Recent progress on nuclear potentials from Lattice QCD

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1. Introduction
Nuclear force is a basis for understanding...

- Structure of ordinary and hyper nuclei
- Structure of neutron star
- Ignition of Type II SuperNova
Phenomenological NN potential
(~40 parameters to fit 5000 phase shift data)

I One-pion exchange
   Yiukawa (1935)

II Multi-pions
   Taketani et al. (1951)

III Repulsive core
   Jastrow (1951)

$v_c(r)$ [MeV]

$r$ [fm]
Plan of my talk

1. Introduction
2. Strategy in (Lattice) QCD
3. Recent Developments
   1. Tensor potential
   2. Full QCD calculation
4. YN and YY interactions in lattice QCD
   1. S=-1 System
   2. S=-2 System
   3. BB interactions in an SU(3) symmetric world
   4. S=-2 Inelastic scattering
   5. H dibaryon
5. Conclusion
2. Strategy in (Lattice) QCD
From Phenomenology to First Principle

“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task.”

• well-defined statistical system (finite $a$ and $L$)
• gauge invariant
• fully non-perturbative

Monte-Carlo simulations

Quenched QCD: neglects creation-annihilation of quark-antiquark pair
Full QCD: includes creation-annihilation of quark-antiquark pair
How to extract NN potentials in (lattice) QCD

Y. Nambu
“Force Potentials in Quantum Field Theory”

K. Nishijima
“How to Extract NN Potentials in (Lattice) QCD”

HAL QCD Collaboration
Sinya Aoki, Takumi Doi, Tetsuo Hatsuda,
Youichi Ikeda, Takashi Inoue, Noriyoshi Ishii,
Keiko Murano, Hidekatsu Nemura, Kenji Sasaki
Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives
  \[ S = e^{2i\delta} \]

- Nambu-Bethe-Salpeter (NBS) Wave function
  \[ \varphi_E(r) = \langle 0 | N(x + r, 0) N(x, 0) | 6q, E \rangle \]

  6 quark QCD eigen-state with energy E

  \[ N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x) : \text{local operator} \]

**Asymptotic behavior**

\[ r = |r| \to \infty \]

\[ \varphi^l_E(r) \to A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \]

**partial wave**

\[ E = \frac{k^2}{2\mu_N} = \frac{k^2}{m_N} \]

\[ \delta_l(k) \text{ is the scattering phase shift} \]
1. Choose your favorite operator: e.g. $N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$
   - observables do not depend on the choice
   - yet the local operator is useful

2. Measure the NBS amplitude: $\varphi_E(r) = \langle 0|N(x + r, 0)N(x, 0)|6q, E\rangle$

3. Define the non-local potential: $[E - H_0]\varphi_E(x) = \int d^3y U(x, y)\varphi_E(y)$

4. Velocity expansion: $U(x, y) = V(x, \nabla)\delta^3(x - y)$

5. Calculate observables: phase shift, binding energy etc.

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89 (arXiv0909.5585)

$S_{12} = \frac{3}{r^2}(\sigma_1 \cdot x)(\sigma_2 \cdot x) - (\sigma_1 \cdot \sigma_2)$

Okubo-Marshak (1958), Tamagaki-Watari (1967)
First (quenched) results

LO Central Potential

\[ ^1S_0, \; ^3S_1 \]

\[ E \simeq 0 \quad m_\pi \simeq 0.53 \text{ GeV} \]

\[ a=0.137 \text{ fm} \quad L=4.4 \text{ fm} \]

Qualitative features of NN potential are reproduced!


This paper has been selected as one of 21 papers in Nature Research Highlights 2007

“The achievement is both a computational tour de force and a triumph for theory.”
Frequently Asked Questions

[Q1] Operator dependence of the potential
[Q2] Energy dependence of the potential

[A1] $(N(x), U(x,y))$ is a combination to define observables

• remember,
  QM: $(\Phi, U) \rightarrow$ observables
  QFT: (asymptotic field, vertices) $\rightarrow$ observables
  EFT: (choice of field, vertices) $\rightarrow$ observables

• local operator = convenient choice for reduction formula

[A2] $U(x,y)$ is $E$-independent by construction

• non-locality can be determined order by order in velocity expansion
  (c.f. ChPT)
Question 3

How good is the velocity expansion of $V$?

