Continuum limit results from 2+1 flavor Domain Wall QCD

(RBC and UKQCD Collaborations)

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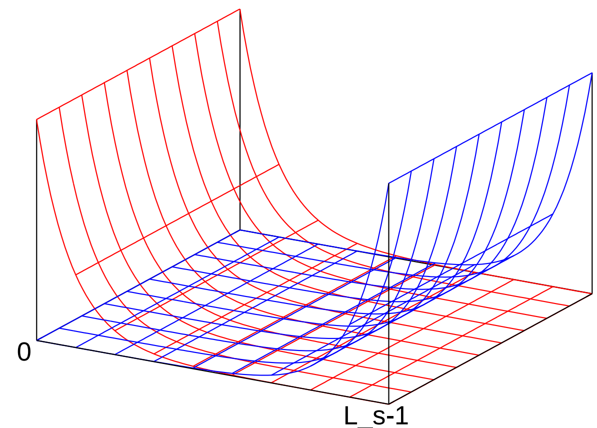
results presented on behalf of

RIKENBrookhavenColumbia and UKQCD Collaborations

- Domain Wall QCD with 2+1 flavors
  (good chiral properties, simulation possible)



- large physics program
  - light meson decay constants
  - quark masses
  - EM splittings
  - neutral kaon mixing
  - semi-leptonic form factors
  - baryon masses
  - . . .
- computational resources RBRC QCDOC, BNL NYBlue, LLNL and ANL resources (USQCD), Edinburgh, . . .

extrapolations in the pion and kaon sector

simulation details

combined chiral/continuum extrapolation

quark masses, decay constants,. . .

neutral kaon mixing (B_K)

renormalization

chiral extrapolation

$K \to \pi$ form factor (Kl_3)

form factors at small q^2

extrapolation ansätze

$K \to \pi\pi$

$N_f = 2 + 1$ Domain Wall Fermion ensembles

- Iwasaki gauge action, 2 lattice spacings

  * $\beta = 2.13$, $1/a = 1.73\,\mathrm{GeV}$
  * $L^3 \times T \times L_s = 24^3 \times 64 \times 16$, $aL \approx 2.7\mathrm{fm}$
  * 2 light dynamical masses, $m_\pi = 330$, 420 MeV

  * $\beta = 2.25$, $1/a = 2.28\,\mathrm{GeV}$
  * $L^3 \times T \times L_s = 32^3 \times 64 \times 16$, $aL \approx 2.7\mathrm{fm}$
  * 3 light dynamical masses, $m_\pi = 290$, 350, 400 MeV

- dynamical strange quark at physical value (tuning + reweighting)

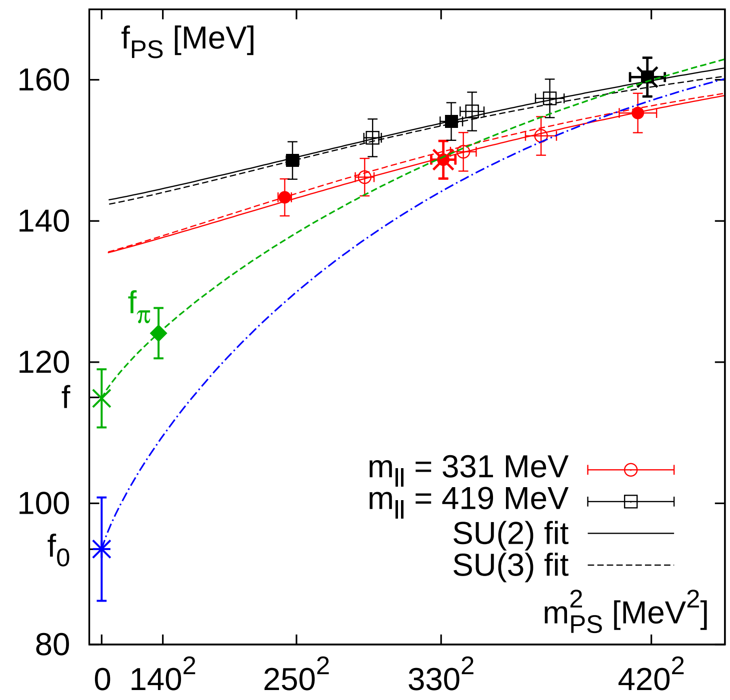
- partially quenched pion masses: 220 MeV

- in preparation: DislocationSuppressingDetRatio(DSDR)-runs (not included in analysis yet)
  * coarser lattices ($1/a \approx 1.4\,\mathrm{GeV}$), larger volumes ($aL \approx 4.5\,\mathrm{fm}$)
  * $m_\pi = 180$, 250 MeV

previous analysis

(Allton et al., Phys. Rev. **D78** (2008) 114509)

- $1/a = 1.73$ GeV
- half data-set
- no continuum limit
- combined fits meson masses/decay constants
- m_{ud}, m_s, $1/a$ from m_π, m_K, m_Ω
- (NLO) SU(2) vs. SU(3) χPT
- SU(2) for kaons (f_K, m_K, B_K)



$$f_\pi = 124.1(3.6)_{\text{stat}}(6.9)_{\text{syst}}\,\text{MeV} \qquad f_K = 149.6(3.6)_{\text{stat}}(6.3)_{\text{syst}}\,\text{MeV}$$

$$f_K/f_\pi = 1.205(18)_{\text{stat}}(62)_{\text{syst}}$$

$$m_{ud} = 3.72(16)_{\text{stat}}(33)_{\text{ren}}(18)_{\text{syst}}\text{MeV} \qquad m_s = 107.3(4.4)_{\text{stat}}(9.7)_{\text{ren}}(4.9)_{\text{syst}}\,\text{MeV}$$

$$m_{ud} : m_s = 1 : 28.8(0.4)_{\text{stat}}(1.6)_{\text{syst}} \qquad (\overline{\text{MS}}, 2\,\text{GeV})$$

adding a 2nd lattice spacing

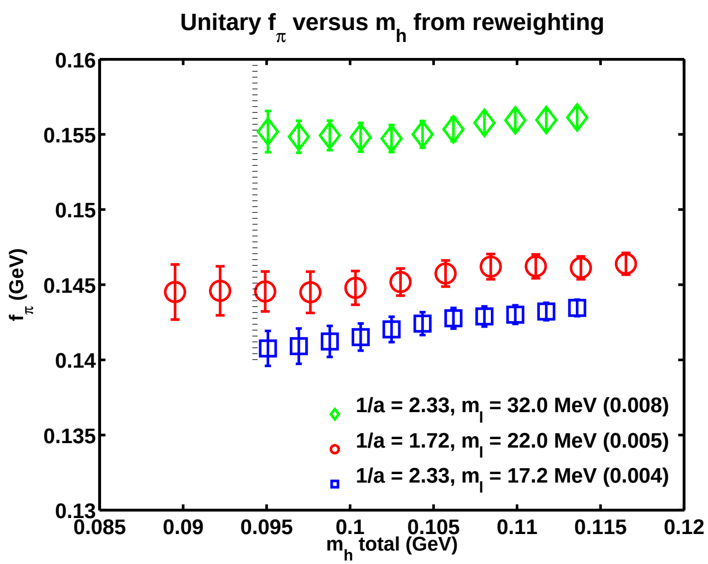
global fits for m_π, m_K, m_Ω, f_π, f_K

- scaling: $1/a_{24c}$, $1/a_{32c}$, quark mass renormalization
  - ∗ match $m^\pi_{ll}/m^\Omega_{hhh}$, m^K_{lh}/m^Ω_{hhh}
  - ∗ m_π, m_K, m_Ω artefact free
- strange quark mass:
  - ∗ know $m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$ *a* posteriori
  - ∗ reweighting 90–110 MeV in global fit
- different fit ansätze
  - ∗ NLO SU(2) χPT
  - ∗ LO polynomial fits
- continuum limit

  a^2-dependence of LO-terms, e.g.

