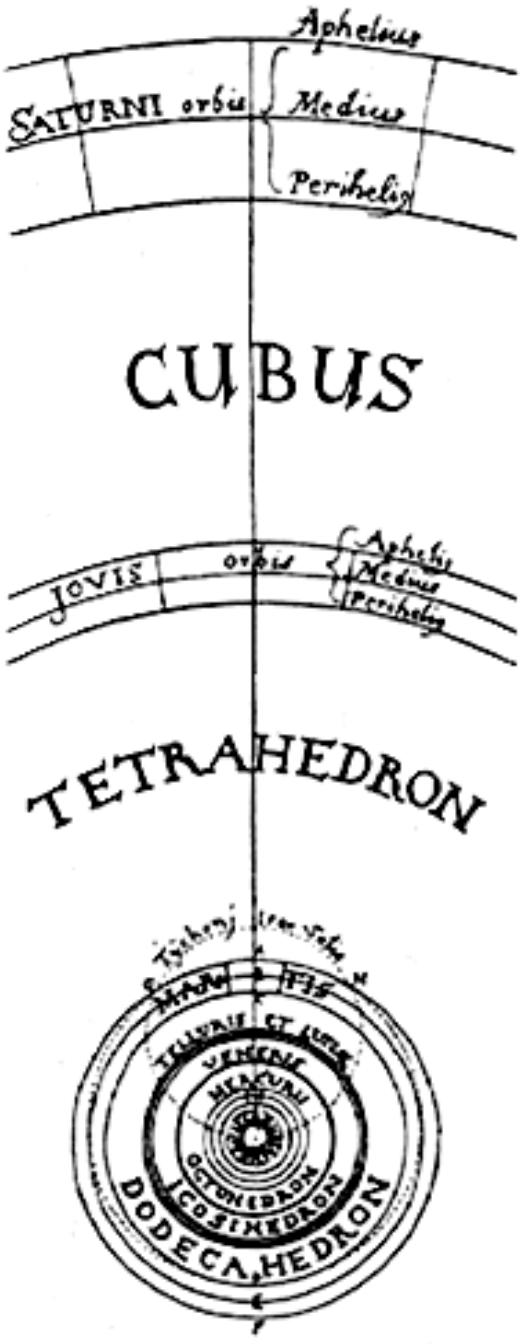
Theoretical Frameworks for Neutrino Masses



[Kepler 1596 Mysterium Cosmographicum]

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	Fogli [NoVe 2008] [0806.2649]	Schwetz et al. [0808.2016]
$\sin^2\vartheta_{12}$	$0.326^{+0.05}_{-0.04}$ $[2\sigma]$	$0.304^{+0.022}_{-0.016}$
$\sin^2\vartheta_{23}$	$0.45^{+0.16}_{-0.09}$ $[2\sigma]$	$0.50^{+0.07}_{-0.06}$
$\sin^2\vartheta_{13}$	0.016 ± 0.010	$0.01^{+0.016}_{-0.011}$
$\Delta m^2_{21}\ (eV^2)$	$(7.66 \pm 0.35)\times 10^{-5}$ $[2\sigma]$	$(7.65^{+0.23}_{-0.20})\times 10^{-5}$
$\lvert\Delta m^2_{31}\ (eV^2)\rvert$	$(2.38 \pm 0.27)\times 10^{-3}$ $[2\sigma]$	$(2.40^{+0.12}_{-0.11})\times 10^{-3}$

$$\vartheta_{12} = \left(34.8^{+3.0}_{-2.5}\right)^0 \quad [2\sigma] \qquad \vartheta_{12} = \left(33.5^{+1.4}_{-1.0}\right)^0$$

$$\vartheta_{23} = \left(42.1^{+9.2}_{-5.3}\right)^0 \quad [2\sigma] \qquad \vartheta_{23} = \left(45.0^{+4.0}_{-3.4}\right)^0$$

Tri-Bimaximal mixing

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

[Harrison, Perkins and Scott]

two opposite interpretations

- mixing angles and mass ratios are O(1)
- there is no hierarchy to explain
- smallness of ϑ_{13} and $\Delta m^2_{21}/\Delta m^2_{31}$ accidental
- no special pattern behind data

[Hall, Murayama, Weiner 1999]

- lepton mixing angles are special and reflect some property of the fundamental theory [this talk]

equally possible at the moment. Experimental errors are still large
some features persistent in the data: all experiments favor ϑ_{23} maximal
[best value of ϑ_{23} is maximal, though sizeable deviations still allowed]

Consider the indication of ϑ_{23} maximal seriously

ϑ_{23} is maximal is not an infrared stable fixed point of RGE
[ϑ_{23} maximal at low energy starting from a small high-energy value
requires either fine-tuned initial conditions or ad hoc threshold effects]

ϑ_{23} maximal cannot arise from an exact symmetry of the whole theory
[if $m_e=m_\mu=0$ in the limit of exact symmetry]

we are left with

ϑ_{23} is maximal by accident

ϑ_{23} is maximal by a broken symmetry

[He, Keum, Volkas 0601001
Lam 0708.3665 + 0804.2622]

$$\begin{array}{ccc} & G_f & \\ \swarrow & & \searrow \\ G_T & & G_S \\ \swarrow & & \searrow \\ (m_e^+ m_e) \text{ diagonal} & & U_{PMNS}^T\, m_\nu\, U_{PMNS} = (m_\nu)_{diag} \end{array}$$

charged lepton sector

neutrino sector

ϑ_{23} maximal from a misalignment between G_T and G_S

if the breaking is spontaneous, induced by $\langle\varphi_T\rangle, \langle\varphi_S\rangle, \ldots$ a special vacuum alignment is needed

Majorana neutrinos $\Rightarrow$ G_S discrete

the most general group leaving $\nu^T m_\nu \nu$ invariant, if ϑ_{ij} do not depend on m_i

$$Z_2 \times Z_2 \times Z_2$$

[go to the basis where m_ν is diagonal: neutrinos can only change by a sign]

