*M*easurement of the decay $B^0 \to \pi^- \ell^+ \nu$ in untagged events and determination of $|V_{ub}|$ at Belle

Hyuncheong Ha

(Korea University, Seoul, Korea)

on behalf of the Belle collaboration

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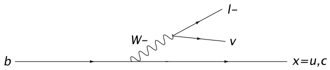
- Outline : 1. Introduction

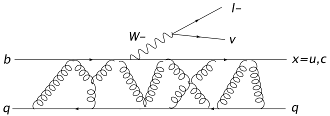
2. Analysis strategy
3. Branching fraction
4. Systematics
5. $|V_{ub}|$ with form factors
6. $|V_{ub}|$ from lattice prediction
7. Summary

1. Introduction (1) : Form factor

- $e^+e^- \to \Upsilon(4S) \to B\bar{B}$, the tree level process of the decay $\Gamma(b \to xl\nu)$ is



- Due to the strong interaction, the real process may be more complicated and "form factors" are introduced.



- For $x = u$, especially for $B \to \pi l\nu$ decay chain,

$$\frac{d\Gamma(B \to \pi l\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_b^3} \lambda(q^2)^{3/2} |f_+^\pi(q^2)|^2,$$

where

$$\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2,$$
$$q^2 = (p_\ell + p_\nu)^2 = (p_B - p_\pi)^2$$

and $f_+(q^2)$ describes semi-leptonic B decays.

1. Introduction (2) : Loose neutrino reconstruction

- Untagged B reconstruction with no strict neutrino quality cuts.

1. Pros.
   - Wide signal region : $M_{bc} > 5.19$ GeV, $|\Delta E| < 1$ GeV
   - High signal efficiency : enough to divide q^2 into many narrow sub-regions, 13-bins(2 GeV) in our case.
   - Direct comparison between experimental data and theoretical predictions. (i.e., shapes and also full q^2 region extrapolation after the unfolding.)

cf. Lattice QCD predicts the form factor for $q^2 > q^2_{max}$ where $q^2_{max} \sim 16$ GeV^2.

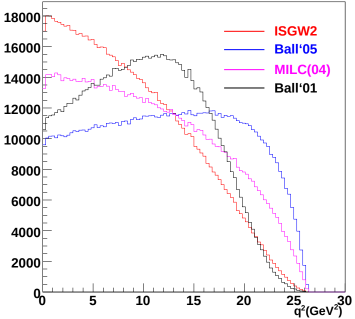
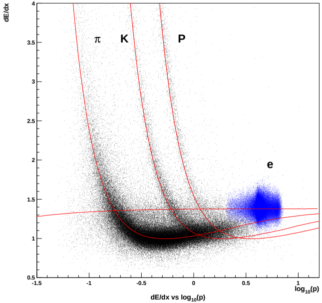


Figure: Generated q^2 distributions.

2. Cons.
   - Larger backgrounds expected
   - Need to understand backgrounds fully (i.e., $b \to c$ in low q^2, $b \to u$ transition bkg. in high q^2)

2. Analysis strategy (1) : Reconstruction of B events

- $B^0 \to \pi^- \ell^+ \nu$ decay has two charged tracks coming from IP.
- Single charged pion and lepton track is identified.
- Neutrino reconstruction from missing momentum in the center-of-mass frame.



$$E_{miss} \equiv 2E_{beam} - \sum_i E_i, \qquad \vec{p}_{miss} \equiv -\sum_i \vec{p}_i$$

, where i refers to all charged tracks and neutral clusters in event, and E_{beam} is the energy of the electron and positron beams in the center-of-mass frame.

And the missing momentum of the event is

$$p_\nu = p_{miss} = (\, \vec{p}_{miss}, \, |\vec{p}_{miss}| \,)$$

- The beam constrained mass, $M_{\rm bc}$, is defined as

$$M_{\rm bc} = \sqrt{E_{beam}^2 - |\, \vec{p}_\pi + \vec{p}_\ell + \vec{p}_\nu|^2}$$

- The energy imbalance in the center of mass frame, the ΔE is,

$$\Delta E = E_{beam} - (\, E_\pi + E_\ell + E_\nu \,)$$

2. Analysis strategy (2) : q^2 resolutions

- The momentum transfer q^2 is defined as,

$$q^2 = (P_\ell + P_\nu)^2 = (P_B - P_\pi)^2$$

- The momentum resolution is measured and it is used to divide full q^2 regions into 13 q^2 bins.

