# Limits on the anomalous $H Z \gamma$ VERTEX ARISing FROM THE PROCESS $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ 

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We study the sensitivity for testing the anomalous triple coupling $H Z \gamma$ via the process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ at high energy linear colliders. Using as an input the data obtained by the L3 and OPAL Collaborations for the reaction $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$, we get limits on the anomalous $H Z \gamma$ vertex of the order $10^{-2}$, which are better by an order of magnitude than the bounds obtained from the known limits on the partial decay widths of the $Z$ boson, but still an order of magnitude above the SM prediction.

## 1 Introduction

In the standard model (SM) of electroweak interactions there are no couplings at the tree level among three neutral bosons such as $\mathrm{HZ} \gamma$ [1]. These couplings only appear at the one-loop level through fermion and charged vector bosons. The interest in this type of couplings lies in the additional contributions that may appear in extensions of the SM. For example, new charged scalar and vector bosons in left-right (L-R) symmetric gauge models, or two Higgs doublet models (THDM), charginos and neutralinos in the minimal supersymmetric standard model (MSSM). The SM and LR symmetric models predict an anomalous $H Z \gamma$ vertex of order $10^{-2}$, the MSSM may induce a suppression effect but an effective Lagrangian approach leave room for an enhancement effect. A measurement of this vertex thus may be used to distinguish among theories beyond the SM.
The sensitivity to the $H Z \gamma$ vertex has been studied in processes like $e^{-} \gamma \rightarrow e^{-} H$ and $e^{+} e^{-} \rightarrow H \gamma$, rare $Z$ and $H$ decays, $p p$ collisions via the basic interaction $q q \rightarrow q q H$ and the annihilation process $e^{+} e^{-} \rightarrow H Z$. It has been found that the latter reaction with polarized beams may lead to the best sensitivity to the $H Z \gamma$ vertex while an anomalous $H Z \gamma$ may enhance partial Higgs decays widths by several orders of magnitude that would lead to measurable effects in Higgs signals at the LHC
The general aim of the present paper is to obtain limits on the $H Z \gamma$ vertex coming from the the LEP-II data on the reaction $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$. We will find limits of order $10^{-2}$, which are better by an order of magnitude than the bounds obtained from the known limits on the partial decay widths of the $Z$ boson, but still an order of magnitue above the SM prediction.
The Feynman diagrams which give the most important contribution to the cross section from $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ are shown in Fig. 1. The total cross section of $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ will be evaluated at the $Z_{1}$-pole in the framework of effective lagrangian

(a)

(b)
(c)

Figure 1: Feynman diagrams induced by the anomalous $H Z \gamma$ vertex of the process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$

The numerical computation on the limits of $h_{1}^{Z \gamma}, h_{2}^{Z \gamma}$ and the total cross section for different values of the Higgs mass are done using the data collected by the L3 and OPAL collaborations at LEP [2, 3].

## 2 Cross Section of the Process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$

The anomalous $V_{1}^{\mu}\left(p_{1}\right)-V_{2}^{\nu}\left(p_{2}\right)-H\left(p_{H}\right)$ vertex function is given by [1]

$$
\begin{equation*}
\Gamma_{\mu \nu}^{H V_{1} V_{2}}\left(p_{H}, p_{1}, p_{2}\right)=g_{Z} M_{Z}^{2}\left[h_{1}^{V_{1} V_{2}} g_{\mu \nu}+\frac{h_{2}^{V_{1} V_{2}}}{M_{Z}^{2}} p_{2 \mu} p_{1 \nu}\right], \tag{1}
\end{equation*}
$$

where $M_{Z}$ is the $Z$ boson mass and $V_{1}, V_{2}$ can be $\left(V_{1} V_{2}\right)=(Z Z),(Z \gamma),(\gamma Z),(\gamma \gamma),\left(W^{+} W^{-}\right)$or $\left(W^{+} W^{-}\right)$. The expression for the respective cross section with anomalous couplings $H Z \gamma$ is given by