**Leading Order**

$$V_C(r) = \frac{(E - H_0)\varphi_E(x)}{\varphi_E(x)}$$

**Local potential approximation**

The local potential obtained at given energy $E$ may depend on $E$.

If the energy dependence of the potential is weak, the local potential approximation is good.

Furthermore one may determine the higher order terms by comparing results among different energies.

$$V(x, \nabla) = V_C(r) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + \{V_D(r), \nabla^2\} + \cdots$$

**Numerical check in quenched QCD**

$$m_\pi \simeq 0.53 \text{ GeV}$$

$$a=0.137\text{fm}$$

K. Murano, S. Aoki, T. Hatsuda, N. Ishii, H. Nemura
- **PBC (E~0 MeV)**
- **APBC (E~46 MeV)**

![PBC BS wave function](image1)

![APBC BS wave function](image2)

![1S0 Vc PBC](image3)

![1S0 Vc APBC](image4)
1. Use phase shifts for $T_{lab} = 0 \sim 350$ MeV ($\sim 4500$ data points) & deuteron properties

2. Fits with $\chi^2$/dof $\sim 1$ by e.g. 18 parameters (Argonne V18)


E dependence of the local potential turns out to be very small at low energy in our choice of wave function.

Quenched QCD

$m_\pi \simeq 0.53$ GeV

$a = 0.137$ fm
3. Recent developments
3-1. Tensor potential

\[(H_0 + V_C + V_T S_{12})|\phi\rangle = E|\phi\rangle\]

mixing between $^3S_1$ and $^3D_1$ through the tensor force

\[|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle\]

\[|\phi_S\rangle = P|\phi\rangle = \frac{1}{24} \sum_{R \in O} R|\phi\rangle \quad \text{“projection” to L=0} \quad ^3S_1\]

\[|\phi_D\rangle = Q|\phi\rangle = (1 - P)|\phi\rangle \quad \text{“projection” to L=2} \quad ^3D_1\]
Wave functions

Quenched

3S_1
3D_1

quenched QCD
E \sim 0 \text{ MeV}

m_\pi = 529 \text{ MeV}
No repulsive core in tensor
Potentials

Tensor Force and Central Force \((t-t_0=5)\)

\[ V_T(r) \]
\[ V_C(r) \]
\[ V_{C,\text{eff}} \]

\[ V_C \approx V_{C,\text{eff}} \]

No repulsive core in tensor

\[ m_\pi \simeq 0.53 \text{ GeV} \]

from
R. Machleidt,
Adv. Nucl. Phys. 19

\[ V_T(r) \]

\[ V_{C,\text{eff}} \]
Quark mass dependence

- Rapid quark mass dependence of tensor potential
- Evidence of one-pion exchange

Fit function

\[
V_T(r) = b_1 (1 - e^{-b_2 r^2})^2 \left( 1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2} \right) \frac{e^{-m_\rho r}}{r} \\
+ b_3 (1 - e^{-b_4 r^2})^2 \left( 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right) \frac{e^{-m_\pi r}}{r},
\]
3-2. Full QCD Calculation

**Full QCD**

\[ m_\pi = 570 \text{ MeV}, \quad L = 2.9 \text{ fm} \]
\[ a = 0.1 \text{ fm} \]

**Quenched QCD**

\[ m_\pi \approx 0.53 \text{ GeV} \quad a = 0.137 \text{ fm} \]

* Large repulsive core than quenched
* Large tensor force than quenched
Phase shift from $V(r)$ in full QCD

$a=0.1 \text{ fm}, L=2.9 \text{ fm}$
4. YN and YY interactions in lattice QCD

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN?
Hyperon Core of Neutron Stars

Radius $\sim 10$ km
Mass $\sim$ solar mass
Central density $\sim 10^{12}$ kg/cm$^3$

Hyperon matter?