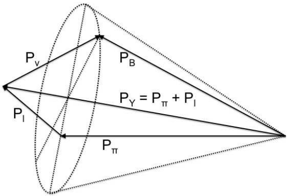


Figure: q^2 is calculated as the weighted average along the cone (Y-average q^2).

- Estimated q^2 resolution is,

$$\sigma_{q^2} \sim 0.5\ \mathrm{GeV}^2/c^4.$$

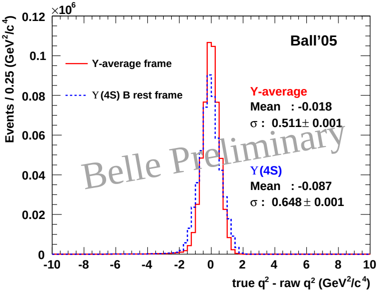


Figure: The comparison of q^2 resolutions between the $\Upsilon(4S)$ rest frame and Y-average frame for $B^0 \to \pi^- \ell^+ \nu$ decay signal Monte-Carlo.

2. Analysis strategy (3) : Yield fit and projections

- The 2+1 (M_{bc}-ΔE, q^2) dimensional simultaneous fit; data sample : 605 fb^{-1}.
- 13 signal, 3 $X_u \ell \nu$ and 4 $X_c \ell \nu$, total 20 parameters are floated. (continuum is fixed.)

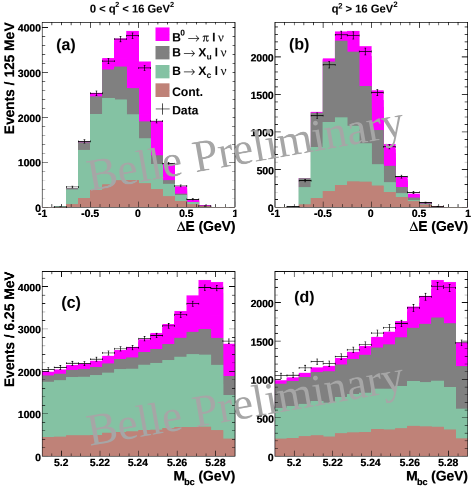


Figure: The fit projections to ΔE in (a) and (b) with $M_{\rm bc} > 5.27$ GeV, and to $M_{\rm bc}$ with $|\Delta E| < 0.125$ GeV in (c) and (d).

3. Branching fraction (1) : Unfolding and efficiencies

- The partial branching fraction, $\Delta\mathcal{B}$ is defined as

$$\Delta\mathcal{B}_i = \frac{D^{-1}Y_i}{0.5 \times 2 \times 2 \times nBB \times effi.}$$

D : response matrix (13×13)
Y_i : signal(e+μ) yield of i-th q^2 bin,
where i = 0,1,2,..,13-th q^2 bins.

- The signal yields are unfolded from direct inversion of the detector response matrix (unregularization).

q^2 bin (GeV$^2/c^4$)	efficiency (%)
0 - 2	7.66 ± 0.04
2 - 4	9.35 ± 0.05
4 - 6	10.44 ± 0.05
6 - 8	11.13 ± 0.05
8 - 10	10.89 ± 0.05
10 - 12	11.89 ± 0.05
12 - 14	11.78 ± 0.05
14 - 16	12.50 ± 0.06
16 - 18	10.33 ± 0.05
18 - 20	10.47 ± 0.05
20 - 22	11.74 ± 0.06
22 - 24	12.63 ± 0.07
24 -	14.98 ± 0.11

Table: The signal efficiencies for 13 q^2 bins.

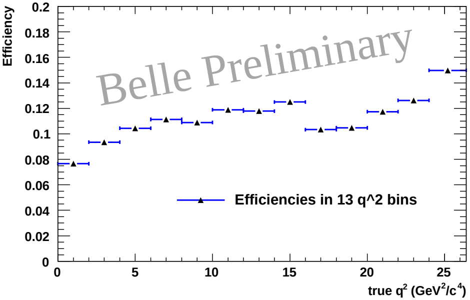


Figure: The signal efficiencies for 13 q^2 bins.

