Power Suppressed Effects in $\bar{B} \rightarrow X_s \gamma$ at $\mathcal{O}(\alpha_s)$

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Based on:
Plan of Talk

- **Motivation:** Rate and Moments
- **Matching (OPE):** Tree and at $\mathcal{O}(\alpha_s)$
- **Results:** $\mathcal{O}(\alpha_s \frac{\Lambda_{QCD}^2}{m_b^2})$ corrections to $\Gamma_{77}(\bar{B} \rightarrow X_s\gamma)$ and moments
- **Summary**
Radiative Decay $b \to s\gamma$: Status

- Sensitive to New Physics
- Moreover, inclusive $\bar{B} \to X_s\gamma$ rate is well approximated by the perturbatively calculable radiative decay rate of the $b$-quark

$$\Gamma(b \to X_s\gamma)_{E_{\gamma}>E_0} = \frac{G_F^2 m_b^5 \alpha_{em}}{32 \pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

Wilson Coefficients $C_i(\mu_b)$ are known at NNLO $\Rightarrow$

$|C_{1,2}(\mu_b)| \sim 1$, $|C_{3,4,5,6}(\mu_b)| < 0.07$, $|C_7(\mu_b)| \sim -0.3$, $|C_8(\mu_b)| \sim -0.15$

$G_{ij}(E_0, \mu_b) \Rightarrow$ Matrix elements of $O_1, \ldots, O_8$

Perturbative NLO $\Rightarrow G_{ij}$ are fully known

Perturbative NNLO $\Rightarrow G_{ij}$ ($i,j = 1,2,7,8$) have been considered so far, $G_{77}$ and $G_{78}$ are known in a complete manner $\Rightarrow$

For Complete NNLO $G_{78}$ see [hep-ph/9903305, arXiv:0805.3911, 1005.5587]

see M. Misiak [arxiv:0808.3134]
The inclusive branching ratio: SM and Exp

\[ \mathcal{B}(\bar{B} \to X_s \gamma)^{NNLO}_{E_\gamma > 1.6 \text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4} \]  
[hep-ph/0609232]

\[ \mathcal{B}(\bar{B} \to X_s \gamma)^{exp}_{E_\gamma > 1.6 \text{ GeV}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4} \]  
[HFAG]

SM prediction is consistent with Experiment ⇒ both have ±7% error

⇒ Useful to constrain many extensions of the SM

**Theoretical error**: 5% (assumed) from non-perturbative dynamics, mainly from

\[ \mathcal{O}(\alpha_s \frac{\Lambda_{QCD}}{m_b}) \]  ⇒ So far quantitative estimate is about 0.7% for detail see [arxiv:0911.1651] & [arxiv:1003.5012]

- Experimental uncertainty is expected to reduce to 5% by the end of \( B \) factory era
- It is desirable to reduce the theoretical uncertainty as much as possible both perturbative and non-perturbative
Photon energy spectrum is largely insensitive to NP
Useful for precision studies of perturbative and non-perturbative strong interaction effects
First moment $\langle E_\gamma \rangle \sim m_b/2 \Rightarrow$ Measured spectrum allows precise measurement of the value of bottom quark mass
Second moment is sensitive to average kinetic energy $\mu_\pi^2$
Measurements of $m_b$ and $\mu_\pi^2$ using $B \to X_s \gamma$ are complementary to the determinations using the inclusive moments of $B \to X_c \ell \bar{\nu}$
Contribute in an important way to the global fits for extraction of $V_{cb}$ and $V_{ub}$

$\Rightarrow \frac{1}{m_b^3}$ corrections were studied
$\Rightarrow$ Perturbative corrections of order $\beta_0 \alpha_s^2$
$\Rightarrow$ All order resummation of $\beta_0^{-1} \alpha_s^n$
$\Rightarrow$ Improved predictions based on QCDF and MSOPE