Example: assume $m_e^+ m_e$ diagonal and take

Z_2 generated by [μ-τ exchange]

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$m_\nu = \begin{pmatrix} x & y & y \\ y & w & z \\ y & z & w \end{pmatrix} \Leftrightarrow \begin{aligned} \vartheta_{13} &= 0 \\ \vartheta_{23} &= \frac{\pi}{4} \end{aligned}$$

G_T can be continuous but the simplest choice is G_f discrete

$G_{T,S}$ may also arise in part as accidental symmetries like B and L in the Standard Model

Example: $G_f = A_4$ generated by T and S [U accidental symmetry, $[S,U]=0$ and $S^2=1$]

[Ma and Rajasekaran 2001, Ma 2002, Babu, Ma and Valle 2003, …]

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{i\frac{2\pi}{3}}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$T^+ (m_e^+ m_e)\, T = (m_e^+ m_e)$$

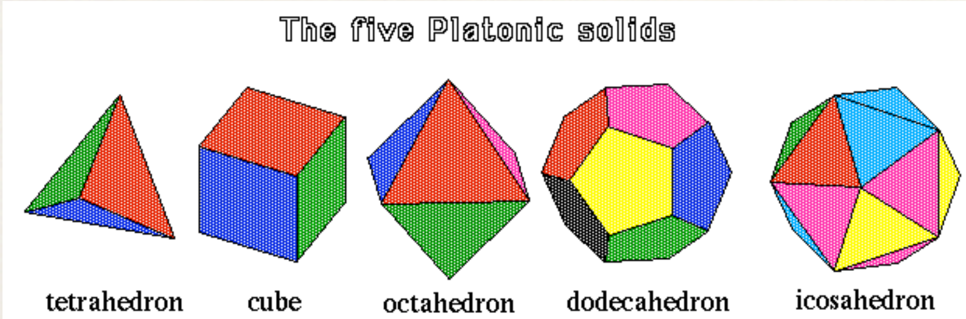
$\Leftrightarrow$ $(m_e^+ m_e)$ diagonal

S and U invariance of m_ν

$\Leftrightarrow$ $U_{TB}^T\, m_\nu\, U_{TB} = (m_\nu)_{diag}$

An intriguing sequence of discrete groups

the (proper) symmetry groups of
the Platonic solids



duality		group	order	n
tetrahedron	tetrahedron	A_4	12	3
cube	octahedron	S_4	24	4
dodecahedron	icosahedron	A_5	60	5

they are all generated by two elements: S and T

$$S^2 = (ST)^3 = 1 \qquad T^n = 1$$

[a longer sequence? The (infinite, discrete) modular group Γ is also generated by S and T satisfying $S^2=(ST)^3=1$ and possesses an infinite serie of finite subgroups Γ/Γ_n (Γ_n being the principal congruence subgroup of level n). For n=3,4,5 we recover the symmetry groups of the Platonic solids]

irreducible representations

A_4	1, 1′, 1″, 3
S_4	1, 1′, 2, 3, 3′
A_5	1, 3, 3′, 4, 5

they all have 3-dimensional representations where the left-handed lepton doublets can be accommodated

models based on these groups have been constructed
U [μ-τ exchange] arise as an accidental symmetry
and guarantees $\vartheta_{23}=45^0$ and $\vartheta_{13}=0$ at the LO

[for a review, see: G. Altarelli and F.F arXiv:1002.0211]

$$\vartheta_{13}=0$$

$$\vartheta_{23}=\frac{\pi}{4}$$

spontaneous breaking of G_f down to G_T (charged leptons) and G_S (neutrinos) leads to

G_f	$\tan\vartheta_{12}$	ϑ_{12}	u
A_4	$1/\sqrt{2}$ [TB]	35.26^0	≈ 0.01
S_4	1 [BM]	45^0	≈ 0.1
A_5	$1/\phi$ [golden ratio]	31.72^0	≈ 0.01

[Everett,Stuart 2008] $\phi\equiv\frac{1+\sqrt{5}}{2}$

these are LO predictions and corrections of order

$$u=\frac{\langle\varphi\rangle}{\Lambda}$$

$\langle\varphi\rangle$ → G_f - breaking VEV

Λ → cutoff

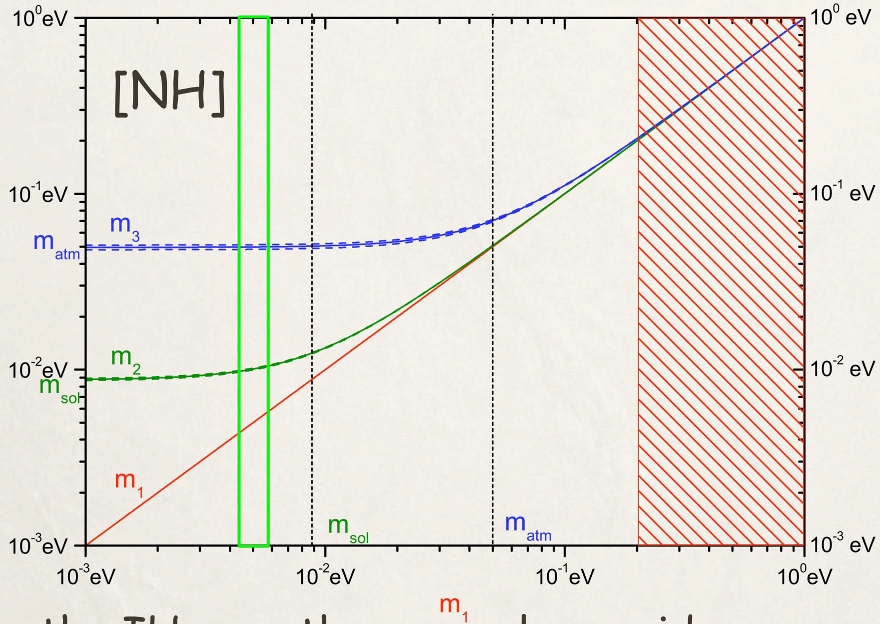
are expected. Then ϑ_{13} becomes of O(u)

