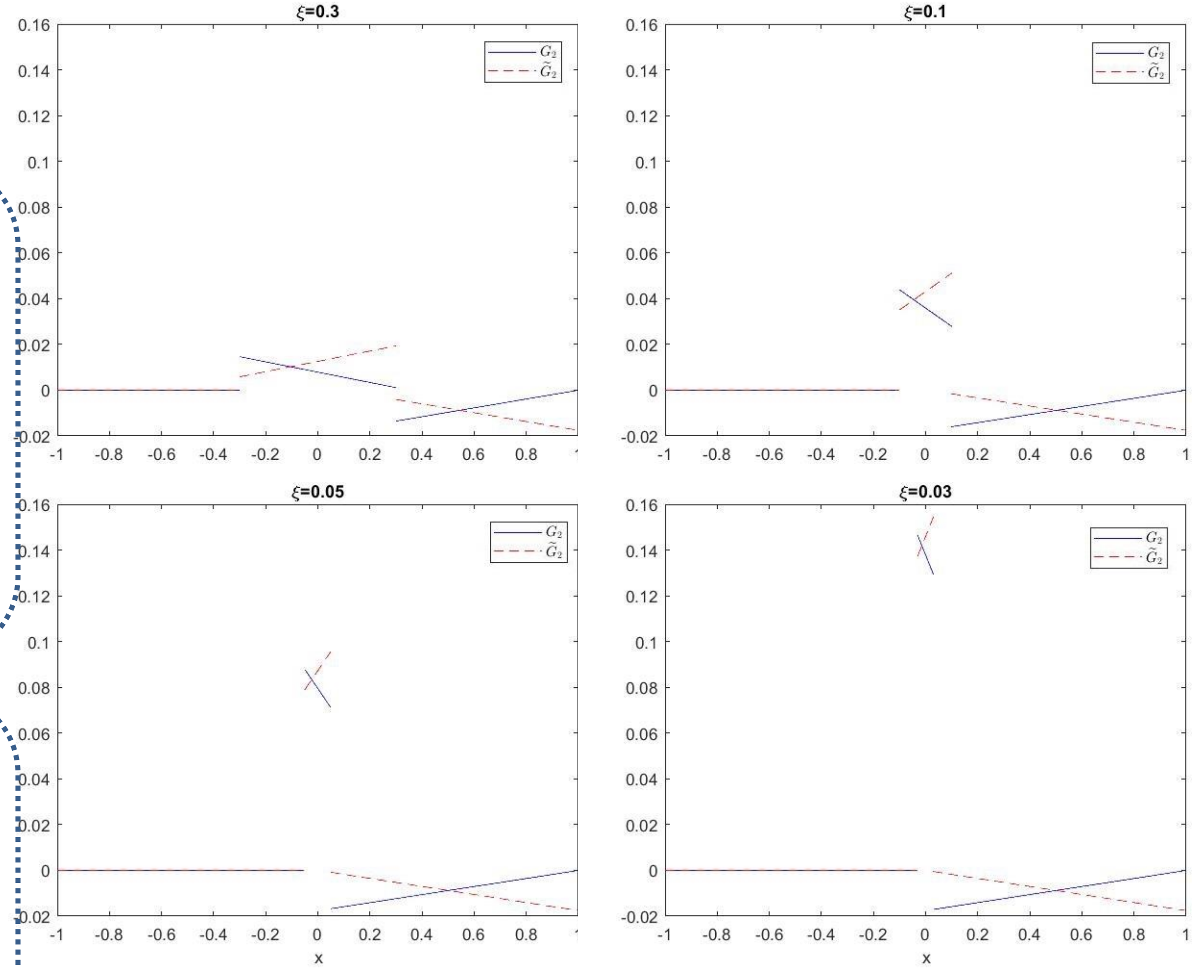


$$G_{2,Div}^{Singular} = \frac{g^2}{8\pi^2}\frac{1}{\xi}\ln\Lambda_{\perp}$$

$\widetilde{G}_2$



$$\widetilde{G}_{2,Div.}^{Singular} = \frac{g^2}{8\pi^2}\frac{x-2\xi^2+1}{\xi(1-x)}\ln\Lambda_{\perp}$$



Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Divergent

The ERBL region becomes a representation of δ(x)

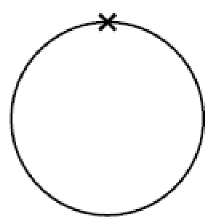
CONCLUSIONS

There are singularities at x=0 in higher twist quark distributions.

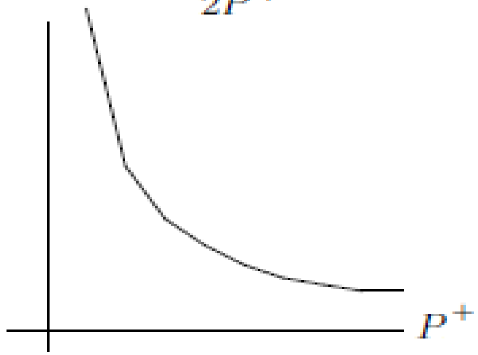
Twist-2 PDF	SDM	QTM	Twist-3 PDF	SDM	QTM
f1	×	×	e	✓	✓
g1	×	×	hL	✓	✓
h1	×	×	gT (g2)	✓	×