$$f_\pi = f\,(1 + c_{af}a^2)\; + \;\mathrm{NLO}$$

- finite volume correction from χPT



plot courtesy of B. Mawhinney

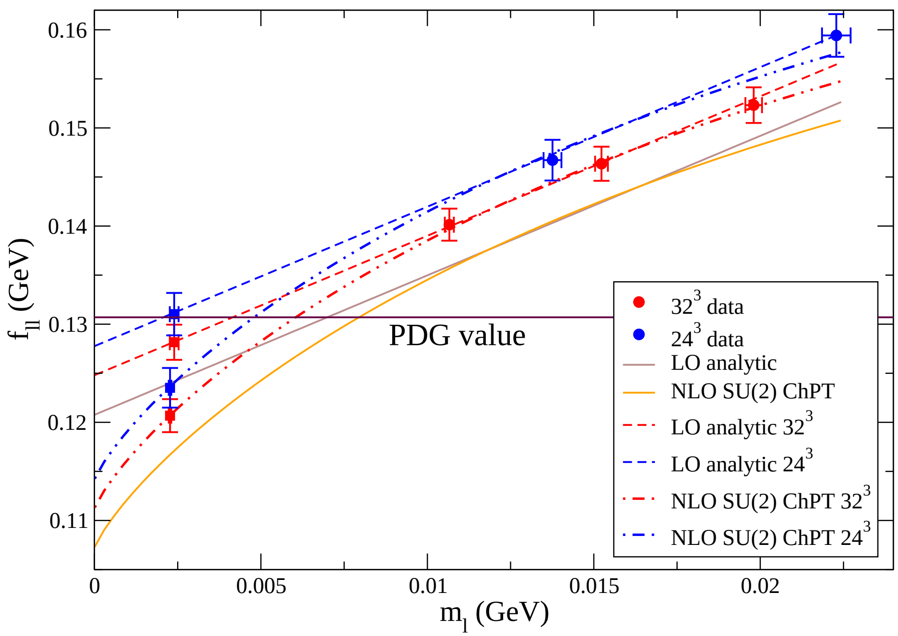
PRELIMINARY **PRELIMINARY**

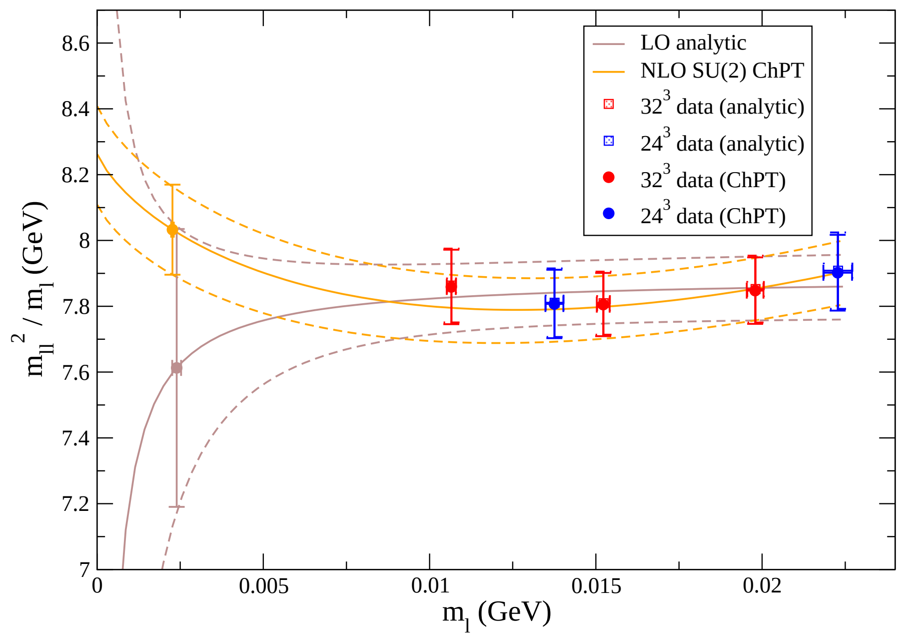
$$f_\pi = 122(2)_{\text{stat}}(5)_\chi(2)_{\text{FV}}\ \text{MeV} \qquad f_K = 147(2)_{\text{stat}}(4)_\chi(1)_{\text{FV}}\ \text{MeV}$$

$$f_K/f_\pi = 1.208(8)_{\text{stat}}(23)_\chi(14)_{\text{FV}}$$

$$m_{ud} = 3.65(20)_{\text{stat}}(8)_{\text{ren}}(13)_{\text{syst}}\ \text{MeV} \qquad m_s = 97.3(1.4)_{\text{stat}}(2.1)_{\text{ren}}(0.2)_{\text{syst}}\ \text{MeV}$$

$(\overline{\text{MS}}$, 2 GeV, NPR, RI − (S)MOM)





(C. Kelly, Lattice 2009, plots courtesy of C. Kelly)

extrapolations in the pion and kaon sector

neutral kaon mixing (B_K)

renormalization

chiral extrapolation

$K \to \pi$ form factor (Kl_3)

$K \to \pi\pi$

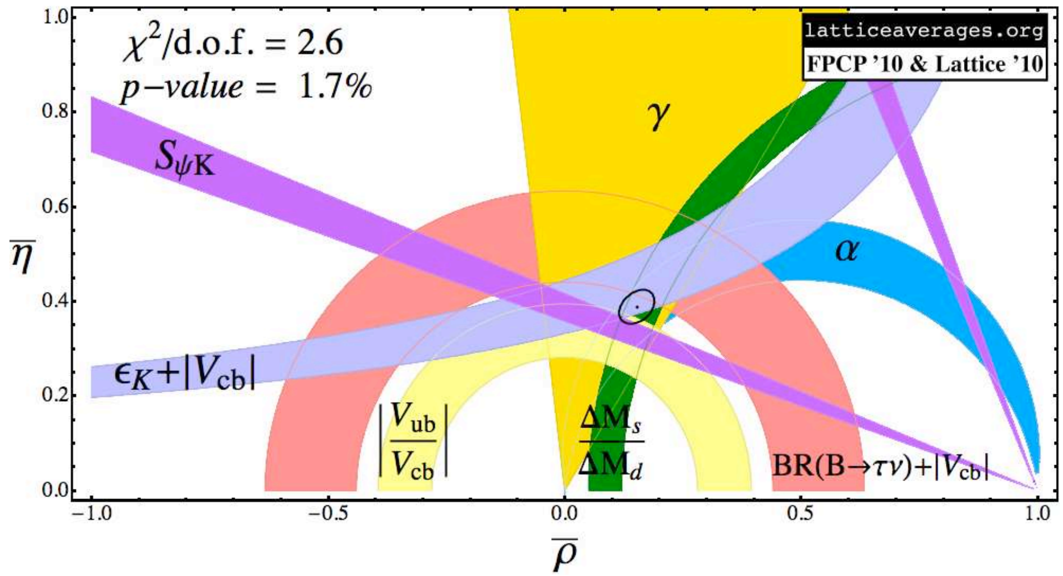
Neutral Kaon Mixing ϵ_K and B_K

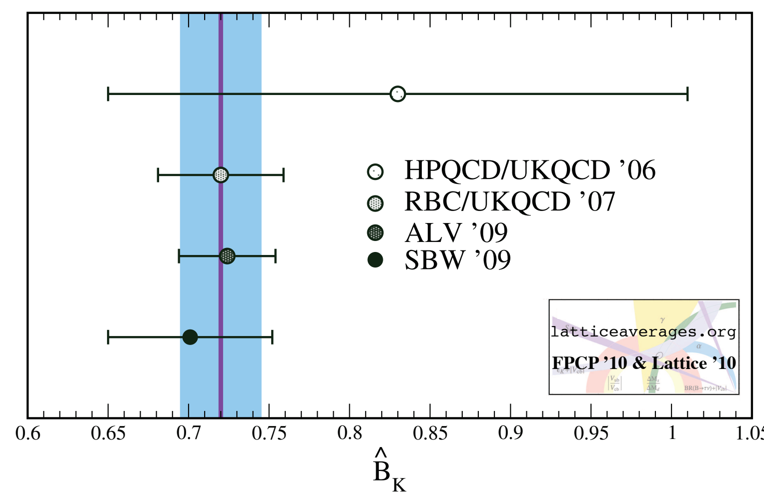
$$B_K(\mu) \;=\; \langle \bar{K}^0 | Q^{\Delta S=2} | K^0 \rangle \;/\; (\tfrac{8}{3} f_K^2 m_K^2)$$

$$|\epsilon_K| \;=\; C_\epsilon \hat{B}_K \lambda^2 \bar{\eta}^2 |V_{cb}|^2 \Big[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} S_0(x_c) \Big]$$