An example based on $G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SUSY + SEE-SAW]

lepton mixing is TB, by construction, plus NLO corrections of order $0.005 < u < 0.05$
at the LO neutrino mass spectrum depends on two complex parameters
there is a sum rule among (complex) mass eigenvalues $m_{1,2,3}$

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

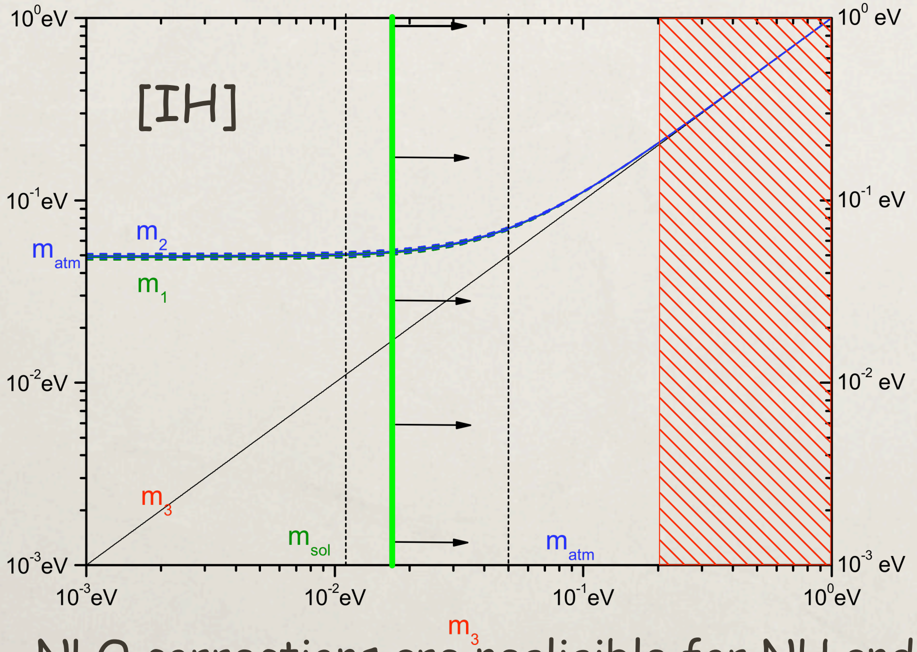
both normal [NH] and inverted [IH] hierarchy are allowed



in the NH case the sum rule
completely determines the spectrum

$$m_1 \approx 0.005\ eV \quad m_2 \approx 0.01\ eV \quad m_3 \approx 0.05\ eV$$

$$|m_{ee}| \approx 0.007\ eV$$



in the IH case the sum rule provides
a lower bound on m_3

$$m_3 \geq 0.017\ eV$$

$$|m_{ee}| \geq 0.017\ eV$$

NLO corrections are negligible for NH and for IH close to the lower bound

Additional tests: LFV from 1-loop SUSY particle exchange

under certain assumptions concerning the SUSY soft breaking terms

$$\frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} = \frac{6 m_W^4 \alpha_{em}}{\pi m_{SUSY}^4} \left[\left| w_{ij}^{(1)} u^2 \right|^2 + \frac{m_j^2}{m_i^2} \left| w_{ij}^{(2)} u \right|^2 \right]$$

$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters

$$BR(\mu \to e\gamma) \approx BR(\tau \to \mu\gamma) \approx BR(\tau \to e\gamma)$$

[up to O(1) coefficients]

independently from $u \approx \vartheta_{13}$

present (expected) sensitivity to m_{SUSY}

Assuming $w^{(1,2)}_{ij} = 1$

BR(μ->eγ) < 1.2×10^{-11} (10^{-13})	
m_{SUSY} > 255 (820) GeV	u=0.005
m_{SUSY} > 0.7 (2.5) TeV	u=0.05

BR(μ->eee) < 10^{-12} (10^{-13})	
m_{SUSY} > 140 (225) GeV	u=0.005
m_{SUSY} > 400 (700) GeV	u=0.05

[F.F. and A. Paris 1005.5526]

CR^{Ti}(μ->e) < (10^{-18})	
m_{SUSY} > (2.3) TeV	u=0.005
m_{SUSY} > (6.6) TeV	u=0.05

m_{susy} in the region of interest for LHC

Leptogenesis

if ν^c_i transform in a 3-dim irreducible representation of G_f then $\epsilon_i=0$ in the exact symmetry limit u=0.

⇒ $\epsilon_i = 0$ at the LO

$\epsilon_i \neq 0$ from the NLO corrections

$$\varepsilon_i \approx \frac{u^2}{16\pi} \qquad [\mathrm{NH}]$$

$$\varepsilon_i \approx \frac{u^2}{16\pi r} \qquad [\mathrm{IH}] \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{30}$$

$\epsilon_i \geq 10^{-6}$ to produce an acceptable baryon asymmetry

$$u \geq \begin{cases} 0.01 & [\mathrm{NH}] \\ 0.002 & [\mathrm{IH}] \end{cases}$$

in agreement with expected range of u

[Jenkins, Manohar 0807.4176
Bertuzzo, Di Bari, FF, Nardi 0908.0161
Hagedorn, Molinari, Petcov 0908.0240]

Main weak points

difficult to extend this description to the quark sector, where mixing angles seem strongly correlated to quark masses
difficult to embed into a GUT
explicit GUT models exist, but the working ones are rather complicated

Conclusions

- do the data suggest a first approximation to lepton mixing angles?