3. Branching fraction (2) : $\Delta\mathcal{B}$ & $\mathcal{B}$

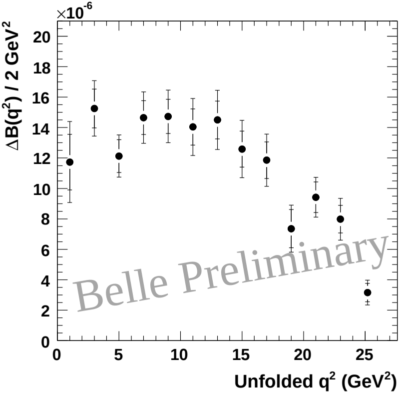
- The total branching fraction and error with 605 fb^{-1} data sample, is calculated from,

$$\mathcal{B} = \sum_{i=1}^{13} \Delta\mathcal{B}_i$$

and

$$\sigma_{\mathcal{B}}^2 = (\sigma_{\Delta\mathcal{B}})_i^T \, V_{ij}^{cor} \, (\sigma_{\Delta\mathcal{B}})_j,$$

using covariance matrix for 13 q^2 bins.



Preliminarily,

$$\mathcal{B} = (1.49 \pm 0.04_{\mathrm{stat}} \pm 0.07_{\mathrm{syst}}) \times 10^{-4}$$

4. Systematics

- Various sources from detector parameters, physical assumptions for Monte-Carlo samples are estimated using pseudo experiments.

q^2(GeV^2/c^4)	0 - 16	16 - 26.4	Total	q^2(GeV^2/c^4)	0 - 16	16 - 26.4	Total
$\Delta\mathcal{B}$ ($\times$ 10^7)	1096	397	1494	$\Delta\mathcal{B}$ ($\times$ 10^7)	1096	397	1494
Lepton ID	2.40	2.49	2.44	$B \to \rho\ell\nu$ BF	0.44	0.42	0.43
Pion ID	1.37	1.08	1.26	$B \to \omega\ell\nu$ BF	0.11	0.31	0.16
Tracking efficiency	2.00	2.09	2.04	$B \to b_1\ell\nu$ BF	0.14	0.14	0.14
γ efficiency	0.37	0.51	0.42	V_{ub} + other $X_u\ell\nu$	0.19	0.15	0.15
Detector effects	3.43	3.46	3.44	$B \to D^*\ell\nu$ BF	0.18	0.13	0.16
				$B \to D\ell\nu$ B	0.07	0.14	0.08
Cont. Correction	2.14	2.62	1.80	$B \to D^{**}\ell\nu$ BF	0.11	0.22	0.13
				Other $X_c\ell\nu$	0.06	0.13	0.06
$\Upsilon(4S) \to B^0\bar{B}^0$ BF	1.56	1.72	1.40	**Physics parameters(BF)**	0.56	0.64	0.55
Signal MC stat. error	0.12	0.39	0.15				
FSR	0.45	0.60	0.37	$B^0 \to \pi^-\ell^+\nu$ FF	0.63	0.86	0.53
B counting	1.36	1.36	1.36	$B^0 \to \rho^-\ell^+\nu$ FF	0.72	0.95	0.60
Other sources	2.12	2.30	1.99	SF parameter	0.71	1.17	0.63
				$B^0 \to D^{*-}\ell^+\nu$ FF	0.81	1.01	0.62
				$B^0 \to D^-\ell^+\nu$ FF	0.11	0.16	0.10
				Physics parameters(FF)	1.28	1.77	1.07
				Total systematics	4.78	5.26	4.53
				Total statistics	3.03	5.31	2.63
				Total error	5.66	7.47	5.23

Table: The partial branching fractions and the relative errors (%) in low,high q^2 ranges.

5. $|V_{ub}|$ with form factors $_{(1)}$: BK parameterization

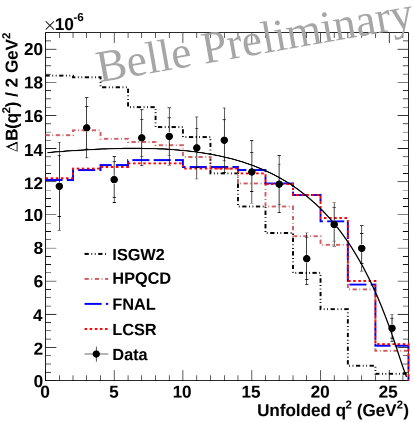
- The BK parameters are fitted to the measured $\Delta\mathcal{B}$ distribution.

$\lvert V_{ub} f_+(0)\rvert$	$(\,9.24 \pm 0.18(stat) \pm 0.20(syst) \pm 0.07(\tau_{B^0})\,) \times 10^{-4}$
α	$0.60 \pm 0.03(stat) \pm 0.02(syst) \pm 0.01(\tau_{B^0})$
Prob.	62 %

- And χ^2 with the measured branching fraction and various theoretical form factor predictions are compared.

Model	χ^2 probability
HPQCD	42 %
FNAL	43 %
LCSR	49 %
ISGW2	0 %

, ISGW2 can be excluded.



