Recent progress in the AdS$_4$/CFT$_3$ correspondence

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Significant progress recently in understanding \(\text{AdS}_4/\text{CFT}_3\) duality in M-theory.

\[
\begin{align*}
\text{Type IIB} & \quad \text{M-theory} \\
\text{AdS}_5 \times Y_5 & \quad \text{AdS}_4 \times Y_7 \\
\int_{Y_5} F_5 = N & \quad \int_{Y_7} *G = N \\
(Y_5, g_{Y_5}) \text{ admits Killing spinor} & \quad (Y_7, g_{Y_7}) \text{ admits } d > 1 \text{ Killing spinors} \\
\Rightarrow \text{Sasaki-Einstein 5-manifold} & \quad \Rightarrow \text{Sasaki-Einstein 7-manifold}
\end{align*}
\]

Equivalently start with \(\text{Mink} \times C(Y)\) where \(C(Y)\) is a Calabi-Yau cone: so

\[
g_{C(Y)} = dr^2 + r^2 g_Y = \frac{\partial^2 r^2}{\partial z_i \partial \bar{z}_j} dz_i d\bar{z}_j
\]

is Ricci-flat and Kähler (\(z_i\) local complex coordinates). AdS backgrounds are near horizon limits of \(N\) D3-branes/M2-branes placed at \(\{r = 0\}\).
Maximally SUSY case:

**Type IIB**

\[ Y_5 = S^5, \text{round metric} \]

\[ \mathcal{N} = 4, \text{(3 + 1)-dim SYM, gauge group } SU(N) \]

\[ W = \text{Tr} [\Phi_1(\Phi_2\Phi_3 - \Phi_3\Phi_2)] \]

**M-theory**

\[ Y_7 = S^7/\mathbb{Z}_k, \text{round metric} \]

\[ \mathcal{N} = 6, \text{(2 + 1)-dim Chern-Simons quiver theory, gauge group } U(N)_k \times U(N)_{-k} \]

\[ W = \text{Tr} [A_1(B_1A_2B_2 - B_2A_2B_1)] \]

M2-brane theory discovered by [Aharony-Bergman-Jafferis-Maldacena] (ABJM).
What about theories with less SUSY (different CY cones/Sasaki-Einstein)? Simplest construction is to orbifold:

**Type IIB**

Finite (abelian) subgroup \( \Gamma \subset \text{SU}(3) \twoheadrightarrow S^5 \subset \mathbb{C}^3 \).

Corresponding action on \( \mathcal{N} = 4 \) SYM with gauge group \( U(|\Gamma|N) \).

Projection onto invariants gives \( \mathcal{N} = 1 \) quiver gauge theory with gauge group \( U(N)|\Gamma| \).

**M-theory**

May similarly take \( \Gamma \subset \text{SU}(4) \twoheadrightarrow S^7 \subset \mathbb{C}^4 \).

However, \(|\Gamma|\) must divide the original CS level \( k \), leading to an additional \( \mathbb{Z}_{|\Gamma|} \) quotient, e.g. \( \Gamma = \mathbb{Z}_2 \) projection of ABJM theory leads to an M2-brane theory on \( \mathbb{C}^4/\mathbb{Z}_2 \times \mathbb{Z}_2 \).

In fact it has been argued that M2-branes on certain \( \mathbb{C}^4/\Gamma \) do not have a Lagrangian description.

In both cases, may Higgs the theories to obtain new theories in the IR, dual to branes on partial resolutions of the orbifolds.
Consider $N$ D3-branes on a CY cone singularity that may be resolved to a smooth non-compact CY 3-fold $X$ (this is not true for all singularities):

- The gauge theory on the D3-branes at the singularity is a quiver gauge theory, with $\chi(X) = \text{Euler number of } X$ nodes. In simple cases, this means a $U(N)^{\chi(X)}$ quiver gauge theory.

- Each node corresponds to a mutually BPS fractional brane: a bound state of D-branes wrapping collapsed cycles at the singularity.

- The bifundamentals (arrows) are open strings between these fractional branes.

The mathematics of this is also rather well-understood: representations of the quiver correspond to coherent sheaves on $X = \text{CY}_3$, which are D-branes.
This understanding of D3-branes can be used to understand some M2-brane theories [Aganagic, Martelli-JFS, Hanany-Zaffaroni].

Start with the T-dual to the D3-branes, namely \( N \) D2-branes on \( \text{Mink}_3 \times \mathbb{R} \times \text{CY}_3 \). The effective gauge theory is the \((2 + 1)\)-dim reduction of the \((3 + 1)\)-dim quiver theory.