- if so, it is rather different from $V_{CKM} \approx 1$
lepton mixing angles look independent from neutrino masses
special values, like $\vartheta_{23}=45^0$, can only be understood in terms of
a broken flavour symmetry

- non-abelian discrete groups like A_4, S_4, A_5,... can provide the basis for
a realistic model of neutrino masses
(SUSY) models based on discrete flavour symmetries offer specific
predictions for the neutrino mass spectrum, for $0\nu\beta\beta$ and for LFV transitions

- extension to the quark sector and embedding into GUTs possible,
but difficult at the moment

back up slides

plan

1. Flavor symmetries: TB mixing and the lepton mixing puzzle
2. TB mixing from symmetry breaking of a flavor symmetry
3. A minimal model based on A_4
4. Lepton Flavour Violation
5. Leptogenesis
6. Conclusion

[Only an example out of many existing possibilities, to illustrate current ideas]

based on
AF1 = Guido Altarelli and F. F. hep-ph/0504165
AF2 = Guido Altarelli and F. F. hep-ph/0512103
AFL = Guido Altarelli, F.F. and Yin Lin hep-ph/0610165
FHLM1 = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo hep-ph/0702194
AFH = Guido Altarelli, F.F. and Claudia Hagedorn hep-ph/0702194
FL = F.F. and Yin Lin hep-ph/07121528
L = Yin Lin hep-ph/08042867

What is the best 1st order approximation to lepton mixing?

in the quark sector

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\vartheta_C)$$

[Wolfenstein 1983]

in the lepton sector

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

agreement of ϑ_{12} suggests that only tiny corrections [$O(\vartheta_C^2)$] are tolerated. If all corrections are of the same order, then

$\vartheta_{13} \approx O(\vartheta_C^2)$ expected

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

can be reconciled with the data through a correction of $O(\vartheta_C)$, for instance a rotation in the 12 sector [from the left side]

$\vartheta_{13} \approx O(\vartheta_C)$ expected

[quark-lepton complementarity ?] [Smirnov; Raidal; Minakata and Smirnov 2004]

$\vartheta_{23} - \pi/4 \approx O(\vartheta_C^2)$

common feature: $\vartheta_{23} \approx \pi/4$ [maximal atm mixing]

… or anarchical U_{PMNS} ? [Hall, Murayama, Weiner 1999]

θ_{23} maximal from some flavour symmetries ?

a no-go theorem
[F. 2004]

$\vartheta_{23} = \pi/4$ can never arise in the limit of an exact realistic symmetry

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects: vanishing when flavour symmetry F is exact

realistic symmetry:

(1) $\left|\delta m_l^0\right| < \left|m_l^0\right|$

(2) m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan\vartheta_{23}^0 = \tan\vartheta_{23}^\nu \cos\vartheta_{12}^e + \left(\frac{\tan\vartheta_{13}^\nu}{\cos\vartheta_{23}^\nu}\right)\sin\vartheta_{12}^e$$

undetermined

$$\vartheta_{23} = \frac{\pi}{4}$$

determined entirely by breaking effects
(different, in general, for ν and e sectors)

Minimal choice

G_f generated by S and T (U can arise as an accidental symmetry) they satisfy

$$S^2 = T^3 = (ST)^3 = 1$$

these are the defining relations of A_4, group of even permutations of 4 objects, subgroup of SO(3) leaving invariant a regular tetrahedron. S and T generate 12 elements

[Ma and Rajasekaran 2001, Ma 2002, Babu, Ma and Valle 2003, ...]

$$A_4 = \left\{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\right\}$$

there are many many non-minimal possibilities: $G_f = S_4$, $\Delta(27)$, $\Delta(108)$, ...

[Medeiros Varzielas,
King and Ross 2005 and 2006;
Luhn, Nasri and Ramond 2007,
Blum, Hagedorn and Lindner 2007,...]

A_4 has 4 irreducible representations: 1, 1′, 1″ and 3

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

$$\begin{array}{lll} 1 & S=1 & T=1 \\ 1' & S=1 & T=\omega^2 \\ 1'' & S=1 & T=\omega \end{array}$$

$$3 \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Minimal Flavor Violation [MFV]

$$G_f = SU(3)_l \times SU(3)_{e^c} \times \ldots$$

[D'Ambrosio, Giudice, Isidori, Strumia 2002
Cirigliano, Grinstein, Isidori, Wise 2005]

the largest G_f

$$l = (\bar{3},1) \qquad e^c = (1,3)$$

$$\varphi \equiv \begin{cases} y_e = (3,\bar{3}) \\ Y = (6,1) \end{cases}$$

G_f broken only by the Yukawa coupling of L_{SM} and L_5

y_e and Y can be expressed in terms of lepton masses and mixing angles

$$y_e = \sqrt{2}\,\frac{m_e^{diag}}{v} \qquad Y = \frac{\Lambda_L}{v^2} U^* m_\nu^{diag} U^+$$

diagonal elements $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$ are of the same size as in A_4x...
similar lower bounds on the scale M

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij} = \beta\,(y_e Y^{+} Y)_{ij} + \ldots$$

$$= \sqrt{2}\beta\,\frac{(m_l)_{ii}}{v}\,\frac{\Lambda_L^2}{v^4}\left[\Delta m^2_{sol} U_{i2} U^*_{j2} \pm \Delta m^2_{atm} U_{i3} U^*_{j3}\right] + \ldots$$