5. $|V_{ub}|$ with form factors (2)

- $|V_{ub}|$ can then be extracted from the measured partial branching fractions, $\Delta\mathcal{B}(q^2)$ and the predicted normalized partial decay rates, $\Delta\zeta = \Gamma/|V_{ub}|^2$

$$|V_{ub}| = \sqrt{\Delta\mathcal{B}(q^2)/(\tau_{B^0}\Delta\zeta)}$$

, where τ_{B^0} is the B^0 lifetime (τ_{B^0} = 1.530 $\pm$ 0.009 ps) and some predictions are given for limited q^2 ranges.

- $|V_{ub}|$ values from form factors.

	q^2 (GeV^2)	$\Delta\zeta$ (ps^{-1})	$\lvert V_{ub}\rvert(10^{-3})$
HPQCD	> 16	2.07 ± 0.57	$3.55 \pm 0.09 \pm 0.09\,^{+0.62}_{-0.41}$
FNAL	> 16	1.83 ± 0.50	$3.78 \pm 0.10 \pm 0.10\,^{+0.65}_{-0.43}$
LCSR	< 16	5.44 ± 1.43	$3.64 \pm 0.06 \pm 0.09\,^{+0.60}_{-0.40}$
ISGW2	all	9.6 ± 4.8	$3.19 \pm 0.04 \pm 0.07\,^{+1.32}_{-0.59}$

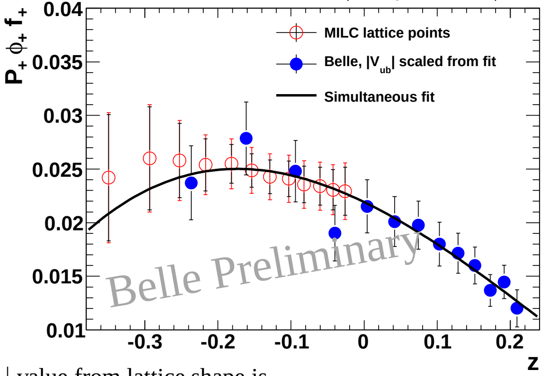
- The uncertainties from form factors are dominant.

6. $|V_{ub}|$ from lattice prediction : Belle + MILC

- With Belle 13 q^2 bins and MILC 12 q^2 bins data (R.Water,PRD79,054507), 5 parameters are free in fit in z-expansion.

$$f(|V_{ub}|; f(z)) = f(|V_{ub}|; a_0 + a_1 z + a_2 z^2 + a_3 z^3)$$

$$\chi^2/n.d.f = 11.78/20$$



- The fitted $|V_{ub}|$ value from lattice shape is,

$$|V_{ub}| \times 10^3 = 3.43 \pm 0.33.$$

7. Summary $_{(1)}$: $|V_{ub}|$, form factor *vs.* lattice prediction

- $|V_{ub}|$ values from form factors.

	q^2 (GeV^2)	$\Delta\zeta$ (ps^{-1})	$\lvert V_{ub}\rvert(10^{-3})$
HPQCD	> 16	2.07 ± 0.57	$3.55 \pm 0.09 \pm 0.09\,^{+0.62}_{-0.41}$
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LCSR	< 16	5.44 ± 1.43	$3.64 \pm 0.06 \pm 0.09\,^{+0.60}_{-0.40}$
ISGW2	all	9.6 ± 4.8	$3.19 \pm 0.04 \pm 0.07\,^{+1.32}_{-0.59}$

- $|V_{ub}|$ value from lattice prediction is, $|V_{ub}| \times 10^3 = 3.43 \pm 0.33$.

- The total uncertainty from two methods,

LCSR model	$\sim$ 20.0 %	Belle
$\lvert V_{ub}\rvert$ extraction fit	$\sim$ 9.6 %	Belle + MILC
	$\sim$ 10.7 %	BABAR + MILC (PRD79, 054507)

7. Summary (2)

- Preliminary branching fraction,

  $\mathcal{B} = (1.49 \pm 0.04_{stat} \pm 0.07_{syst}) \times 10^{-4}$ is obtained.

- Statistical and systematics uncertainties in q^2 bins and its correlations are presented.

- BK parameters from fit to Belle data are,

  $|V_{ub}f_+(0)|$ = (9.24 $\pm$ 0.18(*stat*) $\pm$ 0.20(*syst*) $\pm$ 0.07(τ_{B^0})) $\times 10^{-4}$

  and α = 0.60 $\pm$ 0.03(*stat*) $\pm$ 0.02(*syst*) $\pm$ 0.01(τ_{B^0})

- $|V_{ub}|$ values from form factors are calculated,

  $|V_{ub}| \times 10^3 = 3.64 \pm 0.06 \pm 0.09^{+0.60}_{-0.40}$ for LCSR.