PDG '08: $|V_{cb}| \;=\; 0.0412(11)$ 2.7% $\rightarrow \quad \delta |V_{cb}|^4 \;\simeq\; \delta B_K^{\text{lat}}$

(see Lunghi, Soni (2009) for use of ϵ_K w/o semi-leptonic decays)





(plots courtesy of Laiho, Lunghi, Van De Water)

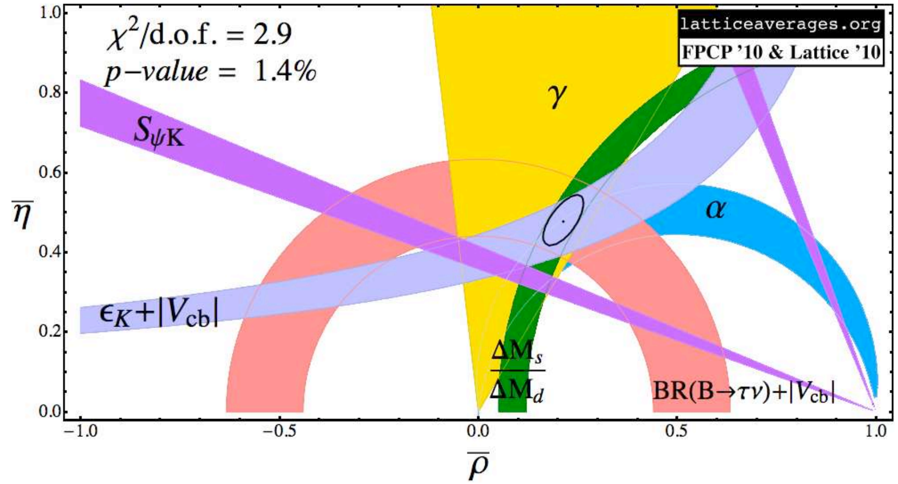
precision B_K

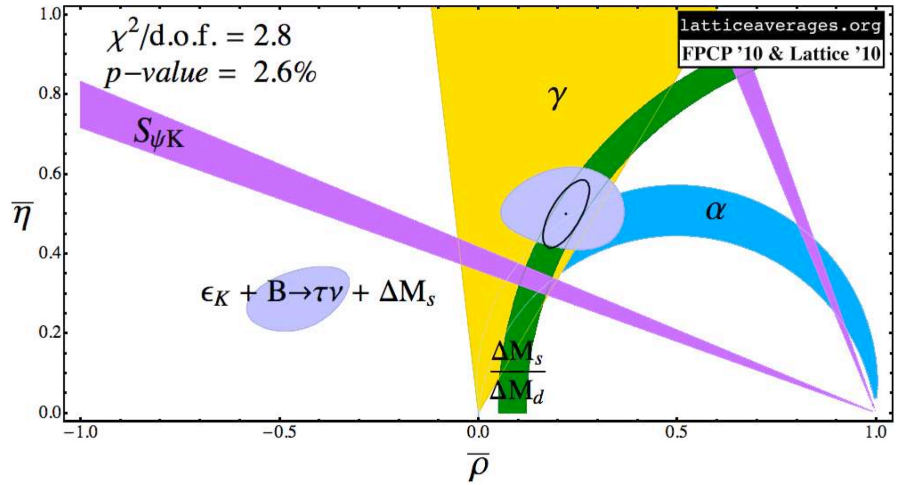
Soni, Lunghi (2008/9)
Laiho, Lunghi, Van de Water (2009)

- NP in K or B-mixing?
- even without $|V_{ub}|$, $|V_{cb}|$

Buras, Guadagnoli (2008)

- $\boldsymbol{\epsilon_K = \bar{\epsilon}_K + i\xi}$
- reaching precision for (lattice) B_K, include $i\xi$
- same effect as lower B_K
- NP in $K - \bar{K}$ and/or $B_d - \bar{B}_d$, $B_s - \bar{B}_s$





(plots courtesy of Lunghi, Laiho, Van De Water)

- ratio of 2- and 3-pt correlators, with 4-quark operator Q_{VV+AA}

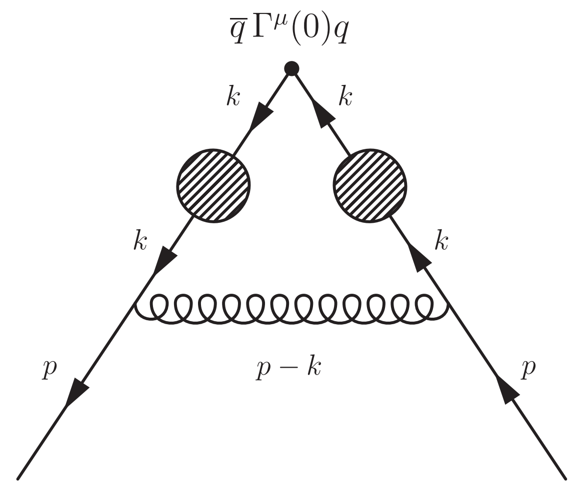
$$B_K(t) \; = \; \frac{3}{8}\frac{\mathcal{C}_{PQP}(t_{\mathsf{src}}, t, t_{\mathsf{snk}})}{\mathcal{C}_{PA}(t_{\mathsf{src}}, t)\mathcal{C}_{AP}(t, t_{\mathsf{snk}})}$$

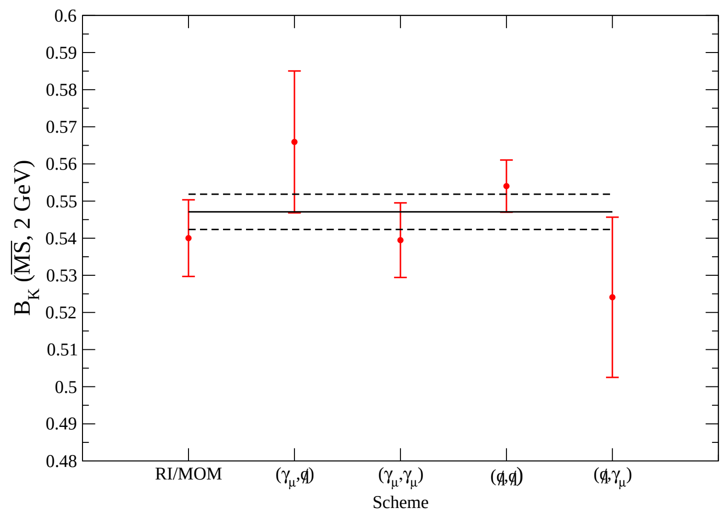
- Q_{VV+AA} mix with Q_{VV-AA}, Q_{SS+PP}, Q_{SS-PP}, O_{TT}
  sufficiently suppressed by chiral properties of Domain-Wall fermions

- previous RBC-result $B_K^{\overline{\mathsf{MS}}}(\mu = 2\,\mathsf{GeV}) \; = \; 0.524(10)_{\mathsf{stat}}(13)_{\mathsf{ren}}(25)_{\mathsf{syst}}$
  ($1/a = 1.73\,$GeV, half data-set, no cont.-extr., RI/MOM-scheme)
  Antonio et al. Phys. Rev. Lett. **100** (2008) 032001, Allton et al. Phys. Rev. **D78** (2008) 114509

- global fit procedure to data $1/a = 1.73$ and 2.28 GeV, various m_l
- reweight strange quark mass to physical m_s
- renormalization of $\langle \bar{K}^0 \left| Q^{\Delta S=2} \right| K^0 \rangle$
  - ∗ global fit
  - ∗ quote result in NDR-scheme ($\overline{\mathsf{MS}}$, $\mu = 2\,$GeV)
  - RI/SMOM-scheme(s)