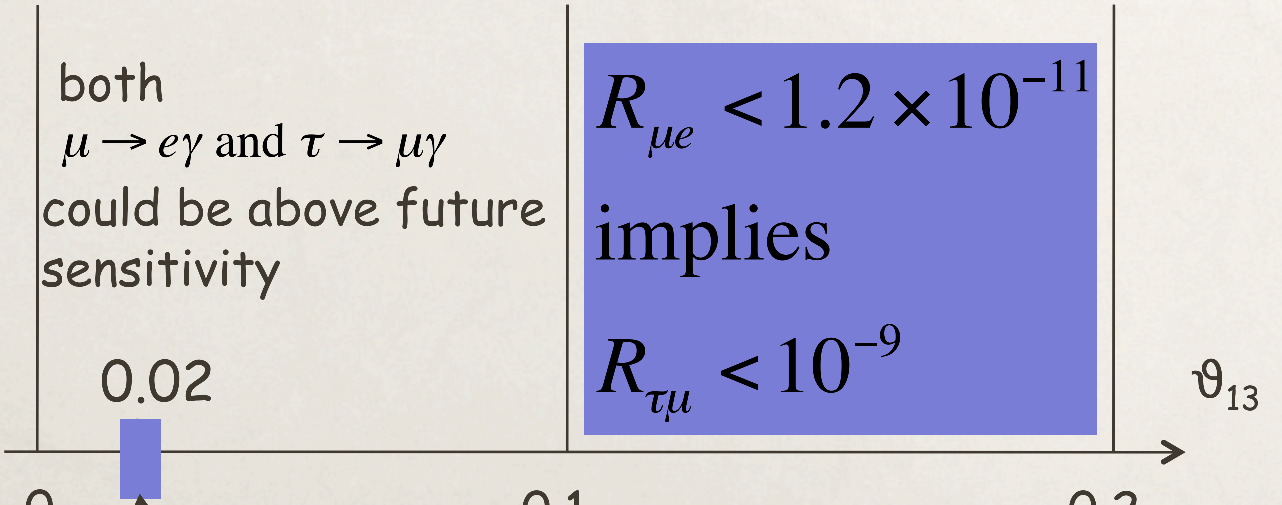
+ for normal hierarchy
- for inverted hierarchy

a positive signal at MEG $10^{-11} < R_{\mu e} < 10^{-13} \div 10^{-14}$ always be accommodated [but for a small interval around $\vartheta_{13} \approx 0.02$ where $R_{\mu e} = 0$]

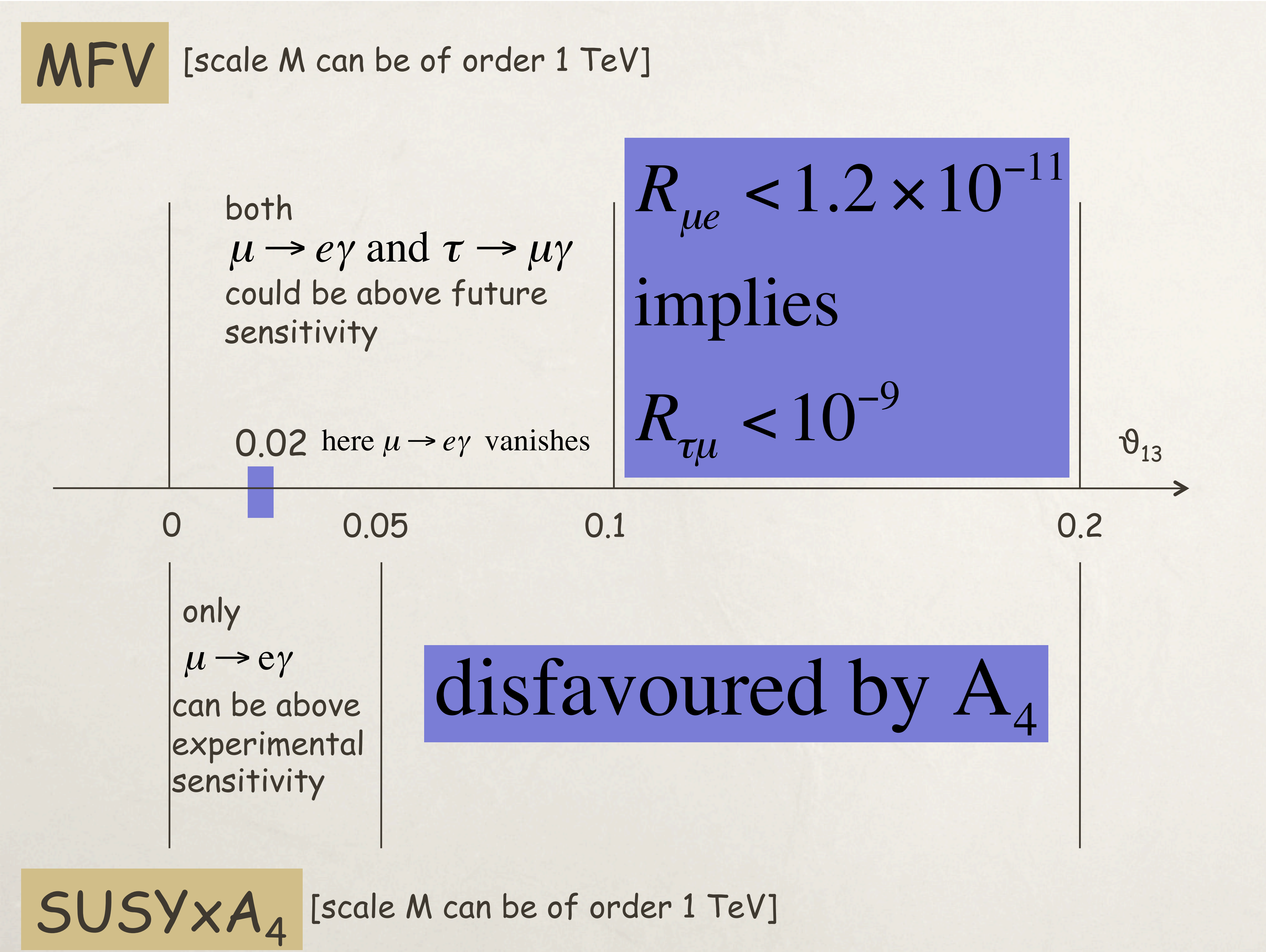
non-observation of R_{ij} can be accommodated by lowering Λ_L

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left|\frac{2}{3} r \pm \sqrt{2}\sin\vartheta_{13} e^{i\delta}\right|^2 < 1 \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