C. W. Bauer [hep-ph/9710513]
Ligeti et. al. [hep-ph/9903305]
Benson et. al [hep-ph/0410080]
M. Neubert [hep-ph/0506245]

Progress on the theory front improved our understanding of photon spectrum $\Rightarrow$ Uncertainties of both perturbative and non-preturbative origin remain $\Rightarrow$ Need further investigations
Non-perturbative corrections to $O_7$ is known through $\frac{1}{m_b^3}$ [hep-ph/9308288] & [hep-ph/9710513]

- We present the first calculation for non-perturbative corrections to the operator $O_7$ at $O(\alpha_s)$ in $\bar{B} \to X_s \gamma$

- We compute the $O(\alpha_s)$ corrections to the Wilson coefficients of the dimension five operators emerging from the OPE of inclusive radiative $B$ decay:
  - off-shell amputated Green functions has been expanded around $b$ quark mass shell
  - matched them onto local operators in Heavy Quark Effective Theory (HQET)

- Finally we discuss the impact of the resulting $O(\alpha_s \frac{\Lambda^2_{QCD}}{m_b^2})$ corrections on the extractions of $m_b$ and $\mu_\pi^2$ from the moments of the photon spectrum
Matching : OPE

Differential decay rate that is induced by $O_7$ self-interference

\[
d\Gamma_{77}(\bar{B} \rightarrow X_s \gamma) = \frac{G_F^2 \alpha_{em} m_b^2(\mu)}{16\pi^3 m_B} |V_{tb}V_{ts}^*|^2 |C_7^{\text{eff}}(\mu)|^2 \frac{d^3 q}{(2\pi)^3 2E_\gamma} W_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta}
\]

\[
P^{\mu\nu\alpha\beta} = \sum_{\lambda = \pm 1} \langle 0 | F^{\mu\nu} | \gamma(q, \lambda) \rangle \langle \gamma(q, \lambda) | F^{\alpha\beta} | 0 \rangle \Rightarrow \text{Photonic Tensor}
\]

Hadronic Tensor:

\[
W_{\mu\nu\alpha\beta} = 2 \text{Im} \left( i \int d^4 x \ e^{-i q \cdot x} \langle \bar{B}(p_B) | T\{ \bar{b}(x) \sigma_{\mu\nu} P_L s(x) \bar{s}(0) \sigma_{\alpha\beta} P_R b(0) \} | \bar{B}(p_B) \rangle \right)
\]

\[
W_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta} = -16\pi m_b \left( c_{\text{dim} 3} O_{\text{dim} 3} + \frac{1}{m_b} c_{\text{dim} 4} O_{\text{dim} 4} + \frac{1}{m_b^2} c_{\text{dim} 5} O_{\text{dim} 5} + \ldots \right)
\]