$\nu, \rho, \Sigma^-, \Lambda, \nu^-$ with $\Sigma^- \leftrightarrow n + e^-$, $\Lambda \leftrightarrow n$
3D Nuclear chart

3 known

40 known

~3000 known
4-1. $S= -1$ System: $\Lambda N$ interaction ($l=1/2$) in full QCD

$a=0.1 \text{ fm}, L=2.9 \text{ fm}$

1. repulsive core + attractive well
2. Large spin dependence
3. Overall attraction

Nemura, Ishii, Aoki, Hatsuda

Full QCD $m_\pi=701 \text{ MeV}$
4-2. BB interactions in an SU(3) symmetric world

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)

\[ m_u = m_d = m_s \]

\[ 8 \times 8 = \begin{array}{c} 27 + 8_s + 1 \end{array} + \begin{array}{c} 10^* + 10 + 8_a \end{array} \]

Symmetric \hspace{2cm} Anti-symmetric

6 independent potential in flavor-basis

\[ V^{(27)}(r), \ V^{(8s)}(r), \ V^{(1)}(r) \]
\[ V^{(10^*)}(r), \ V^{(10)}(r), \ V^{(8a)}(r) \]
Potentials

\[ a = 0.12 \text{ fm}, \ L = 2 \text{ fm} \]

\[ m_{PS} \simeq 840 \text{ MeV} \]

27, 10*: same as before NN channel

8s, 10: strong repulsive core

8a: weak repulsive core, deep attractive pocket

Inoue et al., HAL QCD Collaboration, arXiv:1007.3559[hep-lat]
Bound state in 1(singlet) channel? H-dibaryon?

However, it is difficult to determine $E$ precisely, due to contaminations from excited states.

Singlet potential with a certain value of $E$

Schroedinger eq. predicts a bound state at $E < -30$ MeV

<table>
<thead>
<tr>
<th>$E$ [MeV]</th>
<th>$E_0$ [MeV]</th>
<th>$\sqrt{\langle r^2 \rangle}$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = -30$</td>
<td>$-0.018$</td>
<td>24.7</td>
</tr>
<tr>
<td>$E = -35$</td>
<td>$-0.72$</td>
<td>4.1</td>
</tr>
<tr>
<td>$E = -40$</td>
<td>$-2.49$</td>
<td>2.3</td>
</tr>
</tbody>
</table>

finite size effect is very large on this volume. (consistent with previous results.) simulations on larger volume is needed.

$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$
4-3. S=-2 In-elastic scattering (in real world)

\[ m_N = 939 \text{ MeV}, \quad m_\Lambda = 1116 \text{ MeV}, \quad m_\Sigma = 1193 \text{ MeV}, \quad m_\Xi = 1318 \text{ MeV} \]

**S=-2 System**

\[ M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV} \]

They are so close, the eigen-state of QCD in the finite box is a mixture of them:

\[ |S = -2, E\rangle^{\text{lattice}} = c_1 |\Lambda\Lambda, E\rangle_{\text{in}} + c_2 |\Xi N, E\rangle_{\text{in}} + c_3 |\Sigma\Sigma, E\rangle_{\text{in}} \]

\[ E = 2\sqrt{m_\Lambda^2 + p_1^2} = \sqrt{m_\Xi^2 + p_2^2} + \sqrt{m_N^2 + p_2^2} = 2\sqrt{m_\Sigma^2 + p_3^2} \]

In this situation, we can not directly extract the scattering phase shift in lattice QCD.
HAL’s proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

\[ \Psi^{\Lambda\Lambda}_{\alpha}(x) = \langle 0 | \Lambda(x) \Lambda(0) | E_\alpha \rangle \]
\[ \Psi^{\Xi N}_{\alpha}(x) = \langle 0 | \Xi(x) N(0) | E_\alpha \rangle \]

They satisfy

\[ (\nabla^2 + p^2_\alpha) \Psi^{\Lambda\Lambda}_{\alpha}(x) = 0 \]
\[ (\nabla^2 + q^2_\alpha) \Psi^{\Xi N}_{\alpha}(x) = 0 \]
We define the “potential” from the coupled channel Schrödinger equation:

\[
\left( \frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{p^2_\alpha}{2\mu_{\Lambda\Lambda}} \right) \Psi^\Lambda\Lambda_\alpha(x) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(x) \Psi^\Lambda\Lambda_\alpha(x) + V^{\Lambda\Lambda\leftarrow\Xi N}(x) \Psi^{\Xi N}_\alpha(x)
\]