[Cirigliano, Grinstein, Isidori, Wise 2005]



here $\mu \to e\gamma$ vanishes


MFV
[scale M can be of order 1 TeV]
both
$\mu \to e\gamma$ and $\tau \to \mu\gamma$
could be above future sensitivity
0.02 here $\mu \to e\gamma$ vanishes
$R_{\mu e} < 1.2 \times 10^{-11}$
implies
$R_{\tau\mu} < 10^{-9}$
ϑ_{13}
0
0.05
0.1
0.2
only
$\mu \to e\gamma$
can be above experimental sensitivity
disfavoured by A_4
$SUSY \times A_4$
[scale M can be of order 1 TeV]

[other slides]

conclusion

- additional tests of A_4 models from LFV
  generic prediction

$$R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$$ independently from ϑ_{13} (cfr MFV)

$\tau \to \mu\gamma \quad \tau \to e\gamma$ below expected future sensitivity

- in the generic, non-SUSY, case

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} \propto \left(\frac{u}{M^2}\right)^2$$

0.001 < u < 0.05 requires
M above 10 TeV

$\tau^- \to \mu^+e^-e^- \quad \tau^- \to e^+\mu^-\mu^-$ M above 15 TeV

no match with
M fitting $(g-2)_\mu$

- in the SUSY, case

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} \propto \left(\frac{u^2}{M^2}\right)^2$$

M can be much smaller, in the range of interest for $(g-2)_\mu$

$\tau^- \to \mu^+e^-e^- \quad \tau^- \to e^+\mu^-\mu^-$ bound on M relaxed

O(1) coefficient

$$BR(\mu \to e\gamma) = 0.0014 \times \left(\frac{\delta a_\mu}{30 \times 10^{-10}}\right)^2 [\gamma\vartheta_{13}]^4$$

many models predicts a **large** but **not necessarily maximal** θ_{23}

an example: abelian flavour symmetry group $U(1)_F$

$$F(l) = (\times,0,0) \qquad [\times \neq 0]$$

$$F(e^c) = (\times,\times,0)$$

$$m_e = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \end{pmatrix} v_d \qquad\qquad m_\nu = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \\ \cdot & O(1) & O(1) \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1)$$

maximal only by a fine-tuning!

similarly for all other abelian charge assignements

$$F(l) = (1,-1,-1)$$

$$m_\nu = \begin{pmatrix} \cdot & O(1) & O(1) \\ O(1) & \cdot & \cdot \\ O(1) & \cdot & \cdot \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1) + \text{charged lepton contribution}$$

no help from the see-saw mechanism within abelian symmetries…

θ_{23} maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019]

running effects important only for quasi-degenerate neutrinos

2 flavour case

boundary conditions at Λ>> e.w. scale $\quad m_2, m_3, \vartheta_{23}$

at $Q < \Lambda$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m}\cos 2\vartheta_{23}$$

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log\frac{\Lambda}{Q}$$

[possible only if $\quad \delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$]

gives the scale Q at which
θ_{23}(Q) becomes maximal

$\Rightarrow$

m_2, m_3, ϑ_{23} fine tuned
to obtain Q at the e.w. scale

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2\vartheta_{13}\sin^2 2\vartheta_{23}}{(\sin^2\vartheta_{23}\cos^2\vartheta_{13} + \sin^2\vartheta_{13})^2}$$

infrared stable fixed point

[Chankowski, Pokorski 2002]

if $\vartheta_{23} = \frac{\pi}{4}$

$$\sin^2 2\vartheta_{12} = \frac{4\sin^2\vartheta_{13}}{(1+\sin^2\vartheta_{13})^2} < 0.2 \ \ (\text{Chooz})$$

wrong!

Alignment and mass hierarchies

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda} \right)$$

charged fermion masses are already diagonal

$$\boxed{m_e << m_\mu << m_\tau}$$

can be reproduced by U(1) flavour symmetry

$$\left.\begin{aligned} & Q(e^c) = 4 \quad Q(\mu^c) = 2 \quad Q(\tau^c) = 0 \\ & Q(l) = 0 \end{aligned}\right\} \text{compatible with } A_4$$

$$Q(\vartheta) = -1 \qquad \langle \vartheta \rangle \neq 0$$

$$\Downarrow$$

$$y_e \approx \frac{\langle \vartheta \rangle^4}{\Lambda^4} \qquad y_\mu \approx \frac{\langle \vartheta \rangle^2}{\Lambda^2} \qquad y_\tau \approx 1$$

[see also Lin hep-ph/08042867 for a realization without an additional U(1)]

Quark masses - grand unification

quarks assigned to the same A_4 representations used for leptons?

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

- fermion masses from dim ≥ 5 operators, e.g. $\frac{\tau^c \varphi_T l H_d}{\Lambda}$
  good for leptons, but not for the top quark
- naïve extension to quarks leads diagonal quark mass matrices and to V_{CKM}=1
  departure from this approximation is problematic
  [expansion parameter (VEV/Λ) too small]

possible solution within T', the double covering of A_4

[FHLM1]

$$S^2 = R \quad R^2 = 1 \quad (ST)^3 = T^3 = 1$$

24 elements

representations: 1 1′ 1″ 3 2 2′ 2″

	$\begin{pmatrix} u & d \\ c & s \end{pmatrix}$	$\begin{pmatrix} u^c \\ c^c \end{pmatrix}$	$\begin{pmatrix} d^c \\ s^c \end{pmatrix}$	$\begin{pmatrix} t & b \end{pmatrix}$	t^c	b^c	η	ξ''
T'	2''	2''	2''	1	1	1	2'	1''

[older T' models by
Frampton, Kephard 1994
Aranda, Carone, Lebed 1999, 2000
Carr, Frampton 2007
similar U(2) constructions by
Barbieri, Dvali, Hall 1996
Barbieri, Hall, Raby, Romanino 1997
Barbieri, Hall, Romanino 1997]

- lepton sector as in the A_4 model
- t and b masses at the renormalizable level (τ mass from higher dim operators) at the leading order

$$m_{u,d} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

33>>22,23,32

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$m_t, m_b > m_c, m_s \neq 0$$

$$V_{cb}$$

- masses and mixing angles of 1st generation from higher-order effects
- despite the large number of parameters two relations are predicted