- RI/MOM-scheme: exceptional momenta
- large p^2: Λ^2/p^2-suppression, e.g. $V-A$
- non-execptional momenta $p_1^2 = p_2^2 = (p_1 - p_2)^2$
  RI/SMOM
- large p^2: Λ^6/p^6-suppression
- conversion factor NDR-scheme needed
  1-loop PT
  quark masses, B_K: C. Sturm et al.
- define 4 different SMOM-schemes (projectors)
- volume source technique
- systematic uncertainty
  - $O(4)$-breaking: χ^2-spread
  - non-zero m_s
  - residual χSB
  - truncation error





plot courtesy of C. Kelly

extrapolation to physical m_{ud}

- data at $1/a$=1.73, 2.28 GeV, m_π = 290–420 MeV (dynamical)
- partially quenched data $m_\pi \geq 220$ MeV
- physical m_s via reweighting (dynamical m_h tuned within 10–15%)
- SU(2)-χPT for B_K

$$B_K^{xh} = B_K^0\left[1 + c_a a^2 + c_0\frac{\chi_l}{f^2} + c_1\frac{\chi_x}{f^2} - \frac{\chi_l}{32\pi^2 f^2}\log\frac{\chi_x}{\Lambda_\chi^2}\right]$$

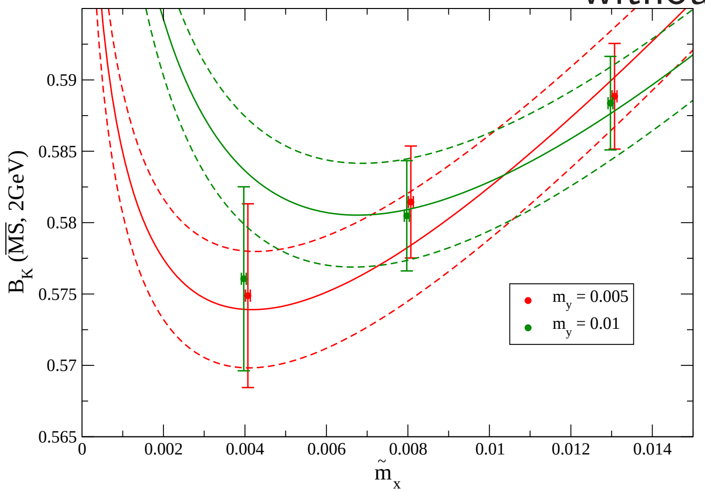
  * $\chi_{x,l} = 2B\tilde{m}_{x,l}$
  * B, f from global SU(2)-χPT fit
  * fit parameters depend on m_s (and Λ_χ)
  * inclusion of finite volume effects ($\log \to \ldots$)
- SU(3)-χPT for B_K

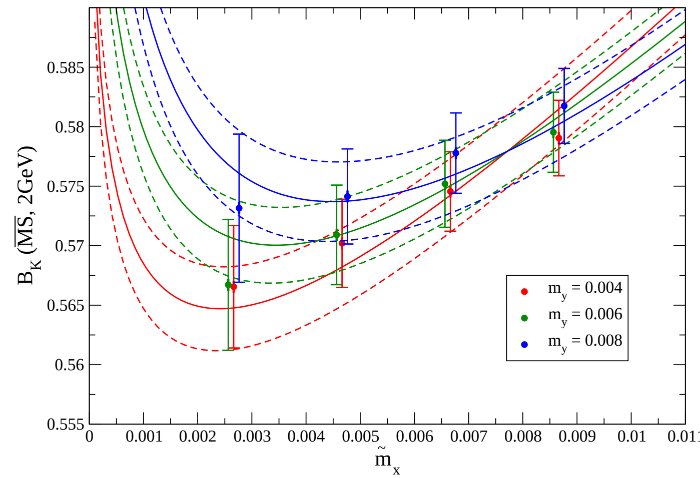
  bad convergence, not needed since physical m_h (tuned, reweighting)
- polynomial extrapolation

$$B_K^{xh} = c_0(1 + c_a a^2) + c_l\tilde{m}_l + c_x\tilde{m}_x$$

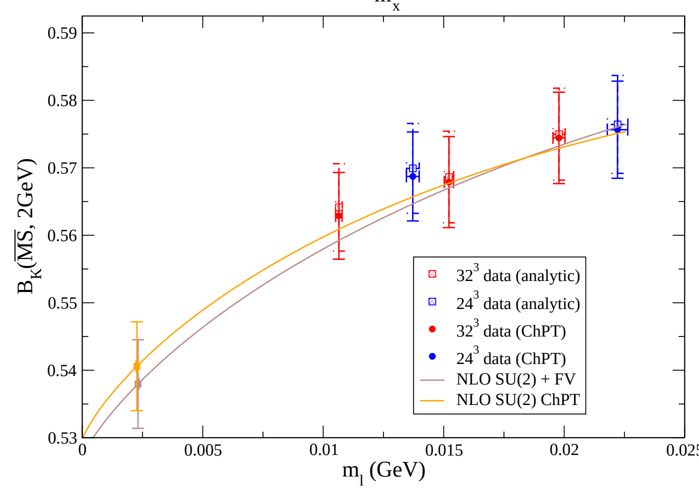
extrapolation with SU(2) χPT

without FV-corrections



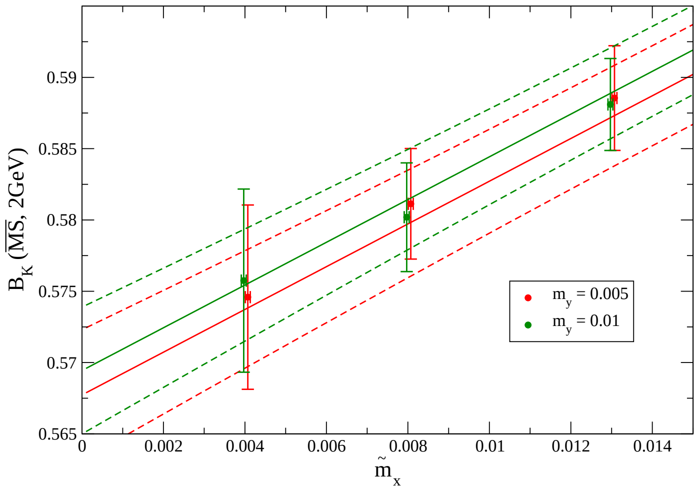


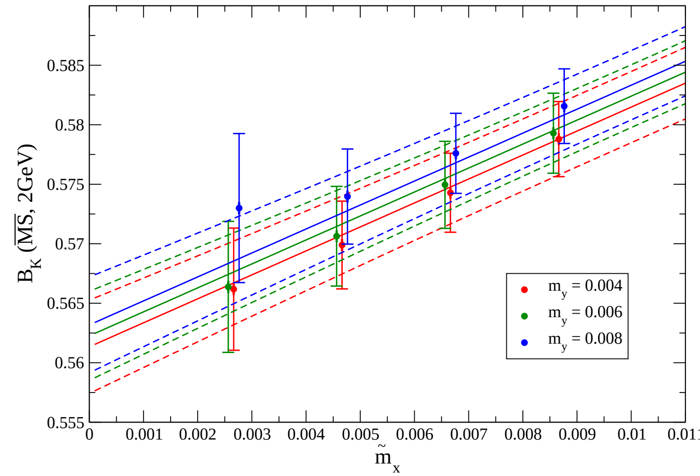
unitary data in continuum limit
SU(2) χPT with and w/o FV-corr.