diagonal \hspace{1cm} \text{off-diagonal}

\[
\left( \frac{\nabla^2}{2\mu_{\Xi N}} + \frac{q^2_\alpha}{2\mu_{\Xi N}} \right) \Psi^{\Xi N}_\alpha(x) = V^{\Xi N\leftarrow\Lambda\Lambda}(x) \Psi^\Lambda\Lambda_\alpha(x) + V^{\Xi N\leftarrow\Xi N}(x) \Psi^{\Xi N}_\alpha(x)
\]

off-diagonal \hspace{1cm} \text{diagonal}

\[\mu: \text{reduced mass}\]

\[
\begin{pmatrix}
(E_1 - H_{0X}^x)\Psi_1^x(x) \\
(E_2 - H_{0X}^x)\Psi_2^x(x)
\end{pmatrix} =
\begin{pmatrix}
\Psi_1^x(x) & \Psi_1^y(x) \\
\Psi_2^x(x) & \Psi_2^y(x)
\end{pmatrix}
\begin{pmatrix}
V^{x\leftarrow x}(x) \\
V^{x\leftarrow y}(x)
\end{pmatrix}
\]

\[X \neq Y\]

\[X, Y = \Lambda\Lambda \text{ or } \Xi N\]

\[
\begin{pmatrix}
V^{x\leftarrow x}(x) \\
V^{x\leftarrow y}(x)
\end{pmatrix} =
\begin{pmatrix}
\Psi_1^x(x) & \Psi_1^y(x) \\
\Psi_2^x(x) & \Psi_2^y(x)
\end{pmatrix}^{-1}
\begin{pmatrix}
(E_1 - H_{0X}^x)\Psi_1^x(x) \\
(E_2 - H_{0X}^x)\Psi_2^x(x)
\end{pmatrix}
\]

\[32\]
Using the potentials:

\[
\begin{pmatrix}
V^{\Lambda\Lambda \leftarrow \Lambda\Lambda}(x) & V^{\Xi N \leftarrow \Lambda\Lambda}(x) \\
V^{\Lambda\Lambda \leftarrow \Xi N}(x) & V^{\Xi N \leftarrow \Xi N}(x)
\end{pmatrix}
\]

we solve the coupled channel Schrödinger equation with **appropriate boundary conditions**.

For example, we take the incoming \(\Lambda\Lambda\) state by hand.

In this way, we can avoid the mixture of several “in”-states.

\[
| S = -2, E \rangle_{\text{lattice}} = c_1 | \Lambda\Lambda, E \rangle_{\text{in}} + c_2 | \Xi N, E \rangle_{\text{in}} + c_3 | \Sigma\Sigma, E \rangle_{\text{in}}
\]

Lattice is a tool to extract the interaction kernel (“T-matrix” or “potential”).
Preliminary results from HAL QCD Collaboration

2+1 flavor full QCD

$a=0.1 \text{ fm, } L=2.9 \text{ fm}$

$\pi \sim 870 \text{ MeV}$
Non-diagonal part of potential matrix

\[ V_{\Lambda \Lambda - N \Xi} \approx V_{B - A} \]

Hermiticity
4-4. Possible scenario for H-dibaryon

1. S=-2 singlet state become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance appears above \( \Lambda \Lambda \) but below \( \Xi N \) threshold.

3. We can check this scenario using the lattice QCD.

   3.1. The potential in SU(3) limit

   3.2. The 3 x 3 potential matrix in real world

4. We may use this type of analysis for other systems such as penta-quark state.
5. Conclusion
QCD meets Nuclei!

Thank you for your attention!
backup slide
• the potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator $N(x)$. (Scheme-dependence)

• Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron

• Is the scheme-dependent potential useful? Yes!
  - useful to understand/describe physics
  - a similar example: running coupling
    - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).
  - “good” scheme?
    - good convergence of the perturbative expansion for the running coupling.
    - good convergence of the derivative expansion for the potential?
      - completely local and energy-independent one is the best and must be unique. (Inverse scattering method)
4-2. $S=-2$ System

$\Xi N$ ($I=1$) potential

Quenched

Nemura, Ishii, Aoki, Hatsuda,

1. repulsive core + attractive well
2. Large spin dependence
3. weaker quark mass dependence

$a=0.137$ fm, $L=4.4$ fm