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2)$$

$0.213 \div 0.243$ $\quad$ 0.2257 ± 0.0021

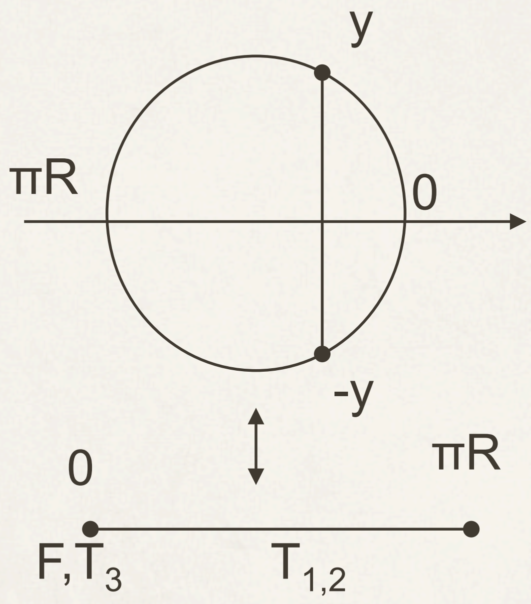
$$\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

$0.208^{+0.008}_{-0.006}$

- vacuum alignment explicitly solved
- lepton sector not spoiled by the corrections coming from the quark sector

other option: SUSY SU(5) in 5D=$M_4 x (S^1 x Z_2)$ + flavour symmetry $A_4 x U(1)$

[AFH]



DT splitting problem solved
via SU(5) breaking induced by compactification

dim 5 B-violating operators forbidden!
p-decay dominated by gauge boson exchange (dim 6)

unwanted minimal SU(5) mass relation $m_e=m_d^T$ avoided by assigning $T_{1,2}$ to the bulk

the construction is compatible with A_4!

	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$
$SU(5)$	1	$\bar{5}$	10	10	10	5	$\bar{5}$
A_4	3	3	1''	1'	1	1	1'

reshuffling of singlet reps.

unsuppressed top Yukawa coupling T_3T_3

realistic quark mass matrices
by an additional U(1) acting on $T_{1,2}$

neutrino masses from see-saw
compatible with both normal and
inverted hierarchy

TB mixing + small corrections

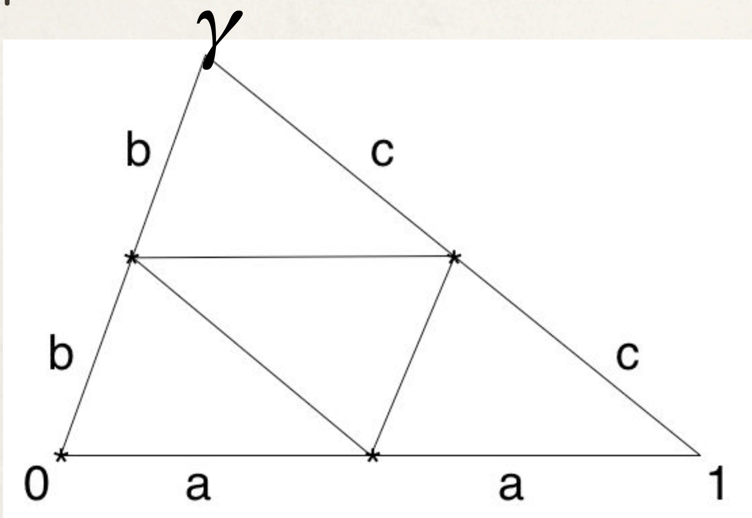
A_4 as a leftover of Poincare symmetry in D>4 [AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

⟺

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x …

a discrete subgroup of the (D-4) euclidean group = translations x rotations can survive in specific geometries



Example: D=6

2 dimensions
compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron invariant under

$$S: \quad z \to z + \frac{1}{2}$$ [translation]

$$T: \quad z \to \gamma^2 z$$ [rotation by 120^0]

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)] the four fixed points (z_1,z_2,z_3,z_4) are permuted under the action of S and T

$$\begin{aligned} S: &\quad (z_1,z_2,z_3,z_4) \to (z_4,z_3,z_2,z_1) \\ T: &\quad (z_1,z_2,z_3,z_4) \to (z_2,z_3,z_1,z_4) \end{aligned}$$

S and T satisfy

$$S^2 = T^3 = (ST)^3 = 1$$

the compact space is invariant under a remnant of 2-translations x SO(2) isomorphic to the A_4 group

Field Theory

brane fields $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$, $\varphi_4(x)$ transform as 3 + (a singlet) under A_4

The previous model can be reproduced by choosing l, e^c, μ^c, τ^c, $H_{u,d}$ as brane fields and φ_T, φ_S and ξ as bulk fields.

String Theory [heterotic string compactified on orbifolds]

in string theory the discrete flavour symmetry is in general bigger than the isometry of the compact space. [Kobayashi, Nilles, Ploger, Raby, Ratz 2006]

orbifolds are defined by the identification

$$(\vartheta\, x) \approx x + l \qquad \begin{cases} l = n_a e_a & \text{translation in a lattice} \\ \vartheta & \text{twist} \end{cases}$$

group generated by (ϑ, l) is called space group

fixed points: special points x_F satisfying

$$x_F \equiv (\vartheta_F^K\, x_F) + l_F \qquad \text{for some} \quad (\vartheta_F^K, l_F)$$

twisted states living at the fixed point $x_F=(\vartheta_F^K,l_F)$ have couplings satisfying space group selection rules [SGSR]. Non-vanishing couplings allowed for

$$\prod_F (\vartheta_F^K, l_F) \equiv (1,0)$$

G_f is the group generated by the orbifold isometry and the SGSR

Example: S^1/Z_2

1 2

Isometry group = S_2 generated by σ^1 in the basis {|1>,|2>}

SGSR = $Z_2 \times Z_2$ generated by $(\sigma^3,-1)$

[allowed couplings when number n_1 of twisted states at |1> and the number n_2 of twisted states at |2> are even]

G_f = semidirect product of S_2 and $(Z_2 \times Z_2) \equiv D_4$

group leaving invariant a square

relation between A_4 and the modular group [AF2]

modular group PSL(2,Z): linear fractional transformation

complex variable

$$\boxed{z \to \frac{a z + b}{c z + d}} \qquad a,b,c,d \in Z \qquad ad - bc = 1$$

discrete, infinite group generated by two elements

$$\boxed{\underbrace{z \to -\frac{1}{z}}_{S}} \qquad \boxed{\underbrace{z \to z+1}_{T}}$$

obeying

$$\boxed{S^2 = (ST)^3 = 1}$$

the modular group is present everywhere in string theory

[any relation to string theory approaches to fermion masses?]