$$
\begin{aligned}
\sigma\left(e^{+} e^{-} \rightarrow \tau \bar{\tau} \gamma\right) & =\int \frac{\alpha^{3}}{96}\left[3 m_{\tau}^{2} C_{1}\left(x_{W}\right)\left[F_{1}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right)\left(h_{1}^{Z \gamma}\right)^{2}+F_{2}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right)\left(h_{2}^{Z \gamma}\right)^{2}\right]\right. \\
& +m_{\tau}^{2} C_{2}\left(x_{W}\right)\left[F_{3}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) h_{1}^{Z \gamma}+F_{4}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) h_{2}^{Z \gamma}\right] \\
& \left.+C_{3}\left(x_{W}\right) F_{5}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right)\right] E_{\gamma} d E_{\gamma} d \cos \theta_{\gamma}
\end{aligned}
$$

where $E_{\gamma}$ and $\cos \theta_{\gamma}$ are the energy and scattering angle of the photon The kinematics is contained in the functions

$$
\begin{align*}
F_{1}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) & \equiv \frac{\left(\frac{1}{2} s-\sqrt{s} E_{\gamma}-2 m_{\tau}^{2}\right)}{\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]\left(s+2 \sqrt{s} E_{\gamma}-M_{H}^{2}\right)^{2}},  \tag{3}\\
F_{2}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) & \equiv \frac{\left(\frac{1}{6} E_{\gamma}^{2}-\frac{1}{3} \frac{E_{\gamma}^{3}}{\sqrt{s}}-\frac{2}{3} \frac{m_{\tau}^{2} E_{\gamma}^{2}}{s}\right)}{\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]\left(s+2 \sqrt{s} E_{\gamma}-M_{H}^{2}\right)^{2}}, \tag{4}
\end{align*}
$$

$$
\begin{align*}
& F_{3}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) \equiv \frac{\left(-1-\frac{4}{\sin ^{2} \theta_{\gamma}}+\frac{2 \sqrt{s}}{E_{\gamma} \sin ^{2} \theta_{\gamma}}+\frac{2 \sqrt{s} E_{\gamma}}{M_{Z}^{2}}-\frac{6 m_{\tau}^{2}}{\sqrt{s} E_{\gamma} \sin ^{2} \theta_{\gamma}}\right)}{\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]\left(s+2 \sqrt{s} E_{\gamma}-M_{H}^{2}\right)},  \tag{5}\\
& F_{4}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) \equiv \frac{\left(-1-\frac{2}{\sin ^{2} \theta_{\gamma}}-\frac{2 \sqrt{s} E_{\gamma}}{M_{Z}^{2}}+\frac{4 \sqrt{s} E_{\gamma}}{M_{Z}^{2} \sin ^{2} \theta_{\gamma}}+\frac{2 s E_{\gamma}^{2}}{M_{Z}^{4}}\right)}{\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]\left(s+2 \sqrt{s} E_{\gamma}-M_{H}^{2}\right)}, \\
& F_{5}\left(s, E_{\gamma}, \cos \theta_{\gamma}\right) \equiv \frac{\left[\left(4-\sin ^{2} \theta_{\gamma}\right) \sqrt{s}-2 E_{\gamma} \sin ^{2} \theta_{\gamma}\right]}{\left[\left(s-M_{Z}^{2}\right)^{2}+M_{Z}^{2} \Gamma_{Z}^{2}\right]\left(\sqrt{s} \sin ^{2} \theta_{\gamma}\right)}, \tag{7}
\end{align*}
$$

while the coefficients $C_{1,2,3}$ are given by

$$
\begin{align*}
C_{1}\left(x_{W}\right) & \equiv \frac{\left(1-4 x_{W}+8 x_{W}^{2}\right)}{x_{W}^{3}\left(1-x_{W}\right)^{3}} \\
C_{2}\left(x_{W}\right) & \equiv \frac{\left(1-4 x_{W}\right)\left(1-4 x_{W}+8 x_{W}^{2}\right)}{x_{W}^{5 / 2}\left(1-x_{W}\right)^{5 / 2}}  \tag{8}\\
C_{3}\left(x_{W}\right) & \equiv \frac{\left(1-4 x_{W}+8 x_{W}^{2}\right)^{2}}{x_{W}^{2}\left(1-x_{W}\right)^{2}}
\end{align*}
$$