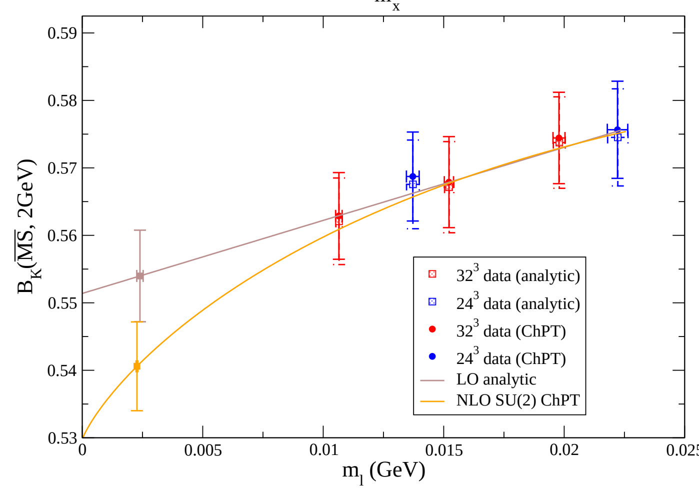


plots courtesy of C. Kelly

extrapolation with polynomial ansatz

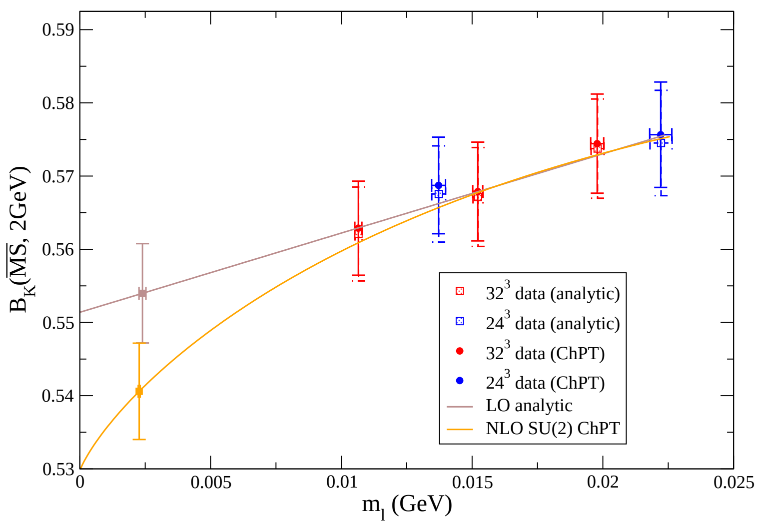






unitary data in continuum limit
SU(2) χPT vs polynomial

plots courtesy of C. Kelly



plot courtesy of C. Kelly

- average central values from SU(2) χ-PT and polyn. extrapolation
- continuum extrapolation
- finite volume systematic error
- renormalization
  - central value from RI/SMOM-scheme best described by PT
  - volume source reduces statistical error
  - other systematics
  - still dominated by truncation in PT

PRELIM. $$B_K^{\overline{\rm MS}}(\mu = 2\,\text{GeV}) \;=\; 0.546(7)_{\text{stat+spread}}(16)_\chi(3)_{\text{FV}}(14)_{\text{ren}}$$ **PRELIM.**

main uncertainty: NPR and chiral extrapolation

C. Kelly, Lattice 2010

previous: $B_K^{\overline{\rm MS}}(\mu = 2\,\text{GeV}) \;=\; 0.524(10)_{\text{stat}}(13)_{\text{ren}}(25)_{\text{syst}}$

- new runs with lighter quark masses (DSDR-action, third $1/a$)

- improve NPR: twisted BCs ($O(4)$-breaking), est. truncation error

extrapolations in the pion and kaon sector

neutral kaon mixing (B_K)

$K \to \pi$ form factor (Kl_3)

form factors at small q^2

extrapolation ansätze

$K \to \pi\pi$

- CKM-unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- $|V_{us}|$ via $|V_{us} f^+(0)|$ from

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I S_{\mathrm{EW}} (1 + 2\Delta_{\mathrm{SU(2)}} + 2\Delta_{\mathrm{EM}}) |V_{us}|^2 |f^+(0)|^2$$

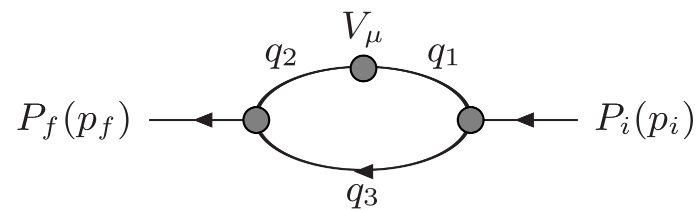
- form factor required

$$\langle \pi(p_\pi) | \bar{u}\gamma_\mu s | \bar{K}(p_K) \rangle = (p_K + p_\pi)_\mu f^+(q^2) + \underbrace{(p_K - p_\pi)_\mu}_{=q_\mu} f^-(q^2)$$

- . . . alternative methods (see C. Sachrajda, LATTICE 2010 (prelim. FLAG-results))
  - ∗ V_{us}/V_{ud} from f_K/f_π — Blucher, Marciano (PDG)
    combined with V_{ud} from nuclear β-decay
  - ∗ $\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right|$, $|V_{us} f^+(0)|$, V_{ud}, unitarity relation
    solve for remaining three ($|V_{ub}|^2 \approx 0$) unknowns

- lattice calculation of K_{l3} form factor
  - $*$ need precision better than 1%
  - $*$ SU(3)-flavor-limit ($m_{ud} = m_s$): $f^+(0) = 1$
  - $*$ $f^+(0) - 1 \; = \; \Delta f \; + \; f_2(f_0, m_\pi^2, m_K^2)$
  - $*$ 20% precision on Δf sufficient
  - $*$ ratios of 2- and 3-pt functions

Becirevic et al. (2004)

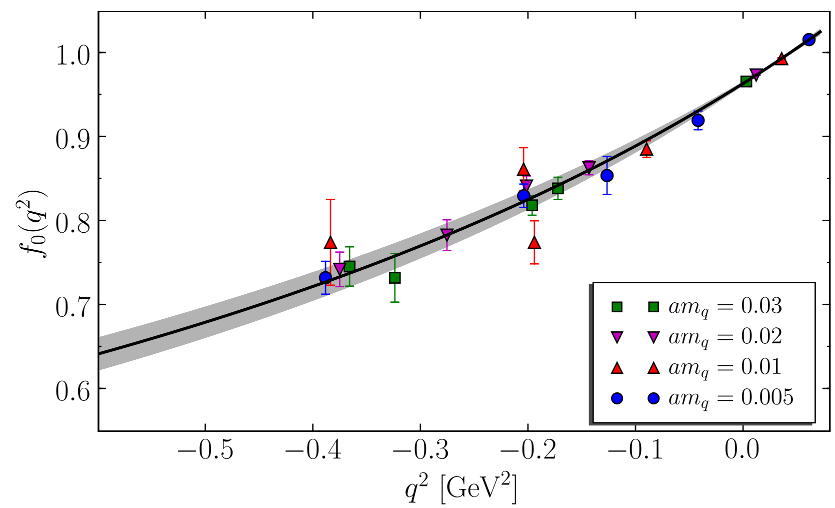


- phenom.: $\Delta f \; = \; -0.016(8)$ Leutwyler and Roos (1984)

- RBC-UKQCD, 2+1 flavor DWF: $\Delta f \; = \; -0.0129(33)_{\text{stat}}(34)_{\text{extrap}}(14)_a$

  - $*$ q^2-interpolation
    $q^2_{\text{max}} = (m_K - m_\pi)^2$, $q^2 < 0$
    lattice (periodic bc) $p = 2\pi/L$
  - $*$ pole-ansatz, model-dependence?

$$f_0(q^2) \; = \; \frac{f_0(0)}{1 - q^2/M^2}$$



Boyle et al., PRL **100** (2008) 141601

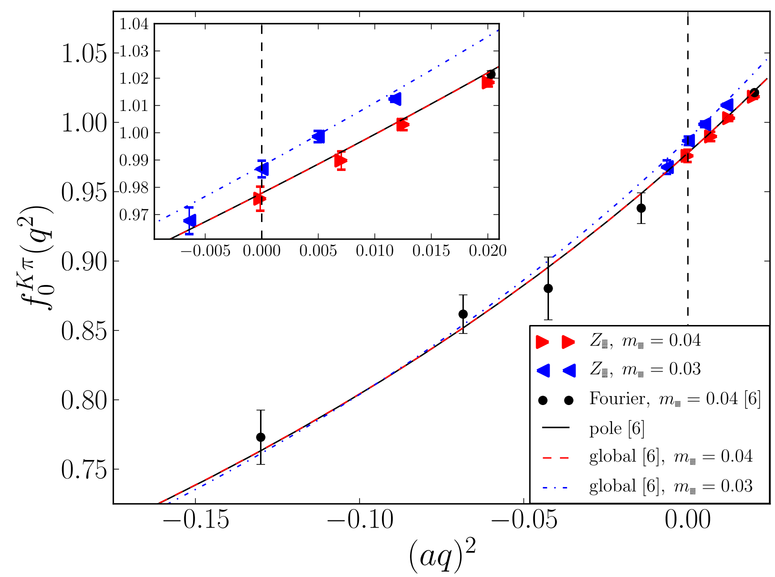
- calculate $\langle \pi(p_\pi) \, |V_\mu| \, K(p_K) \rangle_{q^2}$ at (any) small q^2 Boyle et al., `arXiv:1004.0886[hep-lat]`
- twisted boundary conditions (spatial)

$$\psi(x_k + L) \; = \; e^{\mathrm{i}\theta_k} \psi(x_k)$$

$$\mathbf{p} \; = \; \mathbf{p}_{\mathsf{FT}} \; + \; \theta / L$$

- configurations generated with periodic boundary conditions:
  partially twisted boundary conditions
  (small, negligible finite volume effect, Flynn et al. (2006))
- $q^2 = 0$ (with zero FT-momentum)