- $|V_{ub}|$ extraction fit from Belle and MILC data is

  presented as $|V_{ub}| \times 10^3 = 3.43 \pm 0.33$.

Thanks

Back up

1. Introduction : Extraction of $|V_{ub}|$

- Many theoretical predictions are available from LCSR, LQCD, etc,.
- For example, using the BK(Becirevic, Kaidalov) parameterization, the form factor is parameterized as

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*})(1 - \alpha q^2/m_{B^*})}$$

and two parameters $f_+(0)$ (or, $|V_{ub}\, f_+(0)|$) and α describe the shape.

- Then, the $|V_{ub}|$ can be extracted from the measure partial branching fractions, $\Delta\mathcal{B}(q^2)$ and predicted normalized partial decay rate, $\Delta\zeta = \Gamma/|V_{ub}|^2$

$$|V_{ub}| = \sqrt{\Delta\mathcal{B}(q^2)/(\tau_{B^0}\Delta\zeta)}$$

, where τ_{B^0} is the B^0 lifetime (τ_{B^0} = 1.530 $\pm$ 0.009 ps) and some predictions are dominant in limited q^2 regions.

- $|V_{ub}|$ extraction fit : as normalization, $|V_{ub}| \times f_+$ from experimental data *vs*. f_+ from theoretical calculation.

2. Analysis strategy : Background study

- The backgrounds are categorized as $b \to u(X_u \ell \nu)$, $b \to c(X_c \ell \nu)$ transitions and continuum
- These backgrounds are suppressed throughout analysis cut studies from $S/\sqrt{S+B}$

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1. Good pion and lepton : Belle standard PID

- lepton ID : e and μ track selected
- pion ID : K veto only
- $|dr| < 2.0\ cm, \quad |dz| < 4.0\ cm$
- Vertex Probability > 0.01

2. Physics quality cuts

- $|\cos\theta_{BY}| < 1.$
- $R2 < 0.35$
- $p_e > 0.8, \quad p_\mu > 1.1\ GeV$ (in lab.)
- $3.075 < Y_{mass} < 3.125\ GeV$ vetoed
  (fake leptons from $J/\psi \to \mu\mu$)
- FSR(final state radiation) removal

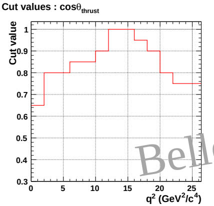
3. And, instead of tight neutrino quality cuts, loose q^2 dependent cuts($cont'd$)

- cos_{thrust}
- $cos\theta_\ell$
- θ_{miss}
- Missing mass squared (M^2_{miss})

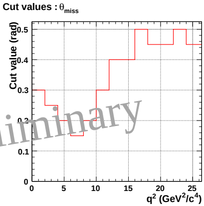
2. Analysis strategy : q^2 dependent cuts

- Four physics variables are studied in 13 q^2 bins
- Loose cuts for neutrino reconstruction, but guarantee to divide many q^2 bins

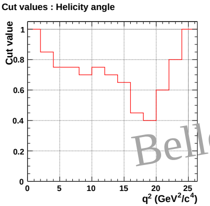
1) thrust angle : $\cos\theta_{trust}$



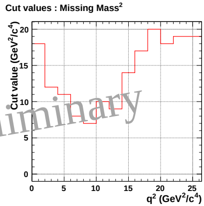
2) helicity angle of lepton : $\cos\theta_{\ell}$



3) missing angle : θ_{miss}

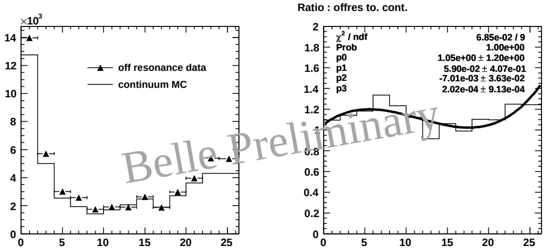


4) and M^2_{miss}



2. Analysis strategy : Correction of continuum MC

- The weight function is obtained by 3rd order polynomial fitting to the ratio between off-resonance and continuum MC.



- Then, this weights are applied to $M_{\rm bc}$-ΔE $p.d.f.$ and the fit is repeated for PULL test and then, for real data sample.

- In result, with this tuned MC, the fitter passed all minimization successfully and we could get correct yields and correlation.