\[
c_{\text{dim} n} = c^{(0)}_{\text{dim} n} + \frac{\alpha_s}{4\pi} c^{(1)}_{\text{dim} n} + \ldots
\]
HQET Lagrangian:
\[
\mathcal{L}_{\text{HQET}} = i \bar{b}_v x \cdot Db_v + \frac{1}{2m_b} \bar{b}_v (iD)^2 b_v - a(\mu) \frac{g_s}{4m_b} \bar{b}_v \sigma_{\mu \nu} G^{\mu \nu} b_v + O \left( \frac{1}{m_b^2} \right)
\]
\[
D^{\mu}_{\perp} = D^{\mu} - \nu^\mu \cdot D,
\quad a(\mu) = 1 + \left[ C_F + C_A \left( 1 + \ln \frac{\mu}{m_b} \right) \right] \frac{\alpha_s}{4\pi} + \ldots
\]
\[
b(x) = e^{-i m_b \nu \cdot x} \left( 1 + \frac{i \not{\!p}}{2m_b} \right) b_v(x) + O \left( \frac{1}{m_b^2} \right)
\]
Taylor expansion of the amputated Green functions corresponding to tree level diagrams gives ⇒
\[
O_b^\mu = \bar{b} \gamma^\mu b
\]
\[
O_2^{\mu \nu} = \bar{b} v \frac{1}{2} \{ i D^\mu, i D^\nu \} b_v
\]
\[
O_1^{\mu} = \bar{b}_v i D^\mu b_v
\]
\[
O_3^{\mu \nu} = \bar{b}_v \frac{g_s}{2} G^{a \mu} \alpha \sigma^{\alpha \nu} T^a b_v
\]
Matrix elements : \[
\langle \bar{B}(p_B) | O_b^{\mu} | \bar{B}(p_B) \rangle = 2m_B v^\mu
\]
\[
\lambda_1 = \frac{1}{2m_B} \langle \bar{B}(v) | \bar{b}_v (iD)^2 b_v | \bar{B}(v) \rangle = -\frac{\mu^2}{\pi} + O(\frac{1}{m_b})
\]
\[
\lambda_2 = -\frac{1}{6m_B} \langle \bar{B}(v) | \bar{b}_v \frac{g_s}{2} G_{\mu \nu} \sigma^{\mu \nu} b_v | \bar{B}(v) \rangle = \frac{\mu^2 \alpha_s}{3} + O(\frac{1}{m_b})
\]
Decay rate for electromagnetic operator ⇒
\[
\frac{d \Gamma^{77}}{dz} = \Gamma^{(0)}_{77} \left[ c_0^{(0)} + c_{\lambda_1}^{(0)} \frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(0)} \frac{\lambda_2(\mu)}{4m_b^2} \right. + c_1^{(1)} \frac{\alpha_s(\mu)}{2m_b^2} \left. \left[ c_0^{(1)} + c_{\lambda_1}^{(1)} \frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(1)} \frac{\lambda_2(\mu)}{2m_b^2} \right] \right]
\]
\[
c_0^{(0)} = \delta(1-z), \quad c_{\lambda_1}^{(0)} = \delta(1-z) - \delta'(1-z) - \frac{1}{3} \delta''(1-z), \quad c_{\lambda_2}^{(0)} = -9 \delta(1-z) - 3 \delta'(1-z)
\]
⇒ One loop diagrams contributing to the Wilson coefficients
⇒ Sixteen additional diagrams with a gluon radiated off an internal line will also contribute

Matching Equation:

**Full side + Counter Term ⇒**

\[ f_{\mu}^{\mu} \left( z, \xi, \mu, \frac{1}{\epsilon_{IR}} \right) v_{\mu} + f_{\lambda_{1}} \left( z, \xi, \mu, \frac{1}{\epsilon_{IR}} \right) \frac{\lambda_{1}}{2m_{b}} + f_{\lambda_{2}} \left( z, \xi, \mu, \frac{1}{\epsilon_{IR}} \right) \frac{\lambda_{2}}{2m_{b}} \]

**Effective side at one loop level ⇒**

\[ -16\pi m_{b} \sum_{n=3}^{\infty} \frac{1}{m_{b}^{n-3}} \left[ c_{\text{dim} n}^{(0)} \langle O_{\text{dim} n} \rangle_{1-\text{loop}} + \left( \frac{\alpha_{s}}{4\pi} c_{\text{dim} n}^{(1)} + \delta Z_{\text{dim} n} c_{\text{dim} n}^{(0)} \right) \langle O_{\text{dim} n} \rangle_{\text{tree}} \right] \]

We need to consider only those operators for which Wilson coefficients are non-zero at tree level.
Renormalization

- Ultraviolet as well as infrared divergences are handled by dimensional regularization.