  * $\theta_\pi = \mathbf{0}$ $|\theta_K| = L \sqrt{\left(\frac{m_K^2 + m_\pi^2}{2 m_\pi}\right)^2 - m_K^2}$
  * $\theta_{\mathbf{K}} = \mathbf{0}$ $|\theta_\pi| = L \sqrt{\left(\frac{m_K^2 + m_\pi^2}{2 m_K}\right)^2 - m_\pi^2}$
  * plus additional values $(q^2 \neq 0)$

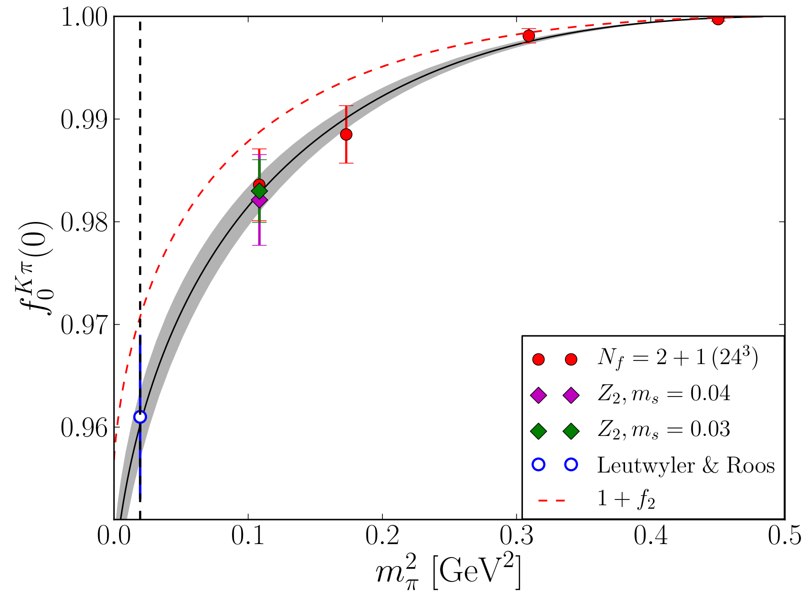


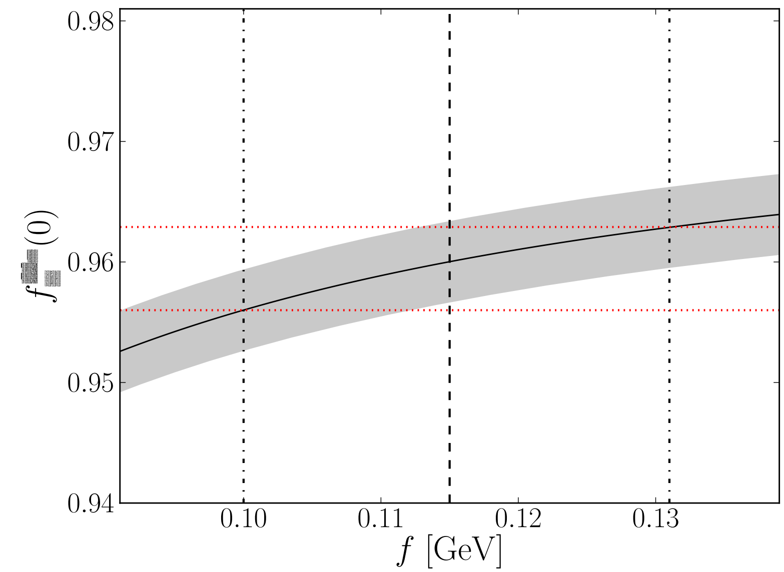
- **completely removes uncertainty due to q^2-extrapolation**

- extrapolation
  - $*$ pole/polynomial-ansatz for q^2
  - $*$ **SU(3)** or SU(2) ChPT
- uncertainty in ChPT:

$$f^+(0) \;=\; 1 + \Delta f + f_2(f_0, m_\pi^2, m_K^2)$$

  - $*$ value of f_0?
  - $*$ $f_0 \to f, f_\pi, \ldots$: reordering (NNLO)
  - $*$ $f_0 =$ 100, **115**, 131 MeV
- fixed lattice spacing: 4% error





$$\boldsymbol{f^+(0) \;=\; 0.9599(34)_{\text{stat}}\left(^{+31}_{-43}\right)_{\text{ChPT}}(14)_a \;=\; 0.960\left(^{+5}_{-6}\right)}$$

previous: $f^+(0) \;=\; 0.9644(33)_{\text{stat}}(34)_{\text{ChPT},q^2}(14)_a$

extrapolations in the pion and kaon sector

neutral kaon mixing (B_K)

$K \to \pi$ form factor (Kl_3)

$K \to \pi\pi$

$K \to \pi\pi,\ \Delta I = 1/2,\ \ldots$

- previous attempts: relate $K \to \pi\pi$ 4-quark operators to $K \to \pi$, $K \to$ vac
  - quenched approximation
  - SU(3)-ChPT required
  - large NLO-corrections
  - LECs unreliable calculated ($> 100\%$ uncertainty) Christ, Li, LATTICE 2008

- current approach
  - directly calculate $\langle \pi\pi | \mathcal{O} | K \rangle$
  - use twisted boundary conditions to impose momentum on $\pi\pi$ states
  - Lellouch-Lüscher approach: Eucl., finite vol. $\to$ physical, infinite vol. matrix-element

  - first **preliminary** results presented at Lattice 2010:
    - $\mathrm{Re}(A_2) = 1.56(07)_{\mathrm{stat}}(25)_{\mathrm{syst}} \cdot 10^{-8}\mathrm{GeV}$ (Lightman)
      phys. kinematics: $m_\pi = 145.6(5)\mathrm{MeV}$, $m_K = 519(2)\mathrm{MeV}$, $E_{\pi\pi} = 516(9)\mathrm{MeV}$
    - $\mathrm{Re}(A_0) = 43(12) \cdot 10^{-8}\mathrm{GeV}$ (Liu)
      unphys. kinematics: $m_\pi = 420\mathrm{MeV}$, $m_K = 778\mathrm{MeV}$, threshold $\pi\pi$ state

extrapolations in the pion and kaon sector

continuum extrapolation from 2 lattice spacings

extrapolations from $m_\pi = 290\text{–}420$ GeV to physical point

results for decay constants, quark masses, LECs

neutral kaon mixing

PRELIM. $$B_K^{\overline{\mathsf{MS}}}(\mu = 2\,\mathsf{GeV}) \;=\; 0.546(7)_{\mathsf{stat+spread}}(16)_\chi(3)_{\mathsf{FV}}(14)_{\mathsf{ren}}$$ **PRELIM.**

$K \to \pi$ form factor (Kl_3)

$$f_+^{K\pi}(0) \;=\; 0.9599(34)_{\mathsf{stat}}\left({}^{+31}_{-43}\right)_{\mathsf{chPT}}(14)_a$$

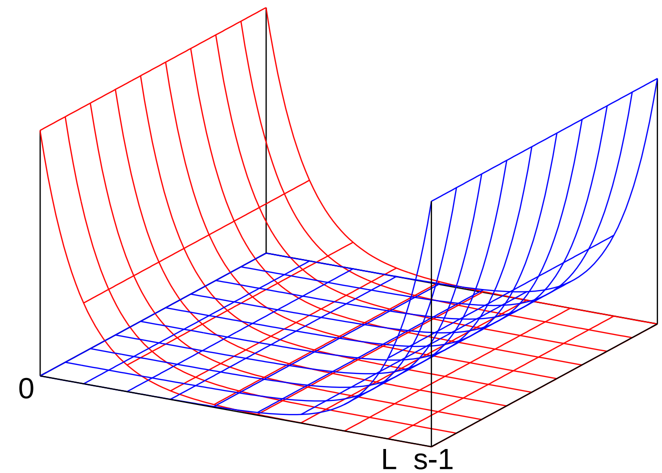
$K \to \pi\pi$

direct computation in progress. . .