where $x_{W} \equiv \sin ^{2} \theta_{W}$

## 3 Results and Conclusions

In practice, detector geometry imposes a cut on the photon polar angle with respect to the electron direction, and further cuts must be applied on the photon energy and minimum opening angle between the photon and tau in order to suppress background from tau decay products. In order to evaluate the integral of the total cross section as a function of the parameters $h_{1}^{Z \gamma}$ and $h_{2}^{Z \gamma}$, we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over $\cos \theta_{\gamma}$ from -0.74 to 0.74 and $E_{\gamma}$ from 5 GeV to 45.5 GeV for various fixed values of the Higgs boson mass $M_{H}$. Using the numerical values $\sin ^{2} \theta_{W}=0.2314$, $M_{Z_{1}}=91.18 \mathrm{GeV}, \Gamma_{Z_{1}}=2.49 \mathrm{GeV}$ and $m_{\tau}=1.776 \mathrm{GeV}$, we obtain the cross section $\sigma=\sigma\left(h_{1}^{Z \gamma}, h_{2}^{Z \gamma}, M_{H}\right)$.
As was discussed in Ref. [3], $N \approx \sigma\left(h_{1}^{Z \gamma}, h_{2}^{Z \gamma}, M_{H}\right)$, using Poisson statistic [3, 4], we require that $N \approx$ $\sigma\left(h_{1}^{Z \gamma}, h_{2}^{Z \gamma}, M_{H}\right)$ be less than 1559 , with $\mathcal{L}=100 \mathrm{pb}^{-1}$, according to the data reported by the L3 collaboration Ref. [3] and references therein. Taking this into consideration, we get limits on $h_{1}^{Z \gamma}$ and $h_{2}^{Z \gamma}$ as a function of $M_{H}$. The values obtained for these limits for several values of $M_{H}$ are included in Table 1.

| $M_{H}$ | $h_{1}^{Z \gamma}$ | $h_{2}^{Z \gamma}$ |
| :---: | :---: | :---: |
| 115 GeV | $[-0.042,0.042]$ | $[-0.045,0.045]$ |
| 130 GeV | $[-0.047,0.047]$ | $[-0.081,0.081]$ |
| 150 GeV | $[-0.14,0.14]$ | $[-0.25,0.25]$ |
| 190 GeV | $[-0.37,0.37]$ | $[-0.742,0.742]$ |

Table 1. Sensitivities achievable at the $95 \%$ C.L. for the $h_{1,2}^{Z \gamma}$ vertices in the process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ with a luminosity of $100 \mathrm{pb}^{-1}$. We have applied the cuts used by L3 for the photon angle and energy.
We plot the total cross section in Fig. 2 as a function of the Higgs boson mass $M_{H}$ for the bounds of $h_{1}^{Z \gamma}=0.047$ and $h_{2}^{Z \gamma}=0.081$ given in Table 1. We observe in this figure that the cross section of the process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ decrease with the increase of the Higgs bosons mass $M_{H}$


FIgURE 2: Cross-section of the process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ as a function of $M_{H}$ with $h_{1}^{Z \gamma}=0.047$ and $h_{2}^{Z \gamma}=0.081$.
In conclusion, we study the sensitivity for the anomalous triple coupling $H Z \gamma$ via the process $e^{+} e^{-} \rightarrow \nu \bar{\nu} \gamma$ at high energy physics linear colliders. We get limits on the anomalous $H Z \gamma$ vertex of the order $10^{-2}$, which are better by an order of magnitude than the bounds obtained from the known limits on the partial decay widths of the $Z$ boson, but still an order of magnitude above the SM prediction. On the other hand, the total cross section of the process $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-} \gamma$ decrease with the increase of the Higgs bosons mass $M_{H}$.

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