- For the self-mixing of the operator $O_7$ we use $\overline{\text{MS}}$-scheme $\Rightarrow Z_{m_b}^{\text{MS}} Z_{77}^{\text{MS}} = 1 + \frac{C_F}{\epsilon} \frac{\alpha_s}{4\pi} + \ldots$

- For the field renormalization constant of the $b$ quark we apply on-shell scheme $\Rightarrow Z_b^{\text{OS}} = 1 - C_F \left( \frac{3}{\epsilon} + 4 + 6 \ln \frac{\mu}{m_b} \right) \frac{\alpha_s}{4\pi} + \ldots$

- We use $\overline{\text{MS}}$ scheme for the operator renormalization $\Rightarrow$

$$\left[ c_{b\mu} O_{b}^{\mu} \right]^{\text{bare}} = Z_b^{\text{OS}} c_{b\mu} O_{b}^{\mu}, \quad \left[ c_{2\mu\nu} O_{2}^{\mu\nu} \right]^{\text{bare}} = Z_{b v}^{\text{OS}} Z_{\text{kin}}^{\overline{\text{MS}}, \mu\nu\alpha\beta} c_{2\mu\nu} O_{2\alpha\beta},$$

$$\left[ c_{1\mu} O_{1}^{\mu} \right]^{\text{bare}} = Z_{b v}^{\text{OS}} c_{1\mu} O_{1}^{\mu}, \quad \left[ c_{3\mu\nu} O_{3}^{\mu\nu} \right]^{\text{bare}} = Z_{b v}^{\text{OS}} Z_{\text{chromo}}^{\overline{\text{MS}}, \mu\nu\alpha\beta} c_{3\mu\nu} O_{3\alpha\beta}. \quad (1)$$

A simple one-loop calculation yields $\Rightarrow$

$$Z_{\text{kin}}^{\overline{\text{MS}}, \mu\nu\alpha\beta} = -C_F \frac{3 - \xi}{\epsilon} (g^{\mu\nu} - 2v^{\mu} v^{\nu}) v^{\alpha} v^{\beta} \frac{\alpha_s}{4\pi} + \ldots$$

$$Z_{\text{chromo}}^{\overline{\text{MS}}, \mu\nu\alpha\beta} = \frac{C_A}{\epsilon} (g^{\mu\alpha} - v^{\mu} v^{\alpha}) g^{\nu\beta} \frac{\alpha_s}{4\pi} + \ldots. \quad (2)$$
Results

Solid Curves ⇒ NLO coefficients of $\lambda_1$
Dashed Curves ⇒ Leading approximation in $\Delta/m_b = (1 - 2 \frac{E_0}{m_b})$

Decay rate ⇒ left panel  First moment ⇒ right panel (lower red)
Second moment ⇒ right panel (upper blue)

⇒ In the cut rate and second moment, the leading approximation deviates by roughly 50% and -35%, respectively, already at $E_0 = 2 GeV$

⇒ In the first moment, it is within roughly 10% of the complete result

In Conclusion ⇒ The range of applicability of leading order approximation [hep-ph/0408179, 0506245] is restricted to the region $E_0 > 2 GeV$
Left panel ⇒ Ratio of NLO to LO coefficient of $\lambda_1$ ⇒ rate (red solid), first moment (blue dashed) and second moment (black dash-dotted)

Right panel ⇒ Ratio of NLO to LO coefficient of $\lambda_2$

- The NLO corrections to $\lambda_2$ are close to 20%
- From second moment we expect to extract a higher value of $\lambda_1$, with our chosen input it is about 10%
- Small correction to the first moment leads to roughly 10 MeV positive shift
Summary

- We present the first calculation of the $\alpha_s$ corrections to $\frac{\Lambda^2}{m_b^2}$ in $\bar{B} \rightarrow X_s \gamma$

- The effect of NLO corrections on $\bar{B} \rightarrow X_s \gamma$ rate is below 1% for $E_0 < 1.8$ GeV

- Our results allow for more precise evaluation of the moments of the photon distribution and will improve the determination of $m_b$ and $\mu^2_\pi$

- Our method is applicable to inclusive semileptonic decay $\Rightarrow \mathcal{O}(\alpha_s \mu^2_\pi / m_b^2)$ corrections to the moments of $B \rightarrow X_c \ell \nu$ have been computed numerically, $\mathcal{O}(\alpha_s \mu^2_G / m_b^2)$ corrections are still unknown $\Rightarrow$ We also believe analytical result might be easier to implement in the fitting codes
Stay Tuned for more results!