BACKUP

(Why) Domain Wall fermions

- different lattice fermions
  - ∗ Wilson fermions and improved versions
  - ∗ staggered fermions
  - ∗ domain wall fermions (DWF)
  - ∗ overlap-fermions
- DWF
  - ∗ fermion fields have a 5th dimension of extent L_s
  - ∗ *left* and *right* handed fermions on slice 0 and $L_s - 1$
  - ∗ propagation through 5th dimension:
    residual chiral symmetry breaking (m_{res})
  - chiral symmetry breaking under control
  - reduces (wrong chirality) operator mixing (B_K)
  - non-perturbative renormalization (quark masses, B_K)

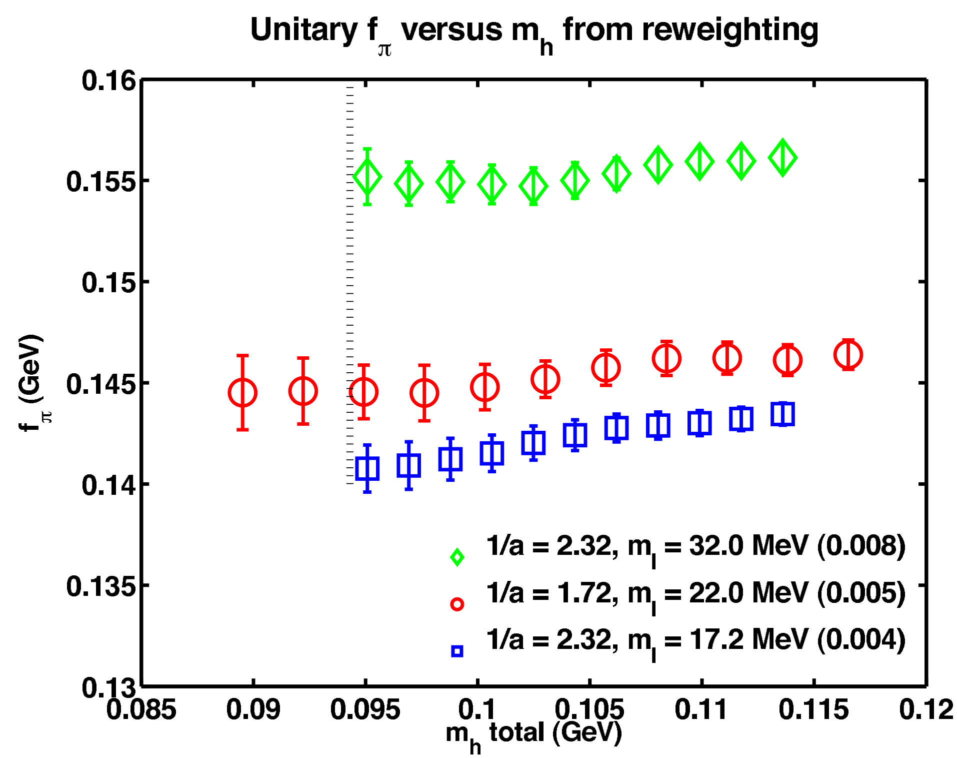


Reweighting

- exact m_s during simulation?
- determined in (global) fit
- stochastically reweight

$$\det\left(\frac{D(m_l,m_h')^\dagger D(m_l,m_h')}{D(m_l,m_h)^\dagger D(m_l,m_h)}\right)^{\frac{1}{2}}$$

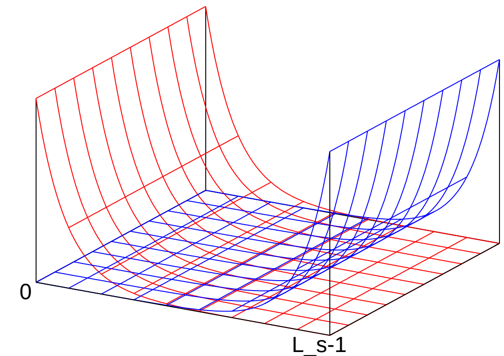
- include reweighting in global fit

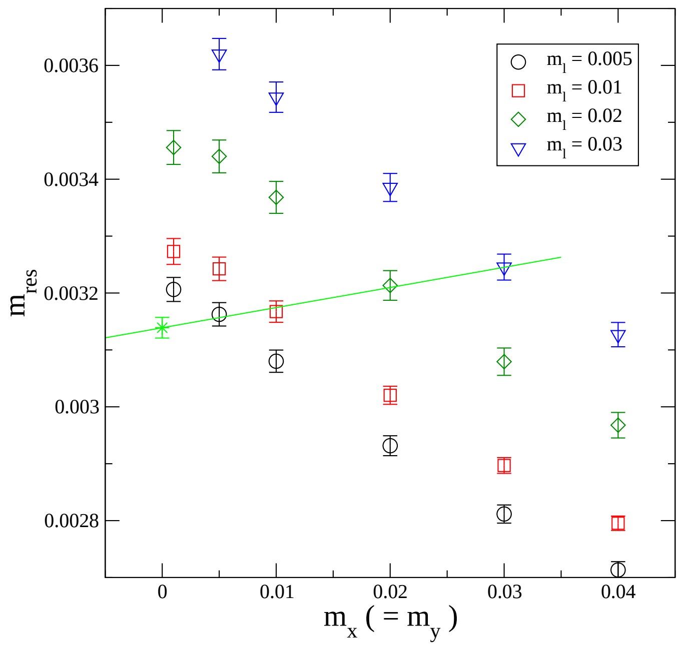


(R. Mawhinney)

residual chiral symmetry breaking

- **Domain Wall Fermions:** good chiral properties (suppress wrong op. mixing, NPR)
- left and right handed fermions separated in 5th dim.
- residual mass term m_{res}





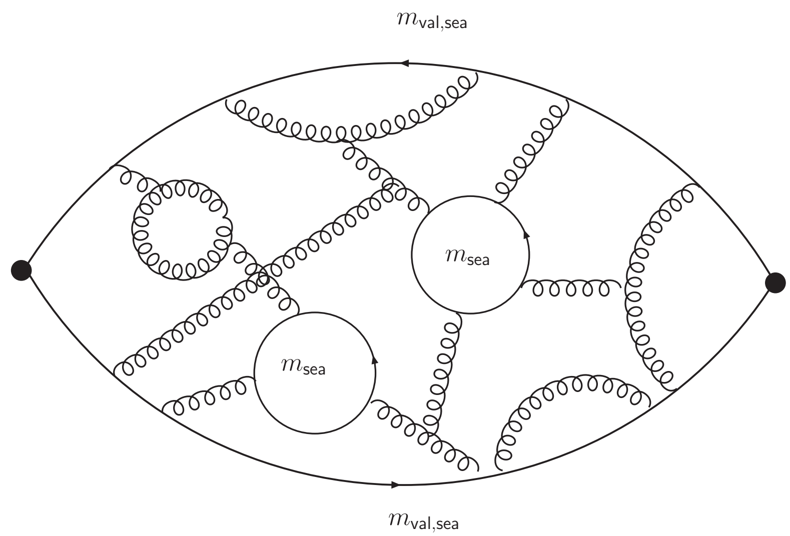
$$R(t) = \frac{\langle \sum_x J^a_{5q}(x,t) P^a(0,0)\rangle}{\langle \sum_x P^a(x,t) P^a(0,0)\rangle} \overset{t\gg 1}{\longrightarrow} m_{\text{res}}(m_x)$$

mid-point operator

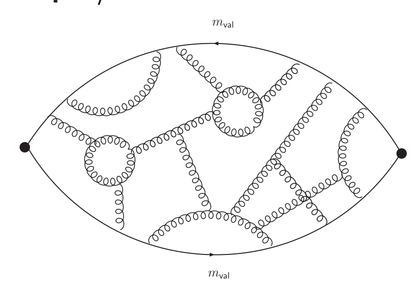
$$J^a_{5q} = \bar{\Psi}_{L_s/2} P_R \tau^a \Psi_{L_s/2-1} - \bar{\Psi}_{L_s/2-1} P_L \tau^a \Psi_{L_s/2}$$

$$m_{\text{res}} = \mathbf{0.00315(02)}$$

Partial Quenching



- dynamically simulated quark masses: m_{sea}
- "measurements" done at different quark masses m_{valence}
- unitary case for $m_{\text{valence}} = m_{\text{sea}}$
- Partially Quenched χPT (Rupak/Shoresh, Sharpe/Shoresh,. . . , Sharpe/van de Water, . . .)



