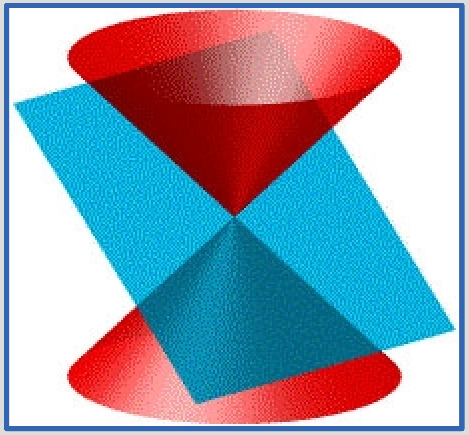
Non-perturbative aspects of light-front quantisation

Peter Lowdon

(Ecole Polytechnique)

Outline

1. Light-front restriction of fields
2. Non-perturbative issues
3. Regularisation resolution
4. Null plane states
5. Summary and outlook



1. Light-front restriction of fields

- A central issue in the construction of light-front quantised field theories is the feasibility of restricting fields to the null plane $\boldsymbol{x^+ = 0}$
- An important finding in this regard was made by Schlieder and Seiler [Schlieder and Seiler, *Commun. Math. Phys.* **25**, 62 (1972)]

$\rightarrow$ Construction of finite-norm states on the null plane requires one to impose additional constraints on the theory!

$$|| \, |\phi; x^+ = 0\rangle \, ||^2 = \int \frac{dp^+ d^2\mathbf{p}_\perp}{(2\pi)^3} \frac{\theta(p^+)}{2p^+} |\widehat{f}(p^+, \mathbf{p}_\perp)|^2$$

Test functions must be restricted!

$$\widetilde{\mathcal{S}}(\mathbb{R}^3) = \left\{ f \in \mathcal{S}(\mathbb{R}^3); \ \widehat{f}(p^+, \mathbf{p}_\perp) = 0, \ \text{for } p^+ = 0 \right\}$$

1. Light-front restriction of fields

- This test function restriction does not occur in instant form

$$|| \, |\phi; x^0 = 0\rangle \, ||^2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2\sqrt{\mathbf{p}^2 + m^2}} |\widehat{f}(\mathbf{p})|^2$$

← ***Finite for any choice of test function, f***

- A manifestation of this issue can already be seen in perturbation theory: vacuum bubble diagrams have more singular properties when working in LF coordinates [Mannheim, PL, Brodsky, 1904.05253]

→ **See P. Mannheim's talk**

- The problem boils down to the fact in LF coordinates the fields no longer satisfy an EOM which is second order in both the spatial *and* temporal coordinates

→ It follows that the fields $\varphi(x^+, x^-, x^T)$ are not continuous in the LF temporal variable x^+

2. Non-perturbative issues

- Does this have any bearing on the non-perturbative characteristics of the theory? **Yes!**

➢ Fields with **different** masses are unitarily equivalent to one another

$\rightarrow$ The space of states are not distinguishable because the test function space is *"too restricted to characterize the local properties of the field operators"* [Schlieder and Seiler, 1972]

- Definition of scattering states becomes more problematic [Suzuki, Tameike, Yamada, *Prog. Theor. Phys.* **55**, 922 (1976)]

$\rightarrow$ Asymptotic ($x^{+} \rightarrow \infty$) fall-off of correlation functions is **less rapid** than in the instant form case

$\rightarrow$ *This undermines the usual arguments for proving the existence of convergent scattering states!*

2. Non-perturbative issues

- Charge operators are defined by: $$Q = \lim_{\substack{d\to 0 \\ R\to\infty}} \int d^4x \; f_{d,R}(x) j^+(x)$$

  $\rightarrow$ Test functions have the form [same as outlined in talk on Monday]:

$$f_{d,R}(x) := \alpha_d(x^+) F_R(x^-, \mathbf{x}_\perp)$$

$$\alpha_d(x^+) \xrightarrow{d\to 0} \delta(x^+), \qquad F_R(x^-, \mathbf{x}_\perp) \xrightarrow{R\to\infty} 1.$$

← ***But this contradicts that test functions vanish at p$^+$=0***

- The Schlieder-Seiler functions are not broad enough to define charge operators in this manner!