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

A_4 is a finite subgroup of the modular group and

$$\boxed{A_4 = \frac{PSL(2,Z)}{H}} \Rightarrow \boxed{\text{representations of } A_4 \text{ are representations of PSL(2,Z)}}$$

infinite discrete normal subgroup of PSL(2,Z)

future improvements
on
atmospheric and reactor angles

$\sin^2\theta_{23}$

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \rightarrow \nu_\mu$ disappearance channel

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

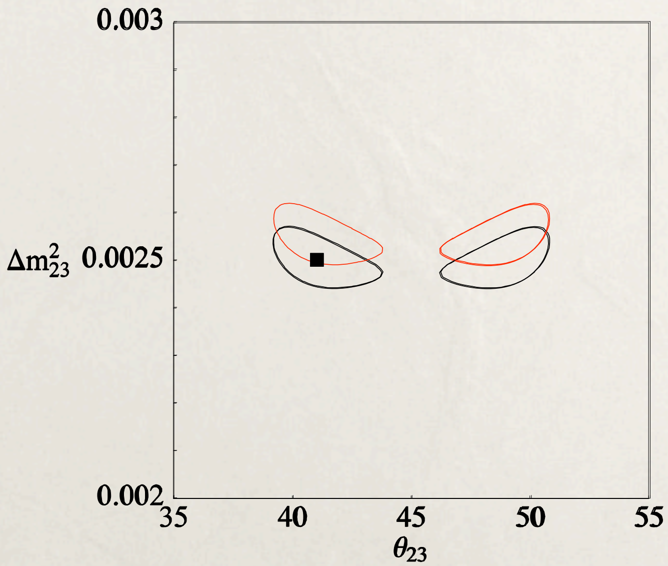
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\,\text{rad} \leftrightarrow 2.9^0$$

improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]

maximal mixing from renormalization group running?

θ $_{23}$ maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019]

running effects important only for quasi-degenerate neutrinos

2 flavour case

boundary conditions at Λ>> e.w. scale $\quad m_2, m_3, \vartheta_{23}$

at $Q < \Lambda$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m}\cos 2\vartheta_{23}$$

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log\frac{\Lambda}{Q}$$

[possible only if $\quad \delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$]

gives the scale Q at which
θ $_{23}$(Q) becomes maximal

$\Rightarrow$

$\boxed{m_2, m_3, \vartheta_{23} \text{ fine tuned to obtain Q at the e.w. scale}}$

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2\vartheta_{13}\sin^2 2\vartheta_{23}}{(\sin^2\vartheta_{23}\cos^2\vartheta_{13} + \sin^2\vartheta_{13})^2}$$

if $\vartheta_{23} = \dfrac{\pi}{4}$

$$\sin^2 2\vartheta_{12} = \frac{4\sin^2\vartheta_{13}}{(1+\sin^2\vartheta_{13})^2} < 0.2 \;\; \text{(Chooz)}$$

wrong!

infrared stable fixed point

[Chankowski, Pokorski 2002]

vacuum alignment from minimization of the scalar potential

(1) natural vacuum alignment

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u$$

$$v_T \approx v_S \approx u$$

it is not a local minimum of the most general renormalizable scalar potential V depending on φ_S , φ_T , ξ and invariant under A_4

a simple solution in 1 extra dimension ≡ ED

[Altarelli, F. 0504165]

$\langle \varphi_T \rangle = (v_T, 0, 0)$

local minimum of V_0

e^c, μ^c, τ^c

$l, h_{u,d}$

0 y L

$\langle \varphi_S \rangle = (v_S, v_S, v_S)$

$\langle \xi \rangle = u$

local minimum of V_L

ν masses arise from local operators at y=L

$$\frac{(\varphi_S l l) h_u h_u}{\Lambda^2} \qquad \frac{\xi (l l) h_u h_u}{\Lambda^2}$$

this explains also the absence of the terms with $\varphi_S \leftrightarrow \varphi_T$

charged lepton masses from non-local operators

bulk fermionY=-1

$$\left.\begin{array}{l} \dfrac{(f^c \varphi_T F)\delta(y)}{\sqrt{\Lambda}} \\ - M F F^c \\ \dfrac{(F^c l) h_d}{\sqrt{\Lambda}} \delta(y-L) \end{array}\right\} \quad E << M$$

$$\frac{(f^c \varphi_T l) h_d}{\Lambda} e^{-ML}$$

a 4D supersymmetric solution ≡ SUSY [Altarelli,F. hep-ph/0512103]

L is identified with the superpotential w_{lepton} in the lepton sector

w_{lepton} is invariant under $A_4 \times Z_3 \times U(1)_R$

	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	1''	1'	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

matter fields · Higgses · A_4 breaking sector · ``driving fields''

absence of $\varphi_S \leftrightarrow \varphi_T$ $x(ll)$ automatic

$$w = w_{lepton} + w_d + \ldots$$

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2$$

minimum of the scalar potential at:

$$\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle &= u \\ \langle \tilde{\xi} \rangle &= 0 \end{aligned}$$

$$v_T = -\frac{3M}{2g} \qquad v_S^2 = -\frac{g_4}{3g_3} u^2$$

u undetermined