These singularities are related to the zero modes (vacuum)

$$\int \frac{dk^-}{(k^2 - m^2 + i\epsilon)^2} = \frac{i\pi}{k_\perp^2 + m^2}\delta(k^+).$$



$$P^- = \frac{m^2 + \vec{P}_\perp^2}{2P^+}$$

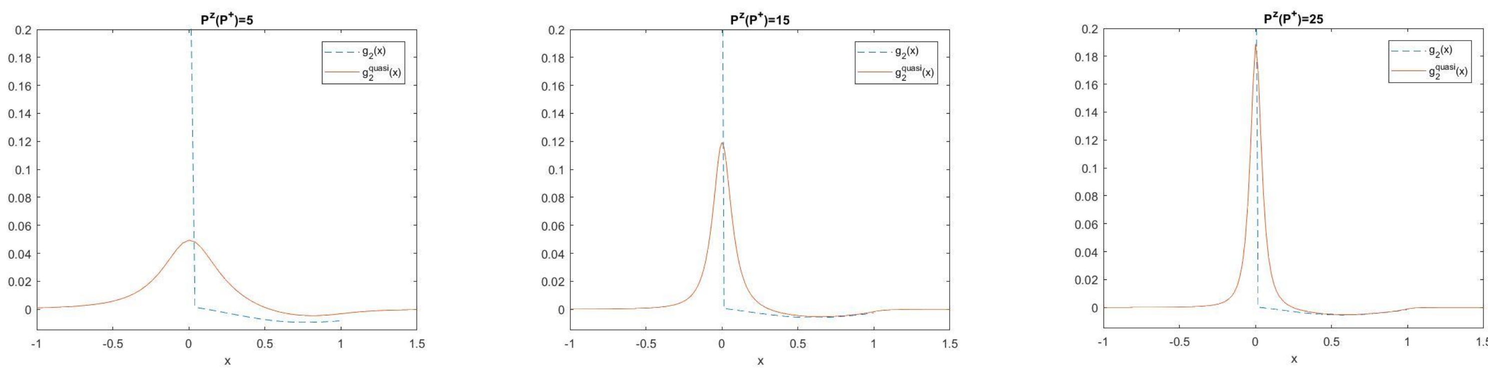


CONCLUSIONS

➢ Using ε-coordinates in LF formalism and the order of taking the continuum and LF limits show that the singularities do not arise from a single zero mode at $k^+=0$ but infinitely many zero modes in the vicinity of $k^+=0$.

1^{st} operation	2^{nd} operation	Results in
Continuum Limit ($L\to\infty$, ε/L fixed)	LF limit ($\varepsilon\to 0$, L fixed)	Infinitely many modes contribute

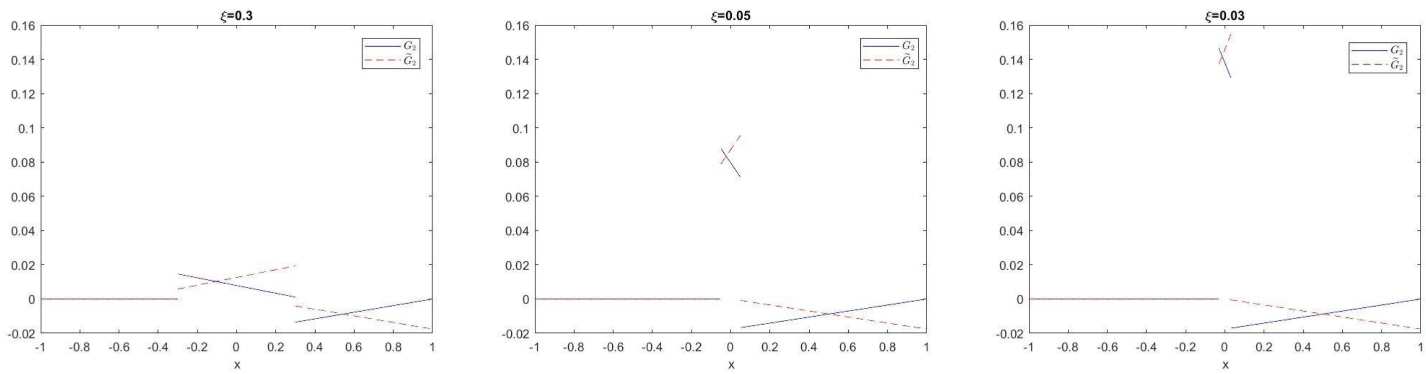
Calculating quasi-pdfs shows that: the zero modes spread out as we go away from the light-cone.



CONCLUSIONS

➢ In the non-forward case, these singularities and the spread of the infinitely zero modes manifest themselves in the ERBL region.

As $\xi \to 0$ the ERBL region becomes a representation of a Delta function.

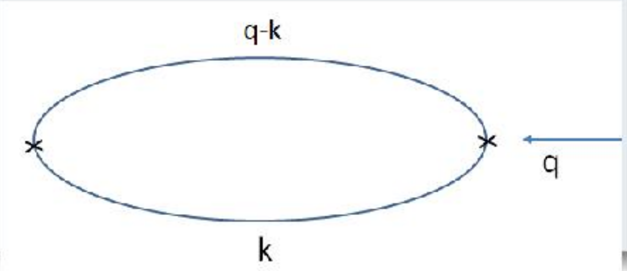


J.Collins, LC workshop 2018

- considered $\int d^2x \langle 0|\phi^2(0)\phi^2(x)|0\rangle e^{iqx}$
- for $q^+ = 0$ same pole structure as generalized tadpoles
- naively vanishes for $q^+ = 0$

$\hookrightarrow$ regulated by taking $q^+ \neq 0$

- support only for $0 < k^+ < q^+$ (k^+ ($q^+ - k^+$) momentum of one of the particles created by $\phi^2|0\rangle$
- $\lim_{q^+\to 0}$ yields finite result
- in terms of k^+, rep. of $\delta(k^+)$



connection of singularities in twist-3 GPDs/PDFs

- pole structure similar to above vacuum corellator
- in GPDs $q^+ \neq 0$, 'regulates' $\delta(x)$ present in PDFs
- rep. of $\delta(x)$ as $q^+ \to 0$