- different from quenched simulations: no dynamical fermions

SU(2) PQχPT

$$m_{xy}^2 = \frac{\chi_x + \chi_y}{2}\Bigg\{1 + \frac{32}{f^2}(2L_6^{(2)} - L_4^{(2)})\chi_l + \frac{8}{f^2}(2L_8^{(2)} - L_5^{(2)})(\chi_x + \chi_y)$$

$$+\Big[\ldots \times \log(\chi_x), \log(\chi_y)\Big]\Bigg\}$$

$$f_{xy} = f\left\{1 + \frac{16}{f^2}L_4^{(2)}\chi_l + \frac{4}{f^2}L_5^{(2)}(\chi_x + \chi_y)\right.$$

$$\left. + \Big[\ldots \times \log(\chi_x + \chi_l), \log(\chi_y + \chi_l), \log(\chi_x), \log(\chi_y)\Big]\right\}$$

$$\chi_X = 2B\Big(m_X + m_{\text{res}}\Big)$$

f, B, $L_i^{(2)}$ depend on (background) m_h

$$\mathcal{L}_{\pi\pi} = \frac{f^2}{8}\mathrm{Tr}\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger + \frac{f^2 B}{4}\mathrm{Tr}\left(M^\dagger\Sigma + M\Sigma^\dagger\right)$$

$$\mathcal{L}_{\pi K} = D_\mu K^\dagger D^\mu K - M_K^2 K^\dagger K$$

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \qquad \Sigma = \xi^2 = \exp\frac{\mathrm{i}}{f}\begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}$$

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger,$$

$$K \to UK, \qquad D_\mu K \to UD_\mu K$$

$$D_\mu K = \partial_\mu K + V_\mu K, \qquad V_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)$$

$$m_{xh}^2 = B^{(K)}(m_h)\widetilde{m}_h\left\{1 + \frac{\lambda_1(m_h)}{f^2}\chi_l + \frac{\lambda_2(m_h)}{f^2}\chi_x\right\}$$

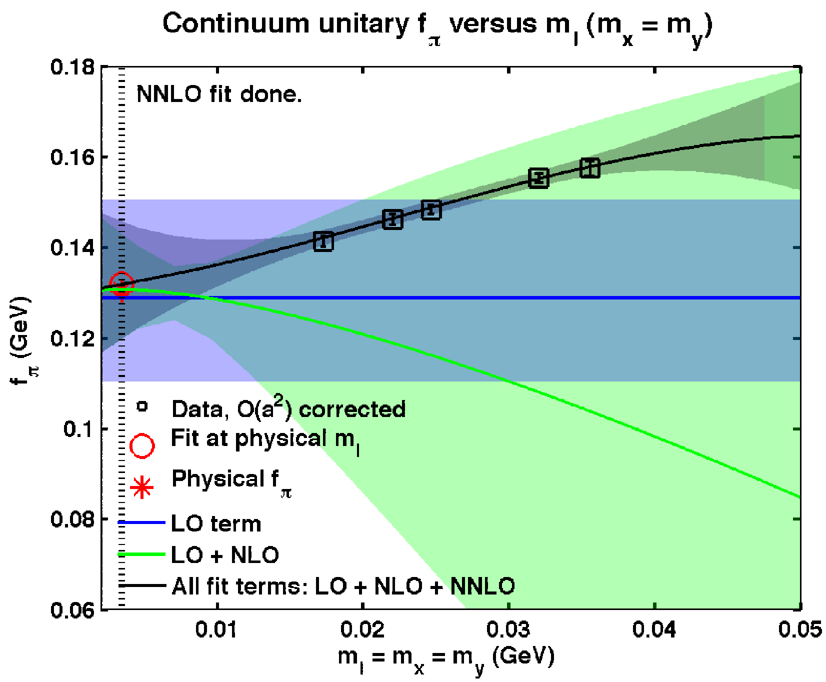
$$f_{xh} = f^{(K)}(m_h)\left\{1 + \frac{\lambda_3(m_h)}{f^2}\chi_l + \frac{\lambda_4(m_h)}{f^2}\chi_x - \frac{1}{(4\pi f)^2}\left[\frac{\chi_x+\chi_l}{2}\log\frac{\chi_x+\chi_l}{2\Lambda_\chi^2} + \frac{\chi_l - 2\chi_x}{4}\log\frac{\chi_x}{\Lambda_\chi^2}\right]\right\}$$

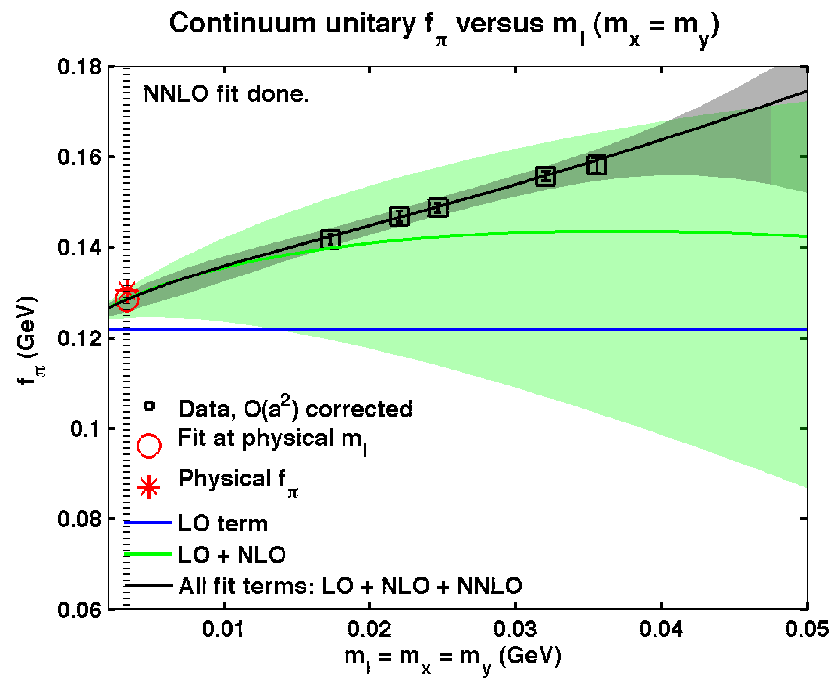
- NLO-fits not working up to the strange quark mass
  ($m_x = 0.001, m_y = 0.04 \;\Rightarrow\; m_{xy} \approx 554$ MeV)
- including NNLO-terms
  - additional LECs (from: $\mathcal{L}_2$, $\mathcal{L}_4$, $\mathcal{L}_6$)
    - SU(3): 4+6 (LO+NLO: 2+4)
    - PQ-SU(3): 5+10 (LO+NLO: 2+4)
    - SU(2): 2+2 (LO+NLO: 2+2)
    - PQ-SU(2): 5+8 (LO+NLO: 2+4)
  - complete formulae available BIJNENS et al.,

    try to apply with 32^3 data
  - just include analytic NNLO-terms
    $(\chi_x + \chi_y)^2$, $(\chi_x - \chi_y)^2$, $\overline{\chi}^2$, $\overline{\chi}(\chi_x + \chi_y)$, $\overline{\chi^2}$
    - still right behaviour in light quark mass region?? non-analytic terms???
    - limited number of data points (sea quark mass)
- chiral symmetry only for up- and down-quarks: SU(2) $\times$ SU(2)

NNLO-SU(2) fits

using the complete χPT up to NNLO from Bijnens, Lahde et al.





f unconstrained vs. $f = 122\,\text{MeV}$ fixed

poor convergence (even worse for masses)

other groups: only constrained fit seem to work at this point