On the resolved \( X = \text{CY}_3 \), there are various two-cycles. Pick one and turn on \( k \) units of RR two-form flux through it. This

- Fibres the M-theory circle over the \( \mathbb{R} \times \text{CY}_3 \) geometry.
- On a fractional D4-brane wrapped on the cycle, this induces the CS coupling

\[
k \int \text{Tr}(A \wedge dA + \frac{2}{3} A^3) \]

via the Wess-Zumino couplings on its worldvolume.

To preserve SUSY, the size of the two-cycle is also fibred over the \( \mathbb{R} \)-direction. Lifting to M-theory, precisely describes \( N \) M2-branes on \( \text{Mink}_3 \times \text{CY}_4 \).
One can similarly add RR four-form flux through (collapsed) four-cycles (M-theory lift not well-understood), and \( m \) units of RR zero-form flux. The latter leads to a massive Type IIA solution (no lift) with CS levels in the quiver satisfying \( \sum_{\text{nodes } i} k_i = m \) [Gaiotto-Tomasiello].

Example [Martelli-JFS]: begin with the quiver

\[
W = \text{Tr}[((-1)^n U^{n+1} + V^{n+1}) \\
+ V(A_1 B_1 + A_2 B_2) + U(B_1 A_1 + B_2 A_2)]
\]

Describes D3-branes on the CY 3-fold \( z_0^{2n} + z_1^2 + z_2^2 + z_3^2 = 0 \).

May be resolved with a single \( \mathbb{CP}^1 \). Adding \( k \) units of \( F_2 \) flux leads to CS levels \((k, -k)\), dual to M2-branes on CY 4-fold \( \{z_0^n + z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0\}/\mathbb{Z}_k \).
Type IIB

For a D3-brane quiver theory, changing the ranks of the gauge groups adds fractional branes.

This is constrained by gauge anomaly cancellation, and breaks conformal invariance.

M-theory

No gauge anomalies in $(2 + 1)$-dims.

In understood examples, such as the previous slide, different ranks correspond to adding different torsion $G$-flux to the conformal theory [ABJ, Martelli-JFS] *i.e.* turning on a discrete “Wilson line” $C$-field through $H^\text{tor}_3(Y_7, \mathbb{Z})$.

The ranks are constrained by SUSY at the quantum level. In the example above, find $U(N + l)_k \times U(N)_{-k}$ where necessarily $0 \leq l \leq nk$. This matches perfectly with the computation of $H_3(Y_7, \mathbb{Z}) \cong \mathbb{Z}_{nk}$.
For the quiver theory for $N$ D3-branes, the central $U(1)$s in $U(N)^\chi$ decouple in the IR, leading to an $SU(N)^\chi$ gauge theory: anomalous $U(1)$s gain a mass via GS mechanism, while the couplings for non-anomalous go to zero in the IR.

For M2-brane theories the corresponding $U(1)$ sector is much more subtle [Benishti-Rodríguez-Gómez-JFS].

$\dim H_2(Y_7, \mathbb{R})$ of the $U(1)$s are dual to abelian gauge fields in $AdS_4$ coming from reduction of $C_3$ on two-cycles. In contrast to the situation in $AdS_5$, there are different quantizations of such abelian gauge fields in $AdS_4$, in fact an $SL(2, \mathbb{Z})$ multiplet for each, and each is generically dual to a different CFT [Witten, Marolf-Ross, etc].

*E.g.* for a given abelian gauge field $A_\mu$ in $AdS_4$, may take conformal boundary conditions where either $E = 0$ or $B = 0$. Interchanged by an action of $S$. [Witten]: start with a CFT with a $U(1)$ current, and then coupling this to a new dynamical $U(1)$ gauge field. Thus M2-brane quiver theories which contain an $SU(N)$ gauge group can be related by an action of $S$ to a theory with this replaced by a $U(1) \times SU(N)$ gauge group.
In general these CFTs, with different quantizations in $\text{AdS}_4$, have quite different properties.

For example: may consider M2-branes wrapped on (non-SUSY) two-cycles in $Y_7$, or M5-branes wrapped on (SUSY) five-cycles in $Y_7$.

An $\text{AdS}_4$ quantization with $E = 0$ does not allow electrically charged states in the bulk, and hence does not allow M2-branes charged under this $U(1)$. Conversely, $B = 0$ does not allow magnetic charges in the bulk, ruling out M5-branes.

The SUSY wrapped M5-branes can be identified with baryonic-type operators for $SU(N)$ gauge groups, as for D3-branes. The $S$-action effectively gauges the $U(1)$, so that these baryonic operators are no longer gauge invariant in the $S$-dual theory. Instead it has been conjectured [Imamura] that the non-BPS M2-branes in this theory are monopole operators for the gauged $U(1)$s.