  $\rightarrow$ **<u>Important:</u>** construction of regularised charge operators is essential for making sense of **Goldstone's Theorem**

- In light of these physical and technical issues, is there a way of restricting fields/states to a fixed LF time whilst avoiding the necessity of Schlieder-Seiler functions?

3. Regularisation resolution

- A clue for the potential resolution of this problem can be seen in other models that possess similar IR singularities
  - *Massless scalar fields in 1+1 dimensions*
  - *Dipole ghost fields in 3+1 dimensions*
  - *Conformal scalar fields with scaling dimension* $\Delta < 1$
- In each case these singularities can be avoided by extending the correlation functions via the introduction of an IR regulator κ
- Similar procedure is **also** applicable for correlation functions of LF restricted fields [Lorcé, PL, *work in preparation*] $u^{+}(\xi^{-},\boldsymbol{\xi}_{\perp}) := \langle 0|\phi(x)\phi(y)|0\rangle_{\xi^{+}=0}$

$$u^{+}_{\kappa}(\xi^{-},\boldsymbol{\xi}_{\perp}) = -\frac{1}{4\pi}\delta(\boldsymbol{\xi}_{\perp})\ln(-\kappa\xi^{-} + i\epsilon)$$

$\rightarrow$ *derived from the formal expression:* $u^{+}_{\kappa}(\xi^{-},\boldsymbol{\xi}_{\perp}) = \int \frac{d^{2}\mathbf{p}_{\perp}}{(2\pi)^{3}} e^{+i\mathbf{p}_{\perp}\cdot\boldsymbol{\xi}_{\perp}} \int_{0}^{\infty} \frac{dp^{+}}{2p^{+}} \left[e^{ip^{+}(-\xi^{-}+i\epsilon)} - \theta(\kappa - p^{+})\right]$

3. Regularisation resolution

- In momentum space this κ-extension has the form:

$$\widehat{u}^+_\kappa(p^+,\mathbf{p}_\perp) = \frac{1}{2}\left[\text{f.p.}\left(\frac{1}{2|p^+|}\right) + \frac{1}{2p^+ + i\epsilon} - \ln(\kappa)\delta(p^+)\right]$$

which is a *regularisation* of the original correlator $\widehat{u}^+(p^+,\mathbf{p}_\perp) = \frac{\theta(p^+)}{2p^+}$

- $\widehat{u}^+_\kappa(p^+,\mathbf{p}_\perp)$ can now be unambiguously integrated with **any** test function $\rightarrow$ *no restriction is required*

- However, unlike the original correlator the κ-extension **violates positivity** due to the second term!

$\rightarrow$ It turns out that positivity violation is a necessary condition for **any** such extension [Schlieder and Seiler, 1972]

Existence of zero or negative norm states is an inevitable consequence of imposing **"light-front restrictability"** of fields

3. Regularisation resolution

- But what effect does this extension have on the dynamical properties of the theory?

Commutator

$$\begin{aligned}\langle 0|\left[\phi_\kappa(x),\phi_\kappa(y)\right]|0\rangle_{x^+=y^+=0} &= u_\kappa^+(\xi^-,\boldsymbol{\xi}_\perp) - u_\kappa^+(-\xi^-,-\boldsymbol{\xi}_\perp) \\ &= -\frac{1}{4\pi}\delta(\boldsymbol{\xi}_\perp)\ln(-\kappa\xi^- + i\epsilon) + \frac{1}{4\pi}\delta(\boldsymbol{\xi}_\perp)\ln(\kappa\xi^- + i\epsilon) \\ &= -\frac{1}{4\pi}\delta(\boldsymbol{\xi}_\perp)\left[\ln|\kappa\xi^-| + i\pi\theta(\kappa\xi^-) - \ln|\kappa\xi^-| - i\pi\theta(-\kappa\xi^-)\right] \\ &= -\frac{i}{4}\delta(\boldsymbol{\xi}_\perp)\epsilon(\xi^-)\end{aligned}$$