2. Analysis strategy : Raw, unfolded yields and $\Delta\mathcal{B}$ and its relative statistical errors

The signal yields, unfolded yields, partial branching fractions, total branching fraction and its errors with corrected *continuum* Monte-Carlo samples are summarized as below,

$q^2(\mathrm{GeV}^2/c^4)$	Raw	Err(%)	Unfolded	Err(%)	$\Delta\mathcal{B}(\times 10^7)$	Err (%)
0-2	1225.8	12.4	1179.9	15.3	117.3	15.3
2-4	1788.5	6.0	1873.6	8.3	152.6	8.3
4-6	1756.5	5.5	1662.6	8.5	121.3	8.5
6-8	2074.8	5.2	2141.6	7.2	146.5	7.2
8-10	2100.6	5.2	2107.3	7.3	147.3	7.3
10-12	2136.4	6.1	2192.3	7.9	140.4	7.9
12-14	2139.0	6.5	2244.0	8.3	145.0	8.3
14-16	1951.7	7.4	2066.7	9.2	125.9	9.2
16-18	1538.0	8.0	1609.1	9.7	118.6	9.7
18-20	1070.3	12.3	1011.9	16.7	73.6	16.7
20-22	1422.8	8.5	1452.8	10.5	94.2	10.5
22-24	1369.8	8.7	1323.4	11.1	79.8	11.1
24-	912.3	11.3	621.3	19.0	31.6	19.0
Total	21486.5	2.6	21486.5	2.6	1494.1	2.6

Table: The signal yields, unfolded yields, partial branching fractions and its relative errors for 13 q^2 bins.

2. Analysis strategy : Correlation matrix for statistics

Table: The correlation matrix between 13 q^2 bins of $\Delta\mathcal{B}(B^0 \to \pi^- \ell^+ \nu)$ signal for statistical uncertainties.

$q^2(\text{GeV}^2)$	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24	24-
0-2	1.000	-0.335	0.149	0.052	0.030	0.032	0.048	-0.008	-0.002	-0.003	-0.004	-0.007	-0.014
2-4	-0.225	1.000	-0.326	0.200	-0.009	0.033	0.037	-0.006	-0.001	-0.003	-0.005	-0.004	-0.006
4-6	0.080	-0.261	1.000	-0.244	0.163	0.056	0.114	-0.017	-0.003	-0.005	-0.005	-0.005	-0.004
6-8	0.025	0.141	-0.215	1.000	-0.250	0.131	0.068	-0.010	-0.003	-0.004	-0.003	-0.003	-0.004
8-10	0.015	-0.006	0.149	-0.261	1.000	-0.170	0.243	-0.037	-0.001	-0.004	-0.004	-0.005	-0.008
10-12	0.013	0.020	0.043	0.115	-0.143	1.000	-0.053	0.024	-0.006	-0.002	-0.006	-0.007	-0.011
12-14	0.020	0.023	0.090	0.061	0.208	-0.054	1.000	-0.254	0.006	-0.025	-0.007	-0.011	-0.016
14-16	-0.003	-0.003	-0.012	-0.008	-0.028	0.021	-0.226	1.000	-0.011	0.120	-0.028	-0.006	-0.011
16-18	-0.001	-0.001	-0.003	-0.003	-0.001	-0.007	0.008	-0.016	1.000	0.102	-0.032	-0.003	-0.006
18-20	-0.001	-0.002	-0.005	-0.004	-0.005	-0.002	-0.031	0.171	0.099	1.000	-0.188	-0.030	-0.057
20-22	-0.002	-0.003	-0.004	-0.002	-0.003	-0.007	-0.007	-0.032	-0.025	-0.149	1.000	-0.038	0.007
22-24	-0.002	-0.002	-0.003	-0.002	-0.003	-0.006	-0.010	-0.006	-0.002	-0.021	-0.033	1.000	-0.132
24-	-0.004	-0.002	-0.002	-0.002	-0.004	-0.007	-0.010	-0.008	-0.003	-0.028	0.004	-0.094	1.000

4. Systematics in 13 q^2 bins

Table: The partial branching fractions and the relative errors (%) from the fit and various systematic sources in 13 q^2 bins and the total branching fraction and the relative (%) error.