$\rightarrow$ *Coincides with the standard LF commutator expression*

LF evolved correlator

$$\langle 0|\phi_\kappa(x)\phi_\kappa(y)|0\rangle = \widetilde{D} * u_\kappa^+ + D * \widetilde{u}_\kappa^+$$

$$D(\xi) = \int \frac{d^4p}{(2\pi)^3} i\epsilon(p^+)\delta(p^2-m^2)e^{-ip\cdot\xi}$$

$\rightarrow$ *κ-dependence drops out after evolution in LF time*

3. Regularisation resolution

States & observables

- Just like in gauge theories, one can define a subsidiary condition to pick out the physical degrees of freedom

$$\phi_\kappa^-(f_0)\mathcal{V}_{\text{phys}} = 0$$

Analogous to the Gupta-Bleuler condition in QED!

$$\partial^\mu A_\mu^{(+)}|\text{phys}\rangle = 0$$

... and a corresponding "charge" operator $Q = i\pi\left[\phi_\kappa^+(f_0) - \phi_\kappa^-(f_0)\right]$

$\rightarrow$ f_0 generates the zero-norm states in the theory

- These physical states are guaranteed to have **positive** norm
- Turns out that the physical states are precisely those smeared with *Schlieder-Seiler* functions $\rightarrow$ "neutral" states wrt this charge
- Like massless 1+1 case [Morchio, Pierotti, Strocchi, *J. Math. Phys.* **31**, 1467 (1990)]

3. Regularisation resolution

Implications of LF-restrictable fields

$\rightarrow$ One can now explore the consequence of these various features

- Consider the scalar TMD defined in the following manner:

$$f_\kappa(k^+, \mathbf{k}_\perp) = \int \frac{d^4w}{(2\pi)^4} e^{-iw\cdot k}\, k^+ \delta(w^+) \langle P|\phi_\kappa^\dagger(w)\phi_\kappa(0)|P\rangle$$

$\rightarrow$ Correlator is evaluated at equal LF time: use κ-extended fields

- From the previous discussion the TMD could potentially violate positivity... but can prove that it doesn't! **What about QCD?**

- Light-front wavefunctions must be Schieder-Seiler functions!

$$|\phi\rangle = \int dp^+ d^2\mathbf{p}_\perp\, \widehat{\phi}_\kappa(p^+, \mathbf{p}_\perp)|0\rangle\, \psi(p^+, \mathbf{p}_\perp)$$

4. Null plane states

- Whenever LF quantised theories are discussed the states are usually restricted to live on $\boldsymbol{x^+ = 0}$
- It is often argued that these are sufficient for characterising *all* states in the theory – this can be traced back to [Leutwyler, Klauder, Streit, *Nuovo Cimento A*, **66**, 536 (1970)]

  $\rightarrow$ *In light of the previous results this seems rather surprising!*

- On closer inspection it turns out that this paper actually proves:

Theorem. *The operator algebra of a scalar field $\phi(x)$ on $\mathcal{O}_{\tau,\varepsilon} = \{x \in \mathbb{R}^{1,3} : |x^+ - \tau| < \varepsilon\}$ is irreducible ($\forall \varepsilon > 0$)*

"Time-slice axiom"

$\rightarrow$ all states can be defined on the light-like **slab**

- This is **not** the same as the null plane: would require taking $\varepsilon \rightarrow 0$

  $\rightarrow$ Still interesting, but requires the absence of zero modes in order to hold...

4. Summary and outlook

- By asserting that fields are meaningful when restricted to a specific LF time this introduces non-perturbative subtleties
- These subtleties can be overcome by introducing an infrared extension $\rightarrow$ but this comes at a price: *negative norm states*
- The physical states can be dealt with in an analogous manner to gauge theories
- This extension imposes constraints on LF-restricted observables: TMDs, GPDs, light-front wavefunctions, ...
- Does one really need strict LF-restrictability? Perhaps light-like slab is enough?

There are still **<u>many</u>** important open questions in LF quantisation!