q^2(GeV^2)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24	24-26.4	0-16	16-26.4	Total
$\Delta\mathcal{B}\,(\times 10^7)$	117.33	152.58	121.28	146.54	147.32	140.39	145.00	125.90	118.57	73.59	94.21	79.80	31.59	1096.34	397.75	1494.09
Lepton ID	2.35	2.41	2.39	2.41	2.38	2.38	2.43	2.47	2.47	2.49	2.45	2.44	2.56	2.40	2.49	2.44
Pion ID	1.32	1.37	1.43	1.49	1.41	1.36	1.32	1.24	1.11	0.97	0.85	1.12	1.38	1.37	1.08	1.26
Tracking efficiency	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.01	2.45	2.00	2.09	2.04
γ efficiency	0.21	0.41	0.14	0.25	0.32	0.45	0.94	0.24	0.81	0.80	0.23	0.26	0.49	0.37	0.51	0.42
$B \to \rho\ell\nu$ BF	0.58	0.60	0.59	0.46	0.74	0.60	0.41	0.57	0.48	0.47	0.41	0.41	0.33	0.44	0.42	0.43
$B \to \omega\ell\nu$ BF	0.28	0.16	0.14	0.12	0.13	0.11	0.16	0.08	0.09	0.12	0.30	0.52	1.19	0.11	0.31	0.16
$B \to b_1\ell\nu$ BF	0.30	0.16	0.14	0.12	0.13	0.12	0.13	0.09	0.11	0.12	0.11	0.14	0.59	0.14	0.14	0.14
V_{ub} + other $X_u\ell\nu$ BF	4.43	1.55	0.96	0.87	0.45	0.35	0.45	0.61	0.13	0.13	0.12	0.17	0.77	0.19	0.15	0.15
$B \to D^*\ell\nu$ BF	0.42	0.40	0.15	0.67	0.09	0.12	0.20	0.09	0.16	0.12	0.12	0.14	0.27	0.18	0.13	0.16
$B \to D\ell\nu$ BF	0.27	0.14	0.14	0.12	0.12	0.09	0.07	0.12	0.09	0.14	0.14	0.18	0.61	0.07	0.14	0.08
$B \to D^{**}\ell\nu$ BF	0.20	0.16	0.16	0.14	0.14	0.12	0.11	0.11	0.42	0.15	0.14	0.20	0.11	0.11	0.22	0.13
Other $X_c\ell\nu$ BF	0.13	0.09	0.13	0.09	0.18	0.12	0.13	0.09	0.14	0.14	0.14	0.20	0.12	0.06	0.13	0.06
$B^0 \to \pi^-\ell^+\nu$ FF	3.58	1.64	1.26	1.27	1.44	1.57	1.67	1.70	1.78	1.97	1.61	1.92	4.03	0.63	0.86	0.53
$B^0 \to \rho^-\ell^+\nu$ FF	3.59	1.73	1.51	1.47	1.64	1.82	2.04	1.89	2.01	2.30	1.99	2.50	4.98	0.72	0.95	0.60
SF parameter	1.44	0.63	1.59	1.07	2.10	2.80	2.52	2.42	2.24	4.02	2.12	3.15	4.66	0.71	1.17	0.63
$B^0 \to D^{*-}\ell^+\nu$ FF	0.51	0.64	0.60	0.89	1.54	1.77	2.51	0.81	1.18	0.56	0.51	0.42	0.98	0.48	0.34	0.36
$B^0 \to D^-\ell^+\nu$ FF	0.33	0.09	0.20	0.30	0.24	0.12	0.26	0.20	0.20	0.09	0.16	0.11	0.11	0.10	0.08	0.08
$\Upsilon(4S) \to B^0B^0$ BF	2.11	1.39	2.41	3.28	3.68	3.90	3.67	2.76	2.83	4.58	3.62	5.21	2.28	1.56	1.72	1.40
Signal MC stat. error	0.48	0.15	0.23	0.27	0.30	0.24	0.28	0.28	0.38	0.52	0.58	0.74	1.97	0.12	0.39	0.15
FSR	0.31	0.58	1.03	0.88	0.93	1.24	1.18	1.43	1.01	1.23	1.15	0.78	0.66	0.45	0.56	0.37
B counting	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36
continuum q^2	13.33	3.64	3.01	4.35	4.91	6.62	6.04	7.42	6.22	6.60	4.67	7.13	5.90	2.14	2.62	1.80
Total systematic error	15.63	6.22	6.14	7.19	8.04	9.56	9.27	9.60	8.71	10.30	7.80	10.63	11.23	4.78	5.26	4.53
Total statistic error	15.35	8.27	8.50	7.24	7.27	7.91	8.32	9.19	9.67	16.68	10.54	11.12	18.98	3.03	5.31	2.63
Total error	21.90	10.35	10.48	10.20	10.84	12.41	12.46	13.29	13.01	19.60	13.12	15.39	22.08	5.66	7.47	5.23

Table: The correlation matrix between 13 q^2 bins of $\Delta\mathcal{B}(B^0 \to \pi^- \ell^+ \nu)$ signal for systematical uncertainties.

q^2(GeV^2)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-24	24-
0-2	1.000	-0.256	0.187	-0.162	0.297	0.181	0.224	0.114	0.104	0.112	0.084	0.020	0.069
2-4	-0.256	1.000	0.142	0.570	0.075	0.163	0.162	0.193	0.202	0.210	0.244	0.303	0.282
4-6	0.187	0.142	1.000	0.202	0.459	0.451	0.469	0.212	0.368	0.329	0.332	0.322	0.336
6-8	-0.162	0.570	0.202	1.000	-0.017	0.240	0.202	0.280	0.256	0.284	0.312	0.333	0.272
8-10	0.297	0.075	0.459	-0.017	1.000	0.375	0.633	0.156	0.321	0.290	0.258	0.244	0.231
10-12	0.181	0.163	0.451	0.240	0.375	1.000	0.433	0.214	0.328	0.332	0.252	0.284	0.230
12-14	0.224	0.162	0.469	0.202	0.633	0.433	1.000	-0.013	0.337	0.278	0.291	0.245	0.246
14-16	0.114	0.193	0.212	0.280	0.156	0.214	-0.013	1.000	0.334	0.500	0.287	0.337	0.322
16-18	0.104	0.202	0.368	0.256	0.321	0.328	0.337	0.334	1.000	0.452	0.334	0.356	0.344
18-20	0.112	0.210	0.329	0.284	0.290	0.332	0.278	0.500	0.452	1.000	0.208	0.430	0.333
20-22	0.084	0.244	0.332	0.312	0.258	0.252	0.291	0.287	0.334	0.208	1.000	0.222	0.402
22-24	0.020	0.303	0.322	0.333	0.244	0.284	0.245	0.337	0.356	0.430	0.222	1.000	0.220
24-	0.069	0.282	0.336	0.272	0.231	0.230	0.246	0.322	0.344	0.333	0.402	0.220	1.000

6. $|V_{ub}|$ from lattice prediction

- Before, $|V_{ub}|$ is dependent to $\Delta\zeta$ values.
- $|V_{ub}|$ extraction fit. (Phys. Rev. D 79, 054507 (2009))

  By combining
  $\rightarrow$ 1) experimentally measured $|V_{ub}| \times f_+(q^2)$ shape,
  $\rightarrow$ 2) theoretically calculated $f_+(q^2)$ shape

  by fitting, $|V_{ub}|$ can be extracted directly.

  Actually, it is done by introducing new variable z,

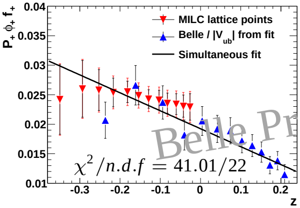
$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}}$$

  , where $t_+ \equiv (m_B + m_\pi)^2$, $t_- \equiv (m_B - m_\pi)^2$, and t_0 is a free parameter. And then, the form-factor $f_+(q^2)$ can be expressed as

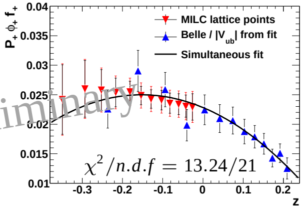
$$f_+(q^2) = \frac{1}{P_+(q^2)\phi_+(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(q^2, t_0)^k$$

6. $|V_{ub}|$ from lattice prediction

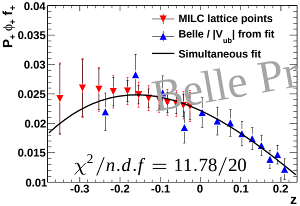
- With various polynomial f_+ functions



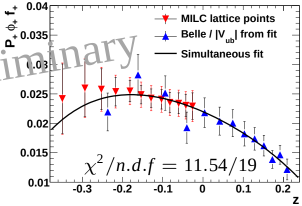
1st order : $|V_{ub}| \times 10^3 = 3.64 \pm 0.36$



2nd order : $|V_{ub}| \times 10^3 = 3.33 \pm 0.31$



3rd order : $|V_{ub}| \times 10^3 = 3.43 \pm 0.33$



4th order : $|V_{ub}| \times 10^3 = 3.43 \pm 0.28$

References

- Model dependent $|V_{ub}|$ calculation

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- $|V_{ub}|$